39th Congress, 2d Session. | HOUSE OF REPRESENTATIVES. | Ex. Doc. No. 87.

# REPORT

OF THE

# SUPERINTENDENT

OF THE

# UNITED STATES COAST SURVEY,

SHOWING

THE PROGRESS OF THE SURVEY

DURING

THE YEAR 1866.



WASHINGTON:
GOVERNMENT PRINTING OFFICE.
1869.

# National Oceanic and Atmospheric Administration

## Annual Report of the Superintendent of the Coast Survey

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IN SENATE, *February* 25, 1867.

*Resolved*, That there be printed two thousand extra copies of the Report of the Superintendent of the Coast Survey for 1866, of which one thousand copies shall be for the use of the Senate, and one thousand copies shall be for distribution by the Superintendent of the Coast Survey.

IN THE HOUSE OF REPRESENTATIVES, *March* 2, 1867.

*Resolved*, That there be printed two thousand five hundred extra copies of the Report of the Superintendent of the Coast Survey for the year eighteen hundred and sixty-six, of which one thousand shall be for distribution by the Superintendent of the Coast Survey, and one thousand five hundred for the use of the members of this House.

# LETTER

FROM

# THE SECRETARY OF THE TREASURY,

TRANSMITTING

A REPORT OF THE SUPERINTENDENT OF THE COAST SURVEY FOR THE YEAR 1866.

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TREASURY DEPARTMENT, *February* 14, 1867.

SIR: I have the honor to transmit, for the information of the House of Representatives, a report made to this department by J. E. Hilgard, Assistant in charge of the Coast Survey Office, stating the operations and progress in the survey of the coast during the year ending November 1, 1866, and the manuscript map of progress brought up to the same date, in accordance with the act of Congress approved March 3, 1853.

I have the honor to be, very respectfully,

HUGH McCULLOCH,
*Secretary of the Treasury.*

Hon. SCHUYLER COLFAX,
*Speaker of the House of Representatives.*

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# REPORT.

WASHINGTON, D. C., *December* 22, 1866.

SIR: In conformity with the law, and with the regulations of the Treasury Department, I have the honor to present the following report on the progress made in the survey of the coast of the United States during the year ending with the 1st of November.

On the coast of all the States bordering on the Atlantic and Gulf of Mexico, excepting three, the survey has been in active operation, and the usual rate of progress has been maintained on the western coast. Such of the results as admit of graphical representation have been added to the large manuscript map which is presented with the report on progress annually. The general progress of the survey, in its principal features, is shown by the accompanying engraved sketch, (No. 25.) Neither of these maps, however, permit the illustration of a most important class of operations pertaining to the work, such as would include the derivation of the laws governing tides on all parts of the coast; generalizations relating to the variation of the magnetic needle; special determinations of longitude, and processes of like character. The interesting results derived within the year from this class, to which the unremitting attention and labor of preceding years have been given, will be referred to more particularly after a brief review of the general operations in the field and in the office, followed, as usual, by estimates for the service to be performed during the next fiscal year.

## PROGRESS DURING THE YEAR.

On the coast of Maine progress has been made in the survey of Passamaquoddy bay and the St. Croix river; the topography of the shore of Muscongus bay, Medomak river, John's bay, Quohog bay, and New Meadow river has been completed, making the survey continuous from Portland eastward to Camden on Penobscot bay. The in-shore hydrography of the same stretch has been prosecuted so far as to admit of its completion in another working season. An examination has been made of the entrance of Saco river, with a view to the improvement of the channel.

On the coast of New Hampshire the topography has been advanced from Great Boar's Head to above Rye. On the coast of Massachusetts, from Plymouth to Sandwich. Progress has been made in the detailed survey of the shores of Narraganset bay.

On the coast of North Carolina the shore between Ocracoke and Cape Lookout has been surveyed; the shoals off Cape Lookout, and the approaches to the coast between Cape Hatteras and Cape Fear, have been sounded. Progress has been made in the survey of Pamlico sound and Neuse river.

On the coast of Georgia the bars and channels of the Savannah river have been completely resurveyed as a preliminary to the removal of obstructions and the re-establishment of the aids to navigation needed below Savannah. Soundings have been continued in the Straits of Florida. Progress has been made in the survey of Charlotte harbor, Florida, and of the coast of the Gulf of Mexico between Pensacola and Mobile Point.

The survey of the passes and delta of the Mississippi has been resumed, and progress has been made in the hydrography of Matagorda bay, and in the topography of the shores of Corpus Christi bay, Texas.

On the western coast of the United States, the topography has been filled in between Point San Pedro and Tunitas creek, completing the coast details between Monterey and Bodega; the triangulation and hydrography of Suisun bay have been completed, as also the off-shore hydrography between Point Año Nuevo and Monterey bay, and the in-shore soundings between Point Reyes and Bodega Head. The survey of Tillamook bay has been commenced, and special examinations have been made of a bank off Cape Flattery, and of the vicinity of Destruction island as a roadstead.

The operations of the Coast Survey office, embracing the computation of observations, the drawing, engraving, and publication of maps and charts, have kept pace with the field-work; six new charts have been published, and eighteen others, issued in advance of their completion, have been brought up to date, and ten charts have been commenced. The entire number worked upon during the year has been forty-eight.

Among the details of office occupation have been the computing and arrangement of tables for predicting the tides at the principal ports of the United States.

Several calls from the engineer department for special surveys have been met in the course of the year, the expenditures for which, as usual, have been defrayed from the appropriations for the several objects.

In Part II of the report, separate notices will be given of the different field operations, and, as usual, a recapitulation showing the distribution of parties on the coast will be found in the Appendix, (No. 1.)

## MAPS AND CHARTS.

The titles of maps and charts completed within the year in the Drawing Division are given in the Appendix, (No. 3,) together with the titles of those now in progress, and also the names of the draughtsmen employed in the office. A list, similar in form, (Appendix No. 4,) contains the titles of plates worked on during the year in the Engraving Division, and the names of the engravers now in employ.

For engraving purposes the pantagraph is now freely used in the office. Much time and expense in the tracing of outlines on copper will be thereby saved, the instrument being applied to copies of the original plane-table sheets, traced on transparent *vellum* cloth, and inverted in order to produce the requisite inversion of the engraving. The processes of reduction heretofore employed, and transfers made by the engraver for his own guidance, are thus dispensed with.

The following list contains the titles of engraved maps, charts, and sketches which accompany this report. As usual, some of the charts named are new editions of those which have already been issued, but none are included that have not been revised in consequence of important changes in hydrography, and in that light vital to the interests of commerce and navigation.

*List of charts and sketches.*

No. 1. Progress Sketch, Section I. Upper part.
2. Winter harbor, Maine.
3. Tenant's harbor, Maine.
4. Sassenow river and passage from Bath to Boothbay, Maine.
5. Portland harbor, (new edition.)
6. Portsmouth harbor, (new edition.)
7. Boston harbor, (new edition,) from survey for harbor commissioners.
8. Sippican harbor, Massachusetts.
9. Warren river, Rhode Island.
10. Primary triangulation between Fire Island and Kent Island base lines.
11. Coast Chart No. 27, from Cape Henlopen to Isle of Wight.
12. Coast Chart No. 28, from Isle of Wight to Chincoteague inlet.
13. Progress Sketch, Section IV.
No. 14. General chart of the coast, No. V, Cape Henry to Cape Lookout.
15. Progress Sketch, Section V.
16. Savannah river and Wassaw sound, Georgia.
17. Gulf-stream soundings.
18. Caloosa entrance, Florida.
19. Progress Sketch, Section IX.
20. Brazos Santiago, Texas.
21. Progress Sketch, Section X.
22. Suisun bay, California.
23. Destruction island, Washington Territory.
24. Washington sound, (new edition.)
25. General Progress Sketch.
26. Thirty-inch theodolite.
27. Twelve-inch theodolite and heliotrope.
28. Zenith telescope.
29. Portable transit.
30. Tides at Cat island.

ESTIMATES FOR THE FISCAL YEAR 1867-'68.

The estimates, as usual, will state with considerable detail the progress contemplated in the several localities and in the operations of the office, and constitute the plan of work which is adhered to as strictly as circumstances will permit. They are the same in amount of the two principal items as those of last year, which were based upon the adopted scale of expenditure immediately before the war. I have no doubt that the work would be done more economically in the aggregate if these amounts were increased by twenty per cent.; that is to say, the time required for completing the survey would be lessened in a greater ratio. In the present state of the public finances, however, I do not feel warranted in urging an increased scale of expenditure.

The item for the survey of the coast and reefs of Florida, of which separate accounts have always been kept, should be increased, as the experience of the present year has shown that the work can be more advantageously prosecuted with a small addition to the estimates of the year previous. With such addition, the item is the same as that appropriated in the year 1861.

For the repairs and maintenance of vessels, I am compelled to increase the estimate, as expenses of this kind have more than doubled, and it becomes necessary to begin to replace some of the older vessels used in the work.

*Estimates in detail.*

| | |
|---|---|
| For general expenses of all the sections, namely; rent, fuel, materials for drawing, engraving and printing, and for transportation of instruments, maps and charts; for miscellaneous office expenses, and for the purchase of new instruments, books, maps, and charts ............ | $19,000 |
| SECTION I. *Coast of Maine, New Hampshire, Massachusetts, and Rhode Island.* FIELD-WORK.—To continue the triangulation and topography of *Passamaquoddy bay* and its estuaries, and to extend the work so as to include the northeastern boundary along the *St. Croix river;* to continue the topography of *Frenchman's bay;* that of the islands at the entrance of *Penobscot bay*, and the western shore of the bay, to include *Belfast;* to continue that of *Saco bay*, and of the coast of *New Hampshire* south of *Portsmouth;* to complete that of the shores of *Massachusetts bay*, between *Scargo* and *Orleans:* and to continue the detailed survey of the shores and islands of *Narraganset bay;* to continue off-shore soundings along the coast of *Maine*, and the hydrography of *Frenchman's bay*, *Goldsborough bay*, *Prospect* and *Winter harbors*, *Penobscot bay* and *Muscongus bay;* to continue tidal and magnetic observations. OFFICE-WORK.—To make the computations required for and computations from the field observations; to continue the drawing of coast chart No. 1, (*Passamaquoddy bay,*) and commence that of No. 3, (*Mooseapeck* to *Mount Desert;*) to continue the drawing and engraving of No. 6 and No. 7, (*Isle au Haut* to *Cape Elizabeth;*) of No. 8 and No. 9, (*Seguin island* to *Cape Ann;*) and of No. 10 and No. 11, which include *Massachusetts bay* and *Cape Cod bay;* to make the drawing and commence the engraving of a chart of *Goldsborough bay*, *Prospect harbor*, and *Belfast bay;* to complete the engraving of the chart of *St. George's river* and *Muscle Ridge channel;* to continue the drawings and engraving of that of *Damariscotta river*, *Medomak river*, and *Muscongus bay;* and those of *Casco bay*, *Saco river entrance*, and *Narraganset bay*, will require ............ | 46,000 |
| SECTION II. *Coast of Connecticut, New York, New Jersey, Pennsylvania, and part of Delaware.* FIELD-WORK.—To make supplementary astronomical observations; to continue verification-work on the coast of *New Jersey;* to continue the topography of the shores of the *Hudson river;* to execute such supplementary hydrography as may be required in *New York bay* and *Delaware bay;* to continue the tidal observations. OFFICE-WORK.—To make the computations and reductions; to continue the drawing and engraving of a chart of *New York harbor* on a large scale; and of coast chart No. 22, (from *Sandy Hook* to *Barnegat,*) will require ............ | 15,000 |
| SECTION III. *Coast of part of Delaware, and that of Maryland, and part of Virginia.* FIELD-WORK.—To continue astronomical and magnetic observations in this section; | |

to complete the topography of the eastern shore of *Virginia*, and of the shores of the *Potomac* and *James rivers;* to make the hydrographic survey of estuaries and inlets remaining unsurveyed in the section; and to continue tidal observations. OFFICE-WORK.—To make the computations from field-work; to continue the drawing and engraving of coast charts No. 29 and No. 30 (from *Chincoteague inlet* to *Cape Henry*) and of general coast chart No. IV, (approaches to *Delaware* and *Chesapeake bays*,) and to make additions of supplementary surveys on the charts of this section heretofore published, will require .......... $23,000

SECTION IV. *Coast of part of Virginia and part of North Carolina.* FIELD-WORK.—To complete, if practicable, the primary triangulation of *Pamlico sound*, and to make the requisite astronomical and magnetic observations; to make the verification of the secondary triangulation between *Cape Lookout* and *Cape Fear;* to continue the triangulation and topography of the western shores and estuaries of *Pamlico sound;* to complete the topography of the outer coast of *North Carolina* between *Beaufort* and *New River inlet;* to continue the in-shore and off-shore hydrography between *Cape Henry* and *Cape Hatteras;* to continue soundings in *Currituck* and *Pamlico sounds* and their estuaries; and to make observations on the tides and currents. OFFICE-WORK.—To make the computations and reductions; to continue the drawing and engraving of general coast chart No. V, (from *Cape Henry* to *Cape Lookout*;) of coast charts No. 46 and No. 47, (from *Cape Lookout* to *Barren inlet*,) and of charts of *Pamlico sound*, *Neuse river*, and *Pamlico river*, will require .......... 33,000

SECTION V. *Coast of South Carolina and Georgia.* FIELD-WORK.—To continue the primary triangulation from *Port Royal* to *Tybee*, and to make the requisite astronomical and magnetic observations; to extend the topography from *Winyah bay* to *Cape Romain;* to continue the topography from *St. Simon's sound* southward to the *St. Mary's river*, and to sound the interior water passages among the sea islands from *Sapelo sound* southward, and continue the off-shore hydrography and the tidal observations. OFFICE-WORK.—To make the computations; to complete the drawing and engraving of coast chart No. 54, (from *Hunting island* to *Wassaw island*;) to continue that of No. 55, (from *Tybee* to *Altamaha*,) and of No. 56, (from *Altamaha* to *St. Mary's*;) to complete the chart of approaches to *Tybee entrance*, including the resurvey of the *Savannah river;* and to continue the drawing and engraving of charts of the inland tide-water communication on the coast of *Georgia*, will require .......... 33,000

SECTION VI. *Coast, keys, and reefs of Florida.*—(See estimates of appropriation for those special objects.)

SECTION VII. *Western coast of Florida peninsula north of Tampa bay, and coast of West Florida.* FIELD-WORK.—To continue the triangulation from *Cedar keys* to the *Suwanee river;* from *St. Andrew's bay* towards *Chattahoochee bay*, and from *Pensacola bay* eastward; to make such astronomical and magnetic observations as may be requisite; to continue the topography to the northward of *Cape San Blas*, and to the westward of *St. Andrew's bay*, and that of the *Gulf coast* adjacent to *Santa Rosa sound;* to survey and sound the entrance to the *Suwanee river;* to complete the hydrography of *St. George's sound*, and to make soundings off *Cape San Blas*, and continue the requisite tidal observations. OFFICE-WORK.—To make the computations from field-work; to continue the drawing and engraving of coast charts No. 84 and No. 85, (from *Ocilla river* to *Cape San Blas*,) and of No. 89, (from *Pensacola* to *Mobile Point*,) and to prepare a chart of the approaches and entrance to the *Suwanee river*, will require .......... 25,000

SECTION VIII. *Coast of Alabama, Mississippi, and part of Louisiana.* FIELD-WORK.—To make the astronomical and magnetic observations required in this section; to extend westward from former limits and complete, if practicable, the survey of the shores of *Isle au Breton sound*, including the adjacent banks of the *Mississippi river*, and the vicinity of the *passes;* to continue the hydrography within the same limits, and complete that of the *Mississippi entrances* in connection with observations on the tides and currents. OFFICE-WORK.—To make the computations pertaining to field-work; to continue the drawing and engraving of the general chart No. XIII, (*Gulf coast* between *Cape San Blas* and the *Southwest Pass*;) to complete coast chart No. 93, (western part of *Mississippi sound*,) and to continue the drawing and engraving of No. 96, (*Mississippi delta*,) will require .......... $28,000

SECTION IX. *Coast of part of Louisiana and coast of Texas.* FIELD-WORK.—To continue the requisite astronomical and magnetic observations, and to measure a primary base-line; to continue the triangulation and topography of *Madre lagoon*, from *Brazos Santiago* northward; to extend the topography south of *Aransas Pass*, and include the shores of *Corpus Christi bay*; to complete the hydrography of *Corpus Christi bay*: and to make the requisite tidal observations. OFFICE-WORK.—To make the office computations; to complete the engraving of coast chart No. 108, (*Matagorda* and *Lavacca bays*;) to continue the drawing and engraving of No. 109, (*Gulf coast* from *Matagorda* to *Aransas Pass*;) to engrave a chart of the *Brazos Santiago entrance*, and to continue the drawing and commence the engraving of general chart No. XVI, (*Gulf coast* from *Galveston* to the *Rio Grande*,) will require .......... 28,000

Total for the *Atlantic coast* and *Gulf of Mexico* .......... 250,000

The estimates for the Florida coast, keys, and reefs, and for the western coast of the United States, are intended to provide for the following progress in the survey:

SECTION VI. *Coast, keys, and reefs of Florida.* FIELD-WORK.—To make such astronomical and magnetic observations as may be requisite in the section; to continue the triangulation and topography of the *Atlantic coast* of the peninsula, south of *Matanzas inlet*; to extend the triangulation and topography northward from *Key Biscayne bay* towards *Jupiter inlet*, and complete the survey of the main shore east of *Cape Sable*, and of the inner keys between it and *Barnes's sound*; to extend the survey of the *Gulf coast* of the peninsula from former limits southward, to include *Clearwater harbor*; to run lines of off-shore soundings northward of *Cape Florida*, and to complete the hydrography of *Florida bay*. OFFICE-WORK.—To compute results from the field observations; to continue the drawing and engraving of the off-shore chart No. XI, (western part of the *Florida reefs*, including the *Tortugas*,) and of coast chart No. 77, (vicinity of *Charlotte harbor*,) to complete a chart of *Caloosa bay*; and to continue the drawing of coast chart No. 64, (*Florida coast* near *Jupiter inlet*,) will require .......... $40,000

SECTION X. *Coast of California.* FIELD-WORK.—To make the required observations for latitude, longitude, and azimuth at stations of the primary triangulation, and to make magnetic observations; to connect the islands of *Santa Cruz*, *Santa Rosa*, and *San Miguel* with the coast triangulation, and to survey the topography of the same; to continue the coast topography from *Buenaventura* to *Santa Barbara*; to make surveys of the entrances to *Eel river* and *Salt river*; to continue the off-shore hydrography of the coast of *California* and the tidal observations. OFFICE-WORK.—To complete the drawing and engraving of a chart of the coast from *Point Pinos* to *Bodega Head*; of the chart of *San Francisco* and *San Pablo bays* in one sheet, and of the chart of *Suisun bay*; to continue the drawing and engraving of a general chart of the coast from *San Diego* to *Point Conception*; also for the operations in—

SECTION XI. *Coast of Oregon and Washington Territory.* FIELD-WORK.—To continue the astronomical and magnetic observations in this section, and the triangulation, topography, and hydrography in *Washington sound* and in *Puget sound*; to make such surveys of special localities as may be called for by public interests on the coast of *Oregon* and of *Washington Territory*, including those of *Tillamook bay*, *Yaquinah river*, *Port Discovery*, and *Possession sound*; and to prepare and engrave maps and charts of the same, will require .......... 130,000

For publishing the observations made in the progress of the survey of the coast of the United States, per act of March 3, 1843 .......... 5,000

| | |
|---|---|
| For repairs and maintenance of the complement of vessels used in the survey of the coast, per act of March 2, 1853 | $30,000 |
| For pay and rations of engineers for the steamers used in the hydrography of the coast survey, no longer supplied by the Navy Department, per act of June 12, 1858 | 10,000 |

## GEODESY.

The primary triangulation of the coast of the New England States having been completed in the preceding year by the occupation of stations Ruland and West Hills which closed the connection between the New York (Fire island) and Maine (Epping plains) base lines, there remained some additional observations of latitude and longitude to be made for the ultimate perfection of the geodetic work. Mount Blue, in New Hampshire, the northwestern station of the series of great triangles extending from the northeastern boundary of the United States to the Hudson, is the northern extremity of an arc of the meridian, of which Nantucket is the southern; and is the western end of an arc of the parallel of 45°, extending at present about 3° eastward to the St. Croix river. The lengths of both these arcs are furnished by the triangulations, and the determinations of their amplitude by the proper astronomical observations will supply the data requisite for ascertaining the actual curvature of the earth's surface in this region. Station Mount Blue itself is not, however, well adapted for making the final observations, being difficult of access, and liable to the effects of local attraction. A station free from these objections has been selected in its vicinity and brought into full connection with the series of triangles, so that it remains only to deduce by computation the lengths of the arcs referred to. Further allusion will be made to this work under the head of Section I, in the body of the report.

In last year's report was given an abstract of the results of the primary triangulation between the Epping and Fire Island base lines, together with a summary statement of the methods of adjustment adopted, by means of which the three measured bases were brought into complete agreement, and the whole chain of triangles made to represent a geometrically perfect figure. In the present report the same subject is continued, and in Appendix No. 8 the adjustment of the primary triangulation between the Fire Island (New York) and Kent Island (Maryland) base lines is given, showing an agreement no less satisfactory than in the former work. Sketch No. 10 shows the scheme of triangulation. It will be seen from the paper in question that the length of the Kent Island base, as measured, differed but four inches from the length derived from the Fire Island base through the triangulation, when in the latter the conditions of form merely were satisfied, and that it required an average correction of less than five hundredths of a second of arc to each angle to bring the measured base lines into perfect agreement. Sketches 26 and 27 illustrate some of the instruments used in measuring the angles of the primary triangulations. The former shows the large theodolite of thirty inches diameter, reading with three microscopes, made for the Coast Survey by Troughton and Simms, of London; the latter shows, above, a heliotrope used for showing to the observer a distant station by the reflection of sunlight from a small mirror, and below, a twelve-inch repeating theodolite by Gambey.

## METHODS OF OBSERVATION AND COMPUTATION.

The methods of observation and computation adopted in the Coast Survey, in different branches of the work, have been heretofore detailed in the discussions of various subjects forming appendices to the annual reports. Engineers, surveyors, and students generally have attached much value to these publications, as has been evinced by the frequent calls upon this office for copies of them, by the use made of them in scientific text-books, and by inquiries for similar information on branches in regard to which no such publication had yet been made. Calls of this nature led to the publication, in last year's report, of a treatise on the use of the plane-table in surveying, which is followed in the present volume by articles on observations of time, of latitude, and of azimuth, (Appendices Nos. 9, 10, and 11, compiled by Assistant C. A. Schott,) the general aim being gradually to embrace the whole range of subjects, so that a collection of the articles will form a complete *Coast Survey Manual.*

## HYDROGRAPHIC AND TIDAL RESEARCHES.

An interesting account is given by Assistant Henry Mitchell, in Appendix No. 5, of deep-sea soundings across the Florida straits between Key West and Havana. It will be seen from the table of results and the diagram that the slope of the bottom is much the steepest on the Cuban shore, and that there is a submarine elevation of some 2,800 feet in the middle of the channel, which is, perhaps, represented by Salt Key bank further eastward.

In Appendix No. 6 Assistant Mitchell presents a preliminary report on the interference of tides in Hell Gate, which have formed a subject of special study for some years past.

Our knowledge of the tides on the coasts of the United States being deemed by long-continued observations at many points sufficiently matured to warrant the computation of predictions, a *Tidal Almanac* has been prepared and published for the year 1867, giving for all the principal ports of the United States the times of high water for every day in the year, and its height above the average low-water level, with reference to which the soundings are given on the charts. A table of tidal constants is added, by means of which the time and height of the tide may be found for all ports intermediate to those for which the full table is given. Appendix No. 7 gives fuller particulars, and an example of the table for one port. All vessels in the government service have been supplied with these Tidal Almanacs, and they are for sale to the public at the cost of printing. The publication of such tables is intended to be continued annually.

## EXTRACTS FROM REPORTS OF FORMER YEARS.

The series of Coast Survey reports printed in quarto form, of which a considerable number of copies have been published, begins with the year 1852. The reports of preceding years were published in octavo form; a small number only were printed, and no copies of the same can now be procured. With the exception of that for 1851, they contained but little matter of permanent interest, being chiefly occupied by reports of progress. There were, however, several papers of general scientific interest contained in them, which are now reprinted as appendices to the present volume, in order that they may again be accessible, and with the view of rendering the quarto series complete as to papers of scientific inquiry published by the Coast Survey. The only matter not now so reprinted is the important table of geographical positions in the report for 1851, the republication of which is reserved for a time when the cardinal longitude will have been permanently settled.

## MAPS PRINTED IN COLORS.

An attempt in color printing has been made with the map marked No. 2, (Winter Harbor,) with a view to show how effectively the distinctive features of the map are brought out in that way. It is true that the necessity of several successive printings, in which each impression must be accurately *registered* or made to fit the preceding ones, renders publication in this form more expensive than in that of the single black impression from the copperplate. Yet the additional expense, although relatively large, is actually not great for each map, being about twelve cents for a sheet of the size presented, and the greatly increased perspicuity of the map seems fully to warrant this enhancement in cost. A novel feature in this map, in the representation of topography, is worthy of attention. The forms and elevations of the hills are accurately indicated by the lines of equal level, which are here retained from the original sheet, and the effect of elevation to the eye is produced by shading in crayon. The usual expensive system of hachuring is thus dispensed with, while an increased effect of solidity is attained by easy means without sacrificing anything of the precision of the data as to the conformation of the ground furnished by the original map of the survey.

## NAVY DEPARTMENT.

The facilities available at the opening of the year for carrying on the hydrographic work of the survey have been increased by the transfer from the Navy Department of a few of the steam launches which were no longer required for naval purposes. These being exactly adapted for in-shore soundings, have proved very effective in the operations of the present season. Of the number assigned by the honorable Secretary of the Navy for use in the hydrography, one has been in service on the coast of Maine, one on the coast of North Carolina, and one on the coast of South Carolina and Georgia.

## OBITUARIES.

Edmund Blunt, the first and oldest assistant in the survey of the coast, died on the 2d of September, at his residence, near the city of New York, in the sixty-seventh year of his age.

The conspicuous services of Mr. Blunt deserve more than a mere expression of personal regret for the loss of an able associate. Since the organization of the Coast Survey he had acted an important part in earning, by the extent of his labors and the accuracy of his results, the reputation which the work has sustained for efficiency and precision. Inheriting from his father a strong inclination for hydrographic pursuits, and commencing in early boyhood the practice of his profession, his entire life may be said to have been devoted to the security and extension of our commerce by determining and describing the dangers in its path.

The law of Congress which provided for the survey of the coast did not take full effect until 1832. Previous to that date the charts of our coast were based upon the early and cursory surveys of Des Barres and others, occasionally corrected by detached surveys in pursuance of special acts of Congress, or by private enterprise. Foremost in this laudable work was the father of the subject of the present notice, Edmund M. Blunt, who, in addition to the Coast Pilot, compiled and published at his suggestion in 1796, undertook hydrographic surveys and examinations. In these latter operations his sons took an active part.

Before he was eighteen years of age, Edmund Blunt made a survey of the harbor of New York. In the years 1819–'20 he assisted in the sounding of the Great Bahama Bank route to the Gulf of Mexico; afterwards in the survey of Nantucket and George's shoal. In 1824 he surveyed the seacoast in the vicinity of New York bay; and between the years 1828 and 1830 the shores and shoals of Long Island sound.

Early in 1833 Blunt was appointed an assistant in the Coast Survey, that work, after a suspension of fifteen years, having been then resumed. This appointment enabled him to bring to the performance of the duties assigned to him, in the systematic operations about to be undertaken, the skill and experience acquired during his previous career.

In subsequent years, as the geodetic survey advanced, the name of Assistant Blunt became in succession identified in its records with the triangulation of Long Island sound and of the adjacent coast; with the triangulation of Delaware bay and river; with the measurement of a base line for verifying the primary triangulation completed previous to 1844; with various detached surveys between New York and Boston; with the triangulation of Chesapeake bay; and with that of the valley of the Hudson between New York city and Albany.

The death of Mr. Blunt was sudden and unexpected. He retained to the last day of his life the vigor and activity which had marked his early manhood. In field operations he laid the basis for the excellent work which he performed by untiring search, and by adopting in all cases the means suggested in a comprehensive review of the ground features, however extended the area might be, designated for triangulation. Concerned chiefly in the primary work, on parts of the coast presenting all the natural difficulties in the way of observing over extended lines of sight, he brought into use many of the expedients now regularly employed in similar localities. The regard for his profession, which seemed to strengthen as time drew on, was befitting in one who had largely shared from the beginning in the labors pertaining to the geodetic survey of the coast.

Prompt, energetic and successful in the field, and at all times devoted to the interests and credit of the work, the example of Mr. Blunt commanded the respect, as his kind and genial disposition gained the regards of all his associates on the survey.

In the Drawing Division of the office we have met with a serious loss in the death of Mr. M. J. McClery, who had been connected for a long period with that branch of the service. His skill as a topographical draughtsman was of the highest order, enabling him to delineate the most intricate details with such clearness and effect as to challenge the art of the engraver in their reproduction. With great manual proficiency were conjoined a refined taste and complete unity of style, sufficient to mark the identity of all his productions.

In private relations Mr. McClery was high in the regards of a large circle of acquaintances. By these, and by his associates in the office, he will be remembered for the steady practice of the virtues and amenities that adorn social life. He died, after a short illness, on the 24th of October.

Mr. M. T. Johnstone, whose death occurred in June, was highly esteemed as an intelligent, industrious, and truly conscientious man. He was called to the office in November, 1861, and was placed in charge of the map room, the issues from which at that period required the constant exercise of forethought and devotion to the welfare of the government. That the publications subject to order for distribution should serve the interests for which they were designed and none other, he realized fully. In the discharge of that duty he acted with scrupulous care. In the performance of others, Mr. Johnstone exercised the resources of his liberal education, to the stores of which his characteristic industry was constantly adding, notwithstanding the approach of age. He was widely known as a man of great moral worth, sound judgment, large experience, and unvarying kindness of manner.

# PART II.*

## SECTION I.

### FROM PASSAMAQUODDY BAY TO POINT JUDITH, INCLUDING THE COAST OF THE STATES OF MAINE, NEW HAMPSHIRE, MASSACHUSETTS, AND RHODE ISLAND. (SKETCHES No. 1 AND No. 2.)

*Telegraphic determination of longitude between America and Europe.*—Soon after the completion of the telegraphic junction between Ireland and Newfoundland, the project of determining the difference of longitude between Valencia and Heart's Content, by means of the Atlantic cable, was carried into successful execution. All the preliminaries had been previously arranged, plans, in fact, having been matured before the year 1858, when the prospect first opened for affording such facilities as had been freely used in the longitude determinations of the Coast Survey.

By the liberality of the Anglo-American Telegraph Company, the early use of their cable had been accorded for passing time signals, and permission had also been given by the New York, Newfoundland, and London Telegraph Company for the free use of their lines connecting Newfoundland with the telegraphic system of the United States.

The purpose of determining the difference of longitude between the ends of the Atlantic cable was carried into effect by the assignment of observers who had had long practice with the telegraphic method—all the important telegraphic points between Calais, in Maine, and New Orleans, in Louisiana, having been fixed by their observations.

In September, Dr. Gould, accompanied by Sub-Assistant A. T. Mosman, proceeded to Ireland, provided with a transit instrument, astronomical clock, and chronograph register. These were used in the observations made at Foilhommerum, on Valencia island. A similar set of instruments was taken by Assistant George W. Dean, and employed in the observations made at Heart's Content by himself and Assistant Edward Goodfellow. A third set was sent to Calais, Maine, for the use of Assistant George Davidson, who was to be aided by Mr. S. C. Chandler, jr., and Mr. F. W. Perkins.

Variable weather and other circumstances presented many difficulties in the intended interchange of signals through the Atlantic cable. The obstacles met were, however, inseparable from the service undertaken at that time, all the facilities possible being afforded for the comfort of the observers by residents nearest to the stations at Foilhommerum and at Heart's Content. Dr. Gould and Assistant Dean succeeded in exchanging satisfactory sets of clock signals on the nights of October 24 and 28, and on the nights of November 5, 6, and 9. The signals on two nights were sent by one of the cables only; but on the other three nights both cables were used in telegraphic

* This part of the report will be made up of short abstracts of the statements turned in at the end of the season by the field assistants. Under separate heads the notices will be so arranged as to describe the work in geographical order and as briefly as possible, mention being made only of the names, localities, and general statistics.

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connection without reference to the earth current. The cable was used during one night in addition for experimenting on the velocity of the magnetic current in transmitting signals.

The astronomer royal of Greenwich observatory, Professor Airy, having concerted arrangements with Dr. Gould, the telegraphic operations in Europe were completed by the exchange of time signals sent respectively from the Royal observatory and Foilhommerum.

Between Heart's Content and Calais, great difficulties were experienced in efforts to pass signals, owing to the condition of the lines. Assistant Davidson remained at Calais until the 4th of December, his services being then due in the prosecution of a special survey across the Isthmus of Darien, arrangements for which had been previously made. During his stay at Calais, all the requisite means for the speedy completion of the work had been provided, dependent, however, for success, upon the repairs which had been for some time in progress along the telegraph lines to Newfoundland. The lines being reported as in working order suitable for longitude purposes, Assistant C. O. Boutelle reached Calais on the 11th and exchanged time signals with Mr. Dean, the observer at Heart's Content, on four nights, closing on the 16th of December the observations required for determining the difference of longitude between Washington and Greenwich by the telegraphic method.

*Geodetic observations.*—The primary triangulation of the coast of New England having been completed, that work has been made available for determining the length of an arc of the meridian by bringing into the series a station near Farmington, New Hampshire. This point was selected by Assistant Boutelle for its easy geodetic connection with Mount Blue, the north end of the arc, and with the adjacent points of the main triangulation, and as being convenient for astronomical observations, and in telegraphic connection with the most eastern astronomical station in the United States.

In the course of the summer, Bannock Hill (see Sketch No. 1) was occupied as a trigonometrical station, and in succession Mount Blue, Mount Sebattis, Farmington, and Stewart in the immediate vicinity of the new geodetic station, of which the latitude was to be determined by special observations. Mr. Boutelle measured with the 12-inch theodolite No. 30, the horizontal angles that connect the new stations with the primary triangulation, recording in the aggregate 1,186 observations. The relative heights of the stations were determined by 276 measurements for vertical angles.

The observations and resulting field computations for latitude are thus reported: "The latitude at Farmington has been determined by 388 observations on 64 pairs of stars upon 22 nights of October and November. Of the 64 pairs, 12 were observed 5 times each, 40 were observed 6 times, 8 pairs 7 times, and 4 pairs 8 times each. The resulting latitude of the zenith telescope at Farmington is $44° 40' 19''.08 \pm 0''.13$."

Assistant Boutelle is now making arrangements for determining the latitude of the south end of the meridian arc which terminates at a geodetic station in the vicinity of Nantucket.

*Triangulation of St. Croix river, Maine.*—The secondary triangulation having embraced Passamaquoddy bay in previous seasons, was this year extended about eighteen miles upward from the entrance of the St. Croix river by the party of Sub-Assistant C. H. Boyd. In its progress, a careful connection was made with the station which had been used for astronomical observations at Calais in 1857. Twelve positions were occupied with the theodolite. The results of the observations have furnished twenty-six points for use in the topographical survey of the shores of the St. Croix river. Mr. Boyd commenced on the 24th of September, and closed work at the end of the following month.

*Shore-line survey of St. Croix river, Maine.*—The shore lines of Passamaquoddy bay and of its islands in the vicinity of the boundary having been traced by Assistant W. H. Dennis, field-work was resumed at the end of August, at the entrance of the St. Croix river. Proceeding upwards, Mr. Dennis traced in the course of the season both shores of the river to a point within two miles of the head of navigation. Thirty-eight miles of shore line are defined on the plane-table sheets, exclusive of the outline of low water, of which there is an aggregate of eleven miles.

*Hydrography of West Quoddy entrance, Maine.*—The progress sketch (No. 1) shows the advance made this season in the hydrography of the approaches to Eastport. In October, Mr. H. L. Marindin, under the immediate direction of Assistant S. A. Gilbert, with a party in the schooner Caswell, sounded the West Quoddy entrance, and the roads between the entrance and Lubec Narrows. Above the Narrows he joined with the inside hydrography previously done by Assistant Boutelle, in the waters known as Johnson's bay.

The great rise and fall made it difficult to observe the tides, yet they were successfully recorded both above and below the Lubec Narrows while the soundings were in progress. Mr. Marindin noted the fact that two hours before high water the tide ebbed through the Narrows, while it was flood near the West Beacon, showing that the level between Johnson's bay and West Quoddy roads must differ materially at different stages of the tide.

The hydrographic party put up nine signals and recorded about eleven thousand soundings. Mr. G. C. Schæffer, jr., was attached to the party as aid.

*Hydrography.*—In order to complete the hydrography which in several localities between Portland harbor and Penobscot entrance was outstanding at the opening of the season, Assistant S. A. Gilbert was placed in charge of that work, and took the general direction of the parties assigned to execute the same. These will be referred to in geographical order. By the arrangement just alluded to unity of method was secured, and completeness in connecting the additional work with the hydrography previously executed, Assistant Gilbert having made a careful study of the data and means requisite in each of the localities.

*Hydrography of Penobscot bay, Maine.*—The soundings at the entrance of Penobscot bay have been extended by the party of Sub-Assistant Charles Junken in a direction northward and eastward from the previous limit of work near Metinic island. Rackley's island on Sketch No. 1 marks the upper limit of the sheet of this season. The approaches to the ledges coming within the limits of work were carefully defined. The soundings, seven thousand five hundred in number, were made with the steamer Endeavor, between the 6th of September and the 1st of November. Messrs. H. M. DeWees and L. A. Sengteller served as hydrographic aids.

Sub-Assistant R. E. Halter, with the boats of the schooner Bailey, sounded Tenant's harbor and the western entrance to Penobscot bay, between Rackley's island and the main, connecting throughout with the hydrography outside, which had been performed in a previous season. Messrs. J. B. Adamson and Eugene Ellicott aided in the service of this season, but the former, taken ill before the close of work, was replaced by F. W. Perkins. The principal lines of soundings were run with the steam launch Barataria. Forty-seven signals were put up, and an aggregate of nearly thirty thousand casts of the lead were recorded. These include the supplementary soundings made in the immediate vicinity of the islands off the entrance to George's river. The work was commenced on the 19th of August and concluded at the end of October.

*Triangulation in Muscongus bay, Maine.*—The subsidiary triangulation required for the survey of the islands in Muscongus bay was taken up by Sub-Assistant J. A. Sullivan, on the 1st of September, with a party in the schooner Hassler. The stations occupied, nine in number, range from Pemaquid Point (Sketch No. 1) in a northeast direction, and include the islands lying in the upper part of the bay. Twenty-nine points were determined in position, by eleven hundred observations with the theodolite. This work was concluded on the 29th of September.

*Topography of Muscongus bay, Maine.*—By the work of the party of Sub-Assistant Cleveland Rockwell, and that which will be noticed under the next head, the shore-line survey of Muscongus bay and its islands has been completed, and the detailed survey nearly so. Mr. Rockwell has returned two plane-table sheets containing the outline and most of the topographical features between Crotch island on the east and Round Pond (see Sketch No. 1) on the west side of the sound. In the latter part of the season his work was furthered by the transfer to him of the schooner Bowditch. The main and islands represented on the two sheets sent in by Mr. Rockwell show an aggregate of sixty-two miles of shore-line, ten miles of road, and about eleven square miles of detailed topography. This work was prosecuted between the 6th of August and the 10th of November.

*Topography of Medomak river, Maine.*—This work has been extended southward from former limits by the party of Sub-Assistant Charles Ferguson. The sheet which at its upper limit included the survey of Waldoboro' now embraces the topographical details of Dutch neck, and the shores and interior adjacent to Broad cove and Delno's cove. For the work of this season Mr. Ferguson determined eighteen points by a subsidiary triangulation in advance of taking the field-work with the plane-table. These served also for the uses of the hydrographic party.

The plane-table work was prosecuted between the 1st of June and the 8th of October, when the schooner Bowditch was turned over to Sub-Assistant Rockwell for use in the topographical survey of Muscongus bay.

*Hydrography of the Medomak river, Maine.*—The upper part of this river was sounded in September and October by a party in charge of Sub-Assistant Horace Anderson. The tides were observed at Waldoboro' and at Howard's wharf in Broad cove. In the intermediate part of the Medomak about twenty-one thousand soundings were made. The numerous ledges which render the navigation intricate in passing up to Waldoboro' were carefully defined. Mr. Anderson closed work on the 1st of November, after completing the sounding of the three difficult passages below Broad cove. The locality of the work is marked on Sketch No. 1.

Mr. H. G. Ogden was attached to the hydrographic party as aid, and Mr. R. B. Palfrey served in that capacity temporarily.

*Topographical survey near Pemaquid Point, Maine.*—The outline and interior features of the peninsula terminating in Pemaquid point were traced by Assistant F. W. Dorr, between the middle of August and the end of October.

At the head of John's river (Sketch No. 1) he joined with previous topographical work done by Sub-Assistant Donn, and passing southward traced the shores of Pemaquid river, and the outlines of the islands at its entrance, as well as the shore of John's bay. Turning northward at Pemaquid light-house the western shore of Muscongus bay was traversed as far as Round Pond harbor, where a junction was made with work of this year done by Sub-Assistant Rockwell. The roads and water-courses of the peninsula appear on the plane-table sheets turned in by Mr. Dorr, and all the details within an area of twenty square miles. The two sheets present about forty-three miles of shore-line, sixty miles of road, and about thirty-seven miles of water-courses. Assistant Dorr was efficiently aided in field duty by Mr. Franklin Platt.

*Hydrography of the Damariscotta river, Maine.*—The hydrographic survey of this river was completed in August by a party in charge of Mr. E. Hergesheimer. The soundings then made develop that part of the river which is included between Damariscotta and the limit of previous work, which had been extended from the entrance upward to a point about eight miles below the town. An aggregate of nearly eight thousand soundings was recorded.

*Hydrography of the Sheepscot river, Maine.*—The supplementary soundings required in the eastern branch for completing the chart of the Sheepscot river were made by Mr. Hergesheimer in September. The sites of work referred to in this and in the preceding notice are shown on the Progress Sketch No. 1. Eight thousand soundings were made in completing the hydrographic survey of the Sheepscot river.

*Topography of Kennebec river, Maine.*—The plane-table survey of the Kennebec has been continued in the vicinity of Merry-meeting bay by Assistant R. M. Bache. In the course of the season, which commenced on the 1st of August and was closed on the 3d of November, ten miles of shore-line were traced, and a margin of topography mapped, making in the aggregate two square miles. The details of survey given on the plane-table sheet are quite intricate and difficult, comprising, among other features, fifteen islands, of which the largest is about a mile in length.

*Triangulation of New Meadow river, Maine.*—Assistant A. W. Longfellow took the field in the latter part of June, and after making a reconnaissance, erected eight signals for the determination of working points on the shores of Quohog bay and New Meadow river. Before the close of July he had occupied the stations of his own selection with the theodolite, his party using the schooner Torrey in going from point to point. After the 26th of July the needful triangulation was continued by Sub-Assistant Sullivan, the plane-table survey being then taken up by Mr. Longfellow. In August working points were furnished for the topographical survey of New Meadow river, and for completing plane-table work at the head of Quohog bay and on the shores of Harpswell sound. The party of Mr. Sullivan erected thirty signals. Over four thousand measurements were made with the theodolite in determining ninety-one positions for the use of the plane-table parties. Mr. Sullivan closed work in this section at the end of October, and after laying up the schooner Hassler at Portland, made arrangements for field service in Section VI.

*Topography and hydrography of New Meadow river, Maine.*—This work was taken up in the first week of August by the party of Sub-Assistant J. W. Donn, and was completed at the end of October. The soundings made, about six thousand in number, complete, with the exception of the upper part of Muscongus bay, the hydrography of the coast of Maine between Cape Elizabeth and Penobscot entrance. Thirteen square miles of area were mapped, showing on the plane-table sheets fifty miles of shore-line and over thirty miles of road.

While sounding in New Meadow river, tidal observations were made regularly at Birch Point. For the work in Quohog bay, with which the soundings required to be connected, the tides were observed by a staff favorably situated in the middle of the upper bay.

Mention will be made, under the head of Section III, of the occupation of Mr. Donn during the earlier part of the working year.

Assistant Longfellow extended the shore-line survey of the New Meadow river northward, from the limit reached in a previous season, to and including the Three islands, and to a junction with the work of Sub-Assistant Donn already mentioned. The plane-table sheet comprises the Phippsburg basin and Winnegance bay, with their islands and ledges. It also includes a portion of the head of Quohog bay, and shows in the aggregate thirty-nine miles of shore-line. For the use of the hydrographic party, Mr. Longfellow furnished a complete tracing of his work of this season. His party used the schooner Torrey for transportation and quarters.

*Hydrography of Quohog bay, Maine.*—This work has been completed by boat soundings made in the upper part of the bay by Mr. H. L. Marindin, with a party in the schooner Caswell. Several localities were also examined within the limits of the work last executed in the vicinity, and where shoal water occurred additional soundings were made. Mr. Paul Mayor was attached as temporary aid to the party of Mr. Marindin.

An examination of the progress sketch (No. 1) will show that the in-shore hydrography is now continuous along the coast of Maine from Cape Elizabeth as far eastward as the entrance to Penobscot bay.

*Saco river entrance, Maine.*—At the request of public authorities communicated by the representative of the first congressional district of Maine, a special examination of the bar and entrance of the Saco was made in May by Assistant George Davidson. The object being to determine the practicability of improving the channel for purposes of commerce, all features having reference or bearing on such an end were noted and communicated to the authorities in the question. In the course of the examination, Mr. Davidson determined the exact position and developed the depth of water on the ledge locally known as the "Pumpkin Rocks." Great changes were found to have occurred in the channel of the Saco since the year 1839. In this special survey, Mr. C. P. Dillaway served as aid.

*Topography of the coast of New Hampshire.*—Assistant Hull Adams took the field on the 11th of June to fill an interval in the detailed topography of the coast north of Hampton river. In the vicinity of Great Boar's Head, a junction was made with the plane-table work done to the southward by Assistant Whiting in a previous season. Mr. Adams, aided by Sub-Assistant T. C. Bowie, advanced the survey towards Portsmouth as far as Rye church, tracing thirty-five miles of water-line and mapping within an area of twenty-two square miles. The topography was made conformable in breadth to the work already done on this part of the coast. When closed for the season, the detailed survey had been extended one mile north of Locke's Point.

*Environs of Boston, Massachusetts.*—On three of the original plane table sheets containing the survey of the vicinity of Boston, Assistant F. W. Dorr has added the lines of railroad constructed since the date of the topographical survey. Seven different lines centering at the city were carefully traced within the limits of the sheets, making in the aggregate a length of fourteen miles.

Mr. Dorr was subsequently employed in topographical duty on the coast of Maine, as already noticed, and also in Section IV.

*Boston harbor.*—Assistant Henry Mitchell has rendered, as heretofore, the aid required in the physical survey by the United States commission, at first in the capacity of consulting engineer and subsequently as a member of the commission. He has collected and promptly furnished data, as needed from time to time, and has prosecuted the physical survey, the details of which are retained in his hands. Under the direction of the commissioners, the topographical and hydrographic work needed in the special inquiries have been performed by Mr. A. Boschke, formerly attached to the Coast Survey.

The tenth report of the United States commission concludes with the following remarks:

"In the further discharge of our obligations we have to say, that without the efficient co-operation of the Coast Survey of the United States, without its vessels, its instruments, its assistants, its books, its training, its methods, and its accumulated resources and stores of information, we should never have ventured to invite the city government to engage in our enterprise. * * * *

"Our debt to the Coast Survey does not consist only of the means and instruments of surveying and of competent persons to use them, but it embraces also the data for making (through its former labors in the same field) comparison maps, showing the changes which have actually taken place, and which are constantly in progress in that part of Boston harbor which we have had under consideration. These maps are of inestimable value, the information they contain is becoming every day of more importance, and in them the engineer will always find a safe instructor and guide."

*Topography of Cape Cod bay, Massachusetts.*—The detailed survey of the shores of Cape Cod bay was taken up by Assistant P. C. F. West, in August. In the vicinity of Plymouth he joined his work with the topographical sheet of that harbor, and completed the survey of the coast between Eel river and Ship Pond. By the end of November he had mapped the details within an area of eight square miles. The plane-table sheet shows over ten miles of the outline of the bay, and twenty-eight miles of road.

Mr. C. S. Hein was attached as aid during part of the working season.

*Topography of Narraganset bay, Rhode Island.*—Under the general direction of Assistant A. M. Harrison, the detailed topography has been continued by Sub-Assistant Charles Hosmer, the impaired health of the chief of the party not permitting him to keep the field during the season. Between the latter part of August and the 3d of November the sheets comprising the shores of Providence river and Prudence island were filled in with details, making an area of about ten square miles. They represent also twenty-five miles of road, five miles of water-courses, and six miles of marsh line.

*Hydrography of Warren river, Rhode Island.*—The hydrography of Narraganset bay has been advanced this season by the thorough development of the channel and bed of the Warren river. For this service a party was organized by Assistant F. P. Webber, at the end of August. The work was completed on the 22d of September, the journal then showing that about ten thousand casts of the lead had been recorded. The numerous rocks found in the river are believed to be all marked in place on the chart.

Assistant Webber was aided in the soundings by Messrs. F. D. Granger and A. L. Ross.

*Inspection of topography.*—The plane-table parties working in this section west of the Penobscot were reviewed in their places in the course of the present season by Assistant H. L. Whiting. In each locality Mr. Whiting observed the natural features, criticised their representation on the plane-table sheets, discussed the expedients for attaining rapidity in execution and precision in contour, and in general advised with reference to the means for securing uniformity in the character of the plane-table details. The result expected from this special co-operation of Assistant Whiting is the return of sheets with details determined by the most judicious selection, when, by reason of redundancy, some of the details must be of necessity generalized. That this object has been to a great extent already attained is due to the past exertions of this accomplished topographer.

In April and May, and again in October and November, Mr. Whiting rendered special service in the line of his profession at the United States Naval Academy, of which mention will be made under the head of Section III.

*Tidal observations.*—The series of tidal observations with a self-registering tide-gauge at Portland was kept up by Mr. H. W. Richardson until April, when he withdrew, and was succeeded by Mr. Horace Anderson, temporarily. Since the beginning of May the observations have been continued by Mr. W. R. Wood.

At the Charlestown navy yard, Massachusetts, Mr. T. E. Ready has continued observations on the tides with an ordinary box gauge. Meteorological observations have been regularly recorded at this station.

## SECTION II.

### FROM POINT JUDITH TO CAPE HENLOPEN, INCLUDING THE COAST OF CONNECTICUT, NEW YORK, NEW JERSEY, PENNSYLVANIA, AND PART OF DELAWARE.

*Hydographic developments near New York harbor.*—At the request of the engineer department, Assistant W. S. Edwards made a special examination of the "Frying Pan," the "Heel Tap," and "Pot Rocks," in October and November. His soundings have been plotted and presented to the department in the form of a chart, so as to admit of ready comparison with previous surveys.

A wreck lying in the vicinity of the main channel into New York bay was determined in position, and soundings were made in its vicinity so as to exhibit its character as an obstacle to the navigation of that channel. In the former part of the season the party of Mr. Edwards had been on duty in Section VI.

While prosecuting the special examination in East river, Assistant Edwards discovered a rock having only ten feet of water on it at low tide. This rock lies about one hundred yards due south of "Holmes' Rock."

*Hell Gate.*—In the course of a thorough survey made of this part of East river, in previous seasons, by Assistant Henry Mitchell, a large amount of information relative to the tides and currents was gathered and placed in the archives. That the data referred to may become available for future purposes, Mr. Mitchell has been directed to collate his results in connection with the former investigations of that locality by Lieutenant Commanding (now Rear-Admiral) Charles H. Davis, and to report thereon. A preliminary report made by him will be found in Appendix No. 6.

*Topography of the coast of New Jersey.*—The coast topography below Shrewsbury inlet has been continued by the party of Assistant C. M. Bache. In the course of the summer and autumn the detailed survey was pushed to a point below Long Branch. The ground passed over includes the most intricate field-work likely to be met south of Navesink, with the survey of which the work of the present year is in connection.

The two sheets returned to the office by Assistant Bache represent twenty-eight miles of shoreline and the surface features within an area of nine square miles. Mr. H. M. De Wees served during part of the season as aid in the plane-table party.

*Triangulation of the coast of New Jersey.*—The coast triangulation has been continued by Assistant John Farley. In the work of revision between Barnegat and Absecom light, eight stations were occupied during the course of the season, at which eleven hundred observations with the theodolite were recorded.

*Topography and hydrography of Barnegat inlet, New Jersey.*—At the request of the Light-house Board, communicated by Colonel Hartman Bache, of the corps of United States engineers, a special examination has been made of the vicinity of the light-house at Barnegat. The object of the board being to determine the rate at which the site of the light-house is suffering encroachment, Sub-Assistant Clarence Fendall made a careful topographical survey of a limited area specified by Colonel Bache, and by continuous tidal observations in May and June, in connection with soundings and observations on the currents, completed the data required for the engineering purposes of the Light-house Board.

The results of the survey made by Mr. Fendall point to the conclusion that the inlet as an opening is moving to the south, the shoals in the vicinity, of course, advancing with it in that direction.

*Special survey at Chester, Pennsylvania.*—For the use of the engineer department a very careful survey has been made of the water front at Chester, in September and October, by Mr. E. Hergesheimer. The outline of the government piers was accurately traced, including the mouth of Chester creek, and the ground surface in their vicinity was surveyed and mapped on a large scale. Lines of level were run, and sections of the piers were drawn on the map, to show their present condition. Mr. Hergesheimer also plotted on the map soundings made in the course of the survey to show the depth of water out to a line one hundred and fifty yards from the piers.

*Tidal observations.*—The self-registering tide gauge at Governor's island, New York harbor, has been successfully kept in operation during the year by Mr. R. T. Bassett. For comparison with the ecord, the usual observations with a box gauge at Brooklyn have been regularly continued by Mr. Bassett in the day time.

Meteorological observations have been steadily recorded at the permanent tidal station on Governor's island.

## SECTION III.

### FROM CAPE HENLOPEN TO CAPE HENRY, INCLUDING THE COAST OF PART OF DELAWARE, THE COAST OF MARYLAND, AND PART OF THE COAST OF VIRGINIA.

*Astronomical observations at Principio station, Maryland.*—The reciprocity treaty between the United States and Great Britain having expired on the 17th of March, 1866, in accordance with notice to that effect given by our government, the commission authorized under the first and second articles of that treaty closed its labors on the 1st of April, and Assistant Richard D. Cutts, who had been detailed as United States surveyor under the treaty, and who had served in that capacity since 1855, and also as colonel and aide-de-camp in the army from November, 1861, until the close of the rebellion, returned to active duty in the Coast Survey.

In July, August and September the geodetic station Principio, near the head of Chesapeake bay, in Maryland, was occupied by Assistant Cutts. His observations there are in continuation of the series of astronomical determinations made between the Fire island base and the base on Kent island, with reference to their probable application in the measurement of an arc of the meridian.

The local time at Principio was determined with transit No. 11 from 127 series of observations on 36 high and low stars. Sidereal chronometer No. 1276, used in the observations, was kindly lent for the purpose by Rear-Admiral Davis, superintendent of the Naval Observatory.

*Latitude.*—For the determination of the latitude of the station 243 observations were made on 40 pairs of close zenith stars with zenith telescope No. 5. The arc value of the micrometer was tested by observations on Polaris at its eastern elongation.

*Azimuth.*—The astronomical meridian and bearing of the primary triangulation line from Principio to Turkey Point were determined by 201 observations on Polaris at and near its eastern elongation, in combination with 228 measurements between the star and azimuth mark, and 20 series of 6 observations each between the mark and the station at Turkey Point. The instrument employed was the 24-inch Troughton theodolite.

At Cape Henry light-house, Virginia, observations of a similar character were commenced by Assistant George Davidson in August. For the astronomical observations an eccentric station was occupied and another determined for the azimuth and magnetic observations, on the line Cape Henry light-house—Cape Charles light-house.

Mr. Davidson made the latitude observations with zenith telescope No. 1, obtaining 187 determinations of the latitude by 40 pairs of stars upon an average of five nights. Some of the stars were observed three times each night: at 30 seconds before meridian passage; at culmination; and at 30 seconds after. In connection with these, observations for time were made with the 26-inch transit No. 11. All the records were duplicated and part of the star places were reduced with the annual precession in north polar distance recalculated with Peters's Elements. For value of the micrometer, observations were made upon $\lambda$ Ursæ Minoris at elongation. To these Assistant Davidson had added the preliminary calculations for azimuth, of $\alpha$ and $\lambda$ Ursæ Minoris at elongation, when it became necessary for him to proceed to Section I to take charge of the Calais end of the telegraph line intended to be used for determining longitude.

The aids in service with Mr. Davidson at Cape Henry were Messrs. W. I. Vinal and M. F. Wright. The instruments for use in the observations were sent from Norfolk to Cape Henry in the government steam tug, by the courtesy of Captain Rodgers of the navy yard.

Near the close of the season Assistant Cutts proceeded to Cape Henry, and in the course of the following month made the remaining observations.

The time at the light-house was determined by 68 series of observations on 25 high and low stars arranged in groups, using transit No. 11 and sidereal chronometer Hutton No. 311.

*Azimuth.*—The azimuth of the primary line from Cape Henry light-house to Cape Charles light-house was determined by the use of the revolving light at Cape Charles as the azimuth mark. For this purpose 90 observations on the light and 106 observations on 51 Cephei at eastern elongation, and 90 observations on the light and 96 on $\lambda$ Ursæ Minoris at eastern elongation, were taken with the Gambey theodolite No. 16.

Assistant Cutts also measured the horizontal angles between Pleasure-house Point and Back River light, and between Back River light and Cape Charles light-house, using for that purpose the Brunner theodolite No. 93. Eight series of observations were made, consisting of 103 repetitions. Mr. A. F. Pearl served as recorder.

*Hydrography of the Patapsco river, Maryland.*—The engineer department having under consideration plans for improving the Brewerton channel of the Patapsco, a resurvey of that river was commenced early in November, at the instance of the department. Assistant F. P. Webber was assigned for that service, and at the present date has completed soundings eastward and southward of the entrance, and has included the southern part of the river. The work is still in progress, the intention being to develop completely the approaches to Baltimore from the Chesapeake. A return made by Assistant Webber on the 19th of December shows that 21,000 soundings had been recorded. He is aided by Messrs. F. D. Granger and W. I. Vinal.

*Topography of the Upper Potomac, Maryland and Virginia.*—The reconnaissance survey of the Potomac river above Harper's Ferry was resumed by Sub-Assistant J. W. Donn, opposite Shepherdstown, on the 25th of November. After tracing a central traverse line the details of ground in the direction toward Harper's Ferry were added during the winter, to include Shepherdstown and its vicinity. The weather was then very unfavorable for field-work. When the spring opened the survey was resumed, and early in June Mr. Donn joined with the limits to which he had extended the work in 1864. A belt of about half a mile on each side of the river was mapped in conformity with the earlier sheets of the survey, and, as before, contour lines to show successive elevations of twenty feet were carried over the entire ground. These lines were referred to the lockage of the canal, the various heights of which were known, and by that means to the line of mean high-water at Georgetown, D. C.

Sub-Assistant Donn was aided during part of the season by Mr. Stehman Forney.

Within the limits of the military district of western Maryland the survey in charge of Mr. Donn was much furthered by details of men and facilities for transportation supplied to him by order of Major General Emory. The two plane-table sheets of this season, completing the intended reconnaissance, present twelve square miles of topographical detail.

*Sharp's Island light-house, (Chesapeake bay.)*—The bluff on which the light at Sharp's island was first erected having washed away to the extent of thirty yards inland of the site, another structure was projected by Captain Newman, the engineer in light-house service, to be secure in position, with reference to probable changes in the shore-line. At the end of the surveying year, the new light-house being then in order, Sub-Assistant Clarence Fendall determined its position, and marked the same on the engraved sheets, furnishing also the bearings needed for the purposes of the Light-house Board.

*Topography and hydrography at Newport News, Virginia.*—In November and December, 1865, this locality was specially surveyed at the request of the Navy Department. Mr. E. Hergesheimer traced the shore-line, and with the plane-table filled in all the details formed within an area of five square miles. Abreast of the ground represented on his topographical sheet, he carefully sounded the James river to a depth of six fathoms, and combined the results of his work on a single sheet for the uses of the department. In surveying, lines of level were run, and the sheet is marked throughout with contour lines showing successive elevations of ten feet. Mr. Hergesheimer was aided in duty by Mr. A. R. Fauntleroy.

*Triangulation at Chesapeake entrance, Virginia.*—For the extension of the system of main triangles southward from the Chesapeake entrance, three points have been determined in position on the shore of Lynn Haven roads, viz: Cape Henry light-house, Pleasure-house Point, and Willoughby's Point. All of these were occupied in the course of the summer and autumn by Assistant S. C. McCorkle, and in connection with them the stations Old Point Comfort light-house and Back river. The results furnish a reliable basis for continuing the main triangulation along the outer coast below Cape Henry.

In determining the horizontal angles over four thousand measurements were recorded.

Mr. McCorkle specially mentions in his report kind offers of assistance from Colonel Bre verton of the corps of engineers, and from Commander Phelps and Lieutenant Commander Grafton, of the navy. The last-named officer rendered material assistance in the work by a detail of men and boats.

*Tidal observations.*—The series of observations at the permanent tidal station at Old Point Comfort, Virginia, has been continued during the present year with a self-registering tide-gauge, as heretofore. Mr. C. Kelley, who had charge of the gauge at the beginning of the year, was succeeded in February by Mr. E. F. Krebs.

*Topography at the Naval Academy, Annapolis, Maryland.*—With the view of giving greater practical value to the course of studies pursued by the first class of naval cadets, graduated under his superintendence in May last, Admiral Porter made a request in April for the services of Assistant H. L. Whiting to give practical illustration of the use of the plane-table as employed in the survey of the coast and harbors of the United States. The detail of the duty desired was arranged by Lieutenant Commander R. L. Phythian, who had charge of the department of astronomy and navigation at the Naval Academy. After a division of the class into sections of sixteen to eighteen students, Mr. Whiting took the sections into the field on successive days, and demonstrated the principles and methods of work by making an actual survey of part of the harbor of Annapolis and of the grounds of the Academy.

These exercises were closed by the annual examination of the cadets, which took place at the close of May. Much interest having been manifested, Admiral Porter renewed his request in October. A class, numbering in the aggregate ninety, was taken into practice by Mr. Whiting; each day, for a period of several weeks, being devoted to a party of about twelve cadets. Of his experience as an instructor in the field during the month of November, Assistant Whiting; remarks: "The interest and attention of the class, and the alacrity with which each cadet took part in and executed all the co-operating details which I assigned to them, were proofs of the merit and advantage of this popular method of instruction."

In connection with the plane-table exercise, a scheme elaborated at the outset in consultation with Captain Phythian, for making hydrographic surveys, was executed by the cadets under his direction.

In addition to the field-work with the class, and as an example for their future practice, Mr. Whiting made an extended survey of interior details, employing as aids a crew of sailors. This survey was desired by Admiral Porter to cover certain grounds north of the city of Annapolis, within limits considered favorable for extending the grounds of the Naval Academy. The sheet was made elaborate in details, and when completed was left for the use of the admiral.

In the interval between the two periods of service at Annapolis, Assistant Whiting inspected the plane-table parties working in Section I.

## SECTION IV.

### FROM CAPE HENRY TO CAPE FEAR, INCLUDING THE COAST OF PART OF VIRGINIA AND PART OF NORTH CAROLINA.—(SKETCH No. 13.)

*Topography between Ocracoke inlet and Cape Lookout, North Carolina.*—The topography of the coast of North Carolina was resumed at Cape Lookout in June, by Assistant W. H. Dennis. Proceeding in the direction towards Ocracoke, the plane-table survey was made to include the peninsula which separates the Atlantic from Pamlico sound and the islands inside of the peninsula. Assistant Dennis being taken seriously ill in July, was relieved by Sub-Assistant Clarence Fendall, who carried forward the topography to a junction with the limits reached by the survey in a previous year at the north side of Ocracoke inlet. The completed sheets were turned in by Mr. Fendall early in September.

*Hydrography of Pamlico sound, North Carolina.*—Sub-Assistant J. S. Bradford, with a party in the schooner Arago, has extended the hydrography of Pamlico sound below Ocracoke inlet and from thence westward to Brant island and Neuse River entrance. The position of signals for use in this work he determined by careful triangulation. Eighteen thousand soundings were recorded before the close of June. At the approach of the sickly season the hydrography was discontinued.

At the request of the Light-house Board a special hydrographic survey was made of the vicinity of Long Shoal, in Pamlico sound, the object of the board being the location of a screw-pile light-house.

*Azimuth and latitude observations near Newbern, North Carolina.*—Early in May, Assistant G. W. Dean proceeded with the requisite instruments to determine the latitude of a station near Newbern, and also the azimuth of lines of the triangulation in that vicinity. Fort Spinola, about a mile and a half from the city, was selected as a favorable point for the proposed operations.

The azimuth was determined by Assistant Dean by means of 236 observations upon Polaris near its lower culmination, and upon Delta and Lambda Ursæ Minoris near their eastern elongation. In determining the bearings of the lines of triangulation 208 observations were made on four of the signals and an elongation mark with a 24-inch theodolite.

The observations for local time and for latitude were made by Assistant Edward Goodfellow, between the 19th of May and the 6th of June. With a 26-inch transit 76 observations were recorded for time, and with zenith telescope No. 5, 165 observations upon 30 sets of stars for determining the latitude. Mr. F. H. Agnew was assigned to act as recorder, but was taken seriously ill at Newbern before the commencement of operations.

The party of Assistant Dean returned to Washington on the 15th of June. Mr. Goodfellow then made the computations, completed the records, and forwarded them to the office. Mr. Dean proceeded to Newfoundland to await the arrival of the Atlantic cable, and to make the arrangements necessary for its use in determining the difference of longitude between Foilhommerum and Heart's Content, of which mention has been made under the head of Section I in this report.

*Topography of Neuse river, North Carolina.*—The plane-table survey of this river was commenced by Assistant F. W. Dorr on the 11th of June at points about two miles above Newbern, the lower part of the Trent river being within the limits of the upper plane-table sheet. The topography of the banks of both rivers was mapped to their junction at the city, and that of the Neuse to points about twelve miles below. Of the obstructions placed in the river during the war, enough have been removed to afford a good beating channel, but everything of the sort which yet shows above water was carefully marked on the plane-table sheets.

Most of the field-work of the lower sheet of the survey was done by Mr. Franklin Platt, whose aptitude and efficiency are again warmly commended by Assistant Dorr. The two sheets represent nearly sixty miles of shore-line and about seventy miles of road within an area of fifty-nine square miles.

The plane-table party used the schooner Dana until the 10th of July, when the vessel was transferred to the charge of Assistant Fairfield.

*Triangulation of Neuse river, North Carolina.*—This work was resumed early in June by Assistant G. A. Fairfield, and continued in the direction towards Pamlico sound. It was brought into connection with the stations used for that survey at Neuse River light-house before the close of September. The triangulation has now included nearly forty miles of the course of the Neuse river. The party of Mr. Fairfield occupied twelve stations this season.

Before taking up the triangulation, Mr. Fairfield joined the party of Assistant Dean and aided in the determinations of latitude and azimuth in the vicinity of Newbern.

Mr. J. G. Spaulding served as aid in the triangulation party. The schooner Joseph Henry was used for transportation.

*Hydrography of Neuse river and Trent river, North Carolina.*—Beginning at Fort Anderson, Sub-Assistant Bradford thoroughly sounded the Neuse river and defined its channel as low down as Johnson's Point. In connection with it the lower part of the Trent was sounded, or so much of that stream as fell within the plane-table limits. Thirty-one thousand soundings were recorded by the party in the Arago before the end of July.

*Off-shore hydrography: Cape Lookout shoals and off-shore soundings between Cape Hatteras and Cape Fear.*—During the months of March, June, July, and August, Acting Master R. Platt, U. S. N., assisted by Mr. G. Bradford and Mr. G. W. Bissell, in the steamer Corwin, completed the survey of Cape Lookout shoals, determining 800 positions, making 8,900 soundings, and running 350 miles of sounding lines in the execution of this work. Afterwards the same party continued the off-shore hydrography between Cape Hatteras and Cape Fear, running 237 miles, in which 1,000 casts of the lead were taken.

## SECTION V.

### FROM CAPE FEAR TO ST. MARY'S RIVER, INCLUDING PART OF THE COAST OF NORTH CAROLINA, AND THE COAST OF SOUTH CAROLINA AND GEORGIA. (SKETCH No. 15.)

*Hydrography of Savannah river, Georgia.*—At the opening of the surveying year, Assistant C. O. Boutelle, with an efficient party in the steamer Bibb, was working in the vicinity of Tybee entrance. After making the triangulation necessary for revising the hydrography of the Savannah, the river was sounded from the bar at Tybee to a point above the city. The vicinity of the obstructions and the channel were carefully defined. The impediments placed in the river having totally changed the direction of proper sailing lines, a copy of the new chart in manuscript, by authority of the department, was furnished to the mayor of Savannah for the use of the river pilots.

Assistant Boutelle during the war having given close attention to lighting and other facilities for navigation, concluded the hydrographic survey of the Savannah by submitting for the consideration of the Light-house Board a report embodying the results of his study in regard to the requirements for that river.

In this section, Mr. Boutelle was aided at intervals by Messrs. A. C. Mitchell, A. M. Wetherill, J. B. Adamson, H. L. Marindin, and J. A. Guldin.

*Tidal observations.*—In connection with the hydrographic survey under his charge, Assistant Boutelle for several months of the present year maintained a series of observations with a self-registering tide-gauge at Bay Point, in Port Royal sound.

## SECTION VI.

### FROM ST. MARY'S RIVER TO ST. JOSEPH'S BAY, INCLUDING THE COAST OF FLORIDA, WITH THE REEFS AND KEYS.

*Straits of Florida.*—A call from the International Telegraph Company in the spring for information relative to the lay of the bottom between Key West and Cuba, was met by the assignment of a hydrographic party, in the steamer Corwin, for making the requisite soundings. Assistant Henry Mitchell was placed in charge of the work, in which he had the co-operation of Acting Master Robert Platt, and of Messrs. Charles Junken and Gershom Bradford, attached to the party as aids. The season proving favorable for deep-sea soundings, the line was soon run satisfactorily.

Thirty casts were made in positions well determined along the proposed track of the telegraph cable; and a variety of incidental observations upon temperature, density, velocity of current, &c., was recorded. Some of the developments of this survey are interesting. It appears that the north bank of this part of the straits of Florida falls away in terraces to the depth of 853 fathoms; and that between this maximum depression and another of 748 fathoms near the Cuban coast, there rises a mountain within 400 fathoms of the surface, over which flows the Gulf stream with a velocity of 2¼ miles per hour.

Full details of the work will be found in Mr. Mitchell's report, (Appendix No. 5.)

*Astronomical and magnetic observations at Punta Rasa, Florida.*—The geographical position of a station at Punta Rasa, the southern entrance to Charlotte harbor, Florida, (Sketch No. 18,) was determined in June and July by Sub-Assistant A. T. Mosman. Its longitude, relative to Key West, was ascertained by transporting, in the schooner Varina, five chronometers, after carefully rating them at Key West. Observations with transit No. 8 had been previously made for time on six successive nights at that port.

On reaching Punta Rasa, the chronometers were at once taken to the intended station, and Mr. Mosman made observations for local time on the night of their arrival. The return of the Varina to Key West in July offering another opportunity, the observations were repeated, but with less success, the vessel being six days at sea between the two places.

At Punta Rasa, the latitude was determined by 115 measures of difference in zenith distance, using 22 pairs of stars, and observing with zenith telescope No. 6.

The azimuth was ascertained by 120 measures of the angle between Polaris and an elongation mark on "Sword Point."

The magnetic declination, dip, and intensity were determined by full sets of observations made in the usual way.

Mr. F. W. Perkins effectively aided in the operations at Punta Rasa and Key West.

Sub-Assistant Mosman is now at Valencia, (Ireland,) and engaged, under the direction of Dr. B. A. Gould, in observations for determining the difference of longitude between the telegraph station there and the station at Heart's Content.

*Hydrography of San Carlos bay, Florida.*—This, which is the southern entrance to Charlotte harbor, was sounded between the middle of June and middle of August by the party of Assistant W. S. Edwards, working with the schooner Varina. The soundings were extended so as to embrace on the hydrographic sheet the mouth of Caloosahatchee river and the approaches to the bay from the Gulf of Mexico. Sixteen thousand soundings were recorded.

Sub-Assistant F. F. Nes and Mr. C. S. Hein were attached to the party in the Varina, but the last named being disabled by illness soon after the arrival of the vessel at her worknig ground, was relieved and assigned to duty at the north.

Assistant Edwards, with the crew of the Varina, co-operated in putting up a temporary observatory and in arranging a shore station, intended to be occupied by Sub-Assistant Mosman, at Punta Rasa.

*Topography and hydrography of Charlotte harbor, Florida.*—The plane-table survey and soundings of the passage between Pine island and the main coast of Florida was resumed on the 3d of June, by a party in charge of Sub-Assistant C. T. Iardella, with the schooner Agassiz. The work was energetically pushed under many disadvantages, the sickly character of the season having disabled all but the chief of the party. Notwithstanding the difficulties thus arising, Mr. Iardella traced eighty-one miles of shore line and mapped twelve square miles of detailed topography. The passage is about twelve miles long. Having set up and determined in position seventeen signals, the passage was carefully sounded out by an aggregate of more than seven thousand casts of the lead.

Mr. E. Ellicott was attached to the party as aid, but being taken seriously ill, was relieved and returned north in August.

Sub-Assistant Iardella, after suspending operations for a few weeks, resumed work at the end of November, and is now prosecuting the survey. While at Key West recently with the schooner Agassiz, he furnished transportation at the request of the engineer of the International Ocean Telegraph Line, who was then making arrangements for laying a cable along the edge of the keys.

## SECTION VII.

### FROM ST. JOSEPH'S BAY TO MOBILE BAY, INCLUDING THE COAST OF PART OF FLORIDA AND ALABAMA.

*Triangulation of Perdido bay, Florida.*—With the view of pushing the triangulation westward from Pensacola, so as to connect with the completed survey of Mobile bay, Assistant J. G. Oltmanns was assigned to duty in this section. During the autumn he examined the stations near Pensacola that had been relied on for continuing the work, but found that most of the marks had been destroyed. Fortunately, however, the city authorities, at his instance, retained in place a screw-pile which had been sunk in the public square in Pensacola, and thus preserved a station mark of considerable importance.

It is to be much regretted that at Barkley's Point no trace whatever remained of the screw-pile and granite post set as marks of the astronomical station in 1856. Two of the subsidiary marks that had been merely displaced were reset by Mr. Oltmanns so as to identify the locality.

Under the circumstances just noted, the stations at Fort Pickens and Fort McCree, not having undergone any change, were deemed the most eligible as starting points in the triangulation intended to be pushed over Perdido bay. Assistant Oltmanns accordingly selected and occupied nine stations, from which he observed with the theodolite on thirteen signals, and by means of the points thus determined traced in the shore-line of the lagoon extended westward from the entrance to Pensacola bay, and part of the adjacent Gulf coasts, making in the aggregate thirty-nine miles.

To insure accuracy in connecting with the survey of Pensacola bay, a subsidiary base-line of about a mile and a half was measured and included within the new triangulation.

## SECTION VIII.

### FROM MOBILE BAY TO VERMILION BAY, INCLUDING THE COAST OF MISSISSIPPI AND PART OF THE COAST OF LOUISIANA.

*Triangulation of the Mississippi delta.*—An efficient party was detailed for the intended hydrographic survey of the delta, to work under the direction of Assistant F. H. Gerdes, but in consequence of detention, from adverse causes, in the passage of the two vessels despatched for the service from Baltimore and Norfolk, the attention of Mr. Gerdes and Sub-Assistant C. H. Boyd was limited to the determination of points to serve for tracing the shore-lines, and for hydrographic signals. Mr. Gerdes identified two of the stations which he had previously occupied, and from them extended the triangulation by observing at seven new stations. He was aided in the field by Sub-Assistant C. H. Boyd, and by Messrs. L. A Sengteller and H. G. Ogden.

## SECTION IX.

FROM VERMILION BAY TO THE RIO GRANDE BOUNDARY, INCLUDING PART OF THE COAST OF LOUISIANA AND THE COAST OF TEXAS. (SKETCH No. 19.)

*Hydrography of Matagorda bay, Texas.*—An interval in the hydrography between the entrance and the port of Matagorda was filled in by the party of Assistant F. P. Webber, in July. For this service the schooner Stevens was assigned, and continued in the work until the 1st of August, when the vessel was laid up at Indianola. Eight thousand soundings were recorded.

Assistant Webber was aided in the hydrography by Messrs. H. Anderson and F. D. Granger.

During the latter part of the season all the members of this party were on duty in other sections of the coast.

*Topography of Corpus Christi bay, Texas.*—The regular topographical survey of the coast of Texas was resumed at Aransas Pass on the 18th of June, by Sub-Assistant Charles Hosmer, with a party in the schooner Peirce. Owing to the damage received in her very stormy passage, the vessel was found to be unavailable for the usual service in moving from place to place as the plane-table work advanced. Signals were however set up and other preliminaries arranged for the successful prosecution of the work. After tracing twenty-four miles of shore-line at the northern end of Corpus Christi bay, the condition of the schooner required that she should be sent to Mobile for repairs. Seven square miles of topographical area were mapped before closing work at the end of July.

Sub-Assistant Hosmer was aided in this section by Mr. A. L. Ross.

## SECTION X.

COAST OF CALIFORNIA FROM THE SOUTHERN BOUNDARY OF THE UNITED STATES ON THE PACIFIC TO THE FORTY-SECOND PARALLEL. (SKETCH No. 21.)

*Topography between Point San Pedro and Pillar Point, California.*—The coast topography between San Francisco entrance and Monterey bay has been completed by the addition of an elaborate sheet of plane-table work turned in by Assistant A. F. Rodgers. The bold features which mark that part of the Pacific coast are expressed on the sheet by contour lines, the directions of which were carefully determined by levelling. Twelve square miles are thus represented, the topography having an average breadth of nearly two miles. Assistant Rodgers was aided in the field by Mr. Alexander Chase.

*Triangulation of Suisun bay, California.*—Assistant W. E. Greenwell resumed this work at the limits to which it had been extended in a previous season by Assistant Lawson, and completed the triangulation of the bay in March and April of the present year. In the upper part of the bay his stations were selected so as to include the entrances of the Sacramento and San Joaquin rivers. The points requisite for the purposes of the hydrographic party were furnished by Assistant Greenwell.

*Hydrography of Suisun bay, California.*—This work was commenced in November, 1865, and was completed in the month of February following, by the party of Assistant Edward Cordell. The soundings, of which an aggregate of twenty thousand were recorded, were made with the schooner Marcy.

At Army Point, (Sketch No. 22,) where the work joins with the hydrography of Captain Alden, day and night observations of the tides were recorded for a complete lunation, and as the work advanced similar observations were made at two stations in the upper bay. In the lower part of the bay the soundings were extended quite into the entrances of the Sacramento and Joaquin rivers. By the heavy rains in January, the water at Sacramento was raised to a level of twenty-two feet above low-water mark.

Mr. W. E. Dennis was attached as aid to the hydrographic party.

*Hydrographic examination of Karquinas strait, California.*—In March, Assistant Cordell examined the strait for the purpose of comparing the soundings with the depth found at particular localities in the survey of 1863.

*Hydrography northward of Point Reyes, California.*—The hydrography of the coast of California has been continued in the vicinity of Point Reyes by Assistant Cordell, and has been kept in connection with the sounding of the approaches to San Francisco entrance. Much of the present season proved unfavorable for outside work. The hydrography, notwithstanding, has been advanced to Bodega Head by using every favorable interval for sounding. The general progress sketch (No. 25) shows the advance which has been made by the party in this vicinity. Mr. Cordell was employed in this work, with the schooner Marcy, between the 1st of June and the 11th of November. The lines of soundings were run broad off from the coast-line to depths varying from 65 to 80 fathoms, or about 16 nautical miles from the land. The in-shore hydrography also was extended from Point Reyes to Tomales Point. All the soundings were reduced for the chart to the mean of the lowest tides derived from day and night observations during two lunations at a station in Drake's bay. The journal shows entries of six thousand four hundred casts of the lead.

*Tidal observations.*—Under the supervision of Captain G. H. Elliot, of the corps of engineers, U. S. A., the tidal stations at San Diego and San Francisco have been kept up very successfully during the present year, and no delay has occurred in the receipt of the records at the office in Washington. The self-registering gauge at the first-named station has been, as heretofore, in charge of Mr. A. Cassidy, and that near San Francisco in charge of Mr. H. E. Uhrlandt. The usual meteorological observations were also recorded by these observers, and have been regularly forwarded with the tidal records.

The third permanent tidal station of the western coast will be referred to under the head of Section XI.

## SECTION XI.

### COAST OF OREGON AND COAST OF WASHINGTON TERRITORY.

*Triangulation of Tillamook bay, Oregon.*—In August, Sub-Assistant Julius Kincheloe measured a base-line of about seven hundred metres, and with it connected a triangulation, which, in the course of that month, was made to include the entire bay. In order to provide for the thorough development of the locality, day and night observations were made of the tides. He thus reports in reference to the bar and entrance of Tillamook bay:

"In coming over the bar I found about fifteen feet of water, and, as near as could be judged, at half tide. The bar is a short one, and the entrance makes in nearly east and west. Inside of the bar there is a depth of ten fathoms, but most of the bay is very shoal, and a large part of it is bare at low water."

Sub-Assistant Kincheloe occupied 16 stations with the theodolite, determined 42 points in position, and traced 26 miles of shore-line with the plane-table. The survey occupied the party until the middle of November, the latter part of the season being employed in the hydrography. Means will be provided as soon as possible for determining the character of the bar, its development not being practicable with the boat used by the party in triangulation.

*Hydrography of Koos bay, Oregon.*—This work was still in progress at the date of my last annual report, but was completed soon after by the party of Assistant J. S. Lawson. His party then returned to San Francisco with the brig Fauntleroy. About five thousand soundings were made and recorded in addition to those included in the previous reports.

*Destruction island and vicinity, Washington Territory.*—In the hope that a close examination of the vicinity of Destruction island might develop a harbor of refuge for the coast of Washington Territory, that neighborhood was carefully examined in July by Assistant Lawson, the brig Fauntleroy with his party being then on their way for duty in Puget sound. The results are given below in extracts from a report forwarded by Mr. Lawson in September:

"Destruction island is three and a quarter miles from the main land. Its surface at the highest point—about ninety feet above the sea—is covered with a very dense growth of bushes. Small patches are cleared, in which the Hooch Indians cultivate meagre crops of potatoes. The length of the island is seven hundred metres. A series of large detached rocks, with deep water between them, extends about fourteen hundred metres from the edge of the bluff at its northern end, and westward from that end of the island a ledge runs out six hundred metres, beyond which are a few detached rocks awash.

"Along the western side the bottom is rocky and very uneven, and entirely unfitted for anchorage with such distance as might afford protection from southerly winds or seas. More than a quarter of a mile seaward of the middle of the island I found a small rock with only thirteen feet of water on it, the depth alongside being four and a half fathoms, and a little to the northward of it another with eighteen feet water. Off the southern end was found a rock having sixteen feet water, in a position less than two hundred metres from the outer reef of the ledge. No other dangers w[illegible]ed outside of the three ledges that make off like fingers.

"The eastern s[illegible]estruction island affords the only proper anchorage, but, if resorted to, vessels must put to s[illegible] at the first indication of a southerly wind. As an anchorage, it is safe only during northerly or northwest winds. From a half to three-quarters of a mile off, the soundings are quite regular in from [illegible] twelve fathoms with hard bottom, but in-shore the bottom is broken, though not so muc[illegible]ere. In places the wall of rock is so bold at low-water mark that vessels might moor [illegible]e sea swell."

In connection w[illegible] survey of the island, the shore-line of the main land of Washington Territory, in the vicinity [illegible]as traced for several miles, and a few lines of soundings were run between the island and the main. The results are given in a sketch (No. 23) accompanying this report. Eleven hundred soundings were recorded in the progress of the survey.

*Tidal observations.*—The tidal station at Astoria has been, as heretofore, in charge of Mr. L. Wilson, under the general direction of Captain G. H. Elliot, United States engineers. The meteorological observations at this station have also been recorded by Mr. Wilson with the usual care and completeness.

Besides the station in Oregon, Captain Elliot has directed the observers employed at the two stations on the coast of California, already mentioned under the head of Section X.

## COAST SURVEY OFFICE.

In the several divisions of the office in Washington the duties allotted to each have been performed with but slight change in the arrangements previously made for the service. As stated in former reports, data and material received from the field, admitting of classification, are appropriately referred, and after being worked up are subsequently conjoined under the direction of the assistant in charge for the intended publications. The divisions are designated as follows:

*Hydrographic Division*, in which, under the direction of Captain C. P. Patterson, hydrographic inspector, all the adjuncts required for the final issue of charts are prepared and arranged, as the inspection and verification of original matter, selec[illegible]on of characteristic soundings, designation of dangers to navigation, sailing lines, and sailing directions. During the year but one draughtsman, Mr. E. Willenb[illegible]r, has been attached to the division.

The equipme[illegible] and care of vessels, and their readiness for service with [illegible]urveying parties, are amongst the duties devolving upon the hydrographic inspector.

*Tidal Division.*—The tidal data previously collected in this division, and retained in the charge of Assistant L. F. Pourtales, have been digested within the present year into the form of tables showing the tides for every day of the year 1867, at ports on the Atlantic, Gulf, and Pacific coasts of the United States. An edition printed and bound in convenient form has been furnished for the use of the naval and revenue service. Mr. R. S. Avery was in charge of the division during the latter part of the year, in the temporary absence of Assistant Pourtales, and much credit is due to him for the good judgment and energy manifested, particularly in the preparation of the manuscript of the tide-tables for publication.

The observations pertaining to this division, maintained at regular stations on the coast, are reported under the sections to which the stations severally belong.

In office details Assistant Pourtales has been aided also by J. Downes, J. Sprandell, D. Schooley, M. Thomas, and F. R. Pendleton.

A specimen showing the arrangement of the tidal ephemeris for 1867, applicable to the port of Eastport, is given in the Appendix, (No. 7.) For the use of the merchant marine an edition of the tables has been provided for sale at the principal ports.

*Computing Division.*—The charge of this division has been continued with Assistant Charles A. Schott, whose personal attention has been given to the refined computations connected with the adjustments of the primary triangulation of the coast, and to other special processes required in the deduction of results from extended series of observations. Amongst these was the establishment of final equations, derived from occultations of the Pleiades for the verification of longitude and for correcting the lunar elements, a work prosecuted under the general direction of Professor Benjamin Peirce, of Harvard.

The routine work, such as office adjustments of the observations made by triangulation parties, reductions required in determining latitude, longitude, azimuth, and the magnetic elements, has been kept up, each part when completed being made the subject of a special report by the chief of the Computing Division.

The computers attached to the office are Messrs. T. W. Werner, Eugene Nulty, G. Rumpf, and E. H. Courtenay. Mr. R. S. Avery was on duty in the division until May, and Messrs. J. G. Spaulding and F. H. Agnew for limited periods previous to their assignment to field service.

*Drawing Division.*—This branch of the service, as during several years past, has remained under the immediate supervision of the assistant in charge of the office. Mr. W. T. Bright, as heretofore, has aided in conducting the office details connected with the division.

The names of the draughtsmen, their employ, and the titles of the drawings on which they have worked during the year, specified as completed or yet in progress, will be found in tabular form in the Appendix, (No. 3.)

*Engraving Division.*—Under the immediate direction of the assistant in charge of the office, the supervision of work in this division has been continued in the care of Mr. Edward Wharton.

The pantagraph has been used with effect during the year in engraving directly on copper at the proper scale, from the outlines of tracings taken from the original sheets; and punches have been further employed for making the figures to express soundings on charts of the second class.

A synopsis of the operations of the division, including the names of the engravers, is given in the Appendix, (No. 4.)

The clerical duty was performed by Mr. George C. Schaeffer, jr., until August, when he was assigned to duty in the field.

*Electrotype and Photographing Division.*—By the electrotype process thirty-two of the most recently engraved plates have been duplicated within the present year by Mr. George Mathiot. For use in the Engraving Division fifteen photographic glass positives and thirty-four glass negatives were made, as reductions from original sheets of the survey. In duplicating maps intended for the engraver, and for other purposes, 184 paper prints have been made by the photographic process. Mr. A. F. Pearl aided in the duties of this division until July, when he was attached to a field party.

*Lithographing.*—Within the year the reconnaissance sheets of the Mississippi river between Cairo and St. Mary's, six in number, have been engraved by Mr. C. G. Krebs, as also nine of the sheets representing the reconnaissance of the Tennessee, and four diagrams. The services of Mr. Krebs have also been in requisition for the preparation of charts issued in colors, and for miscellaneous engraving, as notes and titles of preliminary charts of which only the shore-line and soundings had been engraved on copper. The details of work in this division, as also the chart and map printing, have been directed by Mr. W. W. Cooper.

*Chart and map printing.*—On the copper-plate press 15,820 copies of charts and sketches have been printed in the course of the year ending on the 1st of November. With the lithographic press 5,150 copies have been printed by transfer, of which number about one-third were printed in colors. Early in the year that press was used for final editions of special maps which had been engraved on stone and issued during the war; and of this class 7,922 copies were printed. Exclusive of circulars about 21,000 impressions were printed on the lithographic press during the year. The copper-plate press has been worked by Mr. T. V. Durham, the lithographic press by Mr. A. Brown.

*Distribution of maps and annual reports.*—An aggregate of 10,900 copies of charts have been distributed within the year. The map-room was in the charge of Mr. M. T. Johnstone until a short time before his death, which took place in June. The duties pertaining to it are now performed by Mr. Thomas McDonnell. Of 5,000 copies of annual reports for various years, distributed within the last twelve months, about one-fifth of the number was forwarded to institutions.

CONCLUSION.

The duty of issuing directions for the prosecution of the survey in all its branches, and the responsibility of acting in behalf of the Superintendent was devolved upon me in the autumn of 1864, when he found his health too much impaired to permit him to continue the active direction of the field-work.

It is a pleasure to record that in this arduous service, superadded as it was to the charge of office details, I have had the hearty co-operation of the officers who during previous years acted under the immediate orders of the Superintendent.

My acknowledgments are especially due to the hydrographic inspector, Captain C. P. Patterson, who in addition to his duties in the office has had charge of the direction of the hydrography, and has constantly aided me with his advice in matters of administration.

In the disbursing agent, Samuel Hein, esq., I have ever found the ready and intelligent exponent of the fiscal arrangements best adapted to the interests of the survey, with reference both to economy and efficiency in the operations of the field parties.

I am likewise indebted to W. W. Cooper, esq., for valuable aid in the conduct of the work, in connection with his duties as chief clerk to the Superintendent, which have made him familiar with the details of administration.

Respectfully submitted by

J. E. HILGARD,
*Assistant in Charge for the Superintendent.*

HON. HUGH MCCULLOCH,
*Secretary of the Treasury.*

# APPENDIX.

## APPENDIX No. 1.

*Distribution of the parties of the Coast Survey upon the coasts of the United States during the surveying season of* 1865–'66.

| Limits of sections. | Parties. | Operations. | Persons conducting operations. | Localities of operations. |
|---|---|---|---|---|
| SECTION I. | | | | |
| From Passamaquoddy bay to Point Judith, including the coast of Maine, New Hampshire, Massachussets and Rhode Island. | No. 1 | For longitude by the telegraphic method. | Dr. B. A. Gould, assistant; A. T. Mossman, sub-assistant, (at Valencia, Ireland;) Geo. W. Dean, assistant; Edward Goodfellow, assistant, (at Heart's Content;) Newfoundland. George Davidson, assistant, (part of season;) C. O. Boutelle, assistant, (part of season;) F. W. Perkins and S. C. Chandler, aids, at Calais, Maine. | Observations of time at each station, by astronomical transits, compared by signals made under the direction of the astronomer royal from Greenwich, in exchange with signals from Valencia, by Dr. Gould. From Valencia to Heart's Content, by exchange of signals between Dr. Gould and Mr. Dean; and from Heart's Content to Calais, by exchanges between Mr. Dean and Mr. Boutelle. |
| | 2 | Astronomical observations. | C. O. Boutelle, assistant .... | Bannock hill, N. H., occupied and connected with the primary triangulation at Mount Blue Latitude determined at a station near Farmington, N. H. (See also Section V.) |
| | 3 | Triangulation..... | C. H. Boyd, sub-assistant .. | Triangulation of the St. Croix river, Me., and connected with the astronomical station at Calais, Me. (See also Section VIII.) |
| | 4 | Topography...... | W. H. Dennis, assistant .... | Shore-line survey of the St. Croix river, from its entrance upwards, nearly to the head of navigation. (See also Section IV.) |
| | 5 | Hydrography; in charge of S. A. Gilbert, assist. | H. L. Marindin, in charge; Geo. C. Schaeffer, jr., aid. | Hydrography of West Quoddy bay and approaches, including the Narrows and soundings above, connecting with the survey of Eastport harbor. (See also Section V.) |
| | 6 | .........do........ | Charles Junken, sub-assistant; H. M. De Wees and L. A. Sengteller, aids. | Soundings in the approaches of Penobscot bay, extended northward to Rackley's island. (See also Section VI.) |
| | 7 | .........do........ | R. E. Halter, sub-assistant; J. B. Adamson and Eugene Ellicott, aids. | Tennant's harbor, sounded in connection with Muscle Ridge channel, and supplementary hydrography in the western approaches to St. George's river, Me. |
| | 8 | Triangulation .... | J. A. Sullivan, sub-assistant. | Triangulation for the plane-table survey of islands in Muscongus bay, Me. |
| | 9 | Topography ...... | Charles Ferguson, sub-assistant. | Detailed survey of the shores of the Medomak river, Me., extended southward to the vicinity of Long island. |

APPENDIX No. 1—Continued.

| Limits of sections. | Parties. | Operations. | Persons conducting operations. | Localities of operations. |
|---|---|---|---|---|
| SECTION I—Cont'd. | 10 | Hydrography; in charge of S. A. Gilbert, assist. | Horace Anderson, sub-assistant; H. G. Ogden, aid. | Hydrography of the upper part of the Medomak river, from Waldoboro' southward to Long island. (See also Section IX.) |
| | 11 | Topography...... | Cleveland Rockwell, sub-assistant. | Shore-line survey of the main and islands of Muscongus bay, from Crotch island and Round Pond. |
| | 12 | Topography...... | F. W. Dorr, assistant; Franklin Platt, jr., aid. | Plane-table survey of the peninsula, terminating at Pemaquid Point, Me., including the adjacent islands. (See also Section IV.) |
| | 13 | Hydrography; in charge of S. A. Gilbert, assist. | E. Hergesheimer............ | Supplementary soundings, completing the hydrography of the Damariscotta and Sheepscot rivers, Me. (See also Sections II and III.) |
| | 14 | Topography...... | R. M. Bache, assistant...... | Extension of the plane-table survey of the shores of the Kennebec river, including islands in the vicinity of Merrymeeting bay, Me. |
| | 15 | Triangulation..... | J. A. Sullivan, sub-assistant. | Determination of points for the plane-table survey and hydrography of New Meadows river, in connection with the upper part of Quohog bay. |
| | 16 | Topography and hydrography; in charge of S. A. Gilbert, assist. | J. W. Donn, sub-assistant. .. | Plane-table survey and hydrography of New Meadows river and upper part of Quohog bay, Me. (See also Section III.) |
| | 17 | Topography ..... | A. W. Longfellow, assistant.. | Shore-line survey of Phippsburg basin and Winnegance bay, Me., with intervening islands and ledges. |
| | 18 | Hydrography; in charge of S. A. Gilbert, assist. | H. L. Marindin, aid ........ | Supplementary soundings, completing the hydrography of Quohog bay, Me. (See also Section V.) |
| | 19 | Special survey.... | George Davidson, assistant; C. P. Dillaway, aid. | Hydrography of the bar and approaches to Saco river, Me., and development of changes in shore-line and depth. (See also Section III.) |
| | 20 | Topography...... | Hull Adams, assistant; T. C. Bowie, sub-assistant. | Detailed plane-table survey of the coast of New Hampshire, continued from Great Boar's Head northward to Locke's Point. |
| | 21 | Topography...... | F. W. Dorr, assistant ....... | Plane-table survey and addition to topographical sheets of the lines of railroad centring at Boston, Mass. (See also Section IV.) |
| | 22 | Special survey.... | Henry Mitchell, assistant.... | Surveys and compilation of data continued for the use of the United States commission on Boston harbor. (See also Section VI.) |
| | 23 | Topography...... | P. C. F. West, assistant .... | Topographical survey of the western shore of Cape Cod bay, between Eel river and Ship pond. |
| | 24 | Inspection of topography. | H. L. Whiting, assistant .... | Inspection of field parties on the coast of Maine. |
| | 25 | Topography...... | A. M. Harrison, assistant; Chas. Hosmer, sub-assistant. | Detailed survey of the shores of Providence river and Providence island, in continuation of the topography of Narraganset bay, R. I. (See also Section IX.) |

APPENDIX No. 1—Continued.

| Limits of sections. | Parties. | Operations. | Persons conducting operations. | Localities of operations. |
|---|---|---|---|---|
| SECTION I—Cont'd. | 26 | Hydrography .... | F. P. Webber, assistant; F. D. Granger and A. L. Ross, aids. | Hydrography of Warren river, R. I. (See also Section IX.) |
| | 27 | Tidal observations. | W. R. Wood ............... | Series continued with self-registering tide-gauge at Portland, Maine. |
| | 28 | ........do........ | T. E. Ready................ | Observations continued with box-gauge at Charlestown navy yard, Mass. |
| SECTION II. From Point Judith to Cape Henlopen, including the coast of Connecticut, New York, New Jersey, Pennsylvania, and part of Delaware. | 1 | Hydrography .... | W. S. Edwards, assistant; F. F. Nes, sub-assistant; S. Forney, J. B. Adamson, aids, (part of season;) H. G. Ogden, W. I. Vinal, aids, (part of season) | Development of the "Frying Pan," "Heel Tap," and "Pot Rocks" by soundings in East river, and of the vicinity of a wreck near the main ship channel at the entrance of New York bay. (See also Section VI.) |
| | 2 | Topography ...... | C. M. Bache, assistant; H. M. De Wees, aid. | Detailed plane-table survey of the coast of New Jersey, extended from Shrewsbury inlet southward to and including Long Branch. |
| | 3 | Triangulation .... | John Farley, assistant....... | Triangulation of the outer coast of New Jersey continued in the vicinity of Absecom inlet. |
| | 4 | Topography and hydrography. | Clarence Fendall, sub-assistant. | Plane-table survey and soundings to determine the sea encroachment at the site of Barnegat light-house. (See also Sections III and IV.) |
| | 5 | Topography and hydrography. | E. Hergesheimer............ | Detailed survey of the vicinity and soundings in the Delaware river, off the government wharves at Chester, Pa. (See also Sections I and III.) |
| | 6 | Tidal observations. | R. T. Bassett............... | Series continued with a self-registering tide-gauge at Governor's island, and with a box-gauge at Brooklyn, N. Y. |
| SECTION III. From Cape Henlopen to Cape Henry, including the coast of part of Delaware, the coast of Maryland, and part of the coast of Virginia. | 1 | Astronomical observations. | Richard D. Cutts, assistant; A. F. Pearl, aid. | Principio station, Md., occupied for the determination of latitude and azimuth. |
| | 2 | ........do........ | George Davidson, assistant, (part of season;) Richard D. Cutts, assistant, (part of season:) W. I. Vinal, aid; A. F. Pearl, aid. | Determinations of latitute and azimuth at Cape Henry light-house, Virginia. |
| | 3 | Special survey ... | F. P. Webber, assistant; F. D. Granger, W. I. Vinal, aids. | Soundings in the southern part of the approaches and lower part of the Patapsco river, Md., for the engineer department. (See also Sections I and IX.) |
| | 4 | Topography ...... | J. W. Down, sub-assistant... | Reconnaissance survey of the shore of the Potomac river between Shepherdstown and Harper's Ferry. (See also Section I.) |
| | 5 | Triangulation .... | Clarence Fendall, sub-assistant. | Determination of the position of the new light-house at Sharp's island in Chesapeake bay. (See also Sections II and IV.) |
| | 6 | Topography and hydrography. | E. Hergesheimer in charge; A. R. Fauntleroy, aid. | Plane-table survey at Newport News, and hydrography of the vicinity. (See also Sections I and II.) |
| | 7 | Triangulation .... | S. C. McCorkle, assistant.... | Stations occupied on the south side of Chesapeake entrance for extending the primary triangulation of the coast below Cape Henry. |
| | 8 | Tidal observations. | E. F. Krebs ................ | Series continued with a self-registering gauge at Old Point Comfort, Virginia. |

APPENDIX No. 1—Continued.

| Limits of sections. | Parties. | Operations. | Persons conducting operations. | Localities of operations. |
|---|---|---|---|---|
| SECTION IV. | | | | |
| From Cape Henry to Cape Fear, including the coast of part of Virginia and part of North Carolina. | 1 | Topography ...... | W. H. Dennis, assistant, (part of season;) Clarence Fendall, sub-assistant, (part of season.) | Plane-table survey of the outer coast of North Carolina between Ocracoke inlet and Cape Lookout, including the adjacent islands in Pamlico sound. (See also Sections I, II, and III.) |
| | 2 | Hydrography..... | J. S. Bradford, sub-assistant. | Hydrography of the southern part of Pamlico sound, including the entrance to Neuse river; and special development of the vicinity of Long shoal. |
| | 3 | Astronomical observations. | George W. Dean, assistant; Edward Goodfellow, assistant. | Determination of latitude and azimuth near Newbern, North Carolina. (See also Section I.) |
| | 4 | Topography ...... | F. W. Dorr, assistant; Franklin Platt, jr., aid. | Topography of the shores of the Neuse and Trent rivers in the vicinity of Newbern, North Carolina, and survey of the shores of the Neuse extended southward to Beard's creek. (See also Section I.) |
| | 5 | Triangulation .... | C. A. Fairfield, assistant; J. G. Spaulding, aid. | Triangulation continued in the lower part of Neuse river and nearly completed. |
| | 6 | Hydrography..... | J. S. Bradford, sub-assistant. | Hydrography of Neuse river and Trent river in the vicinity of Newbern, North Carolina, and sounding of the Neuse extended below to Beard's creek. |
| | 7 | Hydrography..... | Robert Platt, acting master U. S. N; Gershon Bradford and G. W. Bissell, aids. | Hydrography of Cape Lookout shoals, and off-shore soundings between Cape Hatteras and Cape Fear. |
| SECTION V. | | | | |
| From Cape Fear to St. Mary's river, including part of the coast of North Carolina and the coast of South Carolina and Georgia. | 1 | Hydrography .... | C. O. Boutelle, assistant; A. C. Mitchell, A. M. Wetherill, J. B. Adamson, H. L. Marindin, and J. A. Guldin, aids. | Hydrography of the Savannah river, including its bar and approaches; and thorough development of the change caused by obstructions in the channel. (See also Section I.) |
| SECTION VI. | | | | |
| From St. Mary's river to St. Joseph's bay, including the coast of Florida, with the reefs and keys. | 1 | Hydrography .... | Henry Mitchell, assistant; Robert Platt, acting master U. S. N.; Chas. Junken, sub-assistant; Gershom Bradford, aid. | Deep-sea soundings in the straits of Florida, made in a line directly between Key West and Havana. (See also Section I.) |
| | 2 | Astronomical observations. | A. T. Mosman, sub-assistant. | Determinations of latitude and azimuth at Punta Rasa, (Charlotte harbor,) and approximate determination of longitude, by means of chronometers. (See also Section I.) |
| | 3 | Hydrography .... | W. S. Edwards, assistant; F. F. Nes, sub-assistant. | Hydrography of the approaches and southern entrance to Charlotte harbor, including the mouth of Caloosahatchie river, Florida. (See also Section II.) |
| | 4 | Topography and Hydrography. | C. T. Iardella, sub-assistant; E. Ellicott, aid, (part of season.) | Plane-table survey of the shores and hydrography of the passage between Pine island, in Charlotte harbor, and the inner coast of Florida. |
| SECTION VII. | | | | |
| From St. Joseph's bay to Mobile bay, including the coast of part of Florida and Alabama. | 1 | Triangulation .... | J. G. Oltmanns, assistant ... | Triangulation extended westward of Pensacola entrance, intended to include Perdido bay. |

APPENDIX No. 1—Continued.

| Limits of sections. | Parties. | Operations. | Persons conducting operations. | Localities of operations. |
|---|---|---|---|---|
| SECTION VIII. | | | | |
| From Mobile bay to Vermilion bay, including the coast of Mississippi and part of the coast of Louisiana. | 1 | Triangulation .... | F. H. Gerdes, assistant; C. H. Boyd, sub-assistant; L. A. Sengteller and H. G. Ogden, aids. | Determination of points for the detailed plane-table survey of the passes of the Mississippi. (See also Section I.) |
| SECTION IX. | | | | |
| From Vermilion bay to the Rio Grande boundary, including part of the coast of Louisiana and the coast of Texas. | 1 | Hydrography .... | F. P. Webber, assistant; H. Anderson, sub-assistant; F. D. Granger, aid. | Supplementary soundings in Matagorda bay, between the entrance and the port of Matagorda. (See also Sections I and III.) |
| | 2 | Topography...... | Chas. Hosmer, sub-assistant; A. L. Ross, aid. | Topography of the coast of Texas resumed at Aransas Pass, and extended to include the north shore of Corpus Christi bay. (See also Section I.) |
| SECTION X. | | | | |
| Coast of California from the southern boundary of the United States, on the Pacific, to the forty-second parallel. | 1 | Topography...... | A. F. Rodgers, assistant; Alex. Chase, aid. | Detailed plane-table survey of the coast of California, between Point Pedro and Pillar Point. |
| | 2 | Triangulation .... | W. E. Greenwell, assistant .. | Triangulation completed in Suisun bay, and made to include the mouths of the Sacramento and Joaquin rivers. |
| | 3 | Hydrography .... | Edward Cordell, assistant; W. E. Dennis, aid. | Hydrography of Suisun bay, including the entrances of the Sacramento and Joaquin rivers. Hydrography of Karquines strait examined and compared with previous survey. In-shore and off-shore soundings extended from Point Reyes, northward, to Bodega Head. |
| | 4 | Tidal observations. | Capt. G. H. Elliot, U. S. engineers; A. Cassiday; H. E. Uhrlandt. | Series continued with self-registering tide-gauges at San Diego and near San Francisco. |
| SECTION XI. | | | | |
| Coast of Oregon and coast of Washington Territory. | 1 | Triangulation .... | Julius Kincheloe, sub-assistant | Triangulation of Tillamook bay, Oregon. |
| | 2 | Hydrography .... | Jas. S. Lawson, assistant.... | Completion of soundings in Koos bay, Oregon. Hydrographic examination of the vicinity of Destruction island, Washington Territory. |
| | 3 | Tidal observations. | Capt. G. H. Elliot, U. S. engineers; L. Wilson. | Series continued with a self-registering tide-gauge at Astoria, Oregon. |

# APPENDIX No. 2.

*Information furnished from the Coast Survey office, by tracings from original sheets, &c., in reply to special calls, during the year ending November 1, 1866.*

| Date. | | Names. | Data furnished. |
|---|---|---|---|
| 1865. | | | |
| November | 10 | Tench F. Tilghman, chief engineer, Maryland and Delaware railroad. | Hydrographic survey of the Delaware river, near Bombay Hook. |
| December | 1 | Navy Department | Map showing site for navy yard, at Port Royal, S. C. |
| | 11 | C. J. Gilman, esq | Hydrographic survey of Blackbird creek and vicinity, Delaware river. |
| 1866. | | | |
| January | 8 | San Francisco Dock Company | Topographic survey of Yerba Buena island, San Francisco bay, Cal. |
| | 8 | San Francisco Dock Company | Hydrographic survey of San Francisco bay, Cal. |
| | 12 | Light-house Board | Hydrographic survey, vicinity of Sharp's island, Chesapeake bay. |
| | 19 | Navy Department | Topographical and hydrographic survey, from Port Royal entrance to Port Royal ferry, |
| | 30 | Light-house Board | Hydrographic resurvey of main entrance to Cape Fear river, N. C. |
| February | 1 | Light-house Board | Hydrographic resurvey of New inlet, Cape Fear river, N. C. |
| | 3 | Joint committee on harbors of Massachusetts | Part of hydrographic survey of Taunton river, Mass. |
| | 3 | J. D. Mitchell, esq | Sketch of Perdido bay, Fla. |
| | 6 | Sir Charles Lyell, England | Copies of surveys of delta, Mississippi river, in 1838 and 1860. |
| | 23 | Navy Department | Topographical and hydrographic resurvey of Newport, News Point, Va. |
| | 24 | Dr. W. Gunton, president Bank of Washington. | Topographical survey, country southeast of and adjoining District of Columbia |
| | 28 | Major C. S. Stewart, corps of engineers | Hydrographic survey of the Delaware river, from New Castle to Reedy Point. |
| March | 3 | Colonel Hartman Bache, corps of engineers | Hydrographic and topographical resurvey of Barnegat inlet, N. J. |
| | 15 | Captain W. P. Craighill, corps of engineers | Tracing of north and south shores of Patapsco river, from Fort McHenry to Fort Carroll. |
| | 22 | Hon. R. J. Walker | Hydrographic survey of eastern branch of Potomac river. |
| | 24 | J. C. Hoadley, esq | Hydrographic survey of Charles river, Boston, Mass. |
| | 31 | Hon. John A. Griswold | Hydrographic survey of the Hudson river from Albany to Troy. |
| April | 3 | Captain P. C. Hains, corps of engineers | Comparative map of Cape Fear River entrances, from surveys of 1852, '57, '58, and '65. |
| | 16 | Lieut. Col. H. W. Benham, corps of engineers. | Comparative map of Cape Cod, Mass. |
| May | 12 | Light-house Board | Hydrographic and topographical survey of Assateague and Chincoteague islands, and entrance to bay. |
| | 28 | Lieut. Col. John Newton, corps of engineers | Topographical and hydrographical survey of Staten and Long islands, vicinity of Narrows, N. Y. |
| June | 5 | Quartermaster General's Office | Road map of approaches to Washington city. |
| | 8 | Light-house Board | Hydrographic survey in the vicinity of Upper and Lower Cedar Points, and Smith's Point, Potomac river. |
| | 9 | Hon Secretary of the Navy | Compiled map of League island, Delaware river. |
| | 21 | Hon. G. V. Fox, Assist. Secretary of the Navy | Compiled map of League island, Delaware river. |
| | 30 | Brvt Major W. P. Craighill, corps of engineers. | Topographical survey of upper Potomac, from Bolivar Heights to Shepherdstown. |
| | 30 | Brvt. Major W. P. Craighill, corps of engineers. | Topographical survey of Baltimore and approaches. |
| July | 26 | Light-house Board | Hydrographic survey of Great harbor, Wood's Hole, Mass. |
| August | 2 | Light-house Board | Hydrographic resurvey of Barnegat inlet, N. J. |
| | 2 | Brvt. Lt. Col. C. S. Stewart, corps of engineers. | Hydrographic survey, vicinity of Chester and Marcus Hook, Delaware river. |
| | 16 | F. Huderhoff, esq., Mississippi | Topographical survey of the Chandeleur islands, La. |
| | 16 | J. W. King, chief engineer U. S. N | Compiled map of League island, Delaware river. |
| | 16 | A. F. Sears, chief engineer New York and Newark railroad. | Hydrographic survey of Newark bay. |
| September | 1 | Oscar Smetberg, esq | Tide-table of the Hudson river. |
| | 17 | Light-house Board | Hydrographic survey of part of Bachelor's bay, entrance to Roanoke river, N. C. |
| | 22 | Brvt. Brig. General H. Brooks, U. S. A | Hydrographic survey of Patapsco river, from Fort McHenry to Sparrow's Point. |
| October | 20 | Hon. Thomas A. Doyle, mayor city of Providence, R. I. | Hydrographic survey of the Seekonk river, R. I. |
| November | 24 | Light-house Board | Hydrographic survey of outer end of Long shoal, Pamlico sound, N. C. |
| December | 1 | Brvt. Col. George Thom, corps of engineers | Hydrographic resurvey, entrance to Saco river, Maine. |
| | 15 | Light-house Board | Hydrographic survey, vicinity of Hooper's Straights light-vessel, Chesapeake bay. |
| | 15 | Light-house Board | Hydrographic survey, vicinity of Jane's Island light-vessel, Chesapeake bay. |

# APPENDIX No. 3.

## DRAWING DIVISION.

*Charts completed, continued, or commenced during the year.*

1. Hydrography. 2. Topography. 3. Drawing for photographic reduction. 4. Tracing for pantagraphic reduction. 5. Pantagraphic engraving. 6. Verification. 7. Lettering.

| Titles of charts. | Scale. | Draughtsman. | Remarks. |
|---|---|---|---|
| COMPLETED. | | | |
| Camden and Rockport harbors, Maine | 1-20,000 | 1. A. Lindenkohl. 7. E. Hergesheimer. | |
| Coast chart No. 3, Cape Small Point to Cape Cod | 1-200,000 | 1, 6. A. Lindenkohl. 6. E. Willenbucher. | Additions. |
| Portland harbor, Maine | 1-20,000 | 1, 2. A. Lindenkohl | ....do. |
| Portsmouth harbor, New Hampshire | 1-30,000 | 2. E. Hergesheimer. | |
| Isles of Shoals, New Hampshire | 1-20,000 | 1, 2. F. Fairfax. 7. E. Hergesheimer. | |
| Boston harbor and approaches | 1-40,000 | 1. A. Lindenkohl. 2. E. Hergesheimer. | New edition. |
| Coast chart No. 12, Monomoy to Muskeget channel | 1-80,000 | 2. A. Lindenkohl | Additions. |
| Sippican harbor, Massachusetts | 1-20,000 | 1. H. Lindenkohl | Corrections. |
| Newport harbor, Rhode Island | 1-20,000 | 1. J. H. Logan. 2. F. Fairfax. | |
| Providence harbor, Rhode Island | 1-10,000 | 1. A. Lindenkohl. 2, 7. E. Hergesheimer. | |
| Bristol harbor, Rhode island | 1-20,000 | 6, 7. E. Hergesheimer. | |
| New York bay and harbor, coast chart No. 21 | 1-80,000 | 3, 6, 7. E. Hergesheimer. | |
| Hudson river, from New York to Teller's Point | 1-60,000 | 1, 2. A. Lindenkohl. 6, 7. E. Hergesheimer. | Additions. |
| Delaware and Chesapeake bays | 1-400,000 | 1. F. Fairfax | ....do. |
| Coast of Maryland, coast chart No. 28 | 1-80,000 | 1. A. Lindenkohl. 7. E. Hergesheimer. | |
| Potomac river, from Indian Head to Georgetown | 1-40,000 | 1, 2. A. Lindenkohl. | |
| Lookout shoals, North Carolina | 1-80,000 | 1. H. Lindenkohl. 1. E. Willenbucher. | |
| Cape Fear river, entrances | 1-30,000 | 1, 2. A. Lindenkohl | New edition. |
| Coast chart No. 48, vicinity of Cape Fear | 1-80,000 | 1. A. Lindenkohl. | |
| Coast chart No. 53, Rattlesnake shoals to St. Helena sound. | 1-80,000 | 1. A. Lindenkohl. 3, 6, 7. E. Hergesheimer. | |
| Port Royal, Broad, and Beaufort harbors, South Carolina. | 1-60,000 | 1, 2. A. Lindenkohl | Additions. |
| Wassaw sound, Georgia | 1-40,000 | 1, 2. A. and H. Lindenkohl. 7. E. Hergesheimer. | |
| Half Moon bay, California | 1-20,000 | 2. F. Fairfax. 6. A. Lindenkohl. | |
| Pacific coast, Point Pinos to Bodega Head | 1-200,000 | 1, 2. A. Lindenkohl. 7. E. Hergesheimer. | |
| San Francisco, upper bay, California | 1-50,000 | 2. A. Lindenkohl. | |
| CONTINUED. | | | |
| General Chart No. 1, Quoddy Head to Cape Cod | 1-400,000 | 1, 2. A. Lindenkohl. | |
| St. George's river and Muscle Ridge channel | 1-40,000 | 1. A. Lindenkohl. 3, 4, 5. A. Molkow. 6, 7. E. Hergesheimer. | |
| Kennebec and Sheepscot rivers, Maine | 1-40,000 | 1. A. Lindenkohl. 3. J. H. Logan. 3, 7. E. Hergesheimer. | |
| Coast chart No. 7, Muscongus bay to Portland | 1-80,000 | 1. A. Lindenkohl. 3, 6, 7. E. Hergesheimer. | |
| Coast chart No. 8, vicinity of Portland harbor | 1-80,000 | 2. F. Fairfax. 3. E. Hergesheimer. | |
| Coast chart No. 10, Boston bay and approaches | 1-80,000 | 1. A. Lindenkohl. 2, 6, 7. E. Hergesheimer. | |
| Coast chart No. 11, Cape Cod bay | 1-80,000 | 1. L. Karcher. 2. M. J. McClery. 6, 7. E. Hergesheimer. | |
| Coast chart No. 14, Narraganset bay | 1-80,000 | 1. A. Lindenkohl. 3. E. Hergesheimer. | |
| Coast chart No. 27, *bis*, entrance to Delaware bay | 1-80,000 | 1. L. Karcher. | |
| Coast of Virginia, coast chart No. 29 | 1-80,000 | 1, 2. F. Fairfax. 7. E. Hergesheimer. | |
| Lake Borgne and Lake Pontchartrain, Louisiana | 1-80,000 | 1. A. Lindenkohl. | |
| San Antonio and Aransas bays, Texas | 1-80,000 | 2. M. J. McClery. | |
| Koos bay, Oregon | 1-30,000 | 1. L. Karcher. 2. F. Fairfax. | |
| COMMENCED. | | | |
| Coast chart No. 6, Isle-au-Haut to Muscongus bay | 1-80,000 | 1. A. Lindenkohl. 3. E. Hergesheimer. | |
| Damariscotta river, Maine | 1-40,000 | 4. A. Molkow. | |
| Casco bay, Maine | 1-40,000 | 4. H. Lindenkohl. 4, 5. A. Molkow. | |
| New York bay and harbor, (lower plate) | 1-40,000 | 4, 5. A. Molkow. | |
| Charleston harbor, South Carolina | 1-30,000 | 1, 2. A. Lindenkohl | New edition. |
| General chart No. XIII, Cape San Blas to Southwest Pass. | 1-400,000 | 2. M. J. McClery. | |
| Washington Sound, Washington Territory | 1-200,000 | 1, 2. A. Lindenkohl | ....do. |

# APPENDIX No. 4.

## ENGRAVING DIVISION.

*Plates completed, continued, or commenced during the year.*

1. Outlines. 2. Topography. 3. Sanding. 4. Lettering.

| Titles of plates, &c. | Scale. | Engravers. | Remarks. |
|---|---|---|---|
| COMPLETED. | | | |
| Camden and Rockport harbors, Maine | 1-20,000 | 3. W. A. Thompson. 4. A. Petersen | Preliminary edition. |
| Portland harbor, Maine | 1-20,000 | 1, 2. A. Maedel. 3. R. F. Bartle. 4. E. A. Maedel. | New edition. |
| Isles of Shoals, New Hampshire | 1-20,000 | 1. A. Maedel. 2. W. A. Thompson. 3. F. W. Benner. 4. A. Petersen. 4. J. G. Thompson. | |
| Sea coast United States, No. 3, Cape Small Point to Cape Cod. | 1-200,000 | 1, 4. J. G. Thompson | Corrections and additions. |
| Coast chart No. 12, Monomoy to Muskeget channel. | 1-80,000 | 3. A. Sengteller. 4. E. A. Maedel | ........do. |
| Coast chart No. 13, Muskeget to Buzzard's bay. | 1-80,000 | 1, 2, 3. A. Sengteller. 4. E. A. Maedel. 4. J. Knight. | ........do. |
| Bristol harbor, Rhode Island | 1-20,000 | 2. W. A. Thompson | With hill curves only. |
| Newport harbor, Rhode Island | 1-20,000 | 1, 4. W. H. Davis. 3. F. W. Benner | Preliminary edition. |
| Potomac river, sheet No. 4, Indian Head to Georgetown. | 1-40,000 | 2. A. Maedel. 3. F. W. Benner. 4. A. Petersen. | |
| Delaware and Chesapeake bays | 1-400,000 | 3, 4. A. Petersen. | |
| Core sound, North Carolina | 1-40,000 | 1, 4. J. G. Thompson | Figures punched. |
| Key West harbor, Florida | 1-50,000 | 4. W. H. Davis | New edition. |
| St. Mark's river, Florida | 1-30,000 | 1, 2, 3, 4. C. Klakeing | On contract. |
| Half Moon bay, California | 1-20,000 | 1, 2. W. A. Thompson | Preliminary edition. |
| CONTINUED. | | | |
| Coast chart No. 7, Muscongus to Portland | 1-80,000 | 1, 2. H. C. Evans. | |
| Coast chart No. 8, Portland harbor and vicinity. | 1-80,000 | 3. H. S. Barnard. 4. A. Petersen. | |
| Coast chart No. 10, Boston bay and approaches. | 1-80,000 | 2. J. Enthoffer. 3. H. S. Barnard. 4. J. Knight. | |
| Coast chart No. 11, Cape Cod bay | 1-80,000 | 2. H. C. Evans. 4. E. A. Maedel. | |
| Coast chart No. 21, New York bay and harbor. | 1-80,000 | 2. J. Enthoffer. 3. H. S. Barnard | Nearly finished. |
| Coast chart No. 28, Isle of Wight to Chincoteague. | 1-80,000 | 2. A. Rolle. 4. E. A. Maedel. | |
| Coast chart No. 29, Chincoteague to Hog Island light. | 1-80,000 | 1, 2. A. Rolle. 4. J. Knight. | |
| Coast chart No. 34, Potomac entrance, Tangier sound. | 1-80,000 | 1, 2, 3. R. F. Bartle. 4. E. A. Maedel. | |
| Coast chart No. 37, Cape Henry to Currituck. | 1-80,000 | 1, 2. A. Sengteller. | |
| Coast chart No. 53, Charleston harbor and St. Helena sound. | 1-80,000 | 1, 2. A. Sengteller. 3. A. Rolle. 4. J. Knight. | |
| Coast chart No. 54, Hunting island to Ossabaw sound. | 1-80,000 | 2. A. Sengteller. | |
| Coast chart No. 93 Lakes Borgne and Pontchartrain. | 1-80,000 | 1, 2. A. Sengteller. | |
| Coast chart No. 106, Galveston to Oyster bay. | 1-80,000 | 2. R. F. Bartle. | |
| Coast chart 108, Matagorda and Lavacca bay. | 1-80,000 | 1, 2. J. C. Kondrup. | |
| Eastport harbor, Maine | 1-40,000 | 2, 3. A. Rolle. 4. E. A. Maedel. | |
| St. George's river and Muscle Ridge channel. | 1-40,000 | 1. W. A. Thompson. 3. H. S. Barnard. 4. E. A. Maedel. | Pantagraph outlines. |
| Kennebec and Sheepscot rivers, Maine | 1-40,000 | 1, 2. H. C. Evans. 3. F. W. Benner. 4. E. A. Maedel. | |
| General coast chart No. IV, Cape May to Cape Henry. | 1-400,000 | 2 A. Maedel. | |
| Upper part San Francisco bay, California | 1-50,000 | 1, 2. J. C. Kondrup. | |
| Pacific coast, Point Pinos to Bodega Head. | 1-200,000 | 1, 2. H. Lindenkohl. | |
| COMMENCED. | | | |
| Port of Providence, Rhode Island | 1-10,000 | 1, 4. J. G. Thompson | Figures punched. |
| New York bay and harbor, lower | 1-40,000 | 1. W. A. Thompson. | Pantagraph outlines. |
| Entrances to Cape Fear river, North Carolina. | 1-30,000 | 4. A. Petersen | New edition. |
| Cape Lookout shoals, North Carolina | 1-80,000 | 1. W. H. Davis. 4. A. Buckle | Figures punched. |
| Koos bay, Oregon | 1-30,000 | 1. W. A. Thompson. 4. A. Buckle | ........do. |
| Diagram tides, western coast | ............ | 1, 4. E. H. Sipe | ........do. |

**Miscellaneous work, such as borders, scales, compasses, sand, curves, lights, and buoys, has been executed, generally, by Messrs. J. G. Thompson, F. W. Benner, E. H. Sipe, and W. H. Davis. Ruling a tint on plates and punching figures, by A. Buckle.**

# APPENDIX No. 5.

REPORT BY HENRY MITCHELL, ASSISTANT UNITED STATES COAST SURVEY, UPON SOUNDINGS ACROSS THE STRAITS OF FLORIDA.

DEAR SIR: By reason of a call for information relative to the form and character of the bottom of the Straits of Florida, along the proposed path of the oceanic telegraph, which it is designed shall connect the United States with Cuba, it became necessary to increase the number of soundings between Key West and Havana.

Several years had elapsed since the Coast Survey had made deep-sea soundings, and therefore no suitable lines and leads remained for this new call. I was obliged, consequently, to prepare an entire outfit, which, in the leisure from other work, occupied me during the month of March of the present year. In the concluding portion of the report I shall comment upon the materials composing the outfit, and discuss briefly the merits of my lines, which were manufactured very carefully and laid up in a different manner from those to be found ready in the market.

When this work was first suggested to me it was proposed that one of the naval vessels from the Gulf squadron should be temporarily detailed for my use with the officers and crew. To this arrangement the Secretary of the Navy was consenting, but after mature inquiry and consideration it appeared that no vessel of suitable character was available from this squadron. The plan was accordingly changed, and Acting Master Robert Platt, commanding the Coast Survey steamer Corwin, was instructed to take me from Savannah to the scene of operations and lend every effort which his vessel and party could afford. I take pains to say in advance of my report upon the result that the officers and the Coast Survey assistants of his party carried out the letter and spirit of their instructions, and not only brought to the aid of the work much practical knowledge which I did not myself possess, but also that earnest personal interest which is not always secured in official relations.

On the 7th of April I left Boston with my freight by packet, and on arriving at Savannah on the evening of the 12th, found the Corwin waiting at her anchor and in readiness for sea. Two days sufficed for transferring the freight and completing necessary purchases, and we dropped down the river to Pulaski roads on the night of the 14th with the intention of going out at daybreak on the following morning. Stormy weather followed, so we did not actually cross the bar till the morning of the 17th, and it is scarcely an exaggeration to say that from this time till our return, nearly a month, we had scarcely a "cap-full of wind."

The Corwin makes but little steam, and with the current often against us, our journey down the coast was slow. I did not regret this, however, because it afforded me opportunities for studying carefully the surface densities and temperatures of the sea. I made very frequent trials with the hydrometer in the vain expectation of finding some of those "fresh-water spots" which have been frequently reported by vessels cruising along the coast of Florida inside the stream. As we crossed and recrossed the edge of the stream, before reaching the latitude of Cape Florida, I was able to acquire a knowledge of those contrasts of temperature which have been made so interesting and useful to navigators by your elaborate investigations in former years. As these observations were not mentioned among the objects of the expedition, I shall not report upon them at this time. I presume they would develop nothing new, but they will prove useful to me in the better comprehension of the former Gulf Stream explorations which you have placed in my hands for compilation.

On the 21st we came to anchor in the harbor of Key West, and after cleaning boilers and coaling we commenced to make soundings on the morning of the 25th, and worked to the southward.

Since it would involve many useless repetitions to describe our operations in the order of dates, I shall refer you at once to the table (annexed) of thirty stations at which successful casts were made, and to two recapitulation tables in which the results are summed up—first, according to number of station, and again by distance along a *section line.*

The positions of the stations were determined by dead reckoning, checked by observations for latitude and longitude, and by horizontal angles when the coast was visible.

Being satisfied that these positions are very close, I have required Mr. Charles Junken, who discussed the ship's reckoning, and Mr. G. Bradford, who observed and computed the altitudes, to compile their notes, over their own signatures, that I might hand them in, as I do, with this report.

In the process of sounding the line with "detaching lead" and "indicator" was suffered to run freely out over a sheave secured to one of the awning stanchions at the stern of the ship. In most cases the lead was secured and the various signals given personally by Captain Platt, while I, with a chronometer by my side, recorded the out-run of every fifty-fathom tag and made notes upon the trend of the line, its direction, and the lay of the ship. When bottom was felt, or when the lead was supposed to have reached it, the line, in the absence of a proper steam reel, was hauled in by a single purchase winch and by hand. In some cases the time of hauling in each fifty fathoms was recorded, with a statement of the number of men employed, the trend of the line, &c. On Diagram I the curves of the out-run and in-come of the line, in feet, per second are plotted. These have proved useful in the discussion of the results.

The track of our soundings was designed to be, as near as practicable a straight line from Sand Key light to El Moro light; and although our work was not continuous, and spaces were filled in our different dates, an inspection of our diagram of positions II, shows that we did not wander often from the right pathway. A very good section line is made possible by the positions of our stations, and upon Diagram III the profile is given.

Our deepest sounding was 853 fathoms, at a station about three miles to the westward of our section line and about 24 miles from the Moro. The interpolations among the stations on either side of our line gives the greatest depression for the profile; 845 fathoms, 37 miles from the Moro. It should be remarked here that the casts in this portion of the straits show that the line of greatest depression has a northeasterly course, perhaps very well marked by stations 28, 14, and 10, Diagram II, at which we have 853, 845, and 794 fathoms. This course is parallel to neither coast and makes an acute angle (about 20°) with our section line, so that our profile is not, strictly speaking, that of a cross-section of the channel way. This course of the main channel does not correspond with that of the surface current, but it would seem to be a natural one for the under-running polar current, so called.

The approaches to the Great Valley from the two coasts are dissimilar in their general features. From the northward the bottom falls away in terraces whose intervening slopes are nowhere abrupt; while from the southward an irregular and hilly approach is developed by our survey, with indications of abrupt, if not precipitous changes of elevation. Above the terraces of the north shore the sea lies almost motionless, while among the banks and cañons of the southern half of the straits flow the Gulf Stream and its counter-currents.

I think I am authorized by these natural distinctions to take up separately the descriptions of these approaches to the deep-channel way.

*Northern approach.*—Leaving Sand island the water deepens to eight fathoms, then shoals again to seven fathoms upon a narrow coast bar, which lies parallel to the reef, and distant from it three-fourths of a mile. Seen from the deck of a ship, upon a pleasant day, this bar is marked by a strip of pale-blue green water, in beautiful contrast to the rich blue-black of the ocean outside. The bottom can be seen on crossing it and appears to be of a pure white rock, *in situ*, strewn over with loose fragments of the weathered and brown reef rocks. Two and three-fourths miles from the "light" carries us to sixty fathoms, (A, Diagram III,) in which neighborhood the bottom is generally hard, but covered here and there with loose debris of shells and corals. From the 7-fathom reef to the 60-fathom curve, the fall is 1 foot in 37. Beyond this the fall is about 1 foot in 103 to the point B, (7½ miles from Sand Key light,) which is a little beyond the 100-fathom curve. Here we reach a plain or terrace which I have deemed worthy of particular notice. In this neighborhood, over a district of eight miles in longitude by one mile in latitude, we made five casts, feeling the hard rock bottom with the hand each time. The respective depths were 117, 105, 117, 116, and 111 fathoms. In one of these casts a few chips of the white coral rock came up in the Berryman specimen cup. In all of them the lead detached. Further on at C (11 miles from Sand Key light) we still find but 129 fathoms, showing a decline of but 1 foot in 195 for 2½ miles; and here one of the specimen cups was broken, as the lead suddenly detached upon the hard rock. We seemed to have reached the brow of the *terrace* for the next 3¼ miles to D, (14 miles from Sand Key light) the decline is 1 foot in 22, and the depth reached 289 fathoms, where the hard rock was still felt by the hand, as the lead detached. We have two soundings from which the depth at D is computed, which lie on either side of section line, about in the same parallel, but differing over four miles in longitude. These two casts gave, respectively, 287½ and 290 fathoms, and from this agreement I argue that the *coral terrace* above referred to lies east and west, or nearly parallel to the adjacent reef, to which formation it certainly belongs. I think we can safely conclude from the many casts we have made and from the yielding nature of the rock, that the *coral terrace* does not terminate in a precipice.

From D to the next point E, 3¾ miles, the decline is more gradual, 1 foot in 48.

At E the rock is no longer felt, but a specimen of bottom is procured from 369 fathoms—a gray mud differing in consistency and color from specimens obtained from the *coral terrace*, and also from those hereafter to be described as characteristic of the bottom beyond; another peculiarity of the specimen is this, instead of setting like the white coral muds, it has a somewhat granulated character on becoming dry. I conceive that the *coral terrace* may once have been a dry reef, covered over like Sand Key with dark fragments of agglomerated reef rock, and that a subsequent submergence has caused all this loose, sand-weathered material to be swept down to the foot of the fore slope. Between D and E, in about 300 fathoms, lies the foot of the swept portion of the Florida reef, if not also the base of the formation.

From E to F, six miles, the slope is still gradual, only 1 foot in 97, while the specimen obtained at F, in 422 fathoms, is a white coral, which sets in drying. From F to C, nearly five miles, the decline increases, becoming 1 foot in 69, with the same kind of bottom, except that at C a few dashes of red appear in the mud brought up from 504 fathoms.

This point C (about 29 miles from Sand Key light) brings us to the 500-fathom curve. We have two casts from which this part of our profile is drawn; station 6, about a mile from the westward of our line, gave 500, and station 12, a mile farther off to the west-northwest., gave 508 fathoms. Between the 400 and 500 contours there seem to be indications of another grand terrace, which might lead us to suspect that the solid coral rock still underlies the mud found by our lead.

Do these features belong to the history of the Gulf Stream or to the geology of the coral reef? As these slopes and terraces are now scarcely within the limits of the stream, I am inclined to regard them as exhibiting the order through which in successive ages the reef alternately subsided and stood still. As far as the *swept* portion of the reef apron extends I see no evidence of any undermining and carrying down of the reef; but, on the contrary, I recognize a connected formation without trace of violence. I hope the examination of specimens of bottom by Mr. Pourtales may throw some light upon these speculations.

Beyond the 500-fathom curve another rapid decline appears; in a distance of 4¾ miles from C to H, it is 1 foot in 47, the material of the bottom remaining the same in 687 fathoms. The next space H to I (four miles) carries us nearly to the 800-fathom curve, along a slope of 1 foot in 38. The depth given in the profile at I is 794 fathoms, from a cast at Station 10, which is at some distance from our section line at the eastward. The specimen obtained at this station is similar to that found at E, and is also from near the foot of a grand slope.

From I to the lowest point on our profile, 7½ miles, the decline is but 1 foot in 150—a slope which the eye would scarcely perceive were the bottom laid bare.

The ample specimens obtained in this maximum depression from 845 to 853 fathoms are white, with a delicate red or rose-colored tint—not unlike the freshly cut stone from the quarries near Havana.

The soundings which I have described along the northern approach to the main channel way were made with great care, and since in all of them the best evidences of bottom were obtained, no doubt rests upon the results. The bottom was felt by the hand as far out as 500 fathoms, beyond this the out-run of line is no criterion of the depth, since our simple purpose was to make sure of reaching the bottom by the most ample supply of line, paid out freely. In the southern approach which I am about to describe, the result was not so completely satisfactory. In the axis of the stream we were obliged to use a heavy lead of faulty construction which did not always detach, even when a specimen found in the cup bore evidence to its reaching the bottom; and in one or two cases, where the lead came home without a specimen, a serious doubt hangs on the results; although we were unsparing of line, having, in some cases, three or four times as much out-run as the Massey indicator showed the depth to require. I shall refer to these matters, however, in detail presently.

*Southern approach.*—The mouth of Havana harbor has rather a low ragged shore upon its southern point, while upon its northern, the rocky cliff of the Moro rises abruptly from the water to heights of 60 to 80 feet. The northwestern extremity of the Moro rock, as seen from the opposite point, is perpendicular at the water-line, but retreats at points higher up, so as to present to the observer a convex profile whose mean dip from the castle wall to the sea is nearly 45°. On near approach the rock is found to be ragged, but I observed no talus at its foot. As might be expected, the waves, acquiring very little motion of translation in the deep approach to this rock, rise and fall upon it with little noise and violence in ordinary weather. I cannot conceive that the undertow or reflex of the sea ever disturbs the bottom at a distance of 100 yards, so that no apprehensions may be entertained by the telegraph company in regard to the safety of their cable once laid along the entrance to the harbor, if it follows the deep water.

Leaving the Moro and advancing to the northward (see profile Diagram III) the bottom declines 1 foot in 7, in our first space of $1\frac{3}{4}$ mile to the point marked *a*. The bottom is rock in 243 fathoms; it was felt by the hand, and the specimen cup brought up only a dead shell. No eddies were observed, but a bodily movement of the water to the northeastward was apparent.

In the next space from *a* to *b* (2 miles) the depth increases at the rate of 1 foot in 6, and the foot of the Moro is passed. At *b* the Berryman specimen cup came up full, although the lead, failing to detach, did not close the cover.

The material is reddish brown mud, which becomes in part granulated in drying; in many respects it resembles the specimen found at the foot of the slope from the *coral terrace* on the north side of the stream; it has, however, the red hue which is not characteristic of the Florida side. It is, no doubt, weathered debris of the Moro rock.

There can be no question that the dip of the rocky part of this space, between *a* and *b*, is much greater than the average we have given; because, 1 foot in 6 is unnatural for the material which was found at *b*. I had hoped to make many casts on this rock, and develop its profile minutely. We could not do this, however, without determining the positions of objects, and putting theodolite observers on shore, and our consul did not seem able to assist us in getting the necessary permits without correspondence with Washington, for which we could not wait.

It is hardly likely that a soft and yielding rock like the Moro can terminate in a precipice, although at these great depths there is no agitation, and the stream is perhaps constant in direction and velocity.

Beyond *b* the slope is gradual, 1 foot in 32, and terminates at *c* in the nearly horizontal bed of a depression, which I shall call the Moro channel. Here at *c*, 9 miles from the "Light," we found 784 fathoms. Our lead did not detach, but one of the hooks was knocked off, showing that bottom had been reached; there was mud also in the specimen cup.

Six miles further on at *d* the depth obtained was 710 fathoms, with the same behavior of the lead, and evidence of dragging on bottom. In this case the specimen cup was empty, but as we had a heavy lead, and over a thousand fathoms of line out, which seemed to run down perpendicularly, no doubt of bottom is admissible, especially as the hook was again knocked off. We have passed across the Moro channel and the water is shoaling.

Further on at *e* and *f* we find ourselves near the summit of a hill or bank, which presents a marked feature in our profile, and, lying so near the axis of the Gulf Stream, may be claimed as a point in the survey of peculiar interest.

This elevation is scarcely 21 miles from the shore of Cuba, whose hills are in view if the weather is fair. We made six casts upon it near our line, three of which were complete failures: first, the line was found snarled about the indicator, which read but 20 fathoms; second, the line was cut, (probably by a shark,) when all but 10 fathoms had been hauled in; third, we tried a drop-line with a linen cord, and its rate of descent proved too slow to yield a sign.

Of the three casts which did give results, only one yielded a specimen, and detached lead. These three casts gave the following depths by indicator: at *e* 380 fathoms; at *f* 461 and 448. The last named yielded a specimen of fine mud, having a light reddish brown hue, stiff when first procured and hardening or setting on becoming dry.

The failure of the two leads to detach I attribute to their lightness, upon the theory that the different rate of the current below prevented their rapid descent, the bight of the line being held back by the stronger surface drift. In the case of detachment we had double the weight used in the two other cases.

The three casts were made upon different dates, using two different indicators and lines. The day on which the specimen was procured there seemed to be less current in this locality than on previous dates, as far as we could judge from the behaviour of the lines on hauling in. We let the ship lie at rest for first 700 fathoms, and paid out the line freely to 1,677 fathoms. Of course, in the current we had no hope of perceiving bottom by the line.

When 600 fathoms were out, the line seemed to run away rapidly, but not by reason of the descent of the lead, for I find in my statement the record that "the line trails out astern 150 feet," the outrun was evidently from the drift of the vessel away from the grounded lead. I am particular to mention these details, because the existence of a hill, composed of the material we obtained, in close proximity to the axis of the stream, might well be doubted, if any evidence of insufficiency or carelessness of observation appeared. My own record shows how suspicious I was of the results, and how I endeavored to explain away the hill, even when no ground for doubting a cast appeared.

On looking over the works of former years, I find that Captain B. F. Sands, in 1858, made some deep-sea soundings between Havana and Tortugas, and that he struck a sounding of only 320 fathoms in the same latitude with our ridge, but some 12 miles further to the westward he had found 802 fathoms in the Moro channel. His shallow soundings I can but regard as the first discovery of this bank, upon which we expended so much labor. Captain Sands, like myself, obtained a specimen of bottom, and in addition made observations upon the temperature of the bottom. It had occurred to me, while suspicious of my results, that the fault might be with the indicator; that a counter drift, the polar current, perhaps, might have caught the lead below a certain point, and caused the premature closing of the lid, which stops the fans. But Captain Sands's observations of temperature removes the least doubt; he found 60° the temperature belonging to that depth in many parts of the stream before explored, and the same as that found by myself in station No. 3—in 287 fathoms, the "polar current" is known to be below 40°.

It might perhaps have been supposed that an obstacle of this kind would have caused the sub-current to ascend and flow over, but when we consider that this hill has not the nature of a bar, and that the deepest channel-way is ample, there appears to be no reason for the ascension of the cold sub-current. The lowest temperature found by myself was 44° in 845 fathoms, north of the bank. It appears to be triangular in its general figure, presenting at its west angle a bold prow to the stream. This form of bank has been familiar to me, in my studies of channels, traversed by strong currents; but it is not that of a *deposit* in a current already rapid and still acquiring velocity. The slope on the northwest side is 1 foot in 14, and on the southwest 1 foot in 17. These are too great for a deposit, too great also for the material found in our specimen cup. This bank must have, like the adjacent coast, a firm constitution. It is an interesting question whether this bank belongs to the mountain system of Cuba, as its line of least water running east and southeast may indicate, or whether it is an ancient reef now wearing and crumbling away in the stream. Its depth on the summit is about that of the swept portion of the reef apron to the north side of the straits. The depth may also be that of the Gulf Stream, and if so, this bank, being only abraded near its base by the counter stream, should have precipitous slopes.

In order to haul in our line from casts near the axis of the stream, we found it necessary to steam up against the current to overtake our lead, from which the ship had drifted away with the surplus line. I took particular pains to note the stage at which the steaming became unnecessary; that is to say, the time when the line came up perpendicularly. At station 9, (748 fathoms,) which lies on the Moro channel, I made the following notes while the line was being hauled in: "On checking the out-run the vessel rode to eastward of the line, showing that the lead was on bottom*****. In pulling in the line, it trailed very much to the westward, wind E. NE. 4. It would seem that the current down below is quite different from the surface****. With 600 fathoms out, the line still tends strong to westward—*i. e.*, we drift to eastward against the wind***. With 300 fathoms out, the line ceases to tend westward—hangs out astern. This is the depth of the stream; its velocity is that of the surface, minus the westwardly drift of the ship." There is a want of logic in this last note, but I give it as made, since the facts show that the depth of the stream is not much over 30 fathoms at any rate.

At station 20, (728 fathoms,) I find similar notes; the line hauled perpendicularly when only 300 fathoms remained, but the station lies due east of our bank, under its lea, if we may so speak, so that the local depth of the stream might properly be that of the water or dam above.

It would seem that the Gulf Stream is an *overflowing* of water—not a profound seaward movement under a contrast of pressures. I am reminded here of some observations made upon similar phenomena in New York harbor, where, although the causes were different, the mode of operation was the same. In making a physical survey for the advisory council of the harbor commissioners, I discovered that at the mouth of the Hudson a constant inflowing stream prevailed along the bed of the channel-way, while upon the surface the seaward flow predominated. On further study I found that the sub-current was salt, while the surface drift was nearly fresh. On running a line of levels up the river, I ascertained that the head of fresh water had so declined during the season that the heavier sea-water was pouring in to restore the balance, and yet the elevation of the river was still sufficient to give to the *surface water* a seaward flow. It was a *change of regimen* in progress—the river was being converted into an arm of the sea. I offer this simply as an illustration.

We made no observations with the express design of measuring the surface velocities of the Gulf Stream, but some incidental information relative to them was gained in the checking of our reckoning as well as in the process of sounding.

Mr. Junken's estimates made in plotting our position are probably very close, especially near the Cuban coast, where he had horizontal angles; and my own estimate from the outrun of the line, with the lead at the bottom, confirms them pretty well.

The strongest current was found in the Moro channel. Mr. Junken makes it 25 miles, and I make it 27 miles, with a surface temperature of $77\frac{1}{2}°$. On the Juroco bank itself Mr. Junken makes it $2\frac{1}{2}$, and I make it $2\frac{1}{4}$, with a surface temperature of $78\frac{3}{4}°$. Beyond the bank, in the main channel, Mr. Junken and myself agree upon $2\frac{1}{4}$, with a surface temperature of 79. As we advance to the northward a very rapid decline of velocities takes place. At station No. 10, (794 fathoms on the north of the main channel-way,) I made the following note: "When the lead reached bottom, the vessel did not incline to ride over to the eastward, indicating no stream." I find, however, there was a light breeze from the eastward, which, I presume, balanced the feeble drift. I find in another note of the same cast, that the trend of the line as it came up indicated the difference of velocity for different depths. In 500 fathoms the drift was quite feeble, so that bottom was felt by us, and I find in my record the following note: "No evidence of strong current; where the lead strikes bottom the lines stop in great measure."

We kept a careful lookout of the westwardly drift, which has been said to prevail near the north shore of the straits. On every occasion except one, we detected only a feeble eastwardly drift, which prevailed in spite of the opposing wind. One very calm day we watched our wake as we steamed south from Sand Key light, and believed we discovered a westwardly drift, but on stopping to sound, $14\frac{1}{4}$ miles from the reef, the ship fell to the eastward, after the lead reached the bottom. If, as we supposed, we met some westwardly drift, I have no doubt it was under the influence of a tidal current. The very quiet period of our survey certainly afforded an opportunity for seeing the Gulf Stream pursuing its natural course.

Our repeated soundings near the axis of the stream made us practically acquainted with the notorious fact that the current has nowhere a *constant velocity*. On the first day of our casts upon the Juroco bank, when we were so perplexed by the stream, a fleet of vessels (as we learned afterward) was prevented from making Havana in the usual time, under similar circumstances of wind and weather. A Cuban pilot informed us that "the stream had been much complained of" for several days by in-bound ship-masters. I questioned the pilot closely as to the periods of these changes in the stream, and endeavored to connect them with the ebb and flow of tides, and with the half-monthly changes of sea-level, but got no clue. I cannot believe that these variations are subject to no law of recurrence. They present points of great interest. It may be that alterations in the width and depth of the moving water stratum are concurrent with the variations of surface velocities, as I once observed in current observation of Fire island.

The proximity of the Gulf Stream to the elevated shores of Cuba offers peculiar advantages for a systematic inquiry upon many interesting points. Two observers acting in concert at different well-determined points on shore could angle upon vessels becalmed within fifteen miles, and not only fix the exact position of the axes of the stream in this important part of its course, but determine the velocities and variations in these. To any one who has visited this locality, and noticed the frequent calms, and the fleets of heavy vessels drifting away, the plan of inquiry that I have proposed will seem reasonable.

The surface temperatures along the 100-fathom line, near either shore of the straits, was found to be 77°, while in the harbor of Havana it was 79°, and in the harbor of Key West it varied from 82½° to 87½°. This variation at Key West depended upon the tidal currents; the ebb from the westward being the warmer.

I have attempted to construct the curves of temperature from twelve hours' observations. The ebb-current, flowing off from shallow coral banks, is heated, while the flood from the ocean is cool. The *night-ebb*, which has not like that of the day been exposed to the sun, is the cooler, but still it is warmer than the *night-flood*. It would seem that the identical water which crosses the reef with the flood does not return with the ebb. A circulation is probable. The flood crosses the harbor with 82½° at 1 a. m., but the ebb at its earliest stage, 4 to 5 a. m., crosses with 84°, while the temperature of the air is 77°.

I will digress a little from my proper subject in this place to say that as we returned home we found the surface temperature in the northern part of the Straits of Florida quite 2° warmer than in the neighborhood of Cuba. The waves crossing portions of the heated Bahama banks may afford by their acquired motion an increase of warm water to this portion of the stream, which would raise its temperature and augment its flow, but this would be very small. Again, this part of the straits is less exposed to the sea, especially in the month of May, than other portions, so that there is less forced circulation among the particles in the upper stratum. Its surface is perhaps heated, as stagnant water might be, notwithstanding its bodily motion to the northward.

The failure of our Saxton deep-sea thermometers, after the first two days' work, was unfortunate. I unwittingly exposed them to injury by fastening them to our lead-lines, and subjecting them to shocks upon the rocky bottom; we had no men to spare for separate lines except occasionally for 100-fathom trials; and our time at any station was limited by the necessity of making up our drift on a diagonal to the next, without losing our reckoning.

Along the track of our soundings between Key West and Havana we saw few of those phenomena which to sailors are the familiar tokens of the Gulf stream. We had usually a smooth and well-defined horizon, a placid sea, and a serene sky. In the latitude of Florida cape, however, I was able to witness some of the palpable signs of the stream: the "serrated horizon," the "breakers ahead," which vanished on near approach, and the quick succession of distant squalls at night, each of which originated in a single black speck in the clear sky, but rapidly swells till it bursts in wind and rain, then vanishes.

I was surprised to find the gulf-weed a rare object in our journeys across to Cuba. A single spray was sometimes all that we saw for an entire day.

No other drift material was seen. The water beyond the reef is very clear. I find in my record frequent statements that our indicator was seen glistening in the sea, 10 fathoms below the surface. In the approach to the shores the telegraph engineers will be able to see the lay of the cable long before it comes within the agitation of the waves. In the event of rupture the ends of the cable can easily be caught and spliced.

## DIFFICULTIES IN THE WAY OF LAYING A TELEGRAPH CABLE.

The proposed crossing of the Florida straits with a telegraph offers to the experience of engineers but one class of difficulties, viz., those to be met with in laying a cable in a current, or system of currents, extending to great depths. The space, however, in which these new difficulties appear is essentially but twenty-four miles, and over the larger part of this distance the hills of Cuba are in full view, so that every step of progress may be accurately measured as the work advances.

Since the course of the surface drift is about east-northeast, and that of the true course from Havana to Sand key north by east ¾ east, there may be a great advantage in starting from the

Cuba side with the somewhat favoring stream. Having considerable confidence in the existence of a westerly sub-drift through the Moro channel, I believe it will be found advantageous in spite of the great length of cable suspended from the ship to stem the current somewhat for a distance of about fifteen miles, then put the ship on a course more acute to that of the stream till the bank is passed.

A great deal may depend upon the lay of the cable on the slopes of the bank, around which eddies and races of great magnitude may be presumed to exist, although none appear on the surface.

I trust the Coast Survey will not lose the opportunity that the laying of the cable will afford for making further inquiries, especially as the knowledge *may* be immediately useful to those practically interested in this noble enterprise.

### REMARKS UPON LINES AND LEADS.

The difficulties that I had in obtaining knowledge from the experience of others, for the preparation of a proper outfit for deep-sea soundings, has induced me to add to this report some brief remarks upon my own experience.

*Lines*.—The lines to be found ready made in the market are defective in many particulars, aside from the bad quality of the raw material. The hemp is not sufficiently cleared of tow and the yarns are not sufficiently twisted, so that in the final laying up, several "after turns" are forced upon the line which weaken it by straining the half-twisted yarns.

These forced "after turns" come out gradually in the use of the line, and in so doing cause a rotation of the lead and erroneous registry upon the indicator. James Shandon, whom I temporarily employed as a quartermaster, is quite an expert in the use and inspection of lead lines, and it was from him that I learned the points I have stated.

Our lines were made by Sewell, Day & Co., of Boston, who use the standing steam machinery, which not only lays up the line in very long pieces, (250 fathoms each,) but gives everywhere an equal turn. In the old style "ropewalk," the yarns are stretched out the entire length, (60 fathoms usually,) and then twisted from one end; of course the twist is very unequal, greatest near the wheel, and least at the further end. In the standing machinery of Sewell, Day & Co., the whole line is laid up and coiled in the space of less than 10 feet. I had supposed that there was an advantage in "hawser-laying" for lead lines, but after full discussion of the matter with the manufacturers, whose sincere personal interest I was so fortunate as to secure, it appeared plain to me that in "hawser-laying" there is no gain of strength, and a decided loss of pliability. Our lines were made of thoroughly combed Italian and Russian hemps. The yarns are well twisted, so that in laying up, the natural back turn sufficed for making a close but pliant rope. On wetting they became somewhat hard, but on again drying, exceedingly soft and limp, without the least disposition to kink or snarl. If I were to undertake deep-sea work again, I should want to be provided with several sets of line, so that I could always use them dry.

Our smallest line used with the detaching lead was of Italian hemp, three-quarters of an inch in circumference—that is, half the size of the deep-sea lines used by the larger class of sea-going steamers. This line was used in 20 casts, beyond the 100-fathom curve. Its entire length of 1,050 fathoms, was run out seven times and on one occasion it brought up from the bottom in 828 fathoms, by registry, the Berryman lead, weighing 96 pounds, besides the indicator, spindle and cup full of mud. A winch turned by four men, and 13 men abaft, were employed at this time in hauling in. It never parted. Of the next larger line, which was one inch in circumference, we had 1,400 fathoms of Italian hemp, and the same length of Russian. In a few casts where great outrun of line was necessary, these two were spliced together.

At the last sounding of the cruise, with the entire length of the Italian line out, 22 men failed for 10 minutes to start it, and the Corwin swung to it as if at anchor by her stern.

*Leads*.—The Brooke sounding apparatus, as improved by Berryman, was the instrument most used by us, although in the matter of detaching the lead it is not so certain as that invented by Captain Sands, which we also carried. (For description and sketches of these instruments, see Coast Survey Report of 1857.)

The chief value of the Berryman-Brooke is its excellent specimen cup, which makes no selection of the material. I find fault with the Sands cup on the ground that it does not open freely, and then when open it receives exclusively, or in great proportion, the fine semi-fluid material—it does not give a fair sample of the material just as it is found in the bottom.

As a matter of experiment I provided myself with a small apparatus of my own, in which the weight consisted of small leaden shot enclosed in a flint-glass flask, which on reaching bottom should be shattered by a piston playing up through a barrel. To the piston Sands's specimen cup was attached. As a lead it was convenient, being simple and easy of mañipulation, and its form, an eccentric ellipsoid, was favorable for rapid descent, but it failed in one trial out of six on hard bottom. It may, no doubt, be improved by using thinner glass, with heavier weight of shot, but I must confess its decided inferiority to Sands's apparatus.

*Indicators.*—We took with us several Massey's, Walker's, (old style,) and Trowbridge's indicators, and the steamer Corwin had a Walker's indicator of the new style. One of the Walker's was selected as the most accurate, and with it most of the casts were made. The Trowbridge indicators furnished to me were rendered useless in deep casts by faulty workmanship. The fans were too heavy and detached prematurely. The graduation seemed accurate.

I was very sorry that Professor Trowbridge's sounding apparatus did not arrive at Key West in time to give me more data for the Jaruco bank. I waited for it several days, when our work was over, but I have learned within a few days that it arrived at Havana about two weeks after we had left for the north.

Very respectfully, yours,

HENRY MITCHELL,
*Assistant United States Coast Survey.*

Professor A. D. BACHE,
*Superintendent U. S. Coast Survey.*

*Soundings across the straits of Florida, from Sand Key to El Moro, April and May, 1866.*

| Station. | Latitude. | Longitude. | Depth by indicator. | Depth by line. | Remarks. | Station. | Latitude. | Longitude. | Depth by indicator. | Depth by line. | Remarks. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | ° ′ ″ | ° ′ ″ | | | | | ° ′ ″ | ° ′ ″ | | | |
| 1 | 24 19 56 | 81 53 45 | 116½ | 119 | Rock bottom felt. | 18 | 23 35 25 | 82 15 10 | 636 | * | |
| 2 | 24 17 00 | 81 55 03 | 129 | 132 | Do. | 20 | 23 33 40 | 81 58 36 | 728 | * | |
| 3 | 24 13 56 | 81 56 29 | 287½ | 304 | Hard bottom felt. | 21 | 23 28 20 | 82 13 40 | 380? | * | |
| 4 | 24 09 57 | 81 58 33 | 369 | 397 | Bottom felt—mud. | 23 | 24 19 00 | 81 52 00 | 105 | 114 | Hard bottom. |
| 5 | 24 06 36 | 82 01 36 | 432 | 466 | Do. | 24 | 24 19 00 | 81 48 30 | 116 | 134 | Rock bottom. |
| 6 | 23 59 27 | 82 04 06 | 500 | 545 | Bottom distinctly felt—mud. | 25 | 24 25 40 | 81 47 00 | 55 | 53 | Coral sand—steamer drifting. |
| 7 | 23 10 45 | 82 22 00 | 243 | 224 | Hard bottom. | | | | 41 | 51 | |
| 8 | 23 12 45 | 82 20 18 | 583 | 620 | Mud bottom. | | | | 37 | 49 | |
| 9 | 23 18 10 | 82 16 00 | 748 | ....... | Do. | | | | ...... | 36 | |
| 10 | 23 51 30 | 82 00 25 | 794 | 850? | Do. | 26 | 24 26 40 | 81 47 00 | 40 | 33 | Steamer drifting—2 casts. |
| 11 | 24 13 35 | 82 00 45 | 290 | 315 | | | | | 38 | 38 | |
| 12 | 24 00 09 | 82 05 26 | 508 | 562 | Do. | | | | 112 | 128 | |
| 13 | 23 55 50 | 82 06 00 | 687 | * | Stiff mud bottom. | 27 | 24 20 30 | 81 56 30 | 121 | ....... | By the different indicators. |
| 14 | 23 43 50 | 82 07 20 | 845 | * | | | | | 111 | 128 | |
| 15 | 23 37 50 | 82 05 33 | 828 | * | | 28 | 23 34 10 | 82 17 55 | 853 | * | Mud. |
| 16 | 23 23 30 | 82 16 58 | 710 | * | Hard bottom. (?) | 29 | 23 29 30 | 82 12 00 | 448 | * | Stiff mud. |
| 17 | 23 29 30 | 82 11 28 | 461 | * | Hard bottom. | 30 | 23 22 36 | 82 10 00 | 469 | * | Mud. |

* No indication of depth by line.

**NOTE.**—Diagrams II and III, referred to above, are combined in Sketch No. 17, appended to this report, showing the positions and depths and the cross-section constructed from the next following table. Diagram I, which has been accidentally omitted in its proper place, is given at the end of the letter-press in a supplementary note. ***Errata.***—On page 36, on line with *Northern approach*, read *thirteen* instead of *eight* fathoms. Page 39, fourth line from bottom, read 300 instead of 600 fathoms. Page 40, lines 23 and 24, read 2.5 and 2.7 miles, instead of 25 and 27.

*Section of soundings across the straits of Florida, from Sand key to El Moro, 1866.*

| Points in profile. | Distance from— Sand key. | Distance from— El Moro. | Depths. By indicator. | Depths. By out-run. | Station used. | Kind of bottom. | Remarks. |
|---|---|---|---|---|---|---|---|
| | *Miles.* | *Miles.* | *Fath's* | *Fath's* | | | |
| *A* | 2¾ | 79½ | ...... | 65 | 123 | Coral ... | Rock bottom, with covering of shells, &c.; lead detached; rock bottom felt. |
| *B* | 7¾ to 8½ | 74¼ | 112 | 125 | 24 and 27 | ..do. | |
| *C* | 11 | 71¼ | 129 | 132 | 2 | Coral (?). | One specimen of coral debris procured; lead detached; no specimen; rock felt. |
| *D* | 14½ | 67¾ | 289 | 309 | 3–11 | Coral ... | One specimen of coral debris; lead detached; rock bottom felt. |
| *E* | 18¼ | 64 | 369 | 397 | 4 | Mud .... | Specimen of gray material; lead detached. |
| *F* | 24¼ | 58 | 432 | 466 | 5 | ..do ..... | Specimen nearly white; lead detached. |
| *G* | 29⅞ | 53⅛ | 504 | 553 | 6 and 12 | ..do ..... | Specimen nearly white, with dashes of red; lead detached. |
| *H* | 34 | 48¼ | 687 | ...... | 13 | ..do ..... | Specimen nearly white, and stiff; lead detached. |
| *I* | 38 | 44¼ | 794 | ...... | 10 | ..do. | |
| *i* | 45½ | 36¾ | 845 | ...... | 14 | ..do ..... | Specimen nearly white, with tinge of red; lead detached. |
| *h* | 51¼ | 31 | 842 | ...... | 14, 15, 28 | ..do ..... | Specimen nearly white, with tinge of red; lead at 28 detached. |
| *g* | 55¾ | 26½ | 813 | ...... | 18 and 28 | ..do ..... | Specimen nearly white, with tinge of red; lead at 28 detached. |
| *f* | 60 | 22¼ | 455 | ...... | 17 and 29 | ..do ..... | Specimen of fine drab mud; lead at 29 detached. |
| *e* | 61½ | 20¾ | 380 | ...... | 21 | .......... | No specimen; some doubt of this cast. |
| *d* | 67¼ | 15 | 710 | ...... | 16 | .......... | No specimen, but evidence of coral bottom; lead came up hanging by one hook. |
| *c* | 73¼ | 9 | 748 | ...... | 9 | Mud .... | Specimen doubtful; lead came up hanging by one hook; the bottom was felt. |
| *b* | 78½ | 3¾ | 583 | 620 | 8 | Sand.... | Or mud of reddish brown color; good specimen, though lead did not detach. |
| *a* | 80¼ | 1¼ | 243 | 244 | 7 | Rock.... | Rock felt; one small shell in specimen cup; lead detached. |

# APPENDIX No. 6.

## PRELIMINARY REPORT ON THE INTERFERENCE TIDES OF HELL GATE, WITH DIRECTIONS FOR REDUCING THE SOUNDINGS.

NEEDHAM, MASSACHUSETTS, *November* 7, 1866.

DEAR SIR: In view of the proposed work in Hell Gate, I submit a preliminary report upon the more prominent features of the tidal interferences in that locality.

The tide wave propagated from the sea by way of Long Island sound, meets that which is propagated from the southward through New York harbor, in the space between Forty-second street and Pot Rock; and in this space the observed tide is a compound of the two waves. No single point can be designated as the meeting place of these two tidal systems, because, since the two waves differ in range and in their laws of change, they combine at different points from day to day. Again, at no single point can the observed tide be said to be a true compound of the two waves, because the different phases of each wave travel at different rates as the depth increases or diminishes, and since their ranges are unequal, the sound wave predominates further westward at and near high water than at lower stages.

The southern tide—that which enters by way of Sandy Hook—scarcely modifies the form of the observed tide of the sound to eastward of Pot Cove; but the sound tide is distinctly traced to Governor's island, where, although much reduced in range, it is frequently found to affect the observed tide mostly in the way of lessening the rise and fall. In other words, the southern tide after coming up through the Narrows meets a small propagation of the sound tide whose *lunar interval* differs nearly six hours; the result is a reduction of range. This is the principal reason why the Governor's island tide is less than that of Sandy Hook.

It is only in the space between Forty-second street and Pot Rock that the two waves struggle for the mastery; and here, although the *intervals* differ several hours, there have never been observed four distinct high waters in a day as at the interference in the Vineyard sound. The currents of Hell Gate restore the differences of elevation in some measure.

In the interference of two tide waves of different range, *the most uncertain phases are those of high and low water.* They are inconstant in interval and elevation, so that as datum planes for soundings they are practically of little value. The most constant plane is that of *mean sea level.* In the Vineyard sound interference this plane is common to both tidal systems; but in Hell Gate the result of the greater westwardly journey of the higher phases of the sound tide is an apparent elevation of the mean sea level in the East river at Blackwell's island. The Ravenswood tidal observations (made in the southern channel nearly opposite the penitentiary) have a mean sea level 0.4 higher than that of the sound, according to our levellings. I feel some doubt of this figure, and would not trust it for engineering purposes without repeating the examination.

I have annexed to this report a schedule of the datum planes about which I feel absolutely certain. It will be seen from this, that *mean level* is essentially a common plane from Hell Gate ferry (foot of Eighty-sixth street) to Bounty's dock, in Pot Cove, and that this can be recovered by levelling down to the sea from any one of the four benches named.

I take occasion to urge upon you the use of mean sea level as the datum plane for all depths and elevations measured and mapped for engineering purposes. Mean low water, however independent of *time* and *epochs*, is not a plane, but a *warped surface.* If I find, for instance, that there is a 4-foot rock upon our chart, among the "Hen and Chickens," and a 7-foot rock on "Way's Reef," I can not conclude that these two objects are 3 feet different in elevation—on the contrary, assuming that they are correctly reduced to local low waters, I compute the difference to be 2.33 feet. When I look at the depths at Hallet's Point, I know they must be *relatively* correct; but if but one gauge has been used in reduction, and the casts made during the same slack water, (which is probable,) the depths upon one side of the point are all wrong, because not what they assume to be.

For the survey of Nantucket and Vineyard sound, I furnished Captain Raymond Rodgers with a map cut up into tidal districts, so that his soundings in each district, reduced by a local gauge, should be essentially correct. In the same way, I purpose to indicate for Hell Gate suitable tidal districts.

A. From Polhemus dock to Way's reef, (inclusive,) use a gauge at Bounty's wharf, in Pot Cove.

B. Shell Drake Rock and Pot Rock should be referred to the mean of tidal observations at Hallet's Point and Bounty's dock, or else to a strictly local gauge on the Astoria shore.

C. Holmes' Rock, Hog's Back, Frying Pan, and east side of Hallet's Point, may be referred to gauge at last-named place.

D. The whole district between Hallet's Point and Horn's Hook, including the group of rocks from the Heel Tap, inclusive, to Flood Rock, may be referred to a gauge on the Astoria shore, opposite Flood Rock.

E. From Horn's Hook to the House of Correction, on Blackwell's island, a gauge at Hell Gate ferry (city side) may be used.

The currents of Hell Gate are interchanges of water between two interfering tides; their periods are referable to the epoch of *restored surface level.* The eastwardly current, usually called the *flood*, commences to run through the Gate about 50 minutes after the restoration of level between adjacent bodies of water on the sound and harbor sides. The westwardly current (*ebb*, as it is called) follows the restoration of level at a smaller interval.

The greatest contrasts of elevations between the waters of the two sides of the Gate occur usually soon after the times of high and low waters in the sound; but the maximum currents occur later. From the relations of these heads to the height of the tide (*i. e.*, to the depth of water off Hallet's Point) a curious effect appears. The flood current flows at a lower stage than the ebb, and is consequently very much stronger than the ebb. Other causes concur in this, but I am inclined to regard the one given as the most important and direct. *At the time when the flood current reaches its maximum flow to the eastward, the depths through Hell Gate are two feet less than they are at the time when maximum ebb prevails.*

Two questions offer themselves for solution, upon which my own mind is not yet made up.

1st. Would the removal of rocks from Hell Gate increase or diminish the rush of water through the dangerous pass at Hallet's Point?

2d. Would the enclosure of Hallet's Point, Hog's Back, &c., by fender piles, increase or diminish the flood current whose bewildering whirls are not so much due to rocks as to the abrupt *change of course at the point of greatest fall.*

I use the terms *flood* and *ebb* as they are applied by pilots, &c. In truth, the words are not applicable to interference currents.

If the above brief statement is not clear, it is not because the data are insufficient, but because, without diagrams, the simplest distinctions between the phenomena of waves and running water are not easily conveyed.

Very respectfully, yours,

H. MITCHELL.

J. E. HILGARD, Esq.,
*Assistant in charge Coast Survey*

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RELATIVE ELEVATIONS OF TIDAL PLANES IN HELL GATE, NEW YORK, FROM OBSERVATIONS MADE UNDER DIRECTION OF ASSISTANT H. MITCHELL.

| | | Datum plane. |
|---|---|---|
| Bench-mark on stone gate-post at Ravenswood | | 00 |
| Bench-mark (copper nail) on east end lower step of Howe's hotel | | 2.13 |
| Bench-mark on stone building at Hell Gate ferry, Eighty-sixth street, New York | | 4.63 |
| Bench-mark (copper nail) on head of pile, Bounty's wharf | | 5.44 |
| Mean high water, | At Bounty's wharf, Pot Cove | 8.50 |
| | East side of Hallet's Point | 8.96 |
| | Hell Gate ferry, Eighty-sixth street, New York city | 9.06 |
| Plane of maximum westward (ebb) current | | 10.50* |
| Mean level, | Hell Gate ferry | 11.30 |
| | Bounty's wharf | 11.35 |
| Plane of maximum eastward (flood) current | | 12.50* |
| Mean low water, | Hell Gate ferry | 13.57 |
| | East side Hallet's Point | 14.04 |
| | Bounty's dock | 14.20 |

* These figures are relatively correct, and apply to the space between Blackwell's island and Hallet's Point. These planes are widely different for a. m. and p. m., and for different phases and declinations of the moon.

*Tides and currents of Hell Gate, New York, from observations of* 1857.

| | TIDES. | | | CURRENTS. | | | |
|---|---|---|---|---|---|---|---|
| | Lun. int. of high water. | Lun. int. of low water. | Range of tide. | Lun. int. of E. to W. | Lun. int. of W. to E. | Velocity, east current. | Velocity, west current. |
| *Observed during one lunation.* | *H. M.* | *H. M.* | *Feet.* | *H. M.* | *H. M.* | *Miles per h.* | *Miles per h.* |
| Hell Gate ferry | 10 07 | 16 20 | 4.44 | | | | |
| Hallet's Point | 11 07 | 17 32 | 5.08 | | | | |
| Bounty's dock | 11 30 | 17 58 | 5.70 | | | | |
| *Observations for semi-lunation.* | | | | | | | |
| North Blackwell's channel, (west of Hell Gate) | | | | 9 32 | 15 49 | 4.2 | 4.3 |
| South Blackwell's channel, (west of Hell Gate) | | | | 9 27 | 15 42 | 4.3 | 4.0 |
| Off Polhemus dock, (east of Hell Gate) | | | | 10 36 | 16 39 | 3.1 | 2.3 |
| In Hell Gate proper, *i. e.*, between Hallet's Point and Hog's Back | | | | 9 26 | 15 50 | 8.5 | 4.4 |

# APPENDIX No. 7.

## TIDE TABLES FOR THE ATLANTIC AND PACIFIC COASTS OF THE UNITED STATES FOR THE YEAR 1867.

PREFACE.—The following tables give for every day of the year 1867 the approximate time and height of the tide at the principal ports on the Atlantic coast of the United States. Their use will be readily understood from the headings and foot notes. For intermediate ports a table of tidal constants is appended, in which the names of the principal ports, or ports of reference, are printed in capitals, and are followed by the names of the ports in the neighborhood, which are to be referred to them respectively. Thus, for instance, Wiscasset is to be referred to Portland, Salem to Boston, &c. If the time and height of the tide are wanted for a given day at one of these intermediate ports, find the time and height for the principal port next preceding it in the table, and add or subtract the figures opposite to the name of the intermediate port, according to the signs + or —.

The columns headed "duration of rise or fall" give the means of obtaining the approximate time of the preceding or following low water, by subtracting in the first case the duration of rise from the time of high water, and in the second case adding the duration of fall.

For the first year of publication the predictions are made simply by means of the tables given in the Coast Survey reports, and entitled "Tide-tables for Navigators," (corrected by the observations made up to date;) they include, therefore, only the half-monthly inequality for the Atlantic coast, and in additional to that the diurnal inequality for the Pacific coast and the western coast of Florida. Hence the results may differ from observations by quantities, dependent on the solar and lunar parallax and declination, which are frequently masked by the irregularities caused by wind, and which, for the general purposes of the navigator, are quite inconsiderable. It is nevertheless intended to obtain in subsequent years a nearer approximation by applying all the corrections which can be satisfactorily deduced from long series of observations. The computations for the purpose are in progress. (*Coast Survey Office, Washington, December*, 1866.)

NOTE.—The predictions for Eastport only are reprinted here as a specimen of the tables.

### EASTPORT.—HIGH WATER, 1867.

| Day of the month. | JANUARY. | | | | FEBRUARY. | | | | MARCH. | | | | APRIL. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A. M. | | P. M. | | A. M. | | P. M. | | A. M. | | P. M. | | A. M. | | P. M. | |
| | Time. | Height. | Time. | Height. | Time. | Height. | Time. | Height. | Time. | Height. | Time. | Height. | Time. | Height. | Time. | Height. |
| | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* |
| 1...... | ........ | ........ | 8 09 | 17.4 | 8 50 | 17.8 | 9 14 | 18.1 | 7 21 | 17.0 | 7 50 | 17.2 | 8 33 | 17.6 | 8 58 | 17.9 |
| 2...... | 8 34 | 17.6 | 8 58 | 17.9 | 9 39 | 18.3 | 10 01 | 18.6 | 8 17 | 17.5 | 8 43 | 17.7 | 9 22 | 18.1 | 9 46 | 18.4 |
| 3...... | 9 21 | 18.2 | 9 43 | 18.4 | 10 23 | 18.9 | 10 43 | 19.2 | 9 07 | 18.0 | 9 32 | 18.2 | 10 08 | 18.7 | 10 30 | 19.0 |
| 4...... | 10 06 | 18.7 | 10 27 | 19.0 | 11 04 | 19.4 | 11 24 | 19.4 | 9 54 | 18.5 | 10 16 | 18.8 | 10 52 | 19.2 | 11 14 | 19.4 |
| 5...... | 10 48 | 19.2 | 11 08 | 19.4 | 11 43 | 19.4 | ........ | ........ | 10 37 | 19.1 | 10 58 | 19.3 | 11 35 | 19.4 | 11 57 | 19.4 |
| 6...... | 11 28 | 19.4 | 11 47 | 19.4 | 0 04 | 19.4 | 0 24 | 19.3 | 11 18 | 19.4 | 11 38 | 19.4 | ........ | ........ | 0 20 | 19.3 |
| 7...... | ........ | ........ | 0 08 | 19.4 | 0 44 | 19.2 | 1 05 | 19.1 | 11 58 | 19.4 | ........ | ........ | 0 43 | 19.2 | 1 07 | 19.1 |
| 8...... | 0 28 | 19.3 | 0 50 | 19.2 | 1 26 | 18.9 | 1 47 | 18.8 | 0 20 | 19.3 | 0 41 | 19.2 | 1 32 | 18.9 | 1 57 | 18.7 |
| 9...... | 1 09 | 19.1 | 1 29 | 18.9 | 2 08 | 18.5 | 2 31 | 18.3 | 1 04 | 19.1 | 1 25 | 18.9 | 2 24 | 18.4 | 2 51 | 18.1 |
| 10...... | 1 49 | 18.8 | 2 10 | 18.5 | 2 54 | 18.0 | 3 19 | 17.7 | 1 48 | 18.7 | 2 12 | 18.5 | 3 19 | 17.7 | 3 51 | 17.5 |
| 11...... | 2 32 | 18.3 | 2 54 | 18.0 | 3 46 | 17.5 | 4 15 | 17.3 | 2 37 | 18.2 | 3 02 | 17.9 | 4 23 | 17.2 | 4 57 | 17.0 |
| 12...... | 3 18 | 17.7 | 3 43 | 17.5 | 4 46 | 17.1 | 5 21 | 16.9 | 3 30 | 17.6 | 4 00 | 17.4 | 5 31 | 16.9 | 6 04 | 16.8 |
| 13...... | 4 09 | 17.3 | 4 38 | 17.1 | 5 55 | 16.8 | 6 30 | 16.9 | 4 33 | 17.2 | 5 08 | 17.0 | 6 38 | 16.9 | 7 11 | 17.0 |
| 14...... | 5 08 | 17.0 | 5 40 | 16.8 | 7 06 | 17.0 | 7 41 | 17.2 | 5 43 | 16.8 | 6 19 | 16.8 | 7 43 | 17.2 | 8 12 | 17.4 |
| 15...... | 6 13 | 16.8 | 6 48 | 16.9 | 8 16 | 17.5 | 8 48 | 17.8 | 6 54 | 16.9 | 7 29 | 17.1 | 8 38 | 17.7 | 9 03 | 17.9 |
| 16...... | 7 22 | 17.0 | 7 57 | 17.3 | 9 17 | 18.1 | 9 45 | 18.4 | 8 03 | 17.3 | 8 32 | 17.6 | 9 27 | 18.2 | 9 50 | 18.5 |
| 17...... | 8 29 | 17.6 | 9 00 | 17.9 | 10 12 | 18.8 | 10 37 | 19.1 | 9 00 | 17.9 | 9 27 | 18.2 | 10 11 | 18.7 | 10 31 | 19.0 |
| 18...... | 9 31 | 18.2 | 9 59 | 18.6 | 11 00 | 19.3 | 11 22 | 19.4 | 9 52 | 18.5 | 10 15 | 18.8 | 10 52 | 19.2 | 11 11 | 19.4 |
| 19...... | 10 27 | 18.9 | 10 53 | 19.3 | 11 43 | 19.4 | ........ | ........ | 10 37 | 19.1 | 10 58 | 19.3 | 11 29 | 19.4 | 11 48 | 19.4 |
| 20...... | 11 19 | 19.4 | 11 42 | 19.4 | 0 05 | 19.4 | 0 27 | 19.3 | 11 18 | 19.4 | 11 37 | 19.4 | ........ | ........ | 0 08 | 19.4 |
| 21...... | ........ | ........ | 0 07 | 19.4 | 0 47 | 19.2 | 1 08 | 19.1 | 11 58 | 19.4 | ........ | ........ | 0 28 | 19.3 | 0 48 | 19.2 |
| 22...... | 0 30 | 19.3 | 0 53 | 19.1 | 1 28 | 18.9 | 1 48 | 18.8 | 0 18 | 19.3 | 0.38 | 19.2 | 1 08 | 19.0 | 1 28 | 18.9 |
| 23...... | 1 15 | 19.0 | 1 37 | 18.8 | 2 09 | 18.6 | 2 30 | 18.3 | 0 58 | 19.1 | 1.17 | 19.0 | 1 48 | 18.7 | 2 10 | 18.5 |
| 24...... | 1 59 | 18.6 | 2 22 | 18.4 | 2 52 | 18.0 | 3 14 | 17.8 | 1 37 | 18.8 | 1.57 | 18.7 | 2 31 | 18.3 | 2 54 | 18.0 |
| 25...... | 2 43 | 18.2 | 3 05 | 17.9 | 3 38 | 17.6 | 4 02 | 17.4 | 2 18 | 18.5 | 2 39 | 18.2 | 3 17 | 17.8 | 3 42 | 17.5 |
| 26...... | 3 28 | 17.7 | 3 53 | 17.4 | 4 28 | 17.2 | 4 56 | 17.0 | 3 01 | 17.9 | 3 25 | 17.7 | 4 07 | 17.3 | 4 34 | 17.2 |
| 27...... | 4 18 | 17.3 | 4 43 | 17.1 | 5 24 | 16.9 | 5 54 | 16.8 | 3 50 | 17.5 | 4 17 | 17.3 | 5 01 | 17.0 | 5 31 | 16.9 |
| 28...... | 5 11 | 17.0 | 5 38 | 16.8 | 6 22 | 16.8 | 6 52 | 16.9 | 4 43 | 17.1 | 5 12 | 16.9 | 6 00 | 16.8 | 6 29 | 16.9 |
| 29...... | 6 07 | 16.8 | 6 35 | 16.9 | ........ | ........ | ........ | ........ | 5 42 | 16.8 | 6 13 | 16.8 | 6 58 | 16.9 | 7 27 | 17.1 |
| 30...... | 7 03 | 17.0 | 7 33 | 17.1 | ........ | ........ | ........ | ........ | 6 40 | 16.9 | 7 10 | 17.0 | 7 56 | 17.3 | 8 22 | 17.5 |
| 31...... | 7 59 | 17.3 | 8 25 | 17.5 | ........ | ........ | ........ | ........ | 7 39 | 17.1 | 8 07 | 17.4 | ........ | ........ | ........ | ........ |

The height of high water is reckoned from the level of average low water, to which the soundings are given on the Coast Survey charts.

## *Eastport.—High water*—Continued.

| Day of the month. | MAY. | | | | JUNE. | | | | JULY. | | | | AUGUST. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A. M. | | P. M. | | A. M. | | P. M. | | A. M. | | P. M. | | A. M. | | P. M. | |
| | Time. | Height. | Time. | Height. | Time. | Height. | Time. | Height. | Time. | Height. | Time. | Height. | Time. | Height. | Time. | Height. |
| | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* |
| 1...... | 8 49 | 17.8 | 9 14 | 18.0 | 10 05 | 18.5 | 10 30 | 19.0 | 10 42 | 19.2 | 11 09 | 19.4 | ........ | ........ | 0 08 | 19.3 |
| 2...... | 9 39 | 18.3 | 10 03 | 18.6 | 10 56 | 19.3 | 11 21 | 19.4 | 11 35 | 19.4 | ........ | ........ | 0 33 | 19.3 | 0 56 | 19.1 |
| 3...... | 10 27 | 18.9 | 10 50 | 19.2 | 11 46 | 19.4 | ........ | ........ | 0 01 | 19.4 | 0 27 | 19.3 | 1 18 | 19.0 | 1 41 | 18.8 |
| 4...... | 11 14 | 19.4 | 11 36 | 19.4 | 0 14 | 19.3 | 0 41 | 19.2 | 0 52 | 19.2 | 1 17 | 19.0 | 2 03 | 18.6 | 2 24 | 18.4 |
| 5...... | ........ | ........ | 0 01 | 19.4 | 1 07 | 19.1 | 1 34 | 18.9 | 1 41 | 18.8 | 2 05 | 18.6 | 2 47 | 18.1 | 3 10 | 17.9 |
| 6...... | 0 27 | 19.3 | 0 52 | 19.1 | 2 00 | 18.6 | 2 26 | 18.4 | 2 30 | 18.3 | 2 54 | 18.0 | 3 34 | 17.6 | 3 59 | 17.4 |
| 7...... | 1 19 | 19.0 | 1 46 | 18.8 | 2 53 | 18.0 | 3 19 | 17.8 | 3 18 | 17.8 | 3 43 | 17.5 | 4 25 | 17.2 | 4 52 | 17.1 |
| 8...... | 2 13 | 18.5 | 2 40 | 18.2 | 3 46 | 17.5 | 4 15 | 17.3 | 4 09 | 17.3 | 4 36 | 17.1 | 5 20 | 16.9 | 5 49 | 16.8 |
| 9...... | 3 08 | 17.9 | 3 38 | 17.6 | 4 43 | 17.1 | 5 12 | 16.9 | 5 04 | 17.0 | 5 32 | 16.8 | 6 17 | 16.8 | 6 45 | 16.9 |
| 10...... | 4 08 | 17.3 | 4 39 | 17.1 | 5 42 | 16.8 | 6 11 | 16.8 | 6 00 | 16.8 | 6 28 | 16.9 | 7 15 | 17.0 | 7 43 | 17.2 |
| 11...... | 5 11 | 17.0 | 5 43 | 16.8 | 6 39 | 16.9 | 7 07 | 17.0 | 6 56 | 16.9 | 7 25 | 17.1 | 8 11 | 17.4 | 8 36 | 17.6 |
| 12...... | 6 14 | 16.8 | 6 44 | 16.9 | 7 34 | 17.1 | 8 00 | 17.3 | 7 52 | 17.2 | 8 18 | 17.5 | 9 01 | 17.9 | 9 25 | 18.2 |
| 13...... | 7 14 | 17.0 | 7 43 | 17.2 | 8 26 | 17.5 | 8 50 | 17.8 | 8 43 | 17.7 | 9 06 | 18.0 | 9 47 | 18.4 | 10 09 | 18.7 |
| 14...... | 8 09 | 17.4 | 8 34 | 17.6 | 9 13 | 18.0 | 9 36 | 18.3 | 9 30 | 18.2 | 9 52 | 18.5 | 10 29 | 19.0 | 10 50 | 19.2 |
| 15...... | 8 58 | 17.9 | 9 21 | 18.1 | 9 57 | 18.6 | 10 18 | 18.8 | 10 14 | 18.8 | 10 35 | 19.1 | 11 10 | 19.4 | 11 28 | 19.4 |
| 16...... | 9 42 | 18.4 | 10 04 | 18.6 | 10 38 | 19.1 | 10 59 | 19.3 | 10 55 | 19.3 | 11 16 | 19.4 | 11 47 | 19.4 | ........ | ........ |
| 17...... | 10 24 | 18.9 | 10 44 | 19.2 | 11 18 | 19.4 | 11 38 | 19.4 | 11 34 | 19.4 | 11 54 | 19.4 | 0 07 | 19.4 | 0 27 | 19.3 |
| 18...... | 11 04 | 19.3 | 11 23 | 19.4 | 11 58 | 19.4 | ........ | ........ | ........ | ........ | 0 14 | 19.3 | 0 46 | 19.2 | 1 07 | 19.1 |
| 19...... | 11 41 | 19.4 | ........ | ........ | 0 18 | 19.3 | 0 38 | 19.2 | 0 34 | 19.2 | 0 53 | 19.1 | 1 27 | 18.9 | 1 47 | 18.8 |
| 20...... | 0 01 | 19.4 | 0 22 | 19.3 | 0 59 | 19.1 | 1 18 | 19.0 | 1 13 | 19.0 | 1 33 | 18.9 | 2 08 | 18.6 | 2 30 | 18.3 |
| 21...... | 0 42 | 19.2 | 1 03 | 19.1 | 1 39 | 18.8 | 1 59 | 18.6 | 1 52 | 18.7 | 2 13 | 18.5 | 2 54 | 18.0 | 3 18 | 17.8 |
| 22...... | 1 23 | 19.0 | 1 43 | 18.8 | 2 20 | 18.4 | 2 40 | 18.2 | 2 34 | 18.3 | 2 56 | 18.0 | 3 45 | 17.5 | 4 13 | 17.3 |
| 23...... | 2 04 | 18.6 | 2 26 | 18.4 | 3 02 | 17.9 | 3 25 | 17.7 | 3 18 | 17.8 | 3 43 | 17.5 | 4 45 | 17.1 | 5 19 | 16.9 |
| 24...... | 2 48 | 18.1 | 3 10 | 17.9 | 3 50 | 17.5 | 4 15 | 17.3 | 4 09 | 17.3 | 4 37 | 17.1 | 5 55 | 16.8 | 6 30 | 16.9 |
| 25...... | 3 33 | 17.6 | 3 58 | 17.4 | 4 41 | 17.1 | 5 09 | 16.9 | 5 08 | 17.0 | 5 40 | 16.8 | 7 06 | 17.0 | 7 43 | 17.2 |
| 26...... | 4 24 | 17.2 | 4 51 | 17.1 | 5 39 | 16.8 | 6 09 | 16.8 | 6 13 | 16.8 | 6 47 | 16.9 | 8 17 | 17.5 | 8 49 | 17.8 |
| 27...... | 5 19 | 16.9 | 5 49 | 16.8 | 6 40 | 16.9 | 7 12 | 17.0 | 7 22 | 17.0 | 7 57 | 17.3 | 9 19 | 18.1 | 9 47 | 18.4 |
| 28...... | 6 17 | 16.8 | 6 47 | 16.9 | 7 45 | 17.2 | 8 17 | 17.5 | 8 30 | 17.6 | 9 02 | 17.9 | 10 14 | 18.8 | 10 38 | 19.1 |
| 29...... | 7 16 | 17.0 | 7 46 | 17.2 | 8 48 | 17.8 | 9 17 | 18.1 | 9 33 | 18.3 | 10 02 | 18.6 | 11 02 | 19.3 | 11 24 | 19.4 |
| 30...... | 8 14 | 17.4 | 8 43 | 17.7 | 9 46 | 18.4 | 10 15 | 18.8 | 10 29 | 19.0 | 10 55 | 19.3 | 11 45 | 19.4 | ........ | ........ |
| 31...... | 9 10 | 18.0 | 9 38 | 18.3 | ........ | ........ | ........ | ........ | 11 20 | 19.4 | 11 45 | 19.4 | 0 08 | 19.4 | 0 29 | 19.3 |

| Day of the month. | SEPTEMBER. | | | | OCTOBER. | | | | NOVEMBER. | | | | DECEMBER. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A. M. | | P. M. | | A. M. | | P. M. | | A. M. | | P. M. | | A. M. | | P. M. | |
| | Time. | Height. | Time. | Height. | Time. | Height. | Time. | Height. | Time. | Height. | Time. | Height. | Time. | Height. | Time. | Height. |
| | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* |
| 1...... | 0 51 | 19.2 | 1 12 | 19.0 | 1 03 | 19.1 | 1 24 | 19.0 | 1 58 | 18.7 | 2 20 | 18.4 | 2 14 | 18.5 | 2 35 | 18.3 |
| 2...... | 1 33 | 18.9 | 1 54 | 18.7 | 1 44 | 18.8 | 2 05 | 18.6 | 2 42 | 18.2 | 3 05 | 17.9 | 2 56 | 18.0 | 3 19 | 17.7 |
| 3...... | 2 15 | 18.5 | 2 37 | 18.2 | 2 27 | 18.3 | 2 51 | 18.1 | 3 28 | 17.7 | 3 54 | 17.4 | 3 42 | 17.5 | 4 07 | 17.4 |
| 4...... | 2 59 | 18.0 | 3 23 | 17.7 | 3 13 | 17.8 | 3 38 | 17.6 | 4 19 | 17.3 | 4 46 | 17.1 | 4 32 | 17.2 | 4 58 | 17.0 |
| 5...... | 3 47 | 17.5 | 4 14 | 17.3 | 4 04 | 17.4 | 4 30 | 17.2 | 5 14 | 16.9 | 5 42 | 16.8 | 5 25 | 16.9 | 5 53 | 16.8 |
| 6...... | 4 40 | 17.1 | 5 08 | 17.0 | 4 58 | 17.0 | 5 28 | 16.9 | 6 11 | 16.8 | 6 39 | 16.9 | 6 22 | 16.9 | 6 50 | 16.9 |
| 7...... | 5 37 | 16.8 | 6 07 | 16.8 | 5 56 | 16.8 | 6 25 | 16.9 | 7 06 | 17.0 | 7 34 | 17.1 | 7 20 | 17.0 | 7 49 | 17.2 |
| 8...... | 6 37 | 16.9 | 7 05 | 17.0 | 6 54 | 16.9 | 7 23 | 17.1 | 8 01 | 17.3 | 8 28 | 17.6 | 8 17 | 17.5 | 8 45 | 17.7 |
| 9...... | 7 34 | 17.1 | 8 03 | 17.3 | 7 51 | 17.2 | 8 17 | 17.5 | 8 53 | 17.8 | 9 17 | 18.1 | 9 12 | 18.0 | 9 40 | 18.3 |
| 10...... | 8 29 | 17.6 | 8 54 | 17.8 | 8 42 | 17.7 | 9 05 | 18.0 | 9 41 | 18.4 | 10 06 | 18.7 | 10 06 | 18.7 | 10 32 | 19.0 |
| 11...... | 9 17 | 18.1 | 9 40 | 18.3 | 9 29 | 18.2 | 9 51 | 18.5 | 10 28 | 19.0 | 10 52 | 19.2 | 10 58 | 19.3 | 11 24 | 19.4 |
| 12...... | 10 01 | 18.6 | 10 21 | 18.9 | 10 13 | 18.8 | 10 34 | 19.0 | 11 16 | 19.4 | 11 39 | 19.4 | 11 49 | 19.4 | ........ | ........ |
| 13...... | 10 43 | 19.2 | 11 02 | 19.3 | 10 55 | 19.3 | 11 17 | 19.4 | ........ | ........ | 0 04 | 19.4 | 0 16 | 19.3 | 0 42 | 19.2 |
| 14...... | 11 22 | 19.4 | 11 41 | 19.4 | 11 36 | 19.4 | 11 58 | 19.4 | 0 29 | 19.3 | 0 55 | 19.1 | 1 09 | 19.0 | 1 35 | 18.8 |
| 15...... | ........ | ........ | 0 01 | 19.4 | ........ | ........ | 0 21 | 19.3 | 1 21 | 19.0 | 1 47 | 18.8 | 2 01 | 18.6 | 2 27 | 18.3 |
| 16...... | 0 21 | 19.3 | 0 42 | 19.2 | 0 44 | 19.2 | 1 08 | 19.1 | 2 14 | 18.5 | 2 41 | 18.2 | 2 54 | 18.0 | 3 20 | 17.7 |
| 17...... | 1 04 | 19.1 | 1 25 | 18.9 | 1 32 | 18.9 | 1 57 | 18.7 | 3 09 | 17.9 | 3 39 | 17.6 | 3 47 | 17.5 | 4 16 | 17.3 |
| 18...... | 1 47 | 18.7 | 2 11 | 18.5 | 2 24 | 18.4 | 2 51 | 18.1 | 4 09 | 17.3 | 4 40 | 17.1 | 4 45 | 17.1 | 5 14 | 16.9 |
| 19...... | 2 36 | 18.2 | 3 01 | 17.9 | 3 19 | 17.8 | 3 52 | 17.5 | 5 11 | 17.0 | 5 43 | 16.8 | 5 43 | 16.8 | 6 13 | 16.8 |
| 20...... | 3 29 | 17.7 | 3 59 | 17.4 | 4 22 | 17.2 | 4 55 | 17.0 | 6 14 | 16.8 | 6 45 | 16.9 | 6 42 | 16.9 | 7 10 | 17.0 |
| 21...... | 4 32 | 17.2 | 5 06 | 17.0 | 5 30 | 16.9 | 6 04 | 16.8 | 7 15 | 17.0 | 7 44 | 17.2 | 7 38 | 17.1 | 8 06 | 17.3 |
| 22...... | 5 42 | 16.8 | 6 18 | 16.8 | 6 38 | 16.9 | 7 10 | 17.0 | 8 12 | 17.4 | 8 36 | 17.6 | 8 31 | 17.6 | 8 55 | 17.8 |
| 23...... | 6 54 | 16.9 | 7 29 | 17.1 | 7 41 | 17.2 | 8 12 | 17.4 | 9 01 | 17.9 | 9 24 | 18.2 | 9 18 | 18.1 | 9 41 | 18.4 |
| 24...... | 8 03 | 17.3 | 8 33 | 17.6 | 8 38 | 17 7 | 9 04 | 17.9 | 9 46 | 18.4 | 10 08 | 18.7 | 10 04 | 18.6 | 10 25 | 18.9 |

The height of high water is reckoned from the level of average low water, to which the soundings are given on the Coast Survey charts.

*Eastport.—High water*—Continued.

| Day of the month. | SEPTEMBER. | | | | OCTOBER. | | | | NOVEMBER. | | | | DECEMBER. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A. M. | | P. M. | | A. M. | | P. M. | | A. M. | | P. M. | | A. M. | | P. M. | |
| | Time. | Height. | Time. | Height. | Time. | Height. | Time. | Height. | Time. | Height. | Time. | Height. | Time. | Height. | Time. | Height. |
| | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* | *h. m.* | *Feet.* |
| 25...... | 9 01 | 17.9 | 9 28 | 18.2 | 9 28 | 18.2 | 9 51 | 18.5 | 10 26 | 19.0 | 10 49 | 19.2 | 10 46 | 19.2 | 11 07 | 19.3 |
| 26...... | 9 53 | 18.5 | 10 16 | 18.8 | 10 13 | 18.8 | 10 34 | 19.0 | 11 11 | 19.4 | 11 29 | 19.4 | 11 26 | 19.4 | 11 45 | 19.4 |
| 27...... | 10 39 | 19.1 | 11 01 | 19.3 | 10 55 | 19.3 | 11 17 | 19.4 | 11 49 | 19.4 | ........ | ........ | ........ | ........ | 0 06 | 19.4 |
| 28...... | 11 21 | 19.4 | 11 41 | 19.4 | 11 34 | 19.4 | 11 54 | 19.4 | 0 09 | 19.3 | 0 30 | 19.3 | 0 26 | 19.3 | 0 45 | 19.2 |
| 29...... | ........ | ........ | 0 02 | 19.4 | ........ | ........ | 0 15 | 19.3 | 0 51 | 19.2 | 1 11 | 19.0 | 1 05 | 19.1 | 1 25 | 18.9 |
| 30...... | 0 23 | 19.3 | 0 43 | 19.2 | 0 35 | 19.2 | 0 56 | 19.1 | 1 32 | 18.9 | 1 53 | 18.7 | 1 45 | 18.8 | 2 04 | 18.6 |
| 31...... | ........ | ........ | ........ | ........ | 1 16 | 19.0 | 1 37 | 18.8 | ........ | ........ | ........ | ........ | 2 25 | 18.4 | 2 45 | 18.1 |

The height of high water is reckoned from the level of average low water, to which the soundings are given on the Coast Survey charts.

# APPENDIX No. 8.

REPORT ON THE GEODETIC CONNECTION OF THE TWO PRIMARY BASE-LINES IN NEW YORK AND MARYLAND, THEIR DEGREE OF ACCORDANCE AND ACCURACY OF THE PRIMARY TRIANGULATION INTERVENING, WITH THE RESULTING ANGLES AND DISTANCES AS FINALLY ADJUSTED. PREPARED BY CHARLES A. SCHOTT, ASSISTANT.

The report on the operations and results of the Coast Survey of last year contains an abstract of the discussion of the lengths of the Fire Island, Massachusetts and Epping base-lines, together with the resulting angles and distances of the primary triangulation, and an estimate of its accuracy, throughout its extent in the New England States. In the present paper it is proposed to give a similar account of the primary triangulation in its continuation to the southward and westward, together with a branch (including subordinate primary triangulation,) reaching Washington city. (Sketch No. 10.)

The distance measured along the triangulation, between the Fire Island and Kent Island base-lines, is 263 miles; the number of triangles connecting them is 40 to the junction-line Pool–Finlay, from which line Kent island is reached directly by five additional triangles. The Washington city branch triangulation, from Kent island to Washington city, is 40 miles in length, and contains 18 primary triangles, (also 11 subordinate ones.)

The reduction by the method of least squares was made in two sections: the first one between the lines West Hills–Ruland and Pool–Finlay; the second between the last-named line and Causten–Seminary. The probable error and weight for each direction were determined and introduced in the adjustment. At stations occupied with the 30-inch and the 24-inch theodolites the probable observing error of a direction was arrived at by the method explained and illustrated by an example in Coast Survey Report of 1864, page 120, paragraph 3. The method given there in paragraph 4, applicable to *repeating* instruments, was, however, not followed, but the shorter method of writing out the results by repetitions, as if they were single measures, was substituted; the results were then obtained precisely as explained in paragraph 3, involving the formation of the diagonal coefficients. We have the following statistics:

| Part. | Locality. | Conditional equations of— | | Equations of correlatives. | Normal equations. |
|---|---|---|---|---|---|
| | | Angles. | Sides. | | |
| V | Fire island to Pool's island. | 29 | 5 | 107 | 34 |
| VI | Washington branch....... | 14 | 4 | 53 | 18 |

The resulting length of the junction Pool–Finlay from the base measures and carried through the triangulation, is as follows:

| | | |
|---|---|---|
| From Fire Island base............................... | 26268.07 metres, | [4.4194281 2] |
| From Kent Island base.............................. | 26267.76 metres, | [4.4194231 1] |

The difference is nearly equal to $\frac{1}{86700}$ of the length. The direct comparison of the base-lines is as follows: Starting from the measured length of the Fire Island base and carrying the distances through the triangulation to the Kent Island base—

| | | |
|---|---|---|
| The length of the latter is found ................. | 8687.645 metres, | [3.9389020 595] |
| Same by direct measure.......................... | 8687.545 metres, | [3.9388970 472] |
| Difference........................................ | 0.100 | [ 50 123] |

This discrepancy amounts to a little less than four inches. In connection with the logarithmic difference, 50 units in the seventh place of decimals, it may be stated that the probable error of the measure of the Fire Island base amounts to $\pm 18$ and that of the Kent Island base to $\pm 19$ such units.

In the report of 1865, Appendix No. 21, the length of the Fire Island base is given as measured and as computed from the two eastern base-lines; to these three values we may now add that derived from the Kent Island base, viz: 14058.809 metres.

To make this difference disappear, an additional equation (length-equation) was introduced between West Hills–Ruland and Pool–Finlay, making the number of normal equations 35. In consequence of this condition the former angles (and sides) were found to change but slightly (less than $0''.05$ on the average), and computing with the newly adjusted angles, the length of the Kent Island base, as derived from the Fire Island base, through the triangulation, was found to agree with the measured length.

The computation of the probable error of the sides of the triangulation was conducted in the same manner as explained at length in last year's report. The probable error of the measure of the Fire Island base is its $\frac{1}{240000}$ part, that of the Kent Island base its $\frac{1}{228000}$ part, of the line Pool–Finlay its $\frac{1}{131200}$ part, and that of the line midway between its $\frac{1}{87600}$ part; the approximate average probable error of the whole intervening triangulation is its $\frac{1}{116000}$ part, or nearly 0.55 inch in a statute mile, which value, when compared with the corresponding one of the northeastern branch of the primary triangulation, indicates a smaller degree of accuracy of the southern work, due principally to the much smaller number of measures of the primary angles, and partially to the use of inferior instruments and a much less number of geometrical conditions in the figure of the triangulation. From the Fire Island base to the side Bethel–Lippencott, the work was executed by the late superintendent, F. R. Hassler, who used the 30-inch and the 24-inch Troughton theodolites; between the line mentioned and the Kent Island base the measures were taken by Assistants James Ferguson and Edmund Blunt, the former using the 24-inch theodolite, the latter a 12-inch Simms repeating theodolite. Notwithstanding the use of these various instruments the character of the triangulation is throughout its extent tolerably uniform. The execution of the work falls between the years 1833 and 1848.

With respect to the Washington branch triangulation, there being an uncertainty of $\frac{1}{228000}$ part in the Kent Island base, and of $\frac{1}{102450}$ part in the terminal line Seminary–Causten, the approximate average uncertainty of the distances is $\frac{1}{141400}$ part. This fraction amounts to 0.45 inch in a statute mile. We have also for the extension of the triangulation south of Kent island the probable uncertainty in the line Marriott–South base $= \pm 0^{m}.165 = \frac{1}{129000}$ part of its length. The triangulation (Part VI) was executed by A. D. Bache, Superintendent, and by Assistants James Ferguson and Edmund Blunt, between the years 1844 and 1851. The instruments used were the 30-inch, 24-inch, and 12-inch theodolites mentioned above.

In conformity with last year's report a few statistical results are added, by which the degree of the accuracy of the measures may further be judged of; designating that portion of the primary triangulation lying between Fire island and Pool–Finlay as Part V, and that between Kent island and the District of Columbia as Part VI, we have:

| No. | No. of triangles. | Greatest error in sum of angles of any triangle. | No. of + errors. | No. of — errors. | Probable error of a direction, derived from △ residuals. | Angles measured apparently too great. | Instruments used. |
|---|---|---|---|---|---|---|---|
| | | ″ | | | ″ | ″ | |
| V. | 35 | 3.351 | 17 | 16 | ±0.412 | —0.226 | A 30-inch, 24-inch, and 12-inch repeating, theodolite. |
| VI. | 18 | 2.964 | 7 | 10 | ±0.428 | —0.145 | A 30-inch, 24-inch, and 12-inch repeating, theodolite. |

The positive and negative residuals in the sum of the angles balance near enough, but there appears to be a slight bias in the angular measures, which appear too small by 0″.185, which may may be due in part to the use of the smaller instruments, (the smallest was but little used,) and in part to accident. A slightly erroneous value of the spherical excess, a diminution of which is highly probable, since the ellipsoid of Bessel, which was employed in our computation, must now be regarded as superseded by the ellipsoid deduced by Captain Clark, though affecting the preceding number, is not sufficient to account for its amount. Thus, in our largest triangle, (Gunstock, Wachusett, Thompson,) with a spherical excess of 26.″040, this value is only changed by 0″.008 by the substitution of Clark's ellipsoid of rotation for that of Bessel's. The length of the sides of the triangles is, of course, not affected by any such change.

If we deduct the probable observing error of a direction at each station from the probable error of a direction resulting from the closing of the triangles we find the following values for the triangle combination error $\varepsilon_{\triangle}$, viz:

Part V ± 0″.328
VI ± 0″.376

values nearly double those given for the preceding parts; the average length of sides, however, is much less, and the probable observing error of a direction reached in Part V ± 0″.250, and in Part VI ± 0″.205.

The above combination error was combined with the special observing error of each direction for the computation of weights to each direction, and employed in the adjustment of the geometrical figure. The extreme weights to a direction are in the proportion of 1 to 5.3 in Part V, and in the proportion of 1 to 2.3 in Part VI.

The corrections to the observed directions, as demanded by the least square adjustment of the geometrical figure of the triangulation, are as follows:

| | Greatest correction to any direction. | Average correction to a direction. |
|---|---|---|
| | ″ | ″ |
| Part V | 2.02 | ±0.39 |
| Part VI | 1.09 | ±0.41 |

The average length of a triangle side (between stations Ruland and Seminary) is 18.0 miles, the longest side 42.6 miles, and the sum total of lengths 1456 miles.

The table of observed and resulting angles and of distances is arranged as for the preceding primary triangulation; all distances are expressed in units of our committee metre, and the numeration of the triangles is continued from last year's report.

*Resulting angles and distances of the primary triangulation extending from New York to the District of Columbia.*

| Number of △. | Name of station. | Observed angles. | Corrections by adjustment. | Resulting angles, (seconds.) | Spherical excess of △. | Log. distances. | Distances, metres, (com.) | Distances, statute miles. |
|---|---|---|---|---|---|---|---|---|
| | | ° ′ ″ | ″ | ″ | ″ | | | |
| | Bald Hill | 36 51 27.977 | −1.590 | 26.387 | | 4.5223968 9 | 33296.370 | 20.69 |
| 112 | Ruland | 53 14 54.171 | +0.048 | 54.219 | 3.759 | 4.6481352 6 | 44476.977 | 27.64 |
| | West Hills | 89 53 43.044 | +0.109 | 43.153 | | 4.7443754 8 | 55510.544 | 34.49 |
| | Bald Hill | 99 31 40.835 | + .395 | 41.230 | | 4.7125612 5 | 51589.492 | 32.05 |
| 113 | Tashua | 58 14 16.937 | − .412 | 16.525 | 2.204 | 4.6481352 5 | 44476.977 | 27.64 |
| | West Hills | 22 14 04.340 | + .109 | 04.449 | | 4.2965406 1 | 19794.321 | 12.30 |
| | Bald Hill | 62 40 12.858 | +1.985 | 14.843 | | 4.6958476 7 | 49641.817 | 30.84 |
| 114 | Tashua | 96 34 59.411 | − .413 | 58.998 | 2.478 | 4.7443754 7 | 55510.543 | 34.49 |
| | Ruland | 20 44 48.684 | − .047 | 48.637 | | 4.2965406 0 | 19794.320 | 12.30 |
| | Round Hill | 93 33 38.594 | −1.539 | 37.055 | | 4.6481352 5 | 44476.977 | 27.64 |
| 115 | Bald Hill | 59 05 13.167 | − .521 | 12.646 | 1.983 | 4.5824337 8 | 38232.595 | 23.76 |
| | West Hills | 27 21 12.705 | − .423 | 12.282 | | 4.3112359 4 | 20475.567 | 12.72 |
| | Harrow | 67 45 27.724 | + .444 | 28.168 | | 4.6481352 5 | 44476.977 | 27.64 |
| 116 | Bald Hill | 22 10 37.774 | −1.389 | 36.385 | 2.048 | 4.2585898 1 | 18138.017 | 11.27 |
| | West Hills | 90 03 57.659 | − .164 | 57.495 | | 4.6817158 7 | 48052.487 | 29.86 |
| | Harrow | 88 58 17.503 | + .284 | 17.787 | | 4.5824337 8 | 38232.595 | 23.76 |
| 117 | Round Hill | 28 18 56.336 | +2.229 | 58.565 | 1.564 | 4.2585898 2 | 18138.017 | 11.27 |
| | West Hills | 62 42 44.954 | + .258 | 45.212 | | 4.5312670 5 | 33983.417 | 21.12 |
| | Harrow | 21 12 49.779 | − .160 | 49.619 | | 4.3112359 4 | 20475.567 | 12.72 |
| 118 | Round Hill | 121 52 34.930 | + .690 | 35.620 | 1.499 | 4.6817158 6 | 48052.485 | 29.86 |
| | Bald Hill | 36 54 35.393 | + .867 | 36.260 | | 4.5312670 4 | 33983.416 | 21.12 |
| | Buttermilk | 63 46 44.026 | + .087 | 44.113 | | 4.5312670 5 | 33983.417 | 21.12 |
| 119 | Round Hill | 98 30 13.760 | −1.146 | 12.614 | 0.984 | 4.5736279 2 | 37465.188 | 23.28 |
| | Harrow | 17 43 04.194 | + .063 | 04.257 | | 4.0617708 8 | 11528.449 | 7.16 |
| | Weasel | 49 54 10.775 | − .135 | 10.640 | | 4.5736279 2 | 37465.188 | 23.28 |
| 120 | Buttermilk | 73 30 35.167 | − .476 | 34.691 | 3.728 | 4.6717522 7 | 46962.615 | 29.18 |
| | Harrow | 56 35 18.472 | − .075 | 18.397 | | 4.6115422 9 | 40882.957 | 25.40 |
| | *Beaconhill* | 42 48 12.225 | − .210 | 12.015 | | 4.6717522 7 | 46962.615 | 29.18 |
| 121 | Weasel | 83 08 60.868 | − .412 | 60.456 | 6.622 | 4.8364663 8 | 68622.476 | 42.64 |
| | Harrow | 54 02 54.270 | − .119 | 54.151 | | 4.7477984 7 | 55949.791 | 34.76 |
| | Springfield | 84 13 25.129 | + .671 | 25.800 | | 4.8364663 8 | 68622.476 | 42.64 |
| 122 | Harrow | 32 07 60.764 | + .378 | 61.142 | 5.726 | 4.5644978 0 | 36685.784 | 22.79 |
| | Beaconhill | 63 38 38.461 | + .323 | 38.784 | | 4.7910095 5 | 61803.000 | 38.40 |
| | Mt. Rose | 47 58 30.192 | − .058 | 30.134 | | 4.5644978 0 | 36685.784 | 22.79 |
| 123 | Springfield | 58 27 49.651 | − .337 | 49.314 | 3.760 | 4.6241925 9 | 42091.324 | 26.15 |
| | Beaconhill | 73 33 44.488 | − .176 | 44.312 | | 4.6754729 7 | 47366.683 | 29.43 |
| | Disborough | 44 09 20.943 | + .197 | 21.140 | | 4.5644978 0 | 36685.784 | 22.79 |
| 124 | *Springfield* | *27 10 (concluded)* | | 15.493 | 2.121 | 4.3810858 5 | 24048.371 | 14.94 |
| | Beaconhill | 108 40 25.771 | − .283 | 25.488 | | 4.6980221 3 | 49890.991 | 31.00 |
| | Disborough | 113 12 55.470 | − .034 | 55.436 | | 4.6241925 9 | 42091.324 | 26.15 |
| 125 | Mt. Rose | 31 40 24.904 | − .038 | 24.866 | 1.478 | 4.3810858 5 | 24048.371 | 14.94 |
| | Beaconhill | 35 06 41.283 | − .107 | 41.176 | | 4.4206564 0 | 26342.464 | 16.37 |
| | Disborough | 69 03 34.527 | − .231 | 34.296 | | 4.6754729 7 | 47366.683 | 29.43 |
| 126 | Mt. Rose | 79 38 55.096 | − .096 | 55.000 | 3.117 | 4.6980221 3 | 49890.991 | 31.00 |
| | Springfield | 31 17 (concluded) | | 33.821 | | 4.4206564 0 | 26342.464 | 16.37 |
| | Stonyhill | 60 40 11.864 | + .137 | 12.001 | | 4.4206564 0 | 26342.464 | 16.37 |
| 127 | Mt. Rose | 35 36 10.085 | + .169 | 10.254 | 1.169 | 4.2452773 0 | 17590.464 | 10.93 |
| | Disborough | 83 43 38.698 | + .216 | 38.914 | | 4.4776257 4 | 30034.868 | 18.66 |
| | Mt. Holly | 45 45 41.983 | + .883 | 42.866 | | 4.4776257 4 | 30034.868 | 18.66 |
| 128 | Mt. Rose | 31 38 20.410 | − .055 | 20.355 | 1.636 | 4.3422405 6 | 21990.776 | 13.66 |
| | *Stonyhill* | 102 35 57.937 | + .478 | 58.415 | | 4.6118566 0 | 40912.555 | 25.42 |
| | Newtown | 41 12 36.256 | +1.152 | 37.408 | | 4.3422405 6 | 21990.776 | 13.66 |
| 129 | Stonyhill | 63 09 51.295 | + .866 | 52.161 | 1.610 | 4.4739845 2 | 29784.103 | 18.51 |
| | Mt. Holly | 75 37 30.708 | +1.333 | 32.041 | | 4.5096575 2 | 32333.858 | 20.09 |
| | Newtown | 105 37 14.249 | + .673 | 14.922 | | 4.6118566 0 | 40912.555 | 25.42 |
| 130 | Mt. Rose | 44 30 57.509 | − .066 | 57.443 | 1.540 | 4.4739845 2 | 29784.103 | 18.51 |
| | Mt. Holly | 29 51 48.725 | + .450 | 49.175 | | 4.3253740 0 | 21153.099 | 13.14 |
| | Newtown | 64 24 37.993 | − .479 | 37.514 | | 4.4776257 4 | 30034.868 | 18.66 |
| 131 | Mt. Rose | 76 09 17.919 | − .121 | 17.798 | 1.566 | 4.5096575 2 | 32333.858 | 20.09 |
| | Stonyhill | 39 26 06.642 | − .388 | 06.254 | | 4.3253740 0 | 21153.099 | 13.14 |

*Resulting angles and distances of the primary triangulation, &c.*—Continued.

| Number of Δ. | Name of station. | Observed angles. | Corrections by adjustment. | Resulting angles, (seconds.) | Spherical excess of Δ. | Log. distances. | Distances, metres, (com.) | Distances, statute miles. |
|---|---|---|---|---|---|---|---|---|
| | | ° ′ ″ | ″ | ″ | ″ | | | |
| | Willowgrove | 67 07 37.536 | —.487 | 37.049 | | 4.4739845 2 | 29784.103 | 18.51 |
| 132 | Newtown | 74 53 42.143 | —1.093 | 41.050 | 1.452 | 4.4942805 7 | 31209.051 | 19.39 |
| | Mt. Holly | 37 58 44.266 | —.913 | 43.353 | | 4.2986856 8 | 19892.331 | 12.36 |
| | Pinehill | 51 44 18.183 | +.476 | 18.659 | | 4.4942805 7 | 31209.051 | 19.39 |
| 133 | Willowgrove | 46 05 54.329 | +.162 | 54.491 | 2.248 | 4.4569576 0 | 28638.984 | 17.79 |
| | Mt. Holly | 82 09 48.442 | +.656 | 49.098 | | 4.5952305 6 | 39375.905 | 24.47 |
| | Yard | 33 40 18.813 | +.204 | 19.017 | | 4.4569576 0 | 28638.984 | 17.79 |
| 134 | Mt. Holly | 48 51 18.622 | +.851 | 19.473 | 2.804 | 4.5899312 9 | 38898.360 | 24.17 |
| | Pinehill | 97 28 24.193 | +.121 | 24.314 | | 4.7094036 0 | 51215.757 | 31.82 |
| | Lippencott | 85 28 48.612 | +1.119 | 49.731 | | 4.5899312 9 | 38898.360 | 24.17 |
| 135 | Yard | 47 24 56.365 | +.778 | 57.143 | 2.078 | 4.4583282 9 | 28729.513 | 17.85 |
| | Pinehill | 47 06 14.531 | +.673 | 15.204 | | 4.4561453 8 | 28585.473 | 17.76 |
| | Bethel | 35 15 32.011 | —.463 | 31.548 | | 4.4583282 9 | 28729.513 | 17.85 |
| 136 | Pinehill | 24 14 38.796 | —.102 | 38.694 | 1.284 | 4.3103935 5 | 20435.890 | 12.70 |
| | Lippencott | 120 29 50.646 | +.396 | 51.042 | | 4.6322825 4 | 42882.742 | 26.65 |
| | Bethel | 100 16 49.450 | +.202 | 49.652 | | 4.4561453 8 | 28585.473 | 17.76 |
| 137 | Yard | 44 42 10.147 | —.259 | 09.888 | 0.851 | 4.3103935 5 | 20435.890 | 12.70 |
| | Lippencott | 35 01 02.034 | —.723 | 01.311 | | 4.2219487 8 | 16670.506 | 10.36 |
| | Bethel | 65 01 17.439 | +.665 | 18.104 | | 4.5899312 9 | 38898.360 | 24.17 |
| 138 | Yard | 92 07 06.512 | +.519 | 07.031 | 1.645 | 4.6322825 5 | 42882.742 | 26.65 |
| | Pinehill | 22 51 35.735 | +.775 | 36.510 | | 4.2219487 8 | 16670.506 | 10.36 |
| | Burden | 30 04 36.799 | —.295 | 36.504 | | 4.3103935 5 | 20435.890 | 12.70 |
| 139 | Bethel | 32 49 26.822 | —.115 | 26.707 | 1.021 | 4.3444654 3 | 22103.723 | 13.73 |
| | Lippencott | 117 05 58.189 | —.379 | 57.810 | | 4.5599145 3 | 36300.660 | 22.56 |
| | Meetinghouse | 37 23 53.256 | —.704 | 52.552 | | 4.3444654 3 | 22103.723 | 13.73 |
| 140 | Lippencott | 73 07 29.710 | —.768 | 28.942 | 1.830 | 4.5419139 1 | 34826.827 | 21.64 |
| | Burden | 69 28 40.923 | —.587 | 40.336 | | 4.5325545 0 | 34084.309 | 21.18 |
| | Buck 2 | 86 45 35.314 | +.312 | 35.626 | | 4.5419139 1 | 34826.827 | 21.64 |
| 141 | Meetinghouse | 59 51 56.885 | +.633 | 57.518 | 1.467 | 4.4795506 9 | 30168.290 | 18.75 |
| | Burden | 33 22 27.846 | +.477 | 28.323 | | 4.2830564 4 | 19189.181 | 11.92 |
| | Principio | 41 47 41.531 | +.383 | 41.914 | | 4.2830564 4 | 19189.181 | 11.92 |
| 142 | Meetinghouse | 56 59 06.222 | +.010 | 06.232 | 1.162 | 4.3827959 7 | 24143.263 | 15.00 |
| | Buck 2 | 81 13 12.906 | +.110 | 13.016 | | 4.4541596 1 | 28455.066 | 17.68 |
| | Turkey Point | 65 21 20.562 | —.033 | 20.529 | | 4.3827959 7 | 24143.263 | 15.00 |
| 143 | Principio | 77 37 28.454 | —.136 | 28.318 | 0.958 | 4.4140630 5 | 25945.560 | 16.12 |
| | Buck 2 | 37 01 12.453 | —.342 | 12.111 | | 4.2039370 4 | 15993.261 | 9.94 |
| | Osborne's Ruin | 35 10 11.669 | —.395 | 11.274 | | 4.2039370 4 | 15993.261 | 9.94 |
| 144 | Principio | 57 36 57.656 | —.258 | 57.398 | 0.951 | 4.3701017 7 | 23447.782 | 14.57 |
| | Turkey Point | 87 12 52.521 | —.242 | 52.279 | | 4.4430009 1 | 27733.259 | 17.23 |
| | Pool's Island | 54 30 55.001 | —.047 | 54.954 | | 4.3701017 7 | 23447.782 | 14.57 |
| 145 | Osborne's Ruin | 81 27 17.526 | —.192 | 17.334 | 1.178 | 4.4544856 2 | 28476.435 | 17.69 |
| | Turkey Point | 44 01 48.723 | +.167 | 48.890 | | 4.3013415 2 | 20014.351 | 12.44 |
| | Finlay | 48 03 33.324 | +1.100 | 34.424 | | 4.3013415 2 | 20014.351 | 12.44 |
| 146 | Osborne's Ruin | 77 29 14.376 | +.875 | 15.251 | 1.086 | 4.4194231 1 | 26267.765 | 16.32 |
| | Pool's Island | 54 27 11.288 | +.123 | 11.411 | | 4.3402947 4 | 21892.469 | 13.60 |
| | Taylor | 38 36 52.373 | —.509 | 51.864 | | 3.9388970 5 | 8687.545 | 5.40 |
| 147 | North Base | 88 35 36.913 | —.233 | 36.680 | 0.244 | 4.1435288 5 | 13916.462 | 8.65 |
| | South Base | 52 47 32.011 | —.311 | 31.700 | | 4.0448164 5 | 11087.061 | 6.89 |
| | Marriott | 18 13 37.318 | +.270 | 37.588 | | 3.9388970 5 | 8687.545 | 5.40 |
| 148 | North Base | 50 05 05.356 | —.702 | 04.654 | 0.437 | 4.3284441 4 | 21303.165 | 13.24 |
| | South Base | 111 41 18.247 | —.052 | 18.195 | | 4.4117658 1 | 25808.681 | 16.04 |
| | Marriott | 40 10 21.280 | +.243 | 21.523 | | 4.1435288 5 | 13916.462 | 8.65 |
| 149 | Taylor | 80 55 51.946 | +.680 | 52.626 | 0.645 | 4.3284441 3 | 21303.165 | 13.24 |
| | South Base | 58 53 46.236 | +.260 | 46.496 | | 4.2664989 1 | 18471.362 | 11.48 |
| | Marriott | 21 56 43.962 | —.027 | 43.935 | | 4.0448164 5 | 11087.061 | 6.89 |
| 150 | Taylor | 119 32 44.319 | +.171 | 44.490 | 0.452 | 4.4117658 0 | 25808.681 | 16.04 |
| | North Base | 38 30 31.557 | +.470 | 32.027 | | 4.2664989 1 | 18471.361 | 11.48 |
| | Linstid | 68 43 33.677 | —.068 | 33.609 | | 4.4117658 1 | 25808.681 | 16.04 |
| 151 | North Base | 70 56 56.975 | +.763 | 59.738 | 1.110 | 4.4179562 3 | 26179.192 | 16.27 |
| | Marriott | 40 19 28.501 | —.738 | 27.763 | | 4.2533973 4 | 17922.448 | 11.14 |

*Resulting angles and distances of the primary triangulation, &c.*—Continued.

| Number of △. | Name of station. | Observed angles. | Corrections by adjustment. | Resulting angles. (seconds.) | Spherical excess of △. | Log. distances. | Distances, metres, (com.) | Distances, statute miles. |
|---|---|---|---|---|---|---|---|---|
| | | ° ′ ″ | ″ | ″ | ″ | | | |
| | Swan Point | 56 08 57.917 | +1.016 | 58.933 | | 4.2533973 4 | 17922.448 | 11.14 |
| 152 | North Base | 60 07 41.140 | —.009 | 41.131 | 0.764 | 4.2721496 9 | 18713.270 | 11.63 |
| | Linstid | 63 43 20.633 | +.067 | 20.700 | | 4.2866876 4 | 19350.297 | 12.02 |
| | Pool's Island | 36 22 15.134 | +.034 | 15.168 | | 4.2721496 9 | 18713.270 | 11.63 |
| 153 | Swan Point | 113 07 27.589 | —.434 | 27.155 | 0.700 | 4.4627139 9 | 29021.108 | 18.03 |
| | Linstid | 30 30 19.239 | —.862 | 18.377 | | 4.2046222 6 | 16018.515 | 9.95 |
| | Finlay | 53 32 21.714 | +1.651 | 23.365 | | 4.4627139 9 | 29021.108 | 18.03 |
| 154 | Pool's Island | 79 44 39.791 | +.509 | 40.300 | 1.905 | 4.5503183 7 | 35507.358 | 22.06 |
| | Linstid | 46 42 57.730 | +.510 | 58.240 | | 4.4194231 1 | 26267.764 | 16.32 |
| | Finlay | 25 43 (conclu | ded) | 37.235 | | 4.2142059 4 | 16375.929 | 10.18 |
| 155 | Linstid | 84 01 06.411 | +.096 | 06.507 | 1.468 | 4.5742634 2 | 37520.051 | 23.31 |
| | Webb | 70 15 16.992 | +.734 | 17.726 | | 4.5503183 6 | 35507.358 | 22.06 |
| | Linstid | 34 46 24.832 | +1.126 | 25.958 | | 4.0448164 5 | 11087.061 | 6.89 |
| 156 | North Base | 32 26 27.418 | +.295 | 27.713 | 0.271 | 4.0181973 7 | 10427.912 | 6.48 |
| | Taylor | 112 47 05.714 | +.886 | 06.600 | | 4.2533973 4 | 17922.448 | 11.14 |
| | Linstid | 33 57 08.845 | —1.195 | 07.650 | | 4.2664989 1 | 18471.361 | 11.48 |
| 157 | Taylor | 127 40 09.967 | —1.057 | 08.910 | 0.387 | 4.4179562 3 | 26179.192 | 16.27 |
| | Marriott | 18 22 44.539 | —.712 | 43.827 | | 4.0181973 7 | 10427.912 | 6.48 |
| | Webb | 76 16 06.190 | —.103 | 06.087 | | 4.4179562 3 | 26179.192 | 16.27 |
| 158 | Linstid | 66 18 42.310 | +.256 | 42.566 | 0.997 | 4.3923246 8 | 24678.836 | 15.33 |
| | Marriott | 37 25 11.128 | +1.216 | 12.344 | | 4.2142059 4 | 16375.929 | 10.18 |
| | Hill | 56 40 32.002 | +.408 | 32.410 | | 4.3923246 8 | 24678.836 | 15.33 |
| 159 | Webb | 53 10 52.339 | —.342 | 51.997 | 1.394 | 4.3737192 9 | 23643.910 | 14.69 |
| | Marriott | 70 08 37.171 | —.184 | 36.987 | | 4.4437204 4 | 27779.245 | 17.26 |
| | Soper | 39 41 37.078 | —.245 | 36.833 | | 4.3923246 8 | 24678.836 | 15.33 |
| 160 | Webb | 102 15 58.886 | +.009 | 58.895 | 1.458 | 4.5770122 1 | 37758.280 | 23.46 |
| | Marriott | 38 02 26.815 | —1.085 | 25.730 | | 4.3767747 6 | 23810.843 | 14.79 |
| | Soper | 75 01 10.921 | —.034 | 10.887 | | 4.4437204 4 | 27779.245 | 17.26 |
| 161 | Webb | 49 05 06.547 | +.351 | 06.898 | 1.269 | 4.3370769 0 | 21730.859 | 13.50 |
| | Hill | 55 53 43.600 | —.116 | 43.484 | | 4.3767747 7 | 23810.843 | 14.79 |
| | Soper | 35 19 33.843 | +.211 | 34.054 | | 4.3737192 9 | 23643.910 | 14.69 |
| 162 | Marriott | 32 06 10.356 | +.901 | 11.257 | 1.205 | 4.3370769 0 | 21730.859 | 13.50 |
| | Hill | 112 34 15.602 | +.292 | 15.894 | | 4.5770122 1 | 37758.280 | 23.46 |
| | Causten | 69 31 00.444 | +1.111 | 01.555 | | 4.3370769 0 | 21730.859 | 13.50 |
| 163 | Soper | 47 08 46.376 | +.925 | 47.301 | 0.839 | 4.2306006 4 | 17005.940 | 10.57 |
| | Hill | 63 20 11.373 | +.610 | 11.983 | | 4.3166124 5 | 20730.628 | 12.88 |
| | Seminary* | 56 42 15.849 | +.892 | 16.741 | | 4.2306006 4 | 17005.940 | 10.57 |
| 164 | Causten | 87 33 12.759 | +.178 | 12.937 | 0.513 | 4.3080755 2 | 20327.104 | 12.63 |
| | Hill | 35 44 30.711 | +.124 | 30.835 | | 4.0749841 9 | 11884.590 | 7.38 |
| Subordinate triangles. | Seaton | 144 03 45.60 | +1.99 | 47.59 | | 4.2306006 | 17005.94 | 10.57 |
| | Causten | 21 00 28.64 | +1.98 | 30.62 | 0.12 | 4.0165390 | 10388.17 | 6.45 |
| | Hill | 14 55 41.41 | +.50 | 41.91 | | 3.8730049 | 7464.57 | 4.64 |
| | Seaton | 140 00 55.28 | —.69 | 54.59 | | 4.3080755 | 20327.10 | 12.63 |
| | Hill | 20 48 49.22 | —.50 | 48.92 | 0.19 | 4.0507747 | 11240.22 | 6.98 |
| | Seminary | 19 10 (conclu | ded) | 16.68 | | 4.0165391 | 10388.17 | 6.45 |
| | Seaton | 75 55 19.12 | —1.30 | 17.82 | | 4.0749842 | 11884.59 | 7.38 |
| | Seminary | 37 32 (conclu | ded) | 00.08 | 0.21 | 3.8730049 | 7464.57 | 4.64 |
| | Causten | 66 32 44.29 | —1.98 | 42.31 | | 4.0507748 | 11240.22 | 6.98 |
| | U. S. Naval Observatory † | 116 46 (conclu | ded) | 55.69 | | 4.2306006 | 17005.94 | 10.57 |
| | Causten | 51 13 26.63 | +.26 | 26.89 | 0.13 | 4.1717550 | 14850.98 | 9.23 |
| | Hill | 11 59 38.39 | —.84 | 37.55 | | 3.5975382 | 3958.57 | 2.46 |
| | U. S. Naval Observatory | 114 38 (conclu | ded) | 27.00 | | 4.3080755 | 20327.10 | 12.63 |
| | Hill | 23 44 52.45 | +.84 | 53.29 | 0.31 | 3.9545400 | 9006.17 | 5.60 |
| | Seminary | 41 36 40.72 | —.70 | 40.02 | | 4.1717551 | 14850.98 | 9.23 |
| | U. S. Naval Observatory | 128 34 (conclu | ded) | 37.31 | | 4.0749842 | 11884.59 | 7.38 |
| | Seminary | 15 05 36.02 | +.70 | 36.72 | 0.07 | 3.5975383 | 3958.57 | 2.46 |
| | Causten | 36 19 46.30 | —.26 | 46.04 | | 3.9545400 | 9006.17 | 5.60 |

* Old steeple. † Station east of dome.

# APPENDIX No. 9.

## DETERMINATION OF TIME BY MEANS OF THE TRANSIT INSTRUMENT.

[Prepared for the Coast Survey Manual by C. A. Schott, assistant.]

(1.) *General remarks on the use of the transit instrument.*—This paper exclusively refers to the portable transit instrument, (as used in the Coast Survey,) and when mounted in the plane of the meridian for the purpose of obtaining local time from observations of transit of stars in connection with an astronomical clock or a chronometer keeping sidereal time. For such modifications and additions to the instrument which will specially adapt it to the determination of the difference of longitude by the electric telegraph, the reader is referred to the article on that subject; for some remarks on the adaptation of the instrument to the determination of latitudes by temporarily converting it into a zenith-telescope, and on the determination by it of a meridian line, see the papers on the zenith-telescope latitudes and on astronomical azimuths. The instrument is not now used on the survey for determining the latitude when mounted in the plane of the prime vertical. The reduction of transit observations is essentially the same whether we note the time by eye and ear or by chronographic registration. In using the former method the observer will, of course, pick up the beat of the chronometer *himself*, and will estimate the time of transit to the nearest tenth of a second.

(2.) *Description of the instrument.*—Two sizes of portable transits are employed on the survey, the larger one (made by Troughton and Simms, of London) being especially used for telegraphic or astronomical longitudes, the smaller one (made by Würdemann, of Washington) to supply the local time for astronomical latitudes and azimuths, or for other minor purposes. In case of necessity, and when we are satisfied with an approximate degree of accuracy, any ordinary astronomical theodolite (altazimuth instrument) may be converted temporarily into and used as a transit. The larger sized transits have a telescope of forty-six inches focal length, with a clear aperture of three inches, and magnifying powers, as generally used, between 80 and 120; they are provided with reversing apparatus, in which the telescope is raised and lowered by means of an eccentric cam, rendering the operation of reversing safe and expeditious. The smaller sized instruments have telescopes of twenty-six inches focal length, with two inches aperture, ordinarily used with a magnifying power of forty. This exceedingly portable instrument, with its folding frame, is shown on the accompanying plate, No. 29.

Either five or seven threads are stretched vertically across the diaphragm with two close horizontal threads at right angles to the former. For telegraphic operations with the chronograph there are five tallies of five threads each. The star is made to traverse the field between the horizontal threads, and the eye is kept directly in front of each thread, in succession, by means of a slide moved by hand. All the instruments are supplied with prismatic eye-pieces.

The finders in the smaller instruments are four inches in diameter, and by means of verniers can be read to single minutes; one generally is graduated for zenith distances, the other for altitudes, to suit the convenience of the observer. The striding level is filled with ether, hermetically closed and supplied with a chamber to regulate the length of the bubble at all temperatures. The sensitiveness of the bubble is such that a change of one second of arc is represented by about one millimetre. It is not customary to observe stars by reflection in mercury; the use of meridian marks, where practicable, is left to the option of the observer.

(3.) *Adjustment of the instrument.*—The stone pier, block of wood, or other support for the transit may be set approximately in position with regard to the meridian by means of a compass needle, the magnetic declination being known and allowed for. The top of the pier is levelled and the frame of the instrument placed in position, so that the transit axis coincides as near as may be with the plane of the prime vertical; the adjusting screws of the Y's, both for horizontal and vertical motion, are placed nearly in the middle of their position; the striding level is carefully adjusted and the transit axis of the telescope levelled. The threads are then placed in the focus of the eye-piece and set vertical. The telescope is adjusted to sidereal focus. The adjustment for collimation may be effected by means of a distant object or by means of a collimating telescope, the axis being reversed in its Y's during the operation; this method suffices for the portable instrument and gives a first approximation to be afterwards tested and perfected by means of transits of stars.

The local time may readily be obtained by the use of a sextant, with an accuracy within a fraction of a minute, and the latitude may be found either by a map or by the same instrument, the nearest minute of arc being sufficient. A small altitude and azimuth instrument may also very advantageously be used for ascertaining with sufficient approximation the local time, latitude, and direction of the meridian, for the purpose of placing the transit in position. To point the telescope to a star when culminating, and supposing the finder to read zenith distances, we have for a star $\left\{ \begin{matrix} S \\ N \end{matrix} \right.$ of the zenith $\zeta = \pm \varphi \mp \delta - r$ where the upper sign refers to southern and the lower sign to northern stars with respect to the zenith; the refraction $r$ may generally be neglected. The index error of the finder may readily be removed by pointing to a known star and keeping it between the horizontal threads when transiting, for which position the finder is to be made to show the correct setting. The chronometer time of the transit of a slow moving (polar) star is next computed, the telescope pointed to it and the star bisected with the middle thread at the computed time of culmination, making use of the slow azimuth motion of the Y, or, if need be, by shifting the frame of the instrument. The axis having been levelled we next set for and observe two close zenith stars, one north, the other south of it, and with clamp east and clamp west, from which we obtain a very close approximation of the chronometer correction on sidereal time. The process of bisecting a circumpolar star may then be repeated, using the azimuth screw only for this adjustment, after which the telescope will generally be found sufficiently near in the plane of the meridian to admit of commencing the regular series of observations.

(4.) *Method of observation.*—Generally a series of observations commences with transits of stars selected to furnish instrumental corrections, then follow transits of so-called time stars; and the night's work is concluded by again observing stars of the character first-named.

The deviation from horizontality of the transit axis is determined by level readings, for each star, if possible, and the inequality of pivots is allowed for. The value of a division of the striding level is ascertained by any of the methods explained in connection with the zenith telescope, and the effect of temperature is to be allowed for, if sensible.

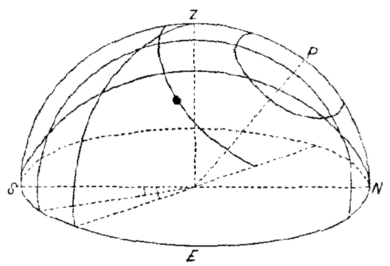
The collimation is ascertained by observing one-half of the number of stars with clamp E, or W; then reversing the telescope and observing the remaining half, clamp W, or E; or we may specially observe for it, noting the transits of a close circumpolar star over one-half of the threads, then reversing the telescope and noting the remaining transits over the same threads, (now presented in the reverse order,) it is well to note the state of the level during each transit.

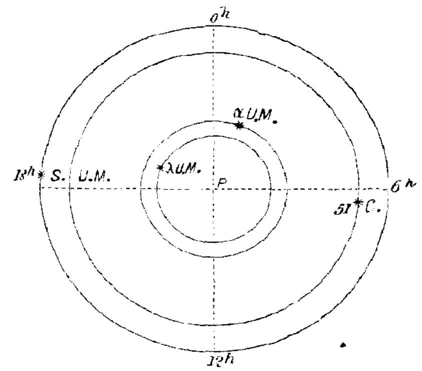
The deviation in azimuth is obtained from observations of stars differing considerably in declination, (sometimes called high and low stars,) but little in right ascension, or from observations of two close circumpolar stars, one culminating above the other below the pole, and having a difference of right ascension not differing much from 12 hours. It is not safe to rely on the stability of the instrument and the constancy of the rate of the chronometer by observing the *same* star at upper and lower culminations immediately succeeding. Knowing the reading of the azimuth screw for the two states of the instrument in which the azimuthal deviation has been determined, the value of one division of its micrometer head becomes also known. It is well if the sum of the azimuthal corrections for the circumzenith stars nearly balance, and for any two zenith stars if the mean of the tangents of their declinations equals the tangent of the latitude, the deduced chronometer correction will then be free of any error in azimuth.

In making up his observing programme the observer, in the first place, selects his stars from the list of standard mean places of circumpolar and time stars, prepared for the use of the United States Coast Survey by Dr. B. A. Gould, (second edition, Washington, 1866,) and in the second place, from the positions given in the American Ephemeris and Nautical Almanac, and in the English Nautical Almanac; should more stars be desirable, they may be selected from the three Greenwich catalogues in preference to others.

If more than one observer engages in the same series of observations, their personal equation must be ascertained by methods explained in the telegraphic determination of longitudes.

(5.) *Method and formulæ of reduction.*—The usual formulæ* for reduction of transit observations with portable instruments are here given in a concise form and in order to facilitate their application, tables of transit factors accompany this paper. The formulæ are arranged with reference to the mean of the threads, and not to the middle thread, which latter may be more convenient in fixed observatories in connection with collimators, and where the instrument is less frequently reversed.




*Equatorial intervals of threads.*

To determine these select complete transits of stars of great declination,
Let $t_1\ t_2\ t_3\ .\ .\ .\ t_n$ be the observed times of transit over the successive threads,
and $i_1\ i_2\ i_3\ .\ .\ .\ i_n$ their equatorial intervals from the mean thread, and $\delta$ the declination of the star,

$$t = \frac{1}{n}(t_1 + t_2 + t_3\ .\ .\ .\ + t_n)$$

$$i_1 = (t_1 - t)\cos\delta$$

$$i_2 = (t_2 - t)\cos\delta$$

etc.,

$$i_n = (t_n - t)\cos\delta$$

also, $o = i_1 + i_2 + i_3\ .\ .\ .\ + i_n$

The intervals of the threads $\left\{\begin{matrix}\text{east}\\ \text{west}\end{matrix}\right\}$ of the mean thread will be $\left\{\begin{matrix}-\\ +\end{matrix}\right\}$ at upper culmination.

For stars within 10° of the pole, (as for $\delta$ Urs. Min., 51 Cephei, Polaris, and $\lambda$ Urs. Min.) use the formulæ:

$$i_1 = (t_1 - t)\cos\delta\ \sqrt[3]{\cos\tau_1}$$

etc.,

$i_n = (t_n - t)\cos\delta\ \sqrt[3]{\cos\tau_n}$ where $\tau_1\ \tau_2\ \tau_3\ .\ .\ .\ \tau_n$ the hour angles of the circumpolar star at the successive threads.

When the chronometer has a large rate the intervals require to be corrected for it.

*Incomplete transits.*

When the star was not observed on some of the threads, the time of transit over the mean of all the threads may, by means of the known intervals of the threads, be found as follows:

$$t = \text{mean of observed times} + \frac{\text{sum of equatorial intervals of missed threads} \times \sec\delta}{\text{number of observed threads.}}$$

If the transit over one, or a few threads only, is observed, we may use the formula

$$t = \text{mean of observed times} + \frac{\text{sum of equatorial intervals of observed threads} \times \sec\delta}{\text{number of observed threads.}}$$

taking care, however, first to change the sign of the intervals.

In reducing broken transits of a circumpolar star use $i_1\ \sqrt[3]{\sec\tau}\ \ i_2\ \sqrt[3]{\sec\tau_2}\ .\ .\ .\ i_n\ \sqrt[3]{\sec\tau_n}$ in the place of the equatorial intervals $i_1\ i_2\ .\ .\ .\ i_n$.

* For a full discussion of the use of the transit instrument, the reader may be referred to Professor W. Chauvenet's Manual on Spherical Astronomy, Philadelphia, 1863, vol. ii., pp. 131–209.

Apply also a correction for rate, if necessary.

| $\tau$ | $\log \sqrt[3]{\cos \tau}$ | $\log \sqrt[3]{\sec \tau}$ | $\tau$ | $\log \sqrt[3]{\cos \tau}$ | $\log \sqrt[3]{\sec \tau}$ | $\tau$ | $\log \sqrt[3]{\cos \tau}$ | $\log \sqrt[3]{\sec \tau}$ |
|---|---|---|---|---|---|---|---|---|
| $1^m$ | 9.99999 | 0.00000 | $16^m$ | 9.99965 | 0.00035 | $31^m$ | 9.99867 | 0.00133 |
| 2 | 99 | 01 | 17 | 960 | 040 | 32 | 858 | 142 |
| 3 | 99 | 01 | 18 | 955 | 045 | 33 | 849 | 151 |
| 4 | 98 | 02 | 19 | 950 | 050 | 34 | 840 | 160 |
| 5 | 97 | 03 | 20 | 945 | 055 | 35 | 831 | 169 |
| 6 | 95 | 05 | 21 | 939 | 061 | 36 | 821 | 179 |
| 7 | 93 | 07 | 22 | 933 | 067 | 37 | 811 | 189 |
| 8 | 91 | 09 | 23 | 927 | 073 | 38 | 800 | 200 |
| 9 | 89 | 11 | 24 | 921 | 079 | 39 | 789 | 211 |
| 10 | 86 | 14 | 25 | 914 | 086 | 40 | 778 | 222 |
| 11 | 83 | 17 | 26 | 907 | 093 | 41 | 767 | 233 |
| 12 | 80 | 20 | 27 | 899 | 101 | 42 | 756 | 244 |
| 13 | 77 | 23 | 28 | 892 | 108 | 43 | 744 | 256 |
| 14 | 73 | 27 | 29 | 884 | 116 | 44 | 732 | 268 |
| 15 | 9.99969 | 0.00031 | 30 | 9.99876 | 0.00124 | 45 | 9.99719 | 0.00281 |

### *Correction for rate of chronometer.*

A correction for rate of chronometer must be applied to the mean of the threads; it is done as follows: Let T be an assumed sidereal time (it is convenient to have it an exact hour) near the middle of the interval of observing the group of stars, $r_h$ the hourly rate of the chronometer, which is known approximately, and $\alpha$ the right ascension of the star observed; then—

Correction for rate $= (\alpha - \mathrm{T})r_h$, the quantity $\alpha - \mathrm{T}$ being expressed in hours.

Rate is $\left\{\begin{matrix}+\\-\end{matrix}\right\}$ when chronometer is $\left\{\begin{matrix}\text{losing.}\\\text{gaining.}\end{matrix}\right\}$

### *Correction for inclination constant.*

Let $w$ $e$ the west and east readings of the level,

$w'$ $e'$ " " " " " when reversed,

$d$ the value of one division of levels, expressed in seconds of arc,

$b$ the level error, or the inclination of the axis of the instrument; it will be $\left\{\begin{matrix}+\\-\end{matrix}\right\}$ when $\left\{\begin{matrix}\text{west}\\\text{east}\end{matrix}\right\}$ is too high.

$$b = \tfrac{1}{4}\left\{(w + w') - (e + e')\right\}\frac{d}{15} \text{ or} = \left\{(w + w') - (e + e')\right\}\frac{d}{60}$$

and correction for level error, in seconds of time, $b \cos(\varphi - \delta) \sec \delta = b\mathrm{B}$,

$\delta$, when north, is $\left\{\begin{matrix}+\\-\end{matrix}\right\}$ for $\left\{\begin{matrix}\text{upper}\\\text{lower}\end{matrix}\right\}$ culmination; when south, it is negative.

For a star reflected in mercury, B changes sign. The factor B is tabulated for various values of $\delta$ and zenith distances $\zeta = \left\{\begin{matrix}+\\-\end{matrix}\right\}(\varphi - \delta)$ for a star $\left\{\begin{matrix}\text{south}\\\text{north}\end{matrix}\right\}$ of the zenith.

### *Correction for inequality of pivots.*

The correction for inequality of pivots, supposing them circular, applies directly to the level constant. If the same V gives level reading too great (is high,) both before and after reversal of instrument, half the difference of the level correction is the effect due to the difference of diameters of the pivots; but if the east V shows high $\left\{\begin{matrix}\text{before}\\\text{after}\end{matrix}\right\}$ and the west V high $\left\{\begin{matrix}\text{after}\\\text{before}\end{matrix}\right\}$ reversal, half the sum of the level corrections gives the effect. The half of the effect is the correction to the level constant for inequality of pivots, (the transit axis passing through the centres of the pivots,) $\left\{\begin{matrix}-\\+\end{matrix}\right\}$ to $\left\{\begin{matrix}\text{large}\\\text{small}\end{matrix}\right\}$ pivot.

### *Correction for collimation of constant.*

Let $c$=error of collimation in seconds of time.

At upper culmination $c$ is $\left\{\begin{matrix}+\\-\end{matrix}\right\}$ when mean of threads is $\left\{\begin{matrix}\text{east}\\\text{west}\end{matrix}\right\}$ of line of collimation.

At lower culmination $c$ is $\left\{\begin{matrix} - \\ + \end{matrix}\right\}$ when mean of threads is $\left\{\begin{matrix} \text{east} \\ \text{west} \end{matrix}\right\}$ of line of collimation.

Correction for error of collimation $= \pm c \sec \delta$. $\left\{\begin{matrix} + \\ - \end{matrix}\right\}$ for $\left\{\begin{matrix} \text{upper} \\ \text{lower} \end{matrix}\right\}$ culmination.

$= \pm c\,C$, where $C =$ collimation factor, for which see foot of the table.

To find the error of collimation of the telescope by means of a close circumpolar star: Note the transit of the star over the first series of threads, including or excluding the middle thread; then reverse the axis and note the transit over the same threads, now in the reverse order. Find the time of transit over the mean of all the threads, both before and after reversal by the method already explained, and correct for rate, inclination, and inequality of pivots, if necessary. The state of the level should be noted for each thread.

Let $t =$ time of transit before reversal and $t'$ after reversal, then for

upper culmination $c = \frac{1}{2}\,(t' - t) \cos \delta$ for position of axis before reversal.
lower " $c = \frac{1}{2}\,(t - t') \cos \delta$ " " " "

*Correction for deviation constant.*

Let $a =$ the azimuthal error in seconds of time.

$a$ is $\left\{\begin{matrix} + \\ - \end{matrix}\right\}$ when plane of telescope is $\left\{\begin{matrix} \text{east} \\ \text{west} \end{matrix}\right\}$ of south.

Correction for azimuthal deviation $c \sin (\varphi - \delta) \sec \delta = a\,A$.

$\delta$, when north, is $\left\{\begin{matrix} + \\ - \end{matrix}\right\}$ for $\left\{\begin{matrix} \text{upper} \\ \text{lower} \end{matrix}\right\}$ culmination; when south, it is negative.

The factor A is tabulated for various values for $\delta$ and $\zeta$.

To find the azimuthal deviation from the transit of two stars, which differ considerably in declination: Let the observed times of transit be corrected for rate, inclination, inequality of pivots, and collimation error, then

$$a = \frac{(a' - a) - (t' - t)}{A' - A}$$

where $a\ t\ A$ and $a'\ t'\ A'$ the apparent right ascension, time of transit, and azimuth factor, respectively for the preceding and following star. It will be seen that A is $\left\{\begin{matrix} + \\ - \end{matrix}\right\}$ when the star culminates $\left\{\begin{matrix} \text{south} \\ \text{north} \end{matrix}\right\}$ of the zenith, and at the lower culmination it is also positive.

For lower culminations the star's right ascension is to be increased by $12^h$. It is desirable that the low star should differ over $50^\circ$ in declination from the high star, and if two close circumpolar stars are observed, their right ascensions should differ 12 hours.

*Correction for diurnal aberration.*

When great precision is required apply to the star's apparent right ascension the effect of the diurnal aberration $= \left\{\begin{matrix} + \\ - \end{matrix}\right\} 0^s.021 \cos \varphi \sec \delta$ for $\left\{\begin{matrix} \text{upper} \\ \text{lower} \end{matrix}\right\}$ culmination.

*Personal equation.*

Let the transits of a star over the first series of threads, including or excluding the middle thread, be noted by one observer, and the remaining transits by the second observer; reduce the observations of each to the mean thread by aid of the known equatorial intervals, the difference in the results will be the personal equation. A number of stars may be observed in this manner, with the observers leading alternately, and the mean of all results must finally be taken. It is desirable that not less than 20 stars be observed.

*Chronometer correction.*

The corrections to the observed time $t$ of the transit of a star, for instrumental deviations, using Mayer's formula as above, becomes $a\,A + b\,B + c\,C$ and consequently the chronometer correction (on local sidereal time) $\Delta t = a - (t + a\,A + b\,B + c\,C.)$

(6.) *Reduction of transit observations by application of the method of least squares.*

Let $\Delta T_0$ = the chronometer correction $\left\{ \begin{matrix} + \\ - \end{matrix} \right\}$ when $\left\{ \begin{matrix} \text{slow} \\ \text{fast} \end{matrix} \right\}$ at an assumed middle time $T_0$

$t_1\ t_2\ t_3 \ldots$ = the observed times of transit of a number of stars forming a group, corrected for rate, inclination, and inequality of pivots.

$\alpha_1\ \alpha_2\ \alpha_3 \ldots$ = their right ascensions.

$A_1\ A_2\ A_3 \ldots$ = their azimuthal factors.

$C_1\ C_2\ C_3 \ldots$ = their collimation factors; let also

$$\alpha_1 - t_1 = \tau_1$$
$$\alpha_2 - t_2 = \tau_2$$
$$\alpha_3 - t_3 = \tau_3$$
&c.,

and $\Delta T_0 = \Delta T + \delta T$ where $\delta T$ is an unknown correction to an assumed chronometer correction $\Delta T$; also let

$$\tau_1 - \Delta T = d_1$$
$$\tau_2 - \Delta T = d_2$$
$$\tau_3 - \Delta T = d_3$$
&c.,

the values of $a$ $c$ and $\delta T$ are then to be found from the conditional equations, from which we form the normal equations:

$$\Sigma \delta T + \Sigma a A + \Sigma c C = \Sigma d$$
$$\Sigma \delta T.A + \Sigma a A^2 + \Sigma c C.A = \Sigma d A$$
$$\Sigma \delta T.C + \Sigma a A.C + \Sigma c C^2 = \Sigma d C$$

It is essential for the proper use of these formulæ that the instrumental deviations should not have changed during the interval of observation of the group. If the equations are specially used for the determination of the instrumental constants, the stars entering into the group should differ widely in declination and the axis should be reversed near the middle of the group; stars observed at their lower culmination answer the same purpose as a reversal, and should, if possible, be included.

Stars observed near the zenith, on both sides of it, and with the axis in a direct and reversed position, answer best when the chronometer correction is sought.

If the collimation error is already known, and the observed times are corrected for it, $a$ and $\delta T$ require to be found; in this case the above equations become

$$\Sigma \delta T + \Sigma a A = \Sigma d$$
$$\Sigma \delta T A + \Sigma a A^2 = \Sigma d A$$

*Probable error of transit observations.*

According to Chauvenet, the weight $w$ of a given observation with respect to the number of threads observed may be represented by $w = \frac{n(N+3)}{N(n+3)}$ where N = the total number of threads and $n$ = number of threads observed. The following table contains the values of $w$ for the two cases of 5 and 7 threads in the diaphragm:

| $n$. | $w$. | $w$. |
|---|---|---|
| 1 | 0.40 | 0.36 |
| 2 | 0.64 | 0.57 |
| 3 | 0.80 | 0.71 |
| 4 | 0.92 | 0.82 |
| 5 | 1.00 | 0.90 |
| 6 | ........ | 0.95 |
| 7 | ........ | 1.00 |

Weights are introduced by multiplying the respective conditional equations with their proper value of $\sqrt{w}$ before we proceed to the formation of the normal equations.

The probable error of the deduced chronometer correction is found as follows: Substitute the values found for the unknown quantities in the conditional equations and form the residuals $[r]$ and the sum of their squares $[rr]$. If $m =$ the number of conditional and $\mu$ the number of normal equations, compute the probable error $\varepsilon$ by the formula

$$\varepsilon = 0.67\sqrt{\frac{[rr]}{m-\mu}}$$

Next find the weight $p$ of the quantity $\delta\,\mathrm{T}$ by aid of the normal equations, as explained by the method of least squares, the probable error $\varepsilon_0$ of the chronometer correction $\Delta\,\mathrm{T}_0$ will be given by

$$\varepsilon_0 = \frac{\varepsilon}{\sqrt{p}}$$

There is, however, a limit, which is soon reached, below which the probable error cannot be reduced by the use of any number of stars, it is perhaps not far from $\pm\,0^s.05$.

If we simply make use of the residuals $[r]$ of the mean chronometer correction, and that deduced from each individual star, the probable error $\varepsilon_0$ of the chronometer correction $\Delta\,\mathrm{T}_0$ may be found by

$$\varepsilon_0 = 0.67\sqrt{\frac{[w.rr]}{(m-\mu)[w]}}$$ where $m =$ the number of stars, and $\mu$ the number of instrumental constants determined. The value $\varepsilon_0$ is limited as above.

The factors $\mathrm{A} = \sin\,(\varphi - \delta)\,\sec\delta$

$\mathrm{B} = \cos\,(\varphi - \delta)\,\sec\delta$

$\mathrm{C} = \sec\delta$

are given in the table.

## (7.) EXAMPLE.

TELEGRAPH HILL, SAN FRANCISCO, CAL., *June* 19, 1853.

$\phi = 37°\ 48'.0$. $\lambda = 122°\ 23'.3$ west of Greenwich. Transit No. 7. Sidereal chronometer No. 5038.

G. D., *Observer.*

| | | | | | | |
|---|---|---|---|---|---|---|
| Object | α Virginis, | η Urs. Maj., | η Bootis, | 1125 T. Y. C., | 1131 T. Y. C., | α Bootis, |
| Appearance | dif. m.—st. | m.—st. | m.—st. thr. cl'ds | m.—st. high wind | m.—st. high wind | m.—st. flam. |
| Declination | − 10° 24′ | + 50° 03′ | + 19° 18′ | + 65° 07′ | − 9° 33′ | + 19° 57′ |
| Position, clamp | W. | W. | W. | E. | E. | E. |
| Level | 22.2 26.6 | 20.7 29.6 | .......... | 26.1 26.3 | .......... | 23.7 29.3 |
| E. and W | 21.4 28.0 | 22.6 28.4 | .......... | 23.7 29.3 | .......... | 26.3 27.0 |
| | *h. m. s.* | *h. m. s.* | *h. m. s.* | *h. m. s.* | *h. m. s.* | *h. m. s.* |
| Threads I | .......... | 39 55.6 | 46 23.6 | 13 62 43.1 | 14 05 56.4 | 14 09 53.2 |
| II | .......... | 40 28.2 | 46 45.6 | 61 53.7 | 05 35.4 | 09 31.2 |
| III | .......... | 41 01.7 | 47 08.3 | 61 02.5 | 05 13.6 | 09 08.2 |
| IV | 17 16.6 | 41 33.5 | 47 30.2 | 60 14.1 | 04 52.7 | 08 46.5 |
| V | 17 37.8 | 42 06.4 | 47 52.2 | 59 23.7 | 04 31.3 | 08 23.7 |
| VI | 17 58.7 | 42 38.2 | 48 14.2 | 58 34.4 | 04 10.2 | 08 02.1 |
| VII | 13 18 20.3 | 13 43 11.2 | 13 48 36.2 | 57 44.7 | 03 49.3 | .......... |
| Mean | 13 17 16.49 | 13 41 33.54 | 13 47 30.04 | 14 00 13.74 | 14 04 52.70 | 14 08 46.38 |
| Corr'n. for rate | − 0.01 | − 0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| inclination | + 0.18 | + 0.54 | + 0.37 | + 0.30 | + 0.10 | + 0.15 |
| collimation | − 0.17 | − 0.27 | − 0.18 | + 0.40 | + 0.17 | + 0.18 |
| deviation | − 0.27 | + 0.12 | − 0.12 | + 0.38 | − 0.26 | − 0.12 |
| Reduced transit | 13 17 16.22 | 13 41 33.92 | 13 47 30.11 | 14 00 14.82 | 14 04 52.71 | 14 08 46.59 |
| Tabular right ascension | 13 17 28.05 | 13 41 45.89 | 13 47 42.15 | 14 00 26.76 | 14 05 04.75 | 14 08 58.54 |
| Correction of chronometer | + 0 00 11.83 | + 0 00 11.97 | + 0 00 12.04 | + 0 00 11.94 | + 0 00 12.04 | + 0 00 11.95 |

Interval of threads, clamp W.:

| | | |
|---|---|---|
| | s. | |
| I. | —62.80 | One division of level scale $0^s.0955$. |
| II. | —42.05 | Approximate hourly rate of chronometer $+0^s.02$. |
| III. | —20.51 | |
| IV. | + 0.01 | |
| V. | + 21.04 | |
| VI. | + 41.72 | |
| VII. | + 62.62 | |

The reduction of the above transits may be proceeded with as follows:

Reduction of imperfect transit of $\alpha$ Virginis:

Sum of equal intervals of missed threads $-125^s.36$

| | | |
|---|---|---|
| log sum | $2.09816n$ | |
| log sec $\delta$ | $0.00719$ | |
| co-log 4 | $9.39794$ | |
| | $1.50329n$ | $-31^s.86$ |
| Mean of observed threads, | | $13^h\ 17^m\ 48\ .35$ |
| Time over mean thread, | | 13 17 16 .49 |

Reduction of imperfect transit of $\alpha$ Bootis:

| | | |
|---|---|---|
| log interval | $1.79671n$ | |
| log sec $\delta$ | $0.02688$ | |
| co-log 6 | $9.22185$ | |
| | $1.04544n$ | $-11^s.10$ |
| Mean of observed threads, | | $14^h\ 08^m\ 57\ .48$ |
| Time over mean thread, | | 14 08 46 .38 |

Correction for inclination:

For $\alpha$ Virginis $b=+0.26$ from the table B=0.68, correction $b\mathrm{B}=+0^s.18$ &c. For $\eta$ Bootis, $b$ interpolated $=+0.37$; and for 1131 T. Y. C. $b=+0.15$.

Azimuthal deviation from a high and low star:

For the stars 1125 and 1131 T. Y. C. we have—

$\alpha'-\alpha=4^m\ 37^s.99$ and $\mathrm{A}'=+0.74$ $\mathrm{A}=-1.08$ by the table.

$t'-t=4\ \ 38\ .76$ supposing for the present the collimation zero,

$a\ \ =-\ 0\ .77:\ 1.82=-0^s.42$. (The value of $a$ will be finally determined by means of the method of least squares.)

For this method we assume $\mathrm{T}_o=14^h$, also $\Delta\mathrm{T}=+0^h\ 0^m\ 12^s$;

we next form $\tau_1=+11^s.39$ &c.......... $\tau_6=+12^s.01$

and $d_1=-\ 0\ .61$ &c.......... $d_6=+\ 0\ .01$

The values of A and C are taken from the table; with respect to the sign of C it has been assumed + for clamp E. and upper culmination.

If we give equal weight to each star the following arrangement will be convenient:

| Star. | $\tau$ | $d$ | A | C | $d$A | $d$C | A² | AC | C² |
|---|---|---|---|---|---|---|---|---|---|
| | h. m. s. | | | | | | | | |
| $\alpha$ Virginis | +0 00 11.39 | —0.61 | +0.76 | —1.02 | —0.46 | +0.62 | 0.58 | —0.78 | 1.04 |
| $\eta$ U. Majoris | 11.82 | —0.18 | —0.33 | —1.56 | +0.06 | +0.28 | 0.11 | +0.51 | 2.43 |
| $\eta$ Bootis | 11.74 | —0.26 | +0.34 | —1.06 | —0.09 | +0.28 | 0.12 | —0.36 | 1.12 |
| 1125 T. Y. C. | 11.72 | +0.72 | —1.08 | +2.38 | —0.78 | +1.71 | 1.17 | —2.57 | 5.66 |
| 1131 T. Y. C. | 11.95 | —0.05 | +0.74 | +1.02 | —0.04 | —0.05 | 0.55 | +0.75 | 1.04 |
| $\alpha$ Bootis | 12.01 | +0.01 | +0.33 | +1.06 | 0.00 | +0.01 | 0.11 | +0.35 | 1.12 |
| | Σ= | —0.37 | +0.76 | +0.82 | —1.31 | +2.85 | 2.64 | —2.10 | 12.41 |

$$6\ \delta T + 0.76\,a + 0.82\,c = -0.37$$
$$+0.76\,\delta T + 2.64\,a - 2.10\,c = -1.31$$
$$+0.82\,\delta T - 2.10\,a + 12.41\,c = +2.85$$

which normal equations give $a = -0^s.35$ $c = +0^s.17$ $\delta T = -0^s.04$, hence, $\Delta T_0 = +0^h\ 0^m\ 11^s.96$.

If we introduce weights on account of the imperfect transits, that of the first star is $w = 0.82$ and of the last 0.95 and the conditional equations of the form.

$\delta T + aA + cC = d$ require to be multiplied by $\sqrt{w}$, but it is more convenient first to form the products as usual, and before summing to multiply by $w$; our normal equations then change to

$$5.77\ \delta T + 0.61\,a + 0.95\,c = -0.26$$
$$+0.61\ \delta T + 2.53\,a - 1.97\,c = -1.23$$
$$+0.95\ \delta T - 1.97\,a + 12.17\,c = +2.73$$

which give $a = -0^s.35$ $c = +0^s.17$ and $\delta T = -0^s.04$. In this case the introduction of weights affects only the third place of decimals in the results.

If we introduce the corrections $aA$ and $cC$ and work out the chronometer corrections for each star, we find the quantity $v$ as follows: $-0^s.13 + 0^s.01 + 0^s.08 - 0^s.02 + 0^s.08$ and $-0^s.01$. Assuming an equal weight for each star, or $w = 1$, we find $\varepsilon_0 = \pm\ 0^s.03$.

The following observations at Telegraph Hill, October 26, 1863, instruments and observer as before, furnish an example of the computation of the collimation constant for transits of a circumpolar star.

867 T. Y. C. at Lower Culmination.
$\delta = 85°\ 00'$

| West. | | East. | |
|---|---|---|---|
| 26.5 | 32.8 | 27.2 | 31.0 |
| 26.5 | 32.7 | 27.6 | 30.7 |

| *h. m. s.* | | *h. m. s.* |
|---|---|---|
| 22 04 25.7 | IV | — — |
| 00 24.8 | V | 08 27.6 |
| — — — | VI | — — |
| 52 26.8 | VII | 22 16 22.2 |

We have $\Delta T_0 = +0^h\ 03^m\ 07^s.9$, also $b = +0.30$ and $+0.17$ for W. and E., and $B = -6.22$, hence correction $b\,B$ to times, clamp W. and E. $-1^s.9$ and $-1^s.1$. We now refer each thread to the mean thread: For thread VII—

| | *h.* | *m.* | | | |
|---|---|---|---|---|---|
| Observed time | 21 | 52.4 | log $\sqrt[3]{sec.\,\tau}$ | 0.0002 | |
| $\Delta T_0$ | + 0 | 03.1 | log $i_{vii}$ | 1.7967 | |
| Observed sid. time | 21 | 55.5 | log $sec\,\delta$ | 1.0597 | |
| $a + 12^h$ | 22 | 07.6 | | 2.8566 | $718^s.8$ |
| $\tau$ | | 12.1 | | | |

| | *h. m. s.* | *h. m. s.* |
|---|---|---|
| Observed time over VII corrected for inclination | 21 52 24.9 | 22 16 21.1 |
| Reduction to mean thread | + 11 58.8 | — 11 58.8 |
| Transit over mean thread | 22 04 23.7 | 22 04 22.3 |
| From thread V we find similarly | 24.3 | 25.1 |
| From thread IV we find similarly | 23.9 | |
| | $t$ = 22 04 24.0 | $t'$ = 22 04 23.7 |

| | |
|---|---|
| log $(t-t')$ | 9.477 |
| log cos $\delta$ | 8.940 |
| co-log 2 | 9.699 |
| log $c$ | 8.116 |

$c = +0^s.013$ for clamp W.

UNITED STATES COAST SURVEY.—TABLE OF FACTORS FOR REDUCTION OF TRANSIT OBSERVATIONS.

Azimuth factor $A = \sin \zeta \sec \delta$. Star's declination $\pm \delta$. Inclination factor $B = \cos \zeta \sec \delta$.

| ζ | 0° | 10° | 15° | 20° | 22° | 24° | 26° | 28° | 30° | 32° | 34° | 36° | 38° | 40° | 41° | 42° | 43° | 44° | 45° | 46° | 47° | 48° | 49° | 50° | ζ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1° | .02 | .02 | .02 | .02 | .02 | .02 | .02 | .02 | .02 | .02 | .02 | .02 | .02 | .02 | .02 | 02 | .02 | .02 | .02 | .02 | .03 | .03 | .03 | .03 | 89° |
| 2 | .04 | .04 | .04 | .04 | .04 | .04 | .04 | .04 | .04 | .04 | .04 | .04 | .04 | .05 | .05 | .05 | .05 | .05 | .05 | .05 | .05 | .05 | .05 | .05 | 88 |
| 3 | .05 | .05 | .05 | .06 | .06 | .06 | .06 | .06 | .06 | .06 | .06 | .06 | .07 | .07 | .07 | .07 | .07 | .07 | .07 | .07 | .08 | .08 | .08 | .08 | 87 |
| 4 | .07 | .07 | .07 | .07 | .08 | .08 | .08 | .08 | .08 | .08 | .08 | .09 | .09 | .09 | .09 | .09 | .10 | .10 | .10 | .10 | .10 | .10 | .11 | .11 | 86 |
| 5 | .09 | .09 | .09 | .09 | .09 | .10 | .10 | .10 | .10 | .10 | .10 | .11 | .11 | .11 | .11 | .12 | .12 | .12 | .12 | .13 | .13 | .13 | .13 | .13 | 85 |
| 6 | .11 | .11 | .11 | .11 | .11 | .11 | .12 | .12 | .12 | .12 | .13 | .13 | .13 | .14 | .14 | .14 | .14 | .15 | .15 | .15 | .15 | .16 | .16 | .16 | 84 |
| 7 | .12 | .12 | .13 | .13 | .13 | .13 | .14 | .14 | .14 | .14 | .15 | .15 | .15 | .16 | .16 | .16 | .17 | .17 | .17 | .18 | .18 | .18 | .19 | .19 | 83 |
| 8 | .14 | .14 | .14 | .15 | .15 | .15 | .16 | .16 | .16 | .16 | .17 | .17 | .18 | .18 | .18 | .19 | .19 | .19 | .20 | .20 | .20 | .21 | .21 | .22 | 82 |
| 9 | .16 | .16 | .16 | .17 | .17 | .17 | .17 | .18 | .18 | .18 | .19 | .19 | .20 | .20 | .21 | .21 | .21 | .22 | .22 | .22 | .23 | .23 | .24 | .24 | 81 |
| **10** | .17 | .18 | .18 | .19 | .19 | .19 | .19 | .20 | .20 | .21 | .21 | .21 | .22 | .23 | .23 | .23 | .24 | .24 | .25 | .25 | .25 | .26 | .26 | .27 | **80** |
| 11 | .19 | .19 | .20 | .20 | .21 | .21 | .21 | .22 | .22 | .23 | .23 | .24 | .24 | .25 | .25 | .26 | .26 | .27 | .27 | .28 | .28 | .28 | .29 | .30 | 79 |
| 12 | .21 | .21 | .22 | .22 | .22 | .23 | .23 | .24 | .24 | .25 | .25 | .26 | .26 | .27 | .27 | .28 | .28 | .29 | .29 | .30 | .30 | .31 | .32 | .32 | 78 |
| 13 | .22 | .23 | .23 | .24 | .24 | .25 | .25 | .26 | .26 | .27 | .27 | .28 | .29 | .29 | .30 | .30 | .31 | .31 | .32 | .32 | .33 | .34 | .34 | .35 | 77 |
| 14 | .24 | .25 | .25 | .26 | .26 | .27 | .27 | .27 | .28 | .29 | .29 | .30 | .31 | .32 | .32 | .33 | .33 | .34 | .34 | .35 | .35 | .36 | .37 | .38 | 76 |
| 15 | .26 | .26 | .27 | .28 | .28 | .28 | .29 | .29 | .30 | .31 | .31 | .32 | .33 | .34 | .34 | .35 | .35 | .36 | .37 | .37 | .38 | .39 | .39 | .40 | 75 |
| 16 | .28 | .28 | .29 | .29 | .30 | .30 | .31 | .31 | .32 | .33 | .33 | .34 | .35 | .36 | .37 | .37 | .38 | .38 | .39 | .40 | .40 | .41 | .42 | .43 | 74 |
| 17 | .29 | .30 | .30 | .31 | .31 | .32 | .33 | .33 | .34 | .34 | .35 | .36 | .37 | .38 | .39 | .39 | .40 | .41 | .41 | .42 | .43 | .44 | .45 | .45 | 73 |
| 18 | .31 | .31 | .32 | .33 | .33 | .33 | .34 | .35 | .36 | .36 | .37 | .38 | .39 | .40 | .41 | .42 | .42 | .43 | .44 | .44 | .45 | .46 | .47 | .48 | 72 |
| 19 | .33 | .33 | .34 | .35 | .35 | .36 | .36 | .37 | .38 | .38 | .39 | .40 | .41 | .42 | .43 | .44 | .45 | .45 | .46 | .47 | .48 | .49 | .50 | .51 | 71 |
| **20** | .34 | .35 | .35 | .36 | .37 | .37 | .38 | .39 | .40 | .40 | .41 | .42 | .43 | .45 | .45 | .46 | .47 | .48 | .48 | .49 | .50 | .51 | .52 | .53 | **70** |
| 21 | .36 | .36 | .37 | .38 | .39 | .39 | .40 | .41 | .41 | .42 | .43 | .44 | .45 | .47 | .47 | .48 | .49 | .50 | .51 | .52 | .52 | .54 | .55 | .56 | 69 |
| 22 | .37 | .38 | .39 | .40 | .40 | .41 | .42 | .42 | .43 | .44 | .45 | .46 | .48 | .49 | .50 | .50 | .51 | .52 | .53 | .54 | .55 | .56 | .57 | .58 | 68 |
| 23 | .39 | .40 | .41 | .42 | .42 | .43 | .44 | .44 | .45 | .46 | .47 | .48 | .50 | .51 | .52 | .53 | .53 | .54 | .55 | .56 | .57 | .58 | .60 | .61 | 67 |
| 24 | .41 | .41 | .42 | .43 | .44 | .45 | .45 | .46 | .47 | .48 | .49 | .50 | .52 | .53 | .54 | .55 | .56 | .57 | .58 | .59 | .60 | .61 | .62 | .63 | 66 |
| 25 | .42 | .43 | .44 | .45 | .46 | .46 | .47 | .48 | .49 | .50 | .51 | .52 | .54 | .55 | .56 | .57 | .58 | .59 | .60 | .61 | .62 | .63 | .64 | .66 | 65 |
| 26 | .44 | .45 | .45 | .47 | .47 | .48 | .49 | .50 | .51 | .52 | .53 | .54 | .56 | .57 | .58 | .59 | .60 | .61 | .62 | .63 | .64 | .65 | .67 | .68 | 64 |
| 27 | .45 | .46 | .47 | .48 | .49 | .50 | .51 | .51 | .52 | .54 | .55 | .56 | .58 | .59 | .60 | .61 | .62 | .63 | .64 | .65 | .67 | .68 | .69 | .71 | 63 |
| 28 | .47 | .48 | .49 | .50 | .51 | .51 | .52 | .53 | .54 | .55 | .57 | .58 | .60 | .61 | .62 | .63 | .64 | .65 | .66 | .68 | .69 | .70 | .72 | .73 | 62 |
| 29 | .48 | .49 | .50 | .52 | .52 | .53 | .54 | .55 | .56 | .57 | .58 | .60 | .61 | .63 | .64 | .65 | .66 | .67 | .69 | .70 | .71 | .72 | .74 | .75 | 61 |
| **30** | .50 | .51 | .52 | .53 | .54 | .55 | .56 | .57 | .58 | .59 | .60 | .62 | .63 | .65 | .66 | .67 | .68 | .69 | .71 | .72 | .73 | .75 | .76 | .78 | **60** |
| 31 | .52 | .52 | .53 | .55 | .56 | .56 | .57 | .58 | .59 | .61 | .62 | .64 | .65 | .67 | .68 | .69 | .70 | .72 | .73 | .74 | .75 | .77 | .78 | .80 | 59 |
| 32 | .53 | .54 | .55 | .56 | .57 | .58 | .59 | .60 | .61 | .63 | .64 | .65 | .67 | .69 | .70 | .71 | .72 | .74 | .75 | .76 | .78 | .79 | .81 | .82 | 58 |
| 33 | .54 | .55 | .56 | .58 | .59 | .60 | .61 | .62 | .63 | .64 | .66 | .67 | .69 | .71 | .72 | .73 | .74 | .76 | .77 | .78 | .80 | .81 | .83 | .85 | 57 |
| 34 | .56 | .57 | .58 | .59 | .60 | .61 | .62 | .63 | .65 | .66 | .67 | .69 | .71 | .73 | .74 | .75 | .76 | .78 | .79 | .80 | .82 | .84 | .85 | .87 | 56 |
| 35 | .57 | .58 | .59 | .61 | .62 | .63 | .64 | .65 | .66 | .68 | .69 | .71 | .73 | .75 | .76 | .77 | .78 | .80 | .81 | .83 | .84 | .86 | .87 | .89 | 55 |
| 36 | .59 | .60 | .61 | .63 | .63 | .64 | .65 | .67 | .68 | .69 | .71 | .73 | .75 | .77 | .78 | .79 | .80 | .82 | .83 | .85 | .86 | .88 | .90 | .91 | 54 |
| 37 | .60 | .61 | .62 | .64 | .65 | .62 | .67 | .68 | .70 | .71 | .73 | .74 | .76 | .79 | .80 | .81 | .82 | .84 | .85 | .87 | .88 | .90 | .92 | .94 | 53 |
| 38 | .62 | .63 | .64 | .66 | .66 | .67 | .69 | .70 | .71 | .73 | .74 | .76 | .78 | .80 | .82 | .83 | .84 | .86 | .87 | .89 | .90 | .92 | .94 | .96 | 52 |
| 39 | .63 | .64 | .65 | .67 | .68 | .69 | .70 | .71 | .73 | .74 | .76 | .78 | .80 | .82 | .83 | .85 | .86 | .87 | .89 | .91 | .92 | .94 | .96 | .98 | 51 |
| **40** | .64 | .65 | .67 | .68 | .69 | .70 | .72 | .73 | .74 | .76 | .77 | .79 | .82 | .84 | .85 | .86 | .88 | .89 | .91 | .93 | .94 | .96 | .98 | 1.00 | **50** |
| 41 | .66 | .67 | .68 | .70 | .71 | .72 | .73 | .74 | .76 | .77 | .79 | .81 | .83 | .86 | .87 | .88 | .90 | .91 | .93 | .94 | .96 | .98 | 1.00 | 1.02 | 49 |
| 42 | .67 | .68 | .69 | .71 | .72 | .73 | .74 | .76 | .77 | .79 | .81 | .83 | .85 | .87 | .89 | .90 | .91 | .93 | .95 | .96 | .98 | 1.00 | 1.02 | 1.04 | 48 |
| 43 | .68 | .69 | .71 | .73 | .74 | .75 | .76 | .77 | .79 | .80 | .82 | .84 | .86 | .89 | .90 | .92 | .93 | .95 | .96 | .98 | 1.00 | 1.02 | 1.04 | 1.06 | 47 |
| 44 | .69 | .71 | .72 | .74 | .75 | .76 | .77 | .79 | .80 | .82 | .84 | .86 | .89 | .90 | .92 | .93 | .95 | .96 | .98 | 1.00 | 1.02 | 1.04 | 1.06 | 1.08 | 46 |
| 45 | .71 | .72 | .73 | .75 | .76 | .77 | .79 | .80 | .82 | .83 | .85 | .87 | .90 | .92 | .94 | .95 | .97 | .98 | 1.00 | 1.02 | 1.04 | 1.06 | 1.08 | 1.10 | 45 |

| ζ | 0° | 10° | 15° | 20° | 22° | 24° | 26° | 28° | 30° | 32° | 34° | 36° | 38° | 40° | 41° | 42° | 43° | 44° | 45° | 46° | 47° | 48° | 49° | 50° | ζ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 46° | .72 | .73 | .74 | .77 | .78 | .79 | .80 | .82 | .83 | .85 | .87 | .89 | .91 | .94 | .95 | .97 | .98 | 1.00 | 1.02 | 1.04 | 1.05 | 1.07 | 1.10 | 1.12 | 44° |
| 47 | .73 | .74 | .76 | .78 | .79 | .80 | .81 | .83 | .84 | .86 | .88 | .90 | .93 | .95 | .97 | .98 | 1.00 | 1.02 | 1.03 | 1.05 | 1.07 | 1.09 | 1.11 | 1.14 | 43 |
| 48 | .74 | .76 | .77 | .79 | .80 | .81 | .83 | .84 | .86 | .88 | .90 | .92 | .94 | .97 | .98 | 1.00 | 1.02 | 1.03 | 1.05 | 1.07 | 1.09 | 1.11 | 1.13 | 1.16 | 42 |
| 49 | .75 | .77 | .78 | .80 | .81 | .83 | .84 | .86 | .87 | .89 | .91 | .93 | .96 | .99 | 1.00 | 1.02 | 1.03 | 1.05 | 1.07 | 1.09 | 1.11 | 1.13 | 1.15 | 1.17 | 41 |
| **50** | .77 | .78 | .79 | .82 | .83 | .84 | .85 | .87 | .89 | .90 | .92 | .95 | .97 | 1.00 | 1.01 | 1.03 | 1.05 | 1.06 | 1.08 | 1.10 | 1.12 | 1.14 | 1.17 | 1.19 | **40** |
| 51 | .78 | .79 | .80 | .83 | .84 | .85 | .87 | .88 | .90 | .92 | .94 | .96 | .99 | 1.01 | 1.03 | 1.05 | 1.06 | 1.08 | 1.10 | 1.12 | 1.14 | 1.16 | 1.18 | 1.21 | 39 |
| 52 | .79 | .80 | .82 | .84 | .85 | .86 | .88 | .89 | .91 | .93 | .95 | .97 | 1.00 | 1.03 | 1.04 | 1.06 | 1.08 | 1.10 | 1.11 | 1.13 | 1.15 | 1.18 | 1.20 | 1.23 | 38 |
| 53 | .80 | .81 | .83 | .85 | .86 | .87 | .89 | .91 | .92 | .94 | .96 | .99 | 1.01 | 1.04 | 1.06 | 1.07 | 1.09 | 1.11 | 1.13 | 1.15 | 1.17 | 1.19 | 1.22 | 1.24 | 37 |
| 54 | .81 | .82 | .84 | .86 | .87 | .89 | .90 | .92 | .93 | .95 | .98 | 1.00 | 1.03 | 1.06 | 1.07 | 1.09 | 1.11 | 1.12 | 1.14 | 1.16 | 1.19 | 1.21 | 1.23 | 1.26 | 36 |
| 55 | .82 | .83 | .85 | .87 | .88 | .90 | .91 | .93 | .95 | .97 | .99 | 1.01 | 1.04 | 1.07 | 1.08 | 1.10 | 1.12 | 1.14 | 1.16 | 1.18 | 1.20 | 1.22 | 1.25 | 1.27 | 35 |
| 56 | .83 | .84 | .86 | .88 | .89 | .91 | .92 | .94 | .96 | .98 | 1.00 | 1.02 | 1.05 | 1.08 | 1.10 | 1.12 | 1.13 | 1.15 | 1.17 | 1.19 | 1.22 | 1.24 | 1.26 | 1.29 | 34 |
| 57 | .84 | .85 | .87 | .89 | .90 | .92 | .93 | .95 | .97 | .99 | 1.01 | 1.04 | 1.06 | 1.09 | 1.11 | 1.13 | 1.15 | 1.17 | 1.19 | 1.21 | 1.23 | 1.25 | 1.28 | 1.31 | 33 |
| 58 | .85 | .86 | .88 | .90 | .91 | .93 | .94 | .91 | .98 | 1.00 | 1.02 | 1.05 | 1.08 | 1.11 | 1.12 | 1.14 | 1.16 | 1.18 | 1.20 | 1.22 | 1.24 | 1.27 | 1.29 | 1.32 | 32 |
| 59 | .86 | .87 | .89 | .91 | .92 | .94 | .95 | .92 | .99 | 1.01 | 1.03 | 1.06 | 1.09 | 1.12 | 1.14 | 1.15 | 1.17 | 1.19 | 1.21 | 1.23 | 1.26 | 1.28 | 1.31 | 1.33 | 31 |
| **60** | .87 | .88 | .90 | .92 | .93 | .95 | .96 | .98 | 1.00 | 1.02 | 1.04 | 1.07 | 1.10 | 1.13 | 1.15 | 1.17 | 1.18 | 1.20 | 1.22 | 1.25 | 1.27 | 1.29 | 1.32 | 1.35 | **30** |
| 61 | .87 | .89 | .91 | .93 | .94 | .96 | .97 | .99 | 1.01 | 1.03 | 1.05 | 1.08 | 1.11 | 1.14 | 1.16 | 1.18 | 1.20 | 1.22 | 1.24 | 1.26 | 1.28 | 1.31 | 1.33 | 1.36 | 29 |
| 62 | .88 | .90 | .91 | .94 | .95 | .97 | .98 | 1.00 | 1.02 | 1.04 | 1.06 | 1.09 | 1.12 | 1.15 | 1.17 | 1.19 | 1.21 | 1.23 | 1.25 | 1.27 | 1.29 | 1.32 | 1.35 | 1.37 | 28 |
| 63 | .89 | .91 | .92 | .95 | .96 | .98 | .99 | 1.01 | 1.03 | 1.05 | 1.07 | 1.10 | 1.13 | 1.16 | 1.18 | 1.20 | 1.22 | 1.24 | 1.26 | 1.28 | 1.31 | 1.33 | 1.36 | 1.39 | 27 |
| 64 | .90 | .91 | .93 | .96 | .97 | .98 | 1.00 | 1.02 | 1.04 | 1.06 | 1.08 | 1.11 | 1.14 | 1.17 | 1.19 | 1.21 | 1.23 | 1.25 | 1.27 | 1.29 | 1.32 | 1.34 | 1.37 | 1.40 | 26 |
| 65 | .91 | .92 | .94 | .96 | .98 | .99 | 1.01 | 1.03 | 1.05 | 1.07 | 1.09 | 1.12 | 1.15 | 1.18 | 1.20 | 1.22 | 1.24 | 1.26 | 1.28 | 1.30 | 1.33 | 1.35 | 1.38 | 1.41 | 25 |
| 66 | .91 | .93 | .95 | .97 | .99 | 1.00 | 1.02 | 1.04 | 1.06 | 1.08 | 1.10 | 1.13 | 1.16 | 1.19 | 1.21 | 1.23 | 1.25 | 1.27 | 1.29 | 1.32 | 1.34 | 1.37 | 1.39 | 1.42 | 24 |
| 67 | .92 | .94 | .95 | .98 | .99 | 1.01 | 1.02 | 1.04 | 1.06 | 1.09 | 1.11 | 1.14 | 1.17 | 1.20 | 1.22 | 1.24 | 1.26 | 1.28 | 1.30 | 1.33 | 1.35 | 1.38 | 1.40 | 1.43 | 23 |
| 68 | .93 | .94 | .96 | .99 | 1.00 | 1.02 | 1.03 | 1.05 | 1.07 | 1.09 | 1.12 | 1.15 | 1.18 | 1.21 | 1.23 | 1.25 | 1.27 | 1.29 | 1.31 | 1.33 | 1.36 | 1.39 | 1.41 | 1.44 | 22 |
| 69 | .93 | .95 | .97 | .99 | 1.01 | 1.02 | 1.04 | 1.06 | 1.08 | 1.10 | 1.13 | 1.15 | 1.18 | 1.22 | 1.24 | 1.26 | 1.28 | 1.30 | 1.32 | 1.34 | 1.37 | 1.40 | 1.42 | 1.45 | 21 |
| **70** | .94 | .95 | .97 | 1.00 | 1.01 | 1.03 | 1.05 | 1.06 | 1.09 | 1.11 | 1.13 | 1.16 | 1.19 | 1.23 | 1.25 | 1.26 | 1.28 | 1.31 | 1.33 | 1.35 | 1.38 | 1.40 | 1.43 | 1.46 | **20** |
| 71 | .95 | .96 | .98 | 1.01 | 1.02 | 1.04 | 1.05 | 1.07 | 1.09 | 1.12 | 1.14 | 1.17 | 1.20 | 1.23 | 1.25 | 1.27 | 1.29 | 1.31 | 1.34 | 1.36 | 1.39 | 1.41 | 1.44 | 1.47 | 19 |
| 72 | .95 | .97 | .98 | 1.01 | 1.03 | 1.04 | 1.06 | 1.08 | 1.10 | 1.12 | 1.15 | 1.17 | 1.21 | 1.24 | 1.26 | 1.28 | 1.30 | 1.32 | 1.34 | 1.37 | 1.39 | 1.42 | 1.45 | 1.48 | 18 |
| 73 | .96 | .97 | .99 | 1.02 | 1.03 | 1.05 | 1.06 | 1.08 | 1.10 | 1.13 | 1.15 | 1.18 | 1.21 | 1.25 | 1.27 | 1.29 | 1.31 | 1.33 | 1.35 | 1.38 | 1.40 | 1.43 | 1.46 | 1.49 | 17 |
| 74 | .96 | .98 | 1.00 | 1.02 | 1.04 | 1.05 | 1.07 | 1.09 | 1.11 | 1.13 | 1.16 | 1.19 | 1.22 | 1.25 | 1.27 | 1.29 | 1.31 | 1.34 | 1.36 | 1.38 | 1.41 | 1.44 | 1.46 | 1.49 | 16 |
| 75 | .97 | .98 | 1.00 | 1.03 | 1.04 | 1.06 | 1.08 | 1.09 | 1.12 | 1.14 | 1.16 | 1.19 | 1.23 | 1.26 | 1.28 | 1.30 | 1.32 | 1.34 | 1.37 | 1.39 | 1.42 | 1.44 | 1.47 | 1.50 | 15 |
| 76 | .97 | .99 | 1.00 | 1.03 | 1.05 | 1.06 | 1.08 | 1.10 | 1.12 | 1.14 | 1.17 | 1.20 | 1.23 | 1.27 | 1.29 | 1.31 | 1.33 | 1.35 | 1.37 | 1.40 | 1.42 | 1.45 | 1.48 | 1.51 | 14 |
| 77 | .97 | .99 | 1.01 | 1.04 | 1.05 | 1.07 | 1.08 | 1.10 | 1.13 | 1.15 | 1.17 | 1.20 | 1.24 | 1.27 | 1.29 | 1.31 | 1.33 | 1.35 | 1.38 | 1.40 | 1.43 | 1.46 | 1.48 | 1.52 | 13 |
| 78 | .98 | .99 | 1.01 | 1.04 | 1.05 | 1.07 | 1.09 | 1.11 | 1.13 | 1.15 | 1.18 | 1.21 | 1.24 | 1.28 | 1.30 | 1.32 | 1.34 | 1.36 | 1.38 | 1.41 | 1.43 | 1.46 | 1.49 | 1.52 | 12 |
| 79 | .98 | 1.00 | 1.02 | 1.04 | 1.06 | 1.08 | 1.09 | 1.11 | 1.13 | 1.16 | 1.18 | 1.21 | 1.25 | 1.28 | 1.30 | 1.32 | 1.34 | 1.36 | 1.39 | 1.41 | 1.44 | 1.47 | 1.50 | 1.53 | 11 |
| **80** | .98 | 1.00 | 1.02 | 1.05 | 1.06 | 1.08 | 1.10 | 1.12 | 1.14 | 1.16 | 1.19 | 1.22 | 1.25 | 1.29 | 1.30 | 1.33 | 1.35 | 1.37 | 1.39 | 1.42 | 1.44 | 1.47 | 1.50 | 1.53 | **10** |
| 81 | .99 | 1.00 | 1.02 | 1.05 | 1.07 | 1.08 | 1.10 | 1.12 | 1.14 | 1.17 | 1.19 | 1.22 | 1.25 | 1.29 | 1.31 | 1.33 | 1.35 | 1.37 | 1.40 | 1.42 | 1.45 | 1.48 | 1.51 | 1.54 | 9 |
| 82 | .99 | 1.01 | 1.03 | 1.05 | 1.07 | 1.08 | 1.10 | 1.12 | 1.14 | 1.17 | 1.19 | 1.22 | 1.26 | 1.29 | 1.31 | 1.33 | 1.35 | 1.38 | 1.40 | 1.43 | 1.45 | 1.48 | 1.51 | 1.54 | 8 |
| 83 | .99 | 1.01 | 1.03 | 1.06 | 1.07 | 1.09 | 1.10 | 1.12 | 1.15 | 1.17 | 1.20 | 1.23 | 1.26 | 1.30 | 1.32 | 1.34 | 1.36 | 1.38 | 1.40 | 1.43 | 1.46 | 1.48 | 1.51 | 1.54 | 7 |
| 84 | .99 | 1.01 | 1.03 | 1.06 | 1.07 | 1.09 | 1.11 | 1.13 | 1.15 | 1.17 | 1.20 | 1.23 | 1.26 | 1.30 | 1.32 | 1.34 | 1.36 | 1.38 | 1.41 | 1.43 | 1.46 | 1.49 | 1.52 | 1.55 | 6 |
| 85 | 1.00 | 1.01 | 1.03 | 1.06 | 1.07 | 1.09 | 1.11 | 1.13 | 1.15 | 1.17 | 1.20 | 1.23 | 1.26 | 1.30 | 1.32 | 1.34 | 1.36 | 1.38 | 1.41 | 1.43 | 1.46 | 1.49 | 1.52 | 1.55 | 5 |
| 86 | 1.00 | 1.01 | 1.03 | 1.06 | 1.08 | 1.09 | 1.11 | 1.13 | 1.15 | 1.18 | 1.20 | 1.23 | 1.27 | 1.30 | 1.32 | 1.34 | 1.36 | 1.39 | 1.41 | 1.44 | 1.46 | 1.49 | 1.52 | 1.55 | 4 |
| 87 | 1.00 | 1.01 | 1.03 | 1.06 | 1.08 | 1.09 | 1.11 | 1.13 | 1.15 | 1.18 | 1.20 | 1.23 | 1.27 | 1.30 | 1.32 | 1.34 | 1.37 | 1.39 | 1.41 | 1.44 | 1.46 | 1.49 | 1.52 | 1.55 | 3 |
| 88 | 1.00 | 1.01 | 1.03 | 1.06 | 1.08 | 1.09 | 1.11 | 1.13 | 1.15 | 1.18 | 1.20 | 1.23 | 1.27 | 1.30 | 1.32 | 1.34 | 1.37 | 1.39 | 1.41 | 1.44 | 1.46 | 1.49 | 1.52 | 1.55 | 2 |
| 89 | 1.00 | 1.02 | 1.04 | 1.06 | 1.08 | 1.09 | 1.11 | 1.13 | 1.15 | 1.18 | 1.21 | 1.24 | 1.27 | 1.31 | 1.32 | 1.35 | 1.37 | 1.39 | 1.41 | 1.44 | 1.47 | 1.49 | 1.52 | 1.56 | 1 |
| **90** | 1.00 | 1.02 | 1.04 | 1.06 | 1.08 | 1.09 | 1.11 | 1.13 | 1.15 | 1.18 | 1.21 | 1.24 | 1.27 | 1.31 | 1.32 | 1.35 | 1.37 | 1.39 | 1.41 | 1.44 | 1.47 | 1.49 | 1.52 | 1.56 | **0** |

**Collimation factor C=sec δ. See last line.**

## UNITED STATES COAST SURVEY.—TABLE OF FACTORS FOR REDUCTION OF TRANSIT OBSERVATIONS.

Azimuth factor $A = \sin \zeta \sec \delta$. Star's declination $\pm \delta$. Inclination factor $B = \cos \zeta \sec \delta$.

| $\zeta$ | 51° | 52° | 53° | 54° | 55° | 56° | 57° | 58° | 59° | 60° | 60½° | 61° | 61½° | 62° | 62½° | 63° | 63½° | 64° | 64½° | 65° | 65½° | 66° | 66½° | 67° | $\zeta$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1° | .03 | .03 | .03 | .03 | .03 | .03 | .03 | .03 | .03 | .03 | .04 | .04 | .04 | .04 | .04 | .04 | .04 | .04 | .04 | .04 | .04 | .04 | .04 | .04 | 89° |
| 2 | .06 | .06 | .06 | .06 | .06 | .06 | .06 | .07 | .07 | .07 | .07 | .07 | .07 | .07 | .08 | .08 | .08 | .08 | .08 | .08 | .08 | .09 | .09 | .09 | 88 |
| 3 | .08 | .08 | .09 | .09 | .09 | .09 | .10 | .10 | .10 | .10 | .11 | .11 | .11 | .11 | .11 | .12 | .12 | .12 | .12 | .12 | .13 | .13 | .13 | .13 | 87 |
| 4 | .11 | .11 | .12 | .12 | .12 | .12 | .13 | .13 | .14 | .14 | .14 | .14 | .15 | .15 | .15 | .15 | .16 | .16 | .16 | .17 | .17 | .17 | .18 | .18 | 86 |
| 5 | .14 | .14 | .14 | .15 | .15 | .16 | .16 | .16 | .17 | .17 | .18 | .18 | .18 | .19 | .19 | .19 | .19 | .20 | .20 | .21 | .21 | .21 | .22 | .22 | 85 |
| 6 | .17 | .17 | .17 | .18 | .18 | .19 | .19 | .20 | .20 | .21 | .21 | .22 | .22 | .22 | .23 | .23 | .23 | .24 | .24 | .25 | .25 | .26 | .26 | .27 | 84 |
| 7 | .19 | .20 | .20 | .21 | .21 | .22 | .22 | .23 | .24 | .24 | .25 | .25 | .26 | .26 | .26 | .27 | .27 | .28 | .28 | .29 | .29 | .30 | .31 | .31 | 83 |
| 8 | .22 | .23 | .23 | .24 | .24 | .25 | .26 | .26 | .27 | .28 | .28 | .29 | .29 | .30 | .30 | .31 | .31 | .32 | .32 | .33 | .34 | .34 | .35 | .36 | 82 |
| 9 | .25 | .25 | .26 | .26 | .27 | .28 | .29 | .29 | .30 | .31 | .32 | .32 | .33 | .33 | .34 | .35 | .35 | .36 | .36 | .37 | .38 | .39 | .39 | .40 | 81 |
| **10** | .28 | .28 | .29 | .30 | .30 | .31 | .32 | .33 | .34 | .35 | .35 | .36 | .36 | .37 | .38 | .38 | .39 | .40 | .40 | .41 | .42 | .43 | .43 | .44 | **80** |
| 11 | .30 | .31 | .32 | .32 | .33 | .34 | .35 | .36 | .37 | .38 | .39 | .39 | .40 | .41 | .41 | .42 | .43 | .44 | .44 | .45 | .46 | .47 | .48 | .49 | 79 |
| 12 | .33 | .34 | .35 | .35 | .36 | .37 | .38 | .39 | .40 | .42 | .42 | .43 | .44 | .44 | .45 | .46 | .47 | .47 | .48 | .49 | .50 | .51 | .52 | .51 | 78 |
| 13 | .36 | .36 | .37 | .38 | .39 | .40 | .41 | .42 | .44 | .45 | .46 | .46 | .47 | .48 | .49 | .50 | .50 | .51 | .52 | .53 | .54 | .55 | .56 | .58 | 77 |
| 14 | .38 | .39 | .40 | .41 | .42 | .43 | .44 | .46 | .47 | .48 | .49 | .50 | .51 | .52 | .52 | .53 | .54 | .55 | .56 | .57 | .58 | .59 | .61 | .62 | 76 |
| 15 | .41 | .42 | .43 | .44 | .45 | .46 | .48 | .49 | .50 | .52 | .53 | .53 | .54 | .55 | .56 | .57 | .58 | .59 | .60 | .61 | .62 | .64 | .65 | .66 | 75 |
| 16 | .44 | .45 | .46 | .47 | .48 | .49 | .51 | .52 | .54 | .55 | .56 | .57 | .58 | .59 | .60 | .61 | .62 | .63 | .64 | .65 | .66 | .68 | .69 | .71 | 74 |
| 17 | .46 | .47 | .49 | .50 | .51 | .52 | .54 | .55 | .57 | .58 | .59 | .60 | .61 | .62 | .63 | .64 | .66 | .67 | .68 | .69 | .70 | .72 | .73 | .75 | 73 |
| 18 | .49 | .50 | .51 | .53 | .54 | .55 | .57 | .58 | .60 | .62 | .63 | .64 | .65 | .66 | .67 | .68 | .69 | .70 | .72 | .73 | .74 | .76 | .77 | .79 | 72 |
| 19 | .52 | .53 | .54 | .55 | .57 | .58 | .60 | .61 | .63 | .65 | .66 | .67 | .68 | .69 | .70 | .72 | .73 | .74 | .76 | .77 | .78 | .80 | .82 | .83 | 71 |
| **20** | .54 | .56 | .57 | .58 | .60 | .61 | .63 | .64 | .66 | .68 | .69 | .70 | .72 | .73 | .74 | .75 | .77 | .78 | .79 | .81 | .83 | .84 | .86 | .88 | **70** |
| 21 | .57 | .58 | .59 | .61 | .62 | .64 | .66 | .68 | .70 | .72 | .73 | .74 | .75 | .76 | .78 | .79 | .80 | .82 | .83 | .85 | .86 | .88 | .90 | .92 | 69 |
| 22 | .60 | .61 | .62 | .64 | .65 | .67 | .69 | .71 | .73 | .75 | .76 | .77 | .78 | .80 | .81 | .82 | .84 | .85 | .87 | .89 | .90 | .92 | .94 | .96 | 68 |
| 23 | .62 | .63 | .65 | .66 | .68 | .70 | .72 | .74 | .76 | .78 | .79 | .81 | .82 | .83 | .85 | .86 | .88 | .89 | .91 | .92 | .94 | .96 | .98 | 1.00 | 67 |
| 24 | .65 | .66 | .68 | .69 | .71 | .73 | .75 | .77 | .79 | .81 | .83 | .84 | .85 | .87 | .88 | .90 | .91 | .93 | .94 | .96 | .98 | 1.00 | 1.02 | 1.04 | 66 |
| 25 | .67 | .69 | .70 | .72 | .74 | .76 | .78 | .80 | .82 | .85 | .86 | .87 | .89 | .90 | .92 | .93 | .95 | .96 | .98 | 1.00 | 1.02 | 1.04 | 1.06 | 1.08 | 65 |
| 26 | .70 | .71 | .73 | .75 | .76 | .78 | .80 | .83 | .85 | .88 | .89 | .90 | .92 | .93 | .95 | .97 | .98 | 1.00 | 1.02 | 1.04 | 1.06 | 1.08 | 1.10 | 1.12 | 64 |
| 27 | .72 | .74 | .75 | .77 | .79 | .81 | .83 | .86 | .88 | .91 | .92 | .94 | .95 | .97 | .98 | 1.00 | 1.02 | 1.04 | 1.05 | 1.07 | 1.09 | 1.12 | 1.14 | 1.16 | 63 |
| 28 | .75 | .76 | .78 | .80 | .82 | .84 | .86 | .89 | .91 | .94 | .95 | .97 | .98 | 1.00 | 1.02 | 1.03 | 1.05 | 1.07 | 1.09 | 1.11 | 1.13 | 1.15 | 1.18 | 1.20 | 62 |
| 29 | .77 | .79 | .81 | .82 | .84 | .87 | .89 | .91 | .94 | .97 | .98 | 1.00 | 1.02 | 1.03 | 1.05 | 1.07 | 1.09 | 1.11 | 1.13 | 1.15 | 1.17 | 1.19 | 1.22 | 1.24 | 61 |
| **30** | .79 | .81 | .83 | .85 | .87 | .89 | .92 | .94 | .97 | 1.00 | 1.01 | 1.03 | 1.05 | 1.07 | 1.08 | 1.10 | 1.12 | 1.14 | 1.16 | 1.18 | 1.21 | 1.23 | 1.25 | 1.28 | **60** |
| 31 | .82 | .84 | .86 | .88 | .90 | .92 | .95 | .97 | 1.00 | 1.03 | 1.05 | 1.06 | 1.08 | 1.10 | 1.11 | 1.13 | 1.15 | 1.17 | 1.20 | 1.22 | 1.24 | 1.27 | 1.29 | 1.32 | 59 |
| 32 | .84 | .86 | .88 | .90 | .92 | .95 | .97 | 1.00 | 1.03 | 1.06 | 1.08 | 1.09 | 1.11 | 1.13 | 1.15 | 1.17 | 1.19 | 1.21 | 1.23 | 1.25 | 1.28 | 1.30 | 1.33 | 1.36 | 58 |
| 33 | .87 | .88 | .91 | .93 | .95 | .97 | 1.00 | 1.03 | 1.06 | 1.09 | 1.11 | 1.12 | 1.14 | 1.16 | 1.18 | 1.20 | 1.22 | 1.24 | 1.26 | 1.29 | 1.31 | 1.34 | 1.37 | 1.39 | 57 |
| 34 | .89 | .91 | .93 | .95 | .97 | 1.00 | 1.03 | 1.05 | 1.09 | 1.12 | 1.14 | 1.15 | 1.17 | 1.19 | 1.21 | 1.23 | 1.25 | 1.27 | 1.30 | 1.32 | 1.35 | 1.37 | 1.40 | 1.43 | 56 |
| 35 | .91 | .93 | .95 | .98 | 1.00 | 1.03 | 1.05 | 1.08 | 1.11 | 1.15 | 1.16 | 1.18 | 1.20 | 1.22 | 1.24 | 1.26 | 1.29 | 1.31 | 1.33 | 1.36 | 1.38 | 1.41 | 1.44 | 1.47 | 55 |
| 36 | .93 | .95 | .98 | 1.00 | 1.03 | 1.05 | 1.08 | 1.11 | 1.14 | 1.18 | 1.19 | 1.21 | 1.23 | 1.25 | 1.27 | 1.30 | 1.32 | 1.34 | 1.37 | 1.39 | 1.42 | 1.45 | 1.47 | 1.51 | 54 |
| 37 | .96 | .98 | 1.00 | 1.02 | 1.05 | 1.08 | 1.10 | 1.14 | 1.17 | 1.20 | 1.22 | 1.24 | 1.26 | 1.28 | 1.30 | 1.33 | 1.35 | 1.37 | 1.40 | 1.42 | 1.45 | 1.48 | 1.51 | 1.54 | 53 |
| 38 | .98 | 1.00 | 1.02 | 1.05 | 1.07 | 1.10 | 1.13 | 1.16 | 1.20 | 1.23 | 1.25 | 1.27 | 1.29 | 1.31 | 1.33 | 1.36 | 1.38 | 1.40 | 1.43 | 1.46 | 1.48 | 1.51 | 1.54 | 1.58 | 52 |
| 39 | 1.00 | 1.02 | 1.05 | 1.07 | 1.10 | 1.12 | 1.15 | 1.19 | 1.22 | 1.26 | 1.28 | 1.30 | 1.32 | 1.34 | 1.36 | 1.39 | 1.41 | 1.43 | 1.46 | 1.49 | 1.52 | 1.55 | 1.58 | 1.61 | 51 |
| **40** | 1.02 | 1.04 | 1.07 | 1.09 | 1.12 | 1.15 | 1.18 | 1.21 | 1.25 | 1.29 | 1.31 | 1.33 | 1.35 | 1.37 | 1.39 | 1.42 | 1.44 | 1.47 | 1.49 | 1.52 | 1.55 | 1.58 | 1.61 | 1.65 | **50** |
| 41 | 1.04 | 1.07 | 1.09 | 1.12 | 1.14 | 1.17 | 1.20 | 1.24 | 1.27 | 1.31 | 1.33 | 1.35 | 1.37 | 1.40 | 1.42 | 1.45 | 1.47 | 1.50 | 1.52 | 1.55 | 1.58 | 1.61 | 1.64 | 1.68 | 49 |
| 42 | 1.06 | 1.09 | 1.11 | 1.14 | 1.17 | 1.20 | 1.23 | 1.26 | 1.30 | 1.34 | 1.36 | 1.38 | 1.40 | 1.42 | 1.45 | 1.47 | 1.50 | 1.53 | 1.55 | 1.58 | 1.61 | 1.64 | 1.68 | 1.71 | 48 |
| 43 | 1.08 | 1.11 | 1.13 | 1.16 | 1.19 | 1.22 | 1.25 | 1.29 | 1.32 | 1.36 | 1.39 | 1.41 | 1.43 | 1.45 | 1.48 | 1.50 | 1.53 | 1.56 | 1.58 | 1.61 | 1.64 | 1.68 | 1.71 | 1.75 | 47 |
| 44 | 1.10 | 1.13 | 1.15 | 1.18 | 1.21 | 1.24 | 1.28 | 1.31 | 1.35 | 1.39 | 1.41 | 1.43 | 1.46 | 1.48 | 1.50 | 1.53 | 1.56 | 1.58 | 1.61 | 1.64 | 1.67 | 1.71 | 1.74 | 1.78 | 46 |
| 45 | 1.12 | 1.15 | 1.17 | 1.20 | 1.23 | 1.26 | 1.30 | 1.33 | 1.37 | 1.41 | 1.44 | 1.46 | 1.48 | 1.51 | 1.53 | 1.56 | 1.58 | 1.61 | 1.64 | 1.67 | 1.70 | 1.74 | 1.77 | 1.81 | 45 |

| ζ | 51° | 52° | 53° | 54° | 55° | 56° | 57° | 58° | 59° | 60° | 60½° | 61° | 61½° | 62° | 62½° | 63° | 63½° | 64° | 64½° | 65° | 65½° | 66° | 66½° | 67° | ζ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 46° | 1.14 | 1.17 | 1.19 | 1.22 | 1.25 | 1.29 | 1.32 | 1.36 | 1.40 | 1.44 | 1.46 | 1.48 | 1.51 | 1.53 | 1.56 | 1.58 | 1.61 | 1.64 | 1.67 | 1.70 | 1.74 | 1.77 | 1.80 | 1.84 | 44° |
| 47 | 1.16 | 1.19 | 1.21 | 1.24 | 1.27 | 1.31 | 1.34 | 1.38 | 1.42 | 1.46 | 1.49 | 1.51 | 1.53 | 1.56 | 1.58 | 1.61 | 1.64 | 1.67 | 1.70 | 1.73 | 1.76 | 1.80 | 1.83 | 1.87 | 43 |
| 48 | 1.18 | 1.21 | 1.23 | 1.26 | 1.30 | 1.33 | 1.36 | 1.40 | 1.44 | 1.48 | 1.50 | 1.53 | 1.55 | 1.58 | 1.60 | 1.63 | 1.66 | 1.69 | 1.72 | 1.75 | 1.79 | 1.82 | 1.86 | 1.90 | 42 |
| 49 | 1.20 | 1.23 | 1.25 | 1.28 | 1.32 | 1.35 | 1.39 | 1.42 | 1.47 | 1.51 | 1.53 | 1.56 | 1.58 | 1.61 | 1.63 | 1.66 | 1.69 | 1.72 | 1.75 | 1.79 | 1.82 | 1.86 | 1.89 | 1.93 | 41 |
| **50** | 1.22 | 1.24 | 1.27 | 1.30 | 1.34 | 1.37 | 1.41 | 1.44 | 1.49 | 1.53 | 1.56 | 1.58 | 1.60 | 1.63 | 1.66 | 1.69 | 1.72 | 1.75 | 1.78 | 1.81 | 1.85 | 1.88 | 1.92 | 1.96 | **40** |
| 51 | 1.23 | 1.26 | 1.29 | 1.32 | 1.35 | 1.39 | 1.43 | 1.47 | 1.51 | 1.55 | 1.58 | 1.60 | 1.63 | 1.66 | 1.68 | 1.71 | 1.74 | 1.77 | 1.80 | 1.84 | 1.87 | 1.91 | 1.95 | 1.99 | 39 |
| 52 | 1.25 | 1.28 | 1.31 | 1.34 | 1.37 | 1.41 | 1.45 | 1.49 | 1.53 | 1.58 | 1.60 | 1.63 | 1.65 | 1.68 | 1.71 | 1.74 | 1.77 | 1.80 | 1.83 | 1.86 | 1.90 | 1.94 | 1.98 | 2.02 | 38 |
| 53 | 1.27 | 1.30 | 1.33 | 1.36 | 1.39 | 1.43 | 1.47 | 1.51 | 1.55 | 1.60 | 1.62 | 1.65 | 1.67 | 1.70 | 1.73 | 1.76 | 1.79 | 1.82 | 1.85 | 1.89 | 1.93 | 1.96 | 2.00 | 2.04 | 37 |
| 54 | 1.29 | 1.31 | 1.34 | 1.38 | 1.41 | 1.45 | 1.49 | 1.53 | 1.57 | 1.62 | 1.64 | 1.67 | 1.69 | 1.72 | 1.75 | 1.78 | 1.81 | 1.85 | 1.88 | 1.91 | 1.95 | 1.99 | 2.03 | 2.07 | 36 |
| 55 | 1.30 | 1.33 | 1.36 | 1.39 | 1.43 | 1.46 | 1.50 | 1.55 | 1.59 | 1.64 | 1.66 | 1.69 | 1.72 | 1.74 | 1.77 | 1.80 | 1.84 | 1.87 | 1.90 | 1.94 | 1.98 | 2.01 | 2.05 | 2.10 | 35 |
| 56 | 1.32 | 1.35 | 1.38 | 1.41 | 1.45 | 1.48 | 1.52 | 1.56 | 1.61 | 1.66 | 1.68 | 1.71 | 1.74 | 1.77 | 1.80 | 1.83 | 1.86 | 1.89 | 1.93 | 1.96 | 2.00 | 2.04 | 2.08 | 2.12 | 34 |
| 57 | 1.33 | 1.36 | 1.39 | 1.43 | 1.46 | 1.50 | 1.54 | 1.58 | 1.63 | 1.68 | 1.70 | 1.73 | 1.76 | 1.79 | 1.82 | 1.85 | 1.88 | 1.91 | 1.95 | 1.98 | 2.02 | 2.06 | 2.10 | 2.15 | 33 |
| 58 | 1.35 | 1.38 | 1.41 | 1.44 | 1.48 | 1.52 | 1.56 | 1.60 | 1.65 | 1.70 | 1.72 | 1.75 | 1.78 | 1.81 | 1.84 | 1.87 | 1.90 | 1.93 | 1.97 | 2.01 | 2.05 | 2.08 | 2.13 | 2.17 | 32 |
| 59 | 1.36 | 1.39 | 1.42 | 1.46 | 1.49 | 1.53 | 1.57 | 1.62 | 1.66 | 1.71 | 1.74 | 1.77 | 1.80 | 1.83 | 1.86 | 1.89 | 1.92 | 1.95 | 1.99 | 2.03 | 2.07 | 2.11 | 2.15 | 2.19 | 31 |
| **60** | 1.38 | 1.41 | 1.44 | 1.47 | 1.51 | 1.55 | 1.59 | 1.63 | 1.68 | 1.73 | 1.76 | 1.79 | 1.81 | 1.84 | 1.88 | 1.91 | 1.94 | 1.97 | 2.01 | 2.05 | 2.09 | 2.13 | 2.17 | 2.22 | **30** |
| 61 | 1.39 | 1.42 | 1.45 | 1.49 | 1.53 | 1.56 | 1.61 | 1.65 | 1.70 | 1.75 | 1.78 | 1.80 | 1.83 | 1.86 | 1.89 | 1.93 | 1.96 | 2.00 | 2.03 | 2.07 | 2.11 | 2.15 | 2.19 | 2.24 | 29 |
| 62 | 1.40 | 1.43 | 1.47 | 1.50 | 1.54 | 1.58 | 1.62 | 1.67 | 1.71 | 1.77 | 1.79 | 1.82 | 1.85 | 1.88 | 1.91 | 1.94 | 1.98 | 2.01 | 2.05 | 2.09 | 2.13 | 2.17 | 2.21 | 2.26 | 28 |
| 63 | 1.42 | 1.45 | 1.48 | 1.52 | 1.55 | 1.59 | 1.64 | 1.68 | 1.73 | 1.78 | 1.81 | 1.84 | 1.87 | 1.90 | 1.93 | 1.96 | 2.00 | 2.03 | 2.07 | 2.11 | 2.15 | 2.19 | 2.23 | 2.28 | 27 |
| 64 | 1.43 | 1.46 | 1.49 | 1.53 | 1.57 | 1.61 | 1.65 | 1.70 | 1.75 | 1.80 | 1.83 | 1.85 | 1.88 | 1.91 | 1.95 | 1.98 | 2.02 | 2.05 | 2.09 | 2.13 | 2.17 | 2.21 | 2.25 | 2.30 | 26 |
| 65 | 1.44 | 1.47 | 1.51 | 1.54 | 1.58 | 1.62 | 1.66 | 1.71 | 1.76 | 1.81 | 1.84 | 1.87 | 1.90 | 1.93 | 1.96 | 2.00 | 2.03 | 2.07 | 2.11 | 2.14 | 2.19 | 2.23 | 2.27 | 2.32 | 25 |
| 66 | 1.45 | 1.48 | 1.52 | 1.55 | 1.59 | 1.63 | 1.68 | 1.72 | 1.77 | 1.83 | 1.85 | 1.88 | 1.91 | 1.95 | 1.98 | 2.01 | 2.05 | 2.08 | 2.12 | 2.16 | 2.20 | 2.25 | 2.29 | 2.34 | 24 |
| 67 | 1.46 | 1.50 | 1.53 | 1.57 | 1.60 | 1.65 | 1.69 | 1.74 | 1.79 | 1.84 | 1.87 | 1.90 | 1.93 | 1.96 | 1.99 | 2.03 | 2.06 | 2.10 | 2.14 | 2.18 | 2.22 | 2.26 | 2.31 | 2.36 | 23 |
| 68 | 1.47 | 1.51 | 1.54 | 1.58 | 1.62 | 1.66 | 1.70 | 1.75 | 1.80 | 1.85 | 1.88 | 1.91 | 1.94 | 1.97 | 2.01 | 2.04 | 2.08 | 2.11 | 2.15 | 2.19 | 2.24 | 2.28 | 2.32 | 2.37 | 22 |
| 69 | 1.48 | 1.52 | 1.55 | 1.59 | 1.63 | 1.67 | 1.71 | 1.76 | 1.81 | 1.87 | 1.90 | 1.93 | 1.96 | 1.99 | 2.02 | 2.06 | 2.09 | 2.13 | 2.17 | 2.21 | 2.25 | 2.30 | 2.34 | 2.39 | 21 |
| **70** | 1.49 | 1.53 | 1.56 | 1.60 | 1.64 | 1.68 | 1.73 | 1.77 | 1.82 | 1.88 | 1.91 | 1.94 | 1.97 | 2.00 | 2.03 | 2.07 | 2.11 | 2.14 | 2.18 | 2.22 | 2.27 | 2.31 | 2.36 | 2.40 | **20** |
| 71 | 1.50 | 1.54 | 1.57 | 1.61 | 1.65 | 1.69 | 1.74 | 1.78 | 1.84 | 1.89 | 1.92 | 1.95 | 1.98 | 2.01 | 2.05 | 2.08 | 2.12 | 2.16 | 2.20 | 2.24 | 2.28 | 2.32 | 2.37 | 2.42 | 19 |
| 72 | 1.51 | 1.54 | 1.58 | 1.62 | 1.66 | 1.70 | 1.75 | 1.80 | 1.85 | 1.90 | 1.93 | 1.96 | 1.99 | 2.03 | 2.06 | 2.09 | 2.13 | 2.17 | 2.21 | 2.25 | 2.29 | 2.34 | 2.38 | 2.43 | 18 |
| 73 | 1.52 | 1.55 | 1.59 | 1.63 | 1.67 | 1.71 | 1.76 | 1.80 | 1.86 | 1.91 | 1.94 | 1.97 | 2.00 | 2.04 | 2.07 | 2.11 | 2.14 | 2.18 | 2.22 | 2.26 | 2.31 | 2.35 | 2.40 | 2.45 | 17 |
| 74 | 1.53 | 1.56 | 1.60 | 1.63 | 1.68 | 1.72 | 1.76 | 1.81 | 1.87 | 1.92 | 1.95 | 1.98 | 2.01 | 2.05 | 2.08 | 2.12 | 2.15 | 2.19 | 2.23 | 2.27 | 2.32 | 2.36 | 2.41 | 2.46 | 16 |
| 75 | 1.53 | 1.57 | 1.60 | 1.64 | 1.68 | 1.73 | 1.77 | 1.82 | 1.88 | 1.93 | 1.96 | 1.99 | 2.02 | 2.06 | 2.09 | 2.13 | 2.16 | 2.20 | 2.24 | 2.29 | 2.33 | 2.37 | 2.42 | 2.47 | 15 |
| 76 | 1.54 | 1.58 | 1.61 | 1.65 | 1.69 | 1.73 | 1.78 | 1.83 | 1.88 | 1.94 | 1.97 | 2.00 | 2.03 | 2.07 | 2.10 | 2.14 | 2.17 | 2.21 | 2.25 | 2.30 | 2.34 | 2.39 | 2.43 | 2.48 | 14 |
| 77 | 1.55 | 1.58 | 1.62 | 1.66 | 1.70 | 1.74 | 1.79 | 1.84 | 1.89 | 1.95 | 1.98 | 2.01 | 2.04 | 2.07 | 2.11 | 2.15 | 2.18 | 2.22 | 2.26 | 2.31 | 2.35 | 2.40 | 2.44 | 2.49 | 13 |
| 78 | 1.55 | 1.59 | 1.62 | 1.66 | 1.70 | 1.75 | 1.80 | 1.85 | 1.90 | 1.96 | 1.99 | 2.02 | 2.05 | 2.08 | 2.12 | 2.15 | 2.19 | 2.23 | 2.27 | 2.31 | 2.36 | 2.40 | 2.45 | 2.50 | 12 |
| 79 | 1.56 | 1.59 | 1.63 | 1.67 | 1.71 | 1.76 | 1.80 | 1.85 | 1.91 | 1.96 | 1.99 | 2.02 | 2.06 | 2.09 | 2.13 | 2.16 | 2.20 | 2.24 | 2.28 | 2.32 | 2.37 | 2.41 | 2.46 | 2.51 | 11 |
| **80** | 1.56 | 1.60 | 1.64 | 1.67 | 1.72 | 1.76 | 1.81 | 1.86 | 1.91 | 1.97 | 2.00 | 2.03 | 2.06 | 2.10 | 2.13 | 2.17 | 2.21 | 2.25 | 2.29 | 2.33 | 2.38 | 2.42 | 2.47 | 2.52 | **10** |
| 81 | 1.57 | 1.60 | 1.64 | 1.68 | 1.72 | 1.77 | 1.81 | 1.86 | 1.92 | 1.98 | 2.01 | 2.04 | 2.07 | 2.10 | 2.14 | 2.18 | 2.21 | 2.25 | 2.29 | 2.34 | 2.38 | 2.43 | 2.48 | 2.53 | 9 |
| 82 | 1.57 | 1.61 | 1.64 | 1.68 | 1.73 | 1.77 | 1.82 | 1.87 | 1.92 | 1.98 | 2.01 | 2.04 | 2.08 | 2.11 | 2.15 | 2.18 | 2.22 | 2.26 | 2.30 | 2.34 | 2.39 | 2.43 | 2.48 | 2.53 | 8 |
| 83 | 1.58 | 1.61 | 1.65 | 1.69 | 1.73 | 1.77 | 1.82 | 1.87 | 1.93 | 1.99 | 2.02 | 2.05 | 2.08 | 2.12 | 2.15 | 2.19 | 2.22 | 2.26 | 2.31 | 2.35 | 2.39 | 2.44 | 2.49 | 2.54 | 7 |
| 84 | 1.58 | 1.62 | 1.65 | 1.69 | 1.73 | 1.78 | 1.83 | 1.88 | 1.93 | 1.99 | 2.02 | 2.05 | 2.08 | 2.12 | 2.15 | 2.19 | 2.23 | 2.27 | 2.31 | 2.35 | 2.40 | 2.45 | 2.49 | 2.55 | 6 |
| 85 | 1.58 | 1.62 | 1.65 | 1.69 | 1.74 | 1.78 | 1.83 | 1.88 | 1.93 | 1.99 | 2.02 | 2.05 | 2.09 | 2.12 | 2.16 | 2.19 | 2.23 | 2.27 | 2.31 | 2.36 | 2.40 | 2.45 | 2.50 | 2.55 | 5 |
| 86 | 1.59 | 1.62 | 1.66 | 1.70 | 1.74 | 1.78 | 1.83 | 1.88 | 1.94 | 2.00 | 2.03 | 2.06 | 2.09 | 2.13 | 2.16 | 2.20 | 2.24 | 2.28 | 2.32 | 2.36 | 2.41 | 2.45 | 2.50 | 2.55 | 4 |
| 87 | 1.59 | 1.62 | 1.66 | 1.70 | 1.74 | 1.79 | 1.83 | 1.88 | 1.94 | 2.00 | 2.03 | 2.06 | 2.09 | 2.13 | 2.16 | 2.20 | 2.24 | 2.28 | 2.32 | 2.36 | 2.41 | 2.46 | 2.50 | 2.56 | 3 |
| 88 | 1.59 | 1.62 | 1.66 | 1.70 | 1.74 | 1.79 | 1.83 | 1.89 | 1.94 | 2.00 | 2.03 | 2.06 | 2.09 | 2.13 | 2.16 | 2.20 | 2.24 | 2.28 | 2.32 | 2.36 | 2.41 | 2.46 | 2.51 | 2.56 | 2 |
| 89 | 1.59 | 1.62 | 1.66 | 1.70 | 1.74 | 1.79 | 1.84 | 1.89 | 1.94 | 2.00 | 2.03 | 2.06 | 2.10 | 2.13 | 2.17 | 2.20 | 2.24 | 2.28 | 2.32 | 2.37 | 2.41 | 2.46 | 2.51 | 2.56 | 1 |
| **90** | 1.59 | 1.62 | 1.66 | 1.70 | 1.74 | 1.79 | 1.84 | 1.89 | 1.94 | 2.00 | 2.03 | 2.06 | 2.10 | 2.13 | 2.17 | 2.20 | 2.24 | 2.28 | 2.32 | 2.37 | 2.41 | 2.46 | 2.51 | 2.56 | **0** |

Collimation factor C = sec δ.      See last line.

Azimuth factor $A = \sin \zeta \sec \delta$     Star's declination $\pm \delta$.     Inclination factor $B = \cos \zeta \sec \delta$.

| $\zeta$ | 67½° | 68° | 68½° | 69° | 69½° | 70° | 70¼° | 70½° | 70¾° | 71° | 71¼° | 71½° | 71¾° | 72° | 72¼° | 72½° | 72¾° | 73° | 73¼° | 73½° | 73¾° | 74° | 74¼° | $\zeta$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1° | .05 | .05 | .05 | .05 | .05 | .05 | .05 | .05 | .05 | .05 | .05 | .05 | .05 | .06 | .06 | .06 | .06 | .06 | .06 | .06 | .06 | .06 | .06 | 89° |
| 2 | .09 | .09 | .10 | .10 | .10 | .10 | .10 | .10 | .11 | .11 | .11 | .11 | .11 | .11 | .11 | .12 | .12 | .12 | .12 | .12 | .12 | .13 | .13 | 88 |
| 3 | .14 | .14 | .14 | .15 | .15 | .15 | .15 | .16 | .16 | .16 | .16 | .16 | .17 | .17 | .17 | .17 | .18 | .18 | .18 | .18 | .19 | .19 | .19 | 87 |
| 4 | .18 | .19 | .19 | .20 | .20 | .20 | .21 | .21 | .21 | .21 | .22 | .22 | .22 | .23 | .23 | .23 | .23 | .24 | .24 | .24 | .25 | .25 | .26 | 86 |
| 5 | .23 | .23 | .24 | .24 | .25 | .25 | .26 | .26 | .26 | .27 | .27 | .27 | .28 | .28 | .29 | .29 | .29 | .30 | .30 | .31 | .31 | .32 | .32 | 85 |
| 6 | .27 | .28 | .28 | .29 | .30 | .31 | .31 | .31 | .32 | .32 | .33 | .33 | .33 | .34 | .34 | .35 | .35 | .36 | .36 | .37 | .37 | .38 | .39 | 84 |
| 7 | .32 | .33 | .33 | .34 | .35 | .36 | .36 | .37 | .37 | .37 | .38 | .38 | .39 | .39 | .40 | .41 | .41 | .42 | .42 | .43 | .44 | .44 | .45 | 83 |
| 8 | .36 | .37 | .38 | .39 | .40 | .41 | .41 | .42 | .42 | .43 | .43 | .44 | .44 | .45 | .46 | .46 | .47 | .48 | .48 | .49 | .50 | .50 | .51 | 82 |
| 9 | .41 | .42 | .43 | .44 | .45 | .46 | .46 | .47 | .47 | .48 | .49 | .49 | .50 | .51 | .51 | .52 | .53 | .53 | .54 | .55 | .56 | .57 | .58 | 81 |
| **10** | .45 | .46 | .47 | .49 | .50 | .51 | .51 | .52 | .53 | .53 | .54 | .55 | .55 | .56 | .57 | .58 | .59 | .60 | .60 | .61 | .62 | .63 | .64 | **80** |
| 11 | .50 | .51 | .52 | .53 | .54 | .56 | .56 | .57 | .58 | .59 | .59 | .60 | .61 | .62 | .63 | .63 | .64 | .65 | .66 | .67 | .68 | .69 | .70 | 79 |
| 12 | .54 | .56 | .57 | .58 | .59 | .61 | .62 | .62 | .63 | .64 | .65 | .66 | .66 | .67 | .68 | .69 | .70 | .71 | .72 | .73 | .74 | .75 | .77 | 78 |
| 13 | .59 | .60 | .61 | .63 | .64 | .66 | .67 | .67 | .68 | .69 | .70 | .71 | .72 | .73 | .74 | .75 | .76 | .77 | .78 | .79 | .80 | .82 | .83 | 77 |
| 14 | .63 | .65 | .66 | .68 | .69 | .71 | .72 | .72 | .73 | .74 | .75 | .76 | .77 | .78 | .79 | .80 | .82 | .83 | .84 | .85 | .87 | .88 | .89 | 76 |
| 15 | .68 | .69 | .71 | .72 | .74 | .76 | .77 | .78 | .78 | .79 | .80 | .81 | .83 | .84 | .85 | .86 | .87 | .89 | .90 | .91 | .93 | .94 | .95 | 75 |
| 16 | .72 | .74 | .75 | .77 | .79 | .81 | .82 | .83 | .84 | .85 | .86 | .87 | .88 | .89 | .91 | .92 | .93 | .94 | .96 | .97 | .99 | 1.00 | 1.02 | 74 |
| 17 | .76 | .78 | .80 | .81 | .83 | .85 | .86 | .88 | .89 | .90 | .91 | .92 | .95 | .95 | .96 | .97 | .99 | 1.00 | 1.01 | 1.03 | 1.05 | 1.06 | 1.08 | 73 |
| 18 | .81 | .83 | .84 | .86 | .88 | .90 | .91 | .93 | .94 | .95 | .96 | .97 | .99 | 1.00 | 1.01 | 1.03 | 1.04 | 1.06 | 1.07 | 1.09 | 1.10 | 1.12 | 1.14 | 72 |
| 19 | .85 | .87 | .89 | .91 | .93 | .95 | .96 | .98 | .99 | 1.00 | 1.01 | 1.03 | 1.04 | 1.05 | 1.07 | 1.08 | 1.10 | 1.11 | 1.13 | 1.15 | 1.16 | 1.18 | 1.20 | 71 |
| **20** | .89 | .91 | .93 | .95 | .98 | 1.00 | 1.01 | 1.02 | 1.04 | 1.05 | 1.06 | 1.08 | 1.09 | 1.11 | 1.12 | 1.14 | 1.15 | 1.17 | 1.19 | 1.20 | 1.22 | 1.24 | 1.26 | **70** |
| 21 | .94 | .96 | .98 | 1.00 | 1.02 | 1.05 | 1.06 | 1.07 | 1.09 | 1.10 | 1.11 | 1.13 | 1.14 | 1.16 | 1.17 | 1.19 | 1.21 | 1.22 | 1.24 | 1.26 | 1.28 | 1.30 | 1.32 | 69 |
| 22 | .98 | 1.00 | 1.02 | 1.05 | 1.07 | 1.09 | 1.11 | 1.12 | 1.14 | 1.15 | 1.17 | 1.18 | 1.20 | 1.21 | 1.23 | 1.25 | 1.26 | 1.28 | 1.30 | 1.32 | 1.34 | 1.36 | 1.38 | 68 |
| 23 | 1.02 | 1.04 | 1.07 | 1.09 | 1.12 | 1.14 | 1.16 | 1.17 | 1.19 | 1.20 | 1.21 | 1.23 | 1.25 | 1.26 | 1.28 | 1.30 | 1.32 | 1.34 | 1.36 | 1.38 | 1.40 | 1.42 | 1.44 | 67 |
| 24 | 1.06 | 1.09 | 1.11 | 1.14 | 1.16 | 1.19 | 1.20 | 1.22 | 1.23 | 1.25 | 1.27 | 1.28 | 1.30 | 1.32 | 1.33 | 1.35 | 1.37 | 1.39 | 1.41 | 1.43 | 1.45 | 1.48 | 1.50 | 66 |
| 25 | 1.10 | 1.13 | 1.15 | 1.18 | 1.21 | 1.24 | 1.25 | 1.27 | 1.28 | 1.30 | 1.31 | 1.33 | 1.35 | 1.37 | 1.39 | 1.41 | 1.42 | 1.45 | 1.47 | 1.49 | 1.51 | 1.53 | 1.56 | 65 |
| 26 | 1.15 | 1.17 | 1.20 | 1.22 | 1.25 | 1.28 | 1.30 | 1.31 | 1.33 | 1.35 | 1.36 | 1.39 | 1.40 | 1.42 | 1.44 | 1.46 | 1.48 | 1.50 | 1.52 | 1.54 | 1.57 | 1.59 | 1.61 | 64 |
| 27 | 1.19 | 1.21 | 1.24 | 1.27 | 1.30 | 1.33 | 1.34 | 1.36 | 1.38 | 1.39 | 1.41 | 1.43 | 1.45 | 1.47 | 1.49 | 1.51 | 1.53 | 1.55 | 1.58 | 1.60 | 1.62 | 1.65 | 1.67 | 63 |
| 28 | 1.23 | 1.25 | 1.28 | 1.31 | 1.34 | 1.37 | 1.39 | 1.41 | 1.42 | 1.44 | 1.46 | 1.48 | 1.50 | 1.52 | 1.54 | 1.56 | 1.58 | 1.60 | 1.63 | 1.65 | 1.68 | 1.70 | 1.73 | 62 |
| 29 | 1.27 | 1.29 | 1.32 | 1.35 | 1.38 | 1.42 | 1.43 | 1.45 | 1.47 | 1.49 | 1.51 | 1.53 | 1.55 | 1.57 | 1.59 | 1.61 | 1.63 | 1.66 | 1.68 | 1.71 | 1.73 | 1.76 | 1.79 | 61 |
| **30** | 1.31 | 1.33 | 1.36 | 1.39 | 1.43 | 1.46 | 1.48 | 1.50 | 1.52 | 1.54 | 1.56 | 1.58 | 1.60 | 1.62 | 1.64 | 1.66 | 1.69 | 1.71 | 1.73 | 1.76 | 1.79 | 1.81 | 1.84 | **60** |
| 31 | 1.35 | 1.38 | 1.40 | 1.44 | 1.47 | 1.51 | 1.52 | 1.54 | 1.56 | 1.58 | 1.60 | 1.62 | 1.64 | 1.67 | 1.69 | 1.71 | 1.74 | 1.76 | 1.79 | 1.81 | 1.84 | 1.87 | 1.90 | 59 |
| 32 | 1.39 | 1.42 | 1.45 | 1.48 | 1.51 | 1.55 | 1.57 | 1.59 | 1.61 | 1.63 | 1.65 | 1.67 | 1.69 | 1.71 | 1.74 | 1.76 | 1.79 | 1.81 | 1.84 | 1.87 | 1.89 | 1.92 | 1.95 | 58 |
| 33 | 1.42 | 1.45 | 1.49 | 1.52 | 1.55 | 1.59 | 1.61 | 1.63 | 1.65 | 1.67 | 1.69 | 1.72 | 1.74 | 1.76 | 1.79 | 1.81 | 1.84 | 1.86 | 1.89 | 1.92 | 1.95 | 1.98 | 2.01 | 57 |
| 34 | 1.46 | 1.49 | 1.53 | 1.56 | 1.60 | 1.63 | 1.65 | 1.68 | 1.70 | 1.72 | 1.74 | 1.76 | 1.79 | 1.81 | 1.83 | 1.86 | 1.89 | 1.91 | 1.94 | 1.97 | 2.00 | 2.03 | 2.06 | 56 |
| 35 | 1.50 | 1.53 | 1.56 | 1.60 | 1.64 | 1.68 | 1.70 | 1.72 | 1.74 | 1.76 | 1.78 | 1.81 | 1.83 | 1.86 | 1.88 | 1.91 | 1.93 | 1.96 | 1.99 | 2.02 | 2.05 | 2.08 | 2.11 | 55 |
| 36 | 1.54 | 1.57 | 1.60 | 1.64 | 1.68 | 1.72 | 1.74 | 1.76 | 1.78 | 1.80 | 1.83 | 1.85 | 1.88 | 1.90 | 1.93 | 1.95 | 1.98 | 2.01 | 2.04 | 2.07 | 2.10 | 2.13 | 2.16 | 54 |
| 37 | 1.57 | 1.61 | 1.64 | 1.68 | 1.72 | 1.76 | 1.78 | 1.80 | 1.83 | 1.85 | 1.87 | 1.90 | 1.92 | 1.95 | 1.97 | 2.00 | 2.03 | 2.06 | 2.09 | 2.12 | 2.15 | 2.18 | 2.22 | 53 |
| 38 | 1.61 | 1.64 | 1.68 | 1.72 | 1.76 | 1.80 | 1.82 | 1.84 | 1.87 | 1.89 | 1.91 | 1.94 | 1.97 | 1.99 | 2.02 | 2.05 | 2.08 | 2.11 | 2.14 | 2.17 | 2.20 | 2.23 | 2.27 | 52 |
| 39 | 1.65 | 1.68 | 1.72 | 1.75 | 1.80 | 1.84 | 1.86 | 1.88 | 1.91 | 1.93 | 1.96 | 1.98 | 2.01 | 2.04 | 2.06 | 2.09 | 2.12 | 2.15 | 2.18 | 2.22 | 2.25 | 2.28 | 2.32 | 51 |
| **40** | 1.68 | 1.72 | 1.75 | 1.79 | 1.84 | 1.88 | 1.90 | 1.93 | 1.95 | 1.97 | 2.00 | 2.03 | 2.05 | 2.08 | 2.11 | 2.14 | 2.17 | 2.20 | 2.23 | 2.26 | 2.30 | 2.33 | 2.37 | **50** |
| 41 | 1.71 | 1.75 | 1.79 | 1.83 | 1.87 | 1.92 | 1.94 | 1.96 | 1.99 | 2.01 | 2.04 | 2.07 | 2.09 | 2.12 | 2.15 | 2.18 | 2.21 | 2.24 | 2.28 | 2.31 | 2.34 | 2.38 | 2.42 | 49 |
| 42 | 1.75 | 1.79 | 1.83 | 1.87 | 1.91 | 1.96 | 1.98 | 2.00 | 2.03 | 2.05 | 2.08 | 2.11 | 2.14 | 2.16 | 2.19 | 2.22 | 2.26 | 2.29 | 2.32 | 2.36 | 2.39 | 2.43 | 2.46 | 48 |
| 43 | 1.78 | 1.82 | 1.86 | 1.90 | 1.95 | 1.99 | 2.02 | 2.04 | 2.07 | 2.09 | 2.12 | 2.15 | 2.18 | 2.21 | 2.24 | 2.27 | 2.30 | 2.33 | 2.37 | 2.40 | 2.44 | 2.47 | 2.51 | 47 |
| 44 | 1.82 | 1.85 | 1.90 | 1.94 | 1.98 | 2.03 | 2.06 | 2.08 | 2.11 | 2.13 | 2.16 | 2.19 | 2.22 | 2.25 | 2.28 | 2.31 | 2.34 | 2.38 | 2.41 | 2.45 | 2.48 | 2.52 | 2.56 | 46 |
| 45 | 1.85 | 1.89 | 1.93 | 1.97 | 2.02 | 2.07 | 2.09 | 2.12 | 2.14 | 2.17 | 2.20 | 2.23 | 2.26 | 2.29 | 2.32 | 2.35 | 2.38 | 2.42 | 2.45 | 2.49 | 2.53 | 2.56 | 2.60 | 45 |

| ζ | 67½° | 68° | 68½° | 69° | 69½° | 70° | 70¼° | 70½° | 70¾° | 71° | 71¼° | 71½° | 71¾° | 72° | 72¼° | 72½° | 72¾° | 73° | 73¼° | 73½° | 73¾° | 74° | 74¼° | ζ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 46° | 1.88 | 1.92 | 1.96 | 2.01 | 2.05 | 2.10 | 2.13 | 2.15 | 2.18 | 2.21 | 2.24 | 2.27 | 2.30 | 3 33 | 2.36 | 2 39 | 2.42 | 2.46 | 2.49 | 2.53 | 2.57 | 2.61 | 2.65 | 44° |
| 47 | 1.91 | 1.95 | 2.00 | 2.04 | 2.09 | 2.14 | 2.16 | 2.19 | 2.22 | 2.25 | 2.27 | 2.30 | 2.33 | 3.37 | 2.40 | 2.43 | 2.47 | 2.50 | 2.54 | 2.57 | 2.61 | 2.65 | 2.69 | 43 |
| 48 | 1.94 | 1.98 | 2.02 | 2.07 | 2.12 | 2.17 | 2.19 | 2.22 | 2.25 | 2.28 | 2.31 | 2.34 | 2.37 | 4.40 | 2.44 | 2.47 | 2.51 | 2.54 | 2.58 | 2.62 | 2.66 | 2.70 | 2.74 | 42 |
| 49 | 1.97 | 2.01 | 2.06 | 2.11 | 2.16 | 2.21 | 2.23 | 2.26 | 2.29 | 2.32 | 2.35 | 2.38 | 2.41 | 4.44 | 2.48 | 2.51 | 2.55 | 2.58 | 2.62 | 2.66 | 2.70 | 2.74 | 2.78 | 41 |
| **50** | 2.00 | 2.04 | 2.09 | 2.14 | 2.19 | 2.24 | 2.27 | 2.29 | 2.32 | 2.35 | 2.38 | 2.41 | 2.45 | 4.48 | 2.51 | 2.55 | 2.58 | 2.62 | 2.66 | 2.70 | 2.74 | 2.78 | 2.82 | **40** |
| 51 | 2.03 | 2.07 | 2.12 | 2.17 | 2.22 | 2.27 | 2.30 | 2.33 | 2.36 | 2.39 | 2.42 | 2.45 | 2.48 | 2.51 | 2.55 | 2.58 | 2.62 | 2.66 | 2.70 | 2.74 | 2.78 | 2.82 | 2.86 | 39 |
| 52 | 2.06 | 2.10 | 2.15 | 2.20 | 2.25 | 2.30 | 2.33 | 2.36 | 2.39 | 2.42 | 2.45 | 2.48 | 2.52 | 2.55 | 2.58 | 2.62 | 2.66 | 2.69 | 2.73 | 2.77 | 2.82 | 2.86 | 2.90 | 38 |
| 53 | 2.09 | 2.13 | 2.18 | 2.23 | 2.28 | 2.33 | 2.36 | 2.39 | 2.42 | 2.45 | 2.48 | 2.52 | 2.55 | 2.58 | 2.62 | 2.66 | 2.69 | 2.73 | 2.77 | 2.81 | 2 85 | 2.90 | 2.94 | 37 |
| 54 | 2.11 | 2.16 | 2.21 | 2.26 | 2.31 | 2.37 | 2.39 | 2.42 | 2.45 | 2.48 | 2.52 | 2.55 | 2.58 | 2.62 | 2.65 | 2.69 | 2.73 | 2.77 | 2.81 | 2.85 | 2.89 | 2.94 | 2.98 | 36 |
| 55 | 2.14 | 2.19 | 2.23 | 2.29 | 2.34 | 2.40 | 2.42 | 2.45 | 2.48 | 2.52 | 2.55 | 2.58 | 2.62 | 2.65 | 2.69 | 2.72 | 2.76 | 2.80 | 2.84 | 2.88 | 2.93 | 2.97 | 3.02 | 35 |
| 56 | 2.17 | 2.21 | 2.26 | 2.31 | 2.37 | 2.42 | 2.45 | 2.48 | 2.51 | 2.55 | 2.58 | 2.61 | 2.65 | 2.68 | 2.72 | 2.76 | 2.80 | 2.84 | 2.88 | 2.92 | 2.96 | 3.01 | 3.05 | 34 |
| 57 | 2.19 | 2.24 | 2.29 | 2.34 | 2.39 | 2.45 | 2.48 | 2.51 | 2.54 | 2.58 | 2.61 | 2.64 | 2.68 | 2.71 | 2.75 | 2.79 | 2.83 | 2.87 | 2.91 | 2.95 | 3.00 | 3.04 | 3.09 | 33 |
| 58 | 2.22 | 2.26 | 2.31 | 2.37 | 2.42 | 2.48 | 2.51 | 2.54 | 2.57 | 2.61 | 2.64 | 2.67 | 2.71 | 2.74 | 2.78 | 2.82 | 2.86 | 2.90 | 2.94 | 2.99 | 3.03 | 3.08 | 3.12 | 32 |
| 59 | 2.24 | 2.29 | 2.34 | 2.39 | 2.45 | 2.51 | 2.54 | 2.57 | 2.60 | 2.63 | 2.67 | 2.70 | 2.74 | 2.77 | 2.81 | 2.85 | 2.89 | 2.93 | 2.97 | 3.02 | 3.06 | 3.11 | 3.16 | 31 |
| **60** | 2.26 | 2.31 | 2.36 | 2.42 | 2.47 | 2.53 | 2.56 | 2.59 | 2.63 | 2.66 | 2.69 | 2.73 | 2.76 | 2.80 | 2.84 | 2.88 | 2.92 | 2.96 | 3.01 | 3.05 | 3.09 | 3.14 | 3.19 | **30** |
| 61 | 2.29 | 2.33 | 2.39 | 2.44 | 2.50 | 2.56 | 2.59 | 2.62 | 2.65 | 2.69 | 2.72 | 2.76 | 2.79 | 2.83 | 2.87 | 2.91 | 2.95 | 2.99 | 3.04 | 3.08 | 3.13 | 3.17 | 3.22 | 29 |
| 62 | 2.31 | 2.36 | 2.41 | 2.46 | 2.52 | 2.58 | 2.61 | 2.64 | 2.68 | 2.71 | 2.75 | 2.78 | 2.82 | 2.86 | 2.90 | 2.94 | 2.98 | 3.02 | 3.06 | 3.11 | 3.16 | 3.20 | 3.25 | 28 |
| 63 | 2.33 | 2.38 | 2.43 | 2.49 | 2.54 | 2.60 | 2.64 | 2.67 | 2.70 | 2.74 | 2.77 | 2.81 | 2.84 | 2.88 | 2.92 | 2.96 | 3.00 | 3.05 | 3.09 | 3.14 | 3.18 | 3.23 | 3.28 | 27 |
| 64 | 2.35 | 2.40 | 2.45 | 2.51 | 2.57 | 2.63 | 2.66 | 2.69 | 2.73 | 2.76 | 2.80 | 2.93 | 2.87 | 2.91 | 2.95 | 2.99 | 3.03 | 3.07 | 3.12 | 3.16 | 3.21 | 3.26 | 3.31 | 26 |
| 65 | 2.37 | 2.42 | 2.47 | 2.53 | 2.59 | 2.65 | 2.68 | 2.71 | 2.75 | 2.78 | 2.82 | 2.86 | 2.89 | 2.93 | 2.87 | 3.01 | 3.06 | 3.10 | 3.14 | 3.19 | 3.24 | 3.29 | 3.34 | 25 |
| 66 | 2.39 | 2.44 | 2.49 | 2.55 | 2.61 | 2.67 | 2.70 | 2.74 | 2.77 | 2.81 | 2.84 | 2.88 | 2.92 | 2.96 | 3.00 | 3.04 | 3.08 | 3.13 | 3.17 | 3.22 | 3.27 | 3.31 | 3.37 | 24 |
| 67 | 2.41 | 2.46 | 2.51 | 2.57 | 2.63 | 2.69 | 2.72 | 2.76 | 2.79 | 2.83 | 2.86 | 2.90 | 2.94 | 2.98 | 3.02 | 3.06 | 3.10 | 3.15 | 3.20 | 3.24 | 3.29 | 3.34 | 3.39 | 23 |
| 68 | 2.42 | 2.47 | 2.53 | 2.59 | 2.65 | 2.71 | 2.74 | 2.78 | 2.81 | 2.85 | 2.88 | 2.92 | 2.96 | 3.00 | 3.04 | 3.08 | 3.13 | 3.17 | 3.22 | 3.26 | 3.31 | 3.36 | 3.42 | 22 |
| 69 | 2.44 | 2.49 | 2.55 | 2.61 | 2.67 | 2.73 | 2.76 | 2.80 | 2.83 | 2.87 | 2.90 | 2.94 | 2.98 | 3.02 | 3.06 | 3.10 | 3.15 | 3.19 | 3.24 | 3.29 | 3.34 | 3.39 | 3.44 | 21 |
| **70** | 2.46 | 2.51 | 2.56 | 2.62 | 2.68 | 2.75 | 2.78 | 2.81 | 2.85 | 2.89 | 2.92 | 2.96 | 3.00 | 3.04 | 3.08 | 3.12 | 3.17 | 3.21 | 3.26 | 3.31 | 3.36 | 3.41 | 3.46 | **20** |
| 71 | 2.47 | 2.52 | 2.58 | 2.64 | 2.70 | 2.77 | 2.80 | 2.83 | 2.87 | 2.90 | 2.94 | 2.98 | 3.02 | 3.06 | 3.10 | 3.14 | 3.19 | 3.24 | 3.28 | 3.33 | 3.38 | 3.43 | 3.48 | 19 |
| 72 | 2.49 | 2.54 | 2.59 | 2.65 | 2.72 | 2.78 | 2.81 | 2.85 | 2.88 | 2.92 | 2.96 | 3.00 | 3.04 | 3.08 | 3.12 | 3.16 | 3.21 | 3.25 | 3.30 | 3.35 | 3.40 | 3.45 | 3.50 | 18 |
| 73 | 2.50 | 2.55 | 2.61 | 2.67 | 2.73 | 2.80 | 2.83 | 2.86 | 2.90 | 2.94 | 2.97 | 3.01 | 3.05 | 3.09 | 3.14 | 3.18 | 3.22 | 3.27 | 3.32 | 3.37 | 3.42 | 3.47 | 3.52 | 17 |
| 74 | 2.51 | 2.57 | 2.62 | 2.68 | 2.74 | 2.81 | 2 84 | 2.88 | 2.92 | 2.95 | 2.99 | 3.03 | 3.07 | 3.11 | 3.15 | 3.20 | 3.24 | 3.29 | 3.33 | 3.38 | 3.44 | 3.49 | 3.54 | 16 |
| 75 | 2.52 | 2.58 | 2.64 | 2.70 | 2.76 | 2.82 | 2.86 | 2.89 | 2.93 | 2.97 | 3.00 | 3.04 | 3.08 | 3.13 | 3.17 | 3.21 | 3.26 | 3.30 | 3.35 | 3.40 | 3.45 | 3.50 | 3.56 | 15 |
| 76 | 2.54 | 2.59 | 2.65 | 2.71 | 2.77 | 2.84 | 2.87 | 2.91 | 2.95 | 2.99 | 3.02 | 3.06 | 3.10 | 3.15 | 3.18 | 3.23 | 3.28 | 3.32 | 3.37 | 3.42 | 3.47 | 3.53 | 3.58 | 14 |
| 77 | 2.55 | 2.60 | 2.66 | 2.72 | 2.78 | 2.85 | 2.88 | 2.92 | 2.95 | 2.99 | 3.03 | 3.07 | 3.11 | 3.15 | 3.19 | 3.24 | 3.29 | 3.33 | 3.38 | 3.43 | 3.48 | 3.54 | 3.59 | 13 |
| 78 | 2.56 | 2.61 | 2.67 | 2.73 | 2.79 | 2.86 | 2.89 | 2.93 | 2.97 | 3.00 | 3.04 | 3.08 | 3.12 | 3.16 | 3.21 | 3.25 | 3.30 | 3.34 | 3.39 | 3.44 | 3.49 | 3.55 | 3.60 | 12 |
| 79 | 2.57 | 2.62 | 2.68 | 2.74 | 2.80 | 2.87 | 2.91 | 2.94 | 2.98 | 3.02 | 3.05 | 3.09 | 3.13 | 3.18 | 3.22 | 3.26 | 3.31 | 3.36 | 3.41 | 3.46 | 3.51 | 3.56 | 3.62 | 11 |
| **80** | 2.57 | 2.63 | 2.69 | 2.75 | 2.81 | 2.88 | 2.91 | 2.95 | 2.99 | 3.02 | 3.06 | 3.10 | 3.14 | 3.19 | 3.23 | 3.27 | 3.32 | 3.37 | 3.42 | 3.47 | 3.52 | 3.57 | 3.63 | **10** |
| 81 | 2.58 | 2.64 | 2.69 | 2.76 | 2.82 | 2.89 | 2.92 | 2.96 | 3.00 | 3.03 | 3.07 | 3.11 | 3.15 | 3.20 | 3.24 | 3.28 | 3.33 | 3.38 | 3.43 | 3.48 | 3.53 | 3.58 | 3.64 | 9 |
| 82 | 2.59 | 2.64 | 2.70 | 2.76 | 2.83 | 2.90 | 2.93 | 2.97 | 3.00 | 3.04 | 3.08 | 3.12 | 3.16 | 3.20 | 3.25 | 3.29 | 3.34 | 3.39 | 3.44 | 3.49 | 3.54 | 3.59 | 3.65 | 8 |
| 83 | 2.59 | 2.65 | 2.71 | 2.77 | 2.83 | 2.90 | 2.94 | 2.97 | 3.01 | 3.05 | 3.09 | 3.13 | 3.17 | 3.21 | 3.26 | 3.30 | 3.35 | 3.40 | 3.45 | 3.49 | 3.55 | 3.60 | 3.66 | 7 |
| 84 | 2.60 | 2.66 | 2.71 | 2.78 | 2.84 | 2.91 | 2.94 | 2.98 | 3.02 | 3.06 | 3.09 | 3.13 | 3.18 | 3.22 | 3.26 | 3.31 | 3.35 | 3.40 | 3.45 | 3.50 | 3.55 | 3.61 | 3.66 | 6 |
| 85 | 2.60 | 2.66 | 2.72 | 2.78 | 2.84 | 2.91 | 2.95 | 2.98 | 3.02 | 3.06 | 3.10 | 3.14 | 3.18 | 3.22 | 3.27 | 3.31 | 3.36 | 3.41 | 3.46 | 3.51 | 3.56 | 3.61 | 3.67 | 5 |
| 86 | 2.61 | 2.66 | 2.72 | 2.78 | 2.85 | 2.92 | 2.95 | 2.99 | 3.03 | 3.06 | 3.10 | 3.14 | 3.19 | 3.23 | 3.27 | 3.32 | 3.36 | 3.41 | 3.46 | 3.51 | 3.57 | 3.62 | 3.68 | 4 |
| 87 | 2.61 | 2.67 | 2.72 | 2.79 | 2.85 | 2.92 | 2.95 | 2.99 | 3.03 | 3.07 | 3.11 | 3.15 | 3.19 | 3.23 | 3.28 | 3.32 | 3.37 | 3.42 | 3.47 | 3.52 | 3.57 | 3.62 | 3.68 | 3 |
| 88 | 2.61 | 2.67 | 2.73 | 2.79 | 2.85 | 2.92 | 2.96 | 2.99 | 3.03 | 3.07 | 3.11 | 3.15 | 3.19 | 3.23 | 3.28 | 3.32 | 3.37 | 3.42 | 3.47 | 3.52 | 3.57 | 3.62 | 3.68 | 2 |
| 89 | 2.61 | 2.67 | 2.73 | 2.79 | 2.86 | 2.92 | 2.96 | 3.00 | 3.03 | 3.07 | 3.11 | 3.15 | 3.19 | 3.24 | 3.28 | 3.33 | 3.37 | 3.42 | 3.47 | 3.52 | 3.57 | 3.63 | 3.68 | 1 |
| **90** | 2.61 | 2.67 | 2.73 | 2.79 | 2.86 | 2.92 | 2.96 | 3.00 | 3.03 | 3.07 | 3.11 | 3.15 | 3.19 | 3.24 | 3.28 | 3.33 | 3.37 | 3.4 | 3.47 | 3.52 | 3.57 | 3.63 | 3.68 | **0** |

Collimation factor C = sec. $\delta$ See last line.

Azimuth factor A = sin ζ sec δ. Star's declination ± δ. Inclination factor B = cos ζ sec δ.

| ζ | 74½° | 74¾° | 75° | 75¼° | 75½° | 75¾° | 76° | 76¼° | 76½° | 76¾° | 77° | 77¼° | 77½° | 77¾° | 78° | 78¼° | 78½° | 78¾° | 79° | 79¼° | 79½° | 79¾° | 80° | ζ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1° | .06 | .07 | .07 | .07 | .07 | .07 | .07 | .07 | .07 | .08 | .08 | .08 | .08 | .08 | .08 | .09 | .09 | .09 | .09 | .09 | .10 | .10 | .10 | 89° |
| 2 | .13 | .13 | .13 | .14 | .14 | .14 | .14 | .15 | .15 | .15 | .16 | .16 | .16 | .16 | .17 | .17 | .18 | .18 | .18 | .19 | .19 | .20 | .20 | 88 |
| 3 | .20 | .20 | .20 | .21 | .21 | .21 | .22 | .22 | .22 | .23 | .23 | .24 | .24 | .25 | .25 | .26 | .26 | .27 | .27 | .28 | .29 | .29 | .30 | 87 |
| 4 | .26 | .27 | .27 | .27 | .28 | .28 | .29 | .29 | .30 | .30 | .31 | .32 | .32 | .33 | .34 | .34 | .35 | .36 | .37 | .37 | .38 | .39 | .40 | 86 |
| 5 | .33 | .33 | .34 | .34 | .35 | .35 | .36 | .37 | .37 | .38 | .39 | .40 | .40 | .41 | .42 | .43 | .44 | .45 | .46 | .47 | .48 | .49 | .50 | 85 |
| 6 | .39 | .40 | .40 | .41 | .42 | .42 | .43 | .44 | .45 | .46 | .46 | .47 | .49 | .49 | .51 | .51 | .52 | .54 | .55 | .56 | .57 | .59 | .60 | 84 |
| 7 | .46 | .46 | .47 | .48 | .49 | .50 | .50 | .51 | .52 | .53 | .54 | .55 | .56 | .57 | .59 | .60 | .61 | .62 | .64 | .65 | .67 | .69 | .70 | 83 |
| 8 | .52 | .53 | .54 | .55 | .56 | .57 | .58 | .59 | .60 | .61 | .62 | .63 | .65 | .66 | .67 | .68 | .70 | .71 | .73 | .75 | .76 | .78 | .80 | 82 |
| 9 | .58 | .59 | .60 | .61 | .62 | .64 | .65 | .66 | .67 | .68 | .70 | .71 | .72 | .74 | .75 | .77 | .78 | .80 | .82 | .84 | .86 | .88 | .90 | 81 |
| **10** | .65 | .66 | .67 | .68 | .69 | .71 | .72 | .73 | .74 | .76 | .77 | .79 | .80 | .82 | .84 | .85 | .87 | .89 | .91 | .93 | .95 | .98 | 1.00 | **80** |
| 11 | .71 | .73 | .74 | .75 | .76 | .77 | .79 | .80 | .82 | .83 | .85 | .86 | .88 | .90 | .92 | .94 | .96 | .98 | 1.00 | 1.02 | 1.05 | 1.07 | 1.10 | 79 |
| 12 | .78 | .79 | .80 | .82 | .83 | .85 | .86 | .88 | .89 | .91 | .92 | .94 | .96 | .98 | 1.00 | 1.02 | 1.04 | 1.07 | 1.09 | 1.11 | 1.14 | 1.17 | 1.20 | 78 |
| 13 | .84 | .86 | .87 | .88 | .90 | .91 | .93 | .95 | .96 | .98 | 1.00 | 1.02 | 1.04 | 1.06 | 1.08 | 1.10 | 1.13 | 1.15 | 1.18 | 1.21 | 1.23 | 1.26 | 1.30 | 77 |
| 14 | .91 | .92 | .94 | .95 | .97 | .98 | 1.00 | 1.02 | 1.04 | 1.06 | 1.08 | 1.10 | 1.12 | 1.14 | 1.16 | 1.19 | 1.21 | 1.24 | 1.27 | 1.30 | 1.33 | 1.36 | 1.39 | 76 |
| 15 | .97 | .98 | 1.00 | 1.02 | 1.03 | 1.05 | 1.07 | 1.09 | 1.11 | 1.13 | 1.15 | 1.17 | 1.20 | 1.22 | 1.25 | 1.27 | 1.30 | 1.33 | 1.36 | 1.39 | 1.42 | 1.46 | 1.49 | 75 |
| 16 | 1.03 | 1.05 | 1.06 | 1.08 | 1.10 | 1.12 | 1.14 | 1.16 | 1.18 | 1.20 | 1.23 | 1.25 | 1.28 | 1.30 | 1.33 | 1.35 | 1.38 | 1.41 | 1.44 | 1.48 | 1.51 | 1.55 | 1.59 | 74 |
| 17 | 1.09 | 1.11 | 1.13 | 1.15 | 1.17 | 1.19 | 1.21 | 1.23 | 1.25 | 1.28 | 1.30 | 1.32 | 1.35 | 1.38 | 1.40 | 1.44 | 1.47 | 1.50 | 1.53 | 1.57 | 1.60 | 1.64 | 1.68 | 73 |
| 18 | 1.16 | 1.17 | 1.19 | 1.21 | 1.23 | 1.25 | 1.28 | 1.30 | 1.32 | 1.35 | 1.37 | 1.40 | 1.43 | 1.46 | 1.49 | 1.52 | 1.55 | 1.58 | 1.62 | 1.66 | 1.70 | 1.74 | 1.78 | 72 |
| 19 | 1.22 | 1.24 | 1.26 | 1.28 | 1.30 | 1.32 | 1.35 | 1.37 | 1.39 | 1.42 | 1.45 | 1.47 | 1.51 | 1.53 | 1.57 | 1.60 | 1.63 | 1.67 | 1.71 | 1.75 | 1.79 | 1.83 | 1.87 | 71 |
| **20** | 1.28 | 1.30 | 1.32 | 1.34 | 1.37 | 1.39 | 1.41 | 1.44 | 1.46 | 1.49 | 1.52 | 1.55 | 1.58 | 1.61 | 1.65 | 1.68 | 1.72 | 1.75 | 1.79 | 1.83 | 1.88 | 1.92 | 1.97 | **70** |
| 21 | 1.34 | 1.36 | 1.38 | 1.41 | 1.43 | 1.46 | 1.48 | 1.51 | 1.54 | 1.56 | 1.59 | 1.62 | 1.65 | 1.69 | 1.72 | 1.76 | 1.80 | 1.84 | 1.88 | 1.92 | 1.97 | 2.01 | 2.06 | 69 |
| 22 | 1.40 | 1.42 | 1.45 | 1.47 | 1.50 | 1.52 | 1.55 | 1.58 | 1.60 | 1.63 | 1.66 | 1.70 | 1.73 | 1.77 | 1.80 | 1.84 | 1.88 | 1.92 | 1.96 | 2.01 | 2.06 | 2.11 | 2.16 | 68 |
| 23 | 1.46 | 1.49 | 1.51 | 1.54 | 1.56 | 1.59 | 1.62 | 1.64 | 1.67 | 1.70 | 1.74 | 1.77 | 1.81 | 1.84 | 1.88 | 1.92 | 1.96 | 2.00 | 2.05 | 2.09 | 2.14 | 2.20 | 2.25 | 67 |
| 24 | 1.52 | 1.55 | 1.57 | 1.60 | 1.63 | 1.65 | 1.68 | 1.71 | 1.74 | 1.77 | 1.81 | 1.84 | 1.88 | 1.92 | 1.96 | 2.00 | 2.04 | 2.08 | 2.13 | 2.18 | 2.23 | 2.29 | 2.34 | 66 |
| 25 | 1.58 | 1.61 | 1.63 | 1.66 | 1.69 | 1.72 | 1.75 | 1.78 | 1.81 | 1.84 | 1.88 | 1.91 | 1.95 | 1.99 | 2.03 | 2.07 | 2.12 | 2.17 | 2.22 | 2.27 | 2.32 | 2.38 | 2.43 | 65 |
| 26 | 1.64 | 1.67 | 1.69 | 1.72 | 1.75 | 1.78 | 1.81 | 1.84 | 1.88 | 1.91 | 1.95 | 1.99 | 2.02 | 2.07 | 2.11 | 2.15 | 2.20 | 2.25 | 2.30 | 2.35 | 2.41 | 2.46 | 2.52 | 64 |
| 27 | 1.70 | 1.73 | 1.75 | 1.78 | 1.81 | 1.85 | 1.88 | 1.91 | 1.95 | 1.98 | 2.02 | 2.06 | 2.10 | 2.14 | 2.18 | 2.23 | 2.28 | 2.33 | 2.38 | 2.43 | 2.49 | 2.55 | 2.61 | 63 |
| 28 | 1.76 | 1.78 | 1.81 | 1.84 | 1.87 | 1.91 | 1.94 | 1.97 | 2.01 | 2.05 | 2.09 | 2.13 | 2.17 | 2.21 | 2.26 | 2.31 | 2.36 | 2.41 | 2.46 | 2.52 | 2.58 | 2.64 | 2.70 | 62 |
| 29 | 1.81 | 1.84 | 1.87 | 1.90 | 1.94 | 1.97 | 2.00 | 2.04 | 2.08 | 2.11 | 2.15 | 2.20 | 2.24 | 2.28 | 2.33 | 2.38 | 2.43 | 2.48 | 2.54 | 2.60 | 2.66 | 2.73 | 2.79 | 61 |
| **30** | 1.87 | 1.90 | 1.93 | 1.96 | 2.00 | 2.03 | 2.07 | 2.10 | 2.14 | 2.18 | 2.22 | 2.27 | 2.31 | 2.36 | 2.40 | 2.46 | 2.51 | 2.56 | 2.62 | 2.68 | 2.74 | 2.81 | 2.88 | **60** |
| 31 | 1.93 | 1.96 | 1.99 | 2.02 | 2.06 | 2.09 | 2.13 | 2.17 | 2.21 | 2.25 | 2.29 | 2.33 | 2.38 | 2.43 | 2.48 | 2.53 | 2.58 | 2.64 | 2.70 | 2.76 | 2.83 | 2.89 | 2.97 | 59 |
| 32 | 1.98 | 2.01 | 2.05 | 2.08 | 2.12 | 2.15 | 2.19 | 2.23 | 2.27 | 2.31 | 2.36 | 2.40 | 2.45 | 2.50 | 2.55 | 2.60 | 2.66 | 2.72 | 2.78 | 2.84 | 2.91 | 2.98 | 3.05 | 58 |
| 33 | 2.04 | 2.07 | 2.10 | 2.14 | 2.18 | 2.21 | 2.25 | 2.29 | 2.33 | 2.38 | 2.42 | 2.47 | 2.52 | 2.57 | 2.62 | 2.67 | 2.73 | 2.79 | 2.85 | 2.92 | 2.99 | 3.06 | 3.14 | 57 |
| 34 | 2.09 | 2.13 | 2.16 | 2.20 | 2.23 | 2.27 | 2.31 | 2.35 | 2.40 | 2.44 | 2.49 | 2.53 | 2.58 | 2.64 | 2.69 | 2.75 | 2.80 | 2.87 | 2.93 | 3.00 | 3.07 | 3.14 | 3.22 | 56 |
| 35 | 2.15 | 2.18 | 2.22 | 2.25 | 2.29 | 2.33 | 2.37 | 2.41 | 2.46 | 2.50 | 2.55 | 2.60 | 2.65 | 2.70 | 2.76 | 2.82 | 2.88 | 2.94 | 3.01 | 3.08 | 3.15 | 3.23 | 3.30 | 55 |
| 36 | 2.20 | 2.24 | 2.27 | 2.31 | 2.35 | 2.39 | 2.43 | 2.47 | 2.52 | 2.56 | 2.61 | 2.66 | 2.72 | 2.77 | 2.83 | 2.89 | 2.95 | 3.01 | 3.08 | 3.15 | 3.23 | 3.30 | 3.38 | 54 |
| 37 | 2.25 | 2.29 | 2.33 | 2.36 | 2.40 | 2.44 | 2.49 | 2.53 | 2.58 | 2.63 | 2.67 | 2.73 | 2.78 | 2.84 | 2.90 | 2.95 | 3.02 | 3.08 | 3.15 | 3.23 | 3.30 | 3.38 | 3.47 | 53 |
| 38 | 2.30 | 2.34 | 2.38 | 2.42 | 2.46 | 2.50 | 2.55 | 2.59 | 2.64 | 2.69 | 2.74 | 2.79 | 2.85 | 2.90 | 2.96 | 3.02 | 3.09 | 3.16 | 3.23 | 3.30 | 3.38 | 3.46 | 3.55 | 52 |
| 39 | 2.35 | 2.39 | 2.43 | 2.47 | 2.51 | 2.56 | 2.60 | 2.65 | 2.70 | 2.75 | 2.80 | 2.85 | 2.91 | 2.97 | 3.03 | 3.09 | 3.16 | 3.23 | 3.30 | 3.37 | 3.45 | 3.53 | 3.62 | 51 |
| **40** | 2.40 | 2.44 | 2.48 | 2.52 | 2.57 | 2.61 | 2.66 | 2.70 | 2.75 | 2.80 | 2.86 | 2.91 | 2.97 | 3.03 | 3.09 | 3.16 | 3.22 | 3.29 | 3.37 | 3.45 | 3.53 | 3.61 | 3.70 | **50** |
| 41 | 2.45 | 2.49 | 2.53 | 2.58 | 2.62 | 2.66 | 2.71 | 2.76 | 2.81 | 2.86 | 2.92 | 2.97 | 3.03 | 3.09 | 3.16 | 3.22 | 3.29 | 3.36 | 3.44 | 3.52 | 3.60 | 3.69 | 3.78 | 49 |
| 42 | 2.50 | 2.54 | 2.58 | 2.63 | 2.67 | 2.72 | 2.77 | 2.81 | 2.87 | 2.92 | 2.97 | 3.03 | 3.09 | 3.15 | 3.22 | 3.29 | 3.36 | 3.43 | 3.51 | 3.59 | 3.67 | 3.76 | 3.85 | 48 |
| 43 | 2.55 | 2.59 | 2.63 | 2.68 | 2.72 | 2.77 | 2.82 | 2.87 | 2.92 | 2.98 | 3.03 | 3.09 | 3.15 | 3.21 | 3.28 | 3.35 | 3.42 | 3.50 | 3.57 | 3.66 | 3.74 | 3.83 | 3.93 | 47 |
| 44 | 2.60 | 2.64 | 2.68 | 2.73 | 2.77 | 2.82 | 2.87 | 2.92 | 2.98 | 3.03 | 3.09 | 3.15 | 3.21 | 3.27 | 3.34 | 3.41 | 3.48 | 3.56 | 3.64 | 3.72 | 3.81 | 3.91 | 4.00 | 46 |
| 45 | 2.65 | 2.69 | 2.73 | 2.78 | 2.82 | 2.87 | 2.92 | 2.97 | 3.03 | 3.08 | 3.14 | 3.20 | 3.27 | 3.33 | 3.49 | 3.47 | 3.55 | 3.62 | 3.71 | 3.79 | 3.88 | 3.97 | 4.07 | 45 |

| ζ | 74½° | 74¾° | 75° | 75¼° | 75½° | 75¾° | 76° | 76¼° | 76½° | 76¾° | 77° | 77¼° | 77½° | 77¾° | 78° | 78¼° | 78½° | 78¾° | 79° | 79¼° | 79½° | 79¾° | 80° | ζ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 46° | 2.69 | 2.73 | 2.78 | 2.82 | 2.87 | 2.92 | 2.97 | 3.03 | 3.08 | 3.14 | 3.20 | 3.26 | 3.32 | 3.39 | 3.46 | 3.53 | 3.61 | 3.69 | 3.77 | 3.86 | 3.95 | 4.04 | 4.14 | 44° |
| 47 | 2.74 | 2.78 | 2.83 | 2.87 | 2.92 | 2.97 | 3.02 | 3.08 | 3.13 | 3.19 | 3.25 | 3.31 | 3.38 | 3.45 | 3.52 | 3.59 | 3.67 | 3.75 | 3.83 | 3.92 | 4.01 | 4.11 | 4.21 | 43 |
| 48 | 2.78 | 2.82 | 2.87 | 2.92 | 2.97 | 3.02 | 3.07 | 3.13 | 3.18 | 3.24 | 3.30 | 3.37 | 3.43 | 3.50 | 3.57 | 3.65 | 3.73 | 3.81 | 3.89 | 3.98 | 4.08 | 4.18 | 4.28 | 42 |
| 49 | 2.82 | 2.87 | 2.92 | 2.96 | 3.01 | 3.07 | 3.12 | 3.18 | 3.23 | 3.29 | 3.35 | 3.42 | 3.49 | 3.56 | 3.63 | 3.71 | 3.79 | 3.87 | 3.96 | 4.05 | 4.14 | 4.24 | 4.35 | 41 |
| **50** | 2.87 | 2.91 | 2.96 | 3.01 | 3.06 | 3.11 | 3.17 | 3.22 | 3.28 | 3.34 | 3.41 | 3.47 | 3.54 | 3.61 | 3.68 | 3.76 | 3.84 | 3.93 | 4.02 | 4.11 | 4.20 | 4.30 | 4.41 | **40** |
| 51 | 2.91 | 2.95 | 3.00 | 3.05 | 3.10 | 3.16 | 3.21 | 3.27 | 3.33 | 3.39 | 3.45 | 3.52 | 3.59 | 3.66 | 3.74 | 3.82 | 3.90 | 3.98 | 4.07 | 4.17 | 4.26 | 4.37 | 4.48 | 39 |
| 52 | 2.95 | 3.00 | 3.04 | 3.09 | 3.15 | 3.20 | 3.26 | 3.31 | 3.38 | 3.44 | 3.50 | 3.57 | 3.64 | 3.71 | 3.79 | 3.87 | 3.95 | 4.04 | 4.13 | 4.22 | 4.32 | 4.43 | 4.54 | 38 |
| 53 | 2.99 | 3.04 | 3.09 | 3.14 | 3.19 | 3.24 | 3.30 | 3.36 | 3.42 | 3.48 | 3.55 | 3.62 | 3.69 | 3.77 | 3.84 | 3.92 | 4.01 | 4.09 | 4.19 | 4.28 | 4.38 | 4.49 | 4.60 | 37 |
| 54 | 3.03 | 3.08 | 3.13 | 3.18 | 3.23 | 3.29 | 3.34 | 3.40 | 3.47 | 3.53 | 3.60 | 3.67 | 3.74 | 3.81 | 3.89 | 3.97 | 4.06 | 4.15 | 4.24 | 4.34 | 4.44 | 4.55 | 4.66 | 36 |
| 55 | 3.07 | 3.11 | 3.16 | 3.22 | 3.27 | 3.33 | 3.39 | 3.45 | 3.51 | 3.57 | 3.64 | 3.71 | 3.78 | 3.86 | 3.94 | 4.02 | 4.11 | 4.20 | 4.29 | 4.39 | 4.50 | 4.60 | 4.72 | 35 |
| 56 | 3.10 | 3.15 | 3.20 | 3.26 | 3.31 | 3.37 | 3.43 | 3.49 | 3.55 | 3.62 | 3.68 | 3.76 | 3.83 | 3.91 | 3.99 | 4.07 | 4.16 | 4.25 | 4.34 | 4.44 | 4.55 | 4.66 | 4.77 | 34 |
| 57 | 3.14 | 3.19 | 3.24 | 3.29 | 3.35 | 3.41 | 3.47 | 3.53 | 3.59 | 3.66 | 3.73 | 3.80 | 3.88 | 3.95 | 4.04 | 4.12 | 4.21 | 4.30 | 4.39 | 4.50 | 4.60 | 4.72 | 4.83 | 33 |
| 58 | 3.17 | 3.22 | 3.28 | 3.33 | 3.39 | 3.45 | 3.51 | 3.57 | 3.63 | 3.70 | 3.77 | 3.84 | 3.92 | 4.00 | 4.08 | 4.16 | 4.25 | 4.35 | 4.44 | 4.55 | 4.65 | 4.77 | 4.88 | 32 |
| 59 | 3.21 | 3.26 | 3.31 | 3.37 | 3.42 | 3.48 | 3.54 | 3.61 | 3.67 | 3.74 | 3.81 | 3.88 | 3.96 | 4.04 | 4.12 | 4.21 | 4.30 | 4.39 | 4.49 | 4.60 | 4.70 | 4.82 | 4.94 | 31 |
| **60** | 3.24 | 3.29 | 3.35 | 3.40 | 3.46 | 3.52 | 3.58 | 3.64 | 3.71 | 3.78 | 3.85 | 3.92 | 4.00 | 4.08 | 4.17 | 4.25 | 4.34 | 4.44 | 4.54 | 4.64 | 4.75 | 4.87 | 4.99 | **30** |
| 61 | 3.27 | 3.33 | 3.38 | 3.44 | 3.49 | 3.55 | 3.62 | 3.68 | 3.75 | 3.82 | 3.89 | 3.96 | 4.04 | 4.12 | 4.21 | 4.29 | 4.39 | 4.48 | 4.58 | 4.69 | 4.80 | 4.92 | 5.04 | 29 |
| 62 | 3.30 | 3.36 | 3.41 | 3.47 | 3.53 | 3.59 | 3.65 | 3.72 | 3.78 | 3.85 | 3.92 | 4.00 | 4.08 | 4.16 | 4.25 | 4.34 | 4.43 | 4.53 | 4.63 | 4.73 | 4.85 | 4.96 | 5.08 | 28 |
| 63 | 3.33 | 3.39 | 3.44 | 3.50 | 3.56 | 3.62 | 3.68 | 3.75 | 3.82 | 3.89 | 3.96 | 4.04 | 4.12 | 4.20 | 4.29 | 4.38 | 4.47 | 4.57 | 4.67 | 4.78 | 4.89 | 5.01 | 5.13 | 27 |
| 64 | 3.36 | 3.42 | 3.47 | 3.53 | 3.59 | 3.65 | 3.72 | 3.78 | 3.85 | 3.92 | 4.00 | 4.07 | 4.15 | 4.24 | 4.32 | 4.41 | 4.51 | 4.61 | 4.71 | 4.82 | 4.93 | 5.05 | 5.18 | 26 |
| 65 | 3.39 | 3.45 | 3.50 | 3.56 | 3.62 | 3.68 | 3.75 | 3.81 | 3.88 | 3.95 | 4.03 | 4.11 | 4.19 | 4.27 | 4.36 | 4.45 | 4.55 | 4.65 | 4.75 | 4.86 | 4.97 | 5.09 | 5.22 | 25 |
| 66 | 3.42 | 3.47 | 3.53 | 3.59 | 3.65 | 3.71 | 3.78 | 3.84 | 3.91 | 3.99 | 4.06 | 4.14 | 4.22 | 4.31 | 4.40 | 4.49 | 4.58 | 4.68 | 4.79 | 4.90 | 5.01 | 5.14 | 5.26 | 24 |
| 67 | 3.44 | 3.50 | 3.56 | 3.62 | 3.68 | 3.74 | 3.81 | 3.87 | 3.94 | 4.02 | 4.09 | 4.17 | 4.26 | 4.34 | 4.43 | 4.52 | 4.62 | 4.72 | 4.82 | 4.94 | 5.05 | 5.18 | 5.30 | 23 |
| 68 | 3.47 | 3.53 | 3.58 | 3.64 | 3.70 | 3.77 | 3.83 | 3.90 | 3.97 | 4.05 | 4.12 | 4.20 | 4.28 | 4.37 | 4.46 | 4.55 | 4.65 | 4.75 | 4.86 | 4.97 | 5.09 | 5.21 | 5.34 | 22 |
| 69 | 3.49 | 3.55 | 3.61 | 3.67 | 3.73 | 3.79 | 3.86 | 3.93 | 4.00 | 4.07 | 4.15 | 4.23 | 4.32 | 4.40 | 4.49 | 4.58 | 4.68 | 4.79 | 4.89 | 5.00 | 5.12 | 5.25 | 5.38 | 21 |
| **70** | 3.52 | 3.57 | 3.63 | 3.69 | 3.75 | 3.82 | 3.89 | 3.95 | 4.03 | 4.10 | 4.18 | 4.25 | 4.34 | 4.43 | 4.52 | 4.61 | 4.71 | 4.82 | 4.93 | 5.04 | 5.16 | 5.28 | 5.41 | **20** |
| 71 | 3.54 | 3.60 | 3.65 | 3.71 | 3.78 | 3.84 | 3.91 | 3.98 | 4.05 | 4.13 | 4.20 | 4.28 | 4.37 | 4.46 | 4.55 | 4.64 | 4.74 | 4.85 | 4.96 | 5.07 | 5.19 | 5.32 | 5.45 | 19 |
| 72 | 3.56 | 3.62 | 3.67 | 3.74 | 3.80 | 3.86 | 3.93 | 4.00 | 4.07 | 4.15 | 4.23 | 4.31 | 4.39 | 4.48 | 4.57 | 4.67 | 4.77 | 4.88 | 4.98 | 5.10 | 5.22 | 5.34 | 5.48 | 18 |
| 73 | 3.58 | 3.64 | 3.69 | 3.76 | 3.82 | 3.89 | 3.95 | 4.02 | 4.10 | 4.17 | 4.25 | 4.33 | 4.42 | 4.51 | 4.60 | 4.70 | 4.80 | 4.90 | 5.01 | 5.13 | 5.25 | 5.37 | 5.51 | 17 |
| 74 | 3.60 | 3.65 | 3.71 | 3.78 | 3.84 | 3.91 | 3.97 | 4.04 | 4.12 | 4.19 | 4.27 | 4.36 | 4.44 | 4.53 | 4.62 | 4.72 | 4.82 | 4.93 | 5.04 | 5.15 | 5.27 | 5.40 | 5.53 | 16 |
| 75 | 3.61 | 3.67 | 3.73 | 3.79 | 3.86 | 3.92 | 3.99 | 4.06 | 4.14 | 4.21 | 4.29 | 4.38 | 4.46 | 4.55 | 4.65 | 4.74 | 4.84 | 4.95 | 5.06 | 5.18 | 5.30 | 5.43 | 5.56 | 15 |
| 76 | 3.64 | 3.69 | 3.75 | 3.82 | 3.88 | 3.94 | 4.01 | 4.08 | 4.16 | 4.23 | 4.31 | 4.40 | 4.48 | 4.57 | 4.67 | 4.76 | 4.87 | 4.97 | 5.09 | 5.20 | 5.32 | 5.45 | 5.59 | 14 |
| 77 | 3.65 | 3.70 | 3.76 | 3.83 | 3.89 | 3.96 | 4.03 | 4.10 | 4.17 | 4.25 | 4.33 | 4.41 | 4.50 | 4.59 | 4.68 | 4.78 | 4.89 | 4.99 | 5.11 | 5.22 | 5.35 | 5.47 | 5.61 | 13 |
| 78 | 3.66 | 3.72 | 3.78 | 3.84 | 3.91 | 3.97 | 4.04 | 4.11 | 4.19 | 4.27 | 4.35 | 4.43 | 4.52 | 4.61 | 4.70 | 4.80 | 4.91 | 5.01 | 5.13 | 5.24 | 5.37 | 5.50 | 5.63 | 12 |
| 79 | 3.67 | 3.73 | 3.79 | 3.86 | 3.92 | 3.99 | 4.06 | 4.13 | 4.21 | 4.28 | 4.36 | 4.45 | 4.54 | 4.63 | 4.72 | 4.82 | 4.92 | 5.03 | 5.14 | 5.26 | 5.39 | 5.52 | 5.65 | 11 |
| **80** | 3.68 | 3.74 | 3.81 | 3.87 | 3.93 | 4.00 | 4.07 | 4.14 | 4.22 | 4.30 | 4.38 | 4.46 | 4.55 | 4.64 | 4.74 | 4.84 | 4.94 | 5.05 | 5.16 | 5.28 | 5.40 | 5.54 | 5.67 | **10** |
| 81 | 3.70 | 3.75 | 3.82 | 3.88 | 3.94 | 4.01 | 4.08 | 4.16 | 4.23 | 4.31 | 4.39 | 4.48 | 4.56 | 4.65 | 4.75 | 4.85 | 4.95 | 5.06 | 5.18 | 5.30 | 5.42 | 5.55 | 5.69 | 9 |
| 82 | 3.71 | 3.76 | 3.83 | 3.89 | 3.96 | 4.02 | 4.09 | 4.17 | 4.24 | 4.32 | 4.40 | 4.49 | 4.57 | 4.67 | 4.76 | 4.86 | 4.97 | 5.08 | 5.19 | 5.31 | 5.43 | 5.56 | 5.70 | 8 |
| 83 | 3.72 | 3.77 | 3.84 | 3.90 | 3.96 | 4.03 | 4.10 | 4.18 | 4.25 | 4.33 | 4.41 | 4.50 | 4.59 | 4.68 | 4.78 | 4.87 | 4.98 | 5.09 | 5.20 | 5.32 | 5.45 | 5.58 | 5.72 | 7 |
| 84 | 3.72 | 3.78 | 3.84 | 3.91 | 3.97 | 4.04 | 4 11 | 4.18 | 4.26 | 4.34 | 4.42 | 4.51 | 4.60 | 4.69 | 4.79 | 4.88 | 4.99 | 5.10 | 5.21 | 5.33 | 5.46 | 5.59 | 5.73 | 6 |
| 85 | 3.73 | 3.79 | 3.85 | 3.91 | 3.98 | 4.05 | 4.12 | 4.19 | 4.27 | 4.35 | 4.43 | 4.51 | 4.60 | 4.69 | 4.79 | 4.89 | 5.00 | 5.11 | 5.22 | 5.34 | 5.47 | 5.60 | 5.74 | 5 |
| 86 | 3.73 | 3.79 | 3.85 | 3.92 | 3.98 | 4.05 | 4.12 | 4.20 | 4.27 | 4.35 | 4.43 | 4.52 | 4.61 | 4.70 | 4.80 | 4.90 | 5.00 | 5.11 | 5.23 | 5.35 | 5.47 | 5.61 | 5.74 | 4 |
| 87 | 3.74 | 3.79 | 3.86 | 3.92 | 3.99 | 4.06 | 4.13 | 4.20 | 4.28 | 4.36 | 4.44 | 4.52 | 4.62 | 4.71 | 4.81 | 4.90 | 5.01 | 5.12 | 5.23 | 5.35 | 5.48 | 5.61 | 5.75 | 3 |
| 88 | 3.74 | 3.80 | 3.86 | 3.92 | 3.99 | 4.06 | 4.13 | 4.20 | 4.28 | 4.36 | 4.44 | 4.53 | 4.62 | 4.71 | 4.81 | 4.91 | 5.01 | 5.12 | 5.24 | 5.36 | 5.48 | 5.61 | 5.75 | 2 |
| 89 | 3.74 | 3.80 | 3.86 | 3.93 | 3.99 | 4.06 | 4.13 | 4.21 | 4.28 | 4.36 | 4.44 | 4.53 | 4.62 | 4.71 | 4.81 | 4.91 | 5.01 | 5.12 | 5.24 | 5.36 | 5.49 | 5.62 | 5.76 | 1 |
| **90** | 3.74 | 3.80 | 3.86 | 3.93 | 3.99 | 4.06 | 4.13 | 4.21 | 4.28 | 4.36 | 4.45 | 4.53 | 4.62 | 4.71 | 4.81 | 4.91 | 5.02 | 5.13 | 5.24 | 5.36 | 5.49 | 5.62 | 5.76 | **0** |

Collimation factor C=sec $\delta$. See last line.

# APPENDIX No. 10.

## DETERMINATION OF THE ASTRONOMICAL LATITUDE OF A STATION BY MEANS OF THE ZENITH TELESCOPE.

[Prepared for the Coast Survey Manual by C. A. Schott, Assistant.]

(1.) *General remarks on Talcott's method.*—The method of determining the latitude or the declination of the zenith by means of the zenith telescope has been repeatedly described;* it was originally devised by Captain Andrew Talcott, late of the United States corps of engineers, and was designed about the year 1834. It substitutes micrometric measures of small differences of zenith distances in the place of measures of large arcs, and it is to this improvement that the accuracy and facility of the method is chiefly due. In its application to latitudes we measure the small difference of zenith distances of two stars culminating on opposite sides of the zenith, at nearly the same altitude, and not far apart in time.

(2.) Since its introduction in the Coast Survey in 1846, the instrument has received some modifications to adapt it more fully to its use for latitude determinations, and it has now superseded all other instruments designed for the same purpose. The telescope turning freely through the zenith, and its horizontal axis being capable of accurate levelling, it was proposed by the late R. H. Fauntleroy, assistant United States Coast Survey, to determine also the local time by means of this instrument, and it has been so used, giving results of sufficient precision for the determination of the latitude. On the other hand, Professor C. S. Lyman, of New Haven, suggested and practically illustrated† the temporary conversion of the ordinary transit instrument into a zenith telescope by attaching to the former the delicate level and micrometer; the great advantage of this combination, both in regard to economy and facility of transportation, need not here be pointed out. Professor Chauvenet also remarks that the instrument may be applied for finding the longitude from equal zenith distances of the moon's limb and a neighboring star.

As the time does not require to be known with great precision, the nearest second or two being ample, it may be obtained either by a sextant, using the method of equal altitudes, by a small portable transit or by the zenith telescope itself; in the latter case a meridian mark with two lamps, differing from the middle line exactly by the distance of the vertical axis of the telescope from its line of collimation, will be found convenient.

(3.) *Description of the instrument.*—Referring to the accompanying plate (No. 28) exhibiting the instrument as now constructed, a brief and general description of it will suffice. The telescope has an aperture of about 3½ inches, and a focal length of about 45 inches, and admits of observing with convenience, stars of the 6½ or 7th magnitude. The magnifying power used varies between 60 and 120. The tube turns fully round a horizontal axis of about 7 inches in length, and is balanced by a weight in such manner as to prevent unequal pressure and flexure of the axis. This axis is supplied with a striding level, and when horizontal the line of collimation of the telescope will move in a vertical plane; it is perforated and polished, and the light of a lamp passes through it for the illumination of the micrometer and other threads.* The vertical axis is supported by a column about 24 inches in height, and at its lower end carries a clamp and vernier in connection with the azimuth circle. The horizontal circle is about 12 inches in diameter, and is graduated to read to half min-

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* See The Journal of the Franklin Institute, (Philadelphia,) of October, 1838. It contains remarks upon the method and an abstract of results, by Professor Courtenay;

A report by Major Emory, United States topographical engineers, in connection with the northeastern boundary survey of the United States;

A pamphlet by Captain T. J. Lee, United States topographical engineers, and assistant United States Coast Survey, dated April, 1848;

A pamphlet by the late A. D. Bache, Superintendent of the United States Coast Survey, New Haven, 1852, being extracted from the American Journal of Science and Arts, vol. xiv, second series. This paper was read before the American Association for the Advancement of Science, at their fifth meeting in Cincinnati—*vide* vol. v, p. 151;

An article in the United States Coast Survey Report for 1857. pp. 324–334; and

An account in Chauvenet's Manual of Spherical and Practical Astronomy: Philadelphia, 1863, vol. ii, pp. 340–366.

† "On the transit instrument as a substitute for the zenith telescope in determining latitude, and on the latitude of New Haven." By Professor C. S. Lyman. American Journal of Science and Arts, vol. xxx, July, 1860.

utes or less; two movable stops can be applied to it, defining on the instrument the position of the plane of the meridian, and without interfering with the motion while reversing. The whole rests upon three foot-screws, by means of which the instrument can be levelled.

A delicate level (value of one division about ¾ of a second) at right angles to the horizontal axis is connected with the telescope, and revolves on a centre so as to indicate the inclination of the telescope; concentric with the level pivot and firmly connected with the tube is a graduated twelve-inch semicircle, or a six-inch full circle, on which zenith distances are read off to within 30″ by means of the arm and vernier attached to the movable level. The telescope can be set to any inclination and clamped; for accurate pointing the bubble is brought into the middle by a fine-motion screw.

The micrometer screw carries a movable thread for the measure of the difference of zenith distances; its head is divided in 100 parts, of which tenths may be estimated; the whole number of turns are read off by means of a rack shown on the side of the field of view. The value of one revolution of the micrometer is about 45″. There are also two fixed threads parallel to the micrometer thread, about 15′ or 20′ apart, indicating the range between which the latter is ordinarily employed. To provide for the case of transit observations there are also five equidistant vertical threads inserted symmetrically over the optical centre. For convenience of observing, the telescope is supplied with a prismatic eye-piece.

The instrument may be mounted on blocks of stone or wood.

(4.) *Adjustment of the instrument.*—When setting up it will be found convenient to place two of the foot-screws in an east and west line; the adjustments of the instrument may then be made as follows: The vertical axis is to be made truly vertical by means of a striding level which should not change when the instrument is made to describe a complete revolution in azimuth. The verticality should also be tested by the more sensitive level of the setting circle in order to avoid large level corrections or a change in the position of the telescope. The horizontality of the transit axis is tested by the reversal of the striding level. The eye-piece is next adjusted to sidereal focus by means of the definition of a circumpolar star and the threads of the diaphragm are properly focused. It is important that this adjustment should not be disturbed during the observations, and to make sure of it a leaden collar is sometimes employed to keep the sliding tube in position. The horizontality of the micrometer thread is proved by an equatorial star running along the thread, or by the same appearance, of a polar star, when the instrument is turned in azimuth, and the verticality of the system of transit threads may either be inferred from this last adjustment or may be tested by the bisection of a distant well-defined terrestrial object, when the telescope is slightly elevated or depressed. The same terrestrial object may be used for the adjustment of the line of collimation, which may be effected by two positions of the instrument exactly 180° apart in the readings of the azimuth circle and making allowance for eccentricity; thus let $d$ = distance of vertical axis from the line of collimation of the telescope, D = distance of object, and $p$ = parallax, then $p = \frac{d}{D \sin 1''}$. A perfect adjustment, however, may be made by means of two collimating telescopes, or by the method employed when using a transit instrument in two positions of the clamp for the same purpose.

The reading on the horizontal circle, of the plane of the meridian, is ascertained by means of the known chronometer time of the culmination of a slow-moving star, which is bisected at that time by the middle thread and the corresponding reading of the circle noted; clamps are then applied to indicate the meridional position with the telescope pointing north or south of the zenith.

(5.) *Selection of stars for observation.*—The observer will next prepare an observing list of pairs of stars, containing the catalogue number of the star, its magnitude, the right ascension and declination, the zenith distance, north or south, and the middle zenith distance of the pair, or the setting. The weak point of the method is want of sufficient accuracy in the catalogue star places; they may deviate as often one way as the other without doing more than increasing the probable error of a resulting latitude, but when the errors are of a constant nature they seriously affect any deduced latitude which may depend upon them; hence too much care cannot be bestowed upon a proper selection of pairs from the catalogues, and only those should be taken which have a satisfactory or, at least, more than one authority. The catalogues usually employed are those of the

British Association and of the Greenwich Observatory;* they will furnish, generally, from one to three dozen of pairs in our latitudes for almost any night in the year. The programme should commence with stars of the earliest right ascensions permitting observation on account of daylight and be continued to as early a morning hour as the observer may find expedient. If there is an abundance of suitable pairs for the period, two lists may be made out covering the same time, but observable on alternate nights. In selecting stars, suitable ones, culminating *sub polo*, should be included, and catalogues should be particularly examined for stars passing so near the zenith as to be within range of the micrometer with the instrument pointing north or south.

The latitude of the station requires to be known only within one or two minutes, which degree of approximation may be had either from a chart, sextant observations, or by means of the finding circle of the transit instrument, or of the zenith telescope itself.

The two stars forming a pair should culminate at nearly equal zenith distances, one north, the other south of the zenith, and their difference of zenith distance should, if possible, not much exceed one-half the breadth of the field of the telescope to avoid observing near its edge; about 15′ (or at most 20′) is the greatest range for our instruments. The interval of time between the culmination of stars forming a pair should not be less than one minute, so as to give time, deliberately, to read the micrometer and to turn the instrument in azimuth for observing the second star, and should not exceed about 20 minutes, to guard against possible changes in the state of the instrument. The interval between any two pairs should afford time for reading the micrometer and level and for setting the instrument preparatory to the next pair, for this three minutes suffice for most observers. If the intervals between the pairs are unavoidably long, they may be filled up by observing transits for time. Stars as low as the 7th magnitude may be selected, their places are, however, generally not so well determined; on the other hand brighter stars are too few in number.

It is desirable to select the pairs with regard to their difference of zenith distances, making the sum of all the positive micrometer corrections equal to the sum of all the negative corrections, which condition leaves the final latitude free of any effect from error in the value of the micrometer screw.

No precise limit can be given of the greatest zenith distance compatible with the requirements of the method, but it may be readily extended to 25° and beyond. The following specimen of a list of selected pairs of stars will serve to show its arrangement.

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*The catalogue of stars of the British Association, &c., &c., reduced to January 1, 1850, &c., &c., by the late Francis Baily, London, 1845.

Catalogue of 2,156 stars formed from the observations made during 12 years, from 1836 to 1847, at the Royal Observatory, Greenwich. London, 1849.

Catalogue of 1,576 stars formed from the observations made during six years, from 1848 to 1853, at the Royal Observatory, Greenwich, and reduced to the epoch 1850. London, 1856.

Seven-year catalogue of 2,022 stars reduced from observations between 1854 and 1860, at the Royal Observatory, Green wich, and reduced to the epoch 1860. Volume for 1862.

Pairs of stars proposed for observation during August and September, 1856, with zenith telescope No. 5, for latitude of station Mount Desert, Maine. Approximate $\varphi = 44^\circ\ 21'.1$.

| Star No. | Catalogue. | Mag. | $\alpha$ | $\delta$ | $\zeta$ | N. or S. | Setting. |
|---|---|---|---|---|---|---|---|
| | | | h. m. s. | ° ′ | ° ′ | | ° ′ |
| 6062 | B. A. C. | 5½ | 17 47 24 | + 40 01 | 4 20 | S. | |
| 6068 | " | 5½ | 48 36 | 40 02 | 4 19 | S. | 4 13 |
| 6129 | " | 5 | 59 23 | 48 28 | 4 07 | N. | |
| 6255 | " | 5 | 18 17 52 | 49 03 | 4 42 | N. | |
| 6268 | " | 5½ | 19 29 | 39 26 | 4 55 | S. | 4 46 |
| 6357 | " | 6 | 33 21 | 39 33 | 4 48 | S. | |
| 6395 | " | 5 | 39 50 | 55 24 | 11 03 | N. | |
| 6429 | " | 3 | 44 47 | 33 12 | 11 09 | S. | 11 07 |
| 6522 | " | 6 | 57 53 | 55 27 | 11 06 | N. | |
| 6553 | " | 6 | 19 01 59 | 32 17 | 12 04 | S. | 12 10 |
| 6583 | " | 5 | 08 57 | 56 37 | 12 16 | N. | |
| 6629 | " | 6½ | 15 28 | 62 57 | 18 36 | N. | 18 29 |
| 6637 | " | 6 | 16 58 | 25 59 | 18 22 | S. | |
| &c. | | | | | | | |

The columns headed $\alpha\ \delta\ \zeta$ contain the approximate right ascension, declination, and zenith distance of each star.

(6.) *Directions for observing.*—The instrument being adjusted and the line of collimation of the telescope placed in the meridian, in which position the azimuthal motion is arrested by the stops, the index of the vertical circle is set to the mean zenith distance of the first pair taken from the list previously prepared, and on which the chronometer time of culmination of each star for the night is noted. The telescope is then directed to that side of the zenith where the first star will culminate, and the bubble of the level is made to play very nearly in the middle. As soon as the star enters the field, and when transiting on one of the vertical wires, or at any convenient number of seconds before the culmination, the observer will pick up the beat of the chronometer and bisect the star with the micrometer thread at the instant of culmination; the level and micrometer is then read, the instrument is revolved 180°, and the second star is observed in the same manner. During these observations the tangent screw of the vertical circle must not be touched, though the tangent screw, which gives a slow motion to the telescope, (and consequently also to the level,) can be used after the reversal of the instrument in the exceptional case where the vertical axis of the instrument is not well adjusted.

(7.) If for some reasons the meridian observation fails, the star may be bisected off the meridian and the time noted, either by moving the telescope in azimuth and bisecting in the line of collimation, or by observing the star off the middle of the field, leaving the telescope undisturbed in the plane of the meridian. The latter method is generally the preferable one, particularly when the star culminates near the zenith. If, however, the meridional distance of the star be considerable the first method had better be followed.

Though the star may be bisected several times while passing through the field, in our experience little is gained in multiplying observations upon the same star under *the same circumstances*. The relative accuracy of a single observation, and of the position of a star assigned by the catalogues, points to the multiplication of stars rather than to that of repeated pointings of the same star.

It is not advisable to combine more than one north star with more than one south star, for the reason that greater accuracy is gained by observing pairs at different nights, and in case of any defect in the position assigned to any of the stars thus combined it would be difficult to detect the faulty one. It is preferable, therefore, to break up combinations into pairs. We have, however, many cases where one star enters as a component of a pair with different stars.

Each pair is generally observed on five or six nights; a greater number of observations would add but very little to the value of the mean result, as will be seen in the discussion of the relative weights.

(8.) *General expression for the latitude.*—Let $\zeta$ and $\zeta'$ equal the true meridional zenith distance of the southern and northern star, and $\delta$ and $\delta'$ the declination of the same, respectively, then the expression for the latitude is

$$\varphi = \tfrac{1}{2}(\delta + \delta') + \tfrac{1}{2}(\zeta - \zeta').$$

Now, if $z$ $z'$ denote the observed zenith distance of the south and north star, $n$ $s$ the north and south reading of the level for the south star, and $n'$ $s'$ the same for the north star, $b$ the value of one division of level, $r$ and $r'$ the refraction correction, and $m$ and $m'$ the reduction to the meridian for the south and north star, respectively, then—

$$\varphi = \tfrac{1}{2}(\delta+\delta') + \tfrac{1}{2}(z-z') + \frac{b}{4}\left\{ (n+n') - (s+s') \right\} + \tfrac{1}{2}(r-r') + \tfrac{1}{2}(m'-m);$$

and if M and M′ be the micrometer readings of the south and north star, the micrometer being supposed to read *from* the zenith, and R the value of one division, then—

$$\tfrac{1}{2}(z-z') = \tfrac{1}{2}(M-M')R.$$

If the micrometer reads *towards* the zenith, (the direction appears, of course, inverted in the astronomical telescope,) change M—M′ into M′—M; and it may be remarked here, that during half of the observations at the station the instrument may be used in the reversed position of the telescope with regard to the vertical axis, thus varying the circumstances under which measures are taken.

(9.) *Determination of the value of a division of the micrometer.*—Different methods have been used for this purpose; the one formerly most employed was by turning the micrometer at right angles to the position in which it is used for making latitude observations and noting the times of the passage of a close circumpolar star near culmination over the micrometer thread placed successively before the star for each turn or half-turn of the screw; now, let $\tau$ = interval (converted into seconds of arc) from culmination, and $\delta$ = the star's declination, then the sine of the angular distance from the meridian = $\sin \tau \cos \delta$, and the differences of quantities thus computed, divided by the corresponding differences in the screw readings, give the value of one division. The treatment of a set of observations by application of the method of least squares is given by J. E. Hilgard, Assistant United States Coast Survey, in Gould's Astronomical Journal, No. 36, (Cambridge, March 13, 1852.) Another method formerly employed was to measure the angular space covered by a well-defined distant terrestrial object by means of a good theodolite, and also by means of the micrometer screw, from which the value of the latter will readily result. The method, however, introduced in 1847 by C. O. Boutelle, Assistant United States Coast Survey, and now almost exclusively used, consists of observing a close circumpolar star near its elongation, when rapidly rising or falling, accompanied with but a slight motion in azimuth; this method avoids the risk of a disturbance in the focal adjustments—it requires the reading of the level in order to allow for possible changes, and necessitates a correction for differential refraction. By $\cos t_e = \cot \delta \tan \varphi$ and $\cos \zeta_e = \operatorname{cosec} \delta \sin \varphi$ we find the star's hour angle $t_e$ and zenith distance $\zeta_e$ at elongation, and if $\alpha$= star's right ascension, and $\Delta T$ = chronometer correction, then—

$$\text{Chronometer time of elongation} = \alpha - \Delta T \pm t_e \text{ where } \left\{ \begin{matrix} + \\ - \end{matrix} \right\} \text{ for } \left\{ \begin{matrix} \text{western} \\ \text{eastern} \end{matrix} \right\} \text{ elongation.}$$

About 40 or more minutes before the elongation the telescope is directed to the star, and transits are noted, the micrometer thread being set in advance, consecutively, by whole or half-turns of the screw, throughout its length. A correction for rate of chronometer should be applied, if sensible. It is well to note the temperature, since the value of the screw may vary with a change of temperature.

Let $t$=difference of time of observation and elongation of the star, and $z''$=number of seconds of arc in the direction of the vertical from elongation, then $z'' = \frac{\cos \delta \sin t}{\sin 1''}$, for which we can write

$$z'' = 15 \cos \delta \left\{ t - \tfrac{1}{6}(15 \sin 1'')^2 t^3 \right\}$$

where $t$ is expressed in seconds of time. It is convenient to apply the term $\frac{1}{6}(15 \sin 1'')^2 t^3$ to the observed time of noting, additive to the observed time before, and subtractive after, either elongation. The following table gives the value of $\frac{1}{6}(15 \sin 1'')^2 t^3$ for every minute of time from elongation up to $44^m$:

| $t$ | Term. | $t$ | Term. | $t$ | Term. | $t$ | Term. |
|---|---|---|---|---|---|---|---|
| m. | s. | m. | s. | m. | s. | m. | s. |
| 5 | 0.0 | 15 | 0.6 | 25 | 3.0 | 35 | 8.2 |
| 6 | 0.0 | 16 | 0.8 | 26 | 3.3 | 36 | 8.9 |
| 7 | 0.0 | 17 | 0.9 | 27 | 3.7 | 37 | 9.6 |
| 8 | 0.1 | 18 | 1.1 | 28 | 4.2 | 38 | 10.4 |
| 9 | 0.1 | 19 | 1.3 | 29 | 4.6 | 39 | 11.3 |
| 10 | 0.2 | 20 | 1.5 | 30 | 5.1 | 40 | 12.2 |
| 11 | 0.2 | 21 | 1.8 | 31 | 5.7 | 41 | 13.1 |
| 12 | 0.3 | 22 | 2.0 | 32 | 6.2 | 42 | 14.1 |
| 13 | 0.4 | 23 | 2.3 | 33 | 6.8 | 43 | 15.1 |
| 14 | 0.5 | 24 | 2.6 | 34 | 7.5 | 44 | 16.2 |

The correction to be applied to the observed times of noting for change of level is given by the formula

$$\pm \left\{ \tfrac{1}{2}(n-s)-\tfrac{1}{2}(n_0-s_0) \right\} \frac{b}{15 \cos \delta},$$

where $n_0$ $s_0$ the north and south readings for a selected state of level, $n$ $s$ the north and south readings for any other state, and $b$ the value of one division of level in seconds of arc; the upper sign is to be used for western, the lower sign for eastern elongation.

After these two corrections have been applied to the observed times of noting, we have in one column the readings of the micrometer, and in another the corresponding times, such as would have been observed if the star had moved uniformly in a vertical line, leaving out of consideration, for the present, the change in refraction. Various methods of combination might be adopted for the determination of the turn of the screw; that followed in the example, where we subtract the values resulting from the first observation from those of the middle one, next those of the second from those of the middle one plus one, and so on, recommends itself for its simplicity, and is probably only inferior to that which employs the method of least squares. We thus obtain a number of values for the time of a given number of turns or half-turns, from which we deduce the value of one turn by the formula given above; the correction for refraction (in seconds of arc) is negative for either eastern or western elongation, and equals the change of refraction for the space equal to one turn = value of one turn times difference of refraction for 1′ at star's altitude, and divided by 60. The probable error of the resulting value of one turn is readily found; see example appended to this paper. If we wish to proceed with the utmost rigor the method of least squares should be applied, a development of which is given in Chauvenet's article on the zenith telescope, above cited, page 363. It is sufficiently explained by the following statement and the example. Let $M_0$= the unknown reading of micrometer for the time of elongation or for the middle time of any one set of observations near elongation, or for $T_0$; also, $M_1$= an assumed approximate value for $M_0$, and $\mu$ its correction; also, $R_1$= an assumed approximate value for R, and $\rho$ its correction; then—

$$M_0=M_1+\mu, \text{ and } R=R_1+\rho.$$

If we now subtract each micrometer reading M from $M_0$, and each corresponding time from $T_0$, and also convert these intervals into differences of zenith distance, or into $z-z_0$, using the first term of our formula, and put $n=z-z_0-(M_1-M)R_1$, we have for each observation the conditional equation $n=R_1\mu+(M_1-M)\rho$, from which we form the normal equations in the usual way and deduce the two quantities $\mu$ and $\rho$. The additional labor is considerable, and since the result differs only from that found by the preceding method by a small fraction of the probable error of R, we may, in all ordinary cases, dispense with its application.*

* In case no special observations for value of micrometer have been made we may still find it from the latitude observations themselves. Let R= approximate value of one turn of the micrometer as used in the computation, and $dR$ its correction; also, $\phi$=the latitude resulting from all the pairs by the use of R and $d\phi$ its correction. Let $M_1$ $M_2$ $M_3$ . . . =half the mean

It is hardly necessary to remark that a number of sets of observations, for value of one turn of screw, are usually taken, and their results are combined to a mean.

(10.) *Determination of the value of one division of the level.*—The value of one division of the level may be found in different ways, according to the means available. The temperature should be noted, since the result may change with a change of temperature. The value may be found directly with a level trier or by attaching the level to a well-divided vertical circle and measuring directly the angular value passed over by a change of inclination of a given number of divisions in the position of the bubble; a distant object may be sighted as a mark, or, better, a second instrument may be used as a collimator and in connection with it; the angular space is measured with the micrometer screw, the value of which is already known. To employ a star for a mark renders the determination unnecessarily complex. In the example appended, the value of a division of level is found in terms of the micrometer screw, the bubble is made to traverse the whole length of graduation, and the micrometer differences corresponding to the displacements of the bubble by a change of inclination are measured by pointing on a collimator; such observations, in particular, should include those divisions of the level which come most commonly into use during the observations for latitude.

(11.) *Correction for differential refraction.*—The difference of refraction for any pair of stars is so small that we can neglect the variation in the state of the atmosphere at the time of the observation from that mean state supposed in the refraction tables. The refraction being nearly proportional to the tangent of the zenith distance, the difference of refraction for the two stars will be given by

$$r - r' = 57''.7 \sin(z - z') \sec^2 z;$$

and since the difference of zenith distances is measured by the micrometer, the following table of correction to the latitude for differential refraction has been prepared for the argument ½ difference of zenith distance, or ½ difference of micrometer reading on the side, and the argument "zenith distance" on the top. The sign of the correction is the same as that of the micrometer difference.

| ½ diff. in zenith distance. | Zenith distance. | | | | | | ½ diff. in zenith distance. | Zenith distance. | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0° | 10° | 20° | 25° | 30° | 35° | | 0° | 10° | 20° | 25° | 30° | 35° |
| ′ | ″ | ″ | ″ | ″ | ″ | ″ | ′ | ″ | ″ | ″ | ″ | ″ | ″ |
| 0 | .00 | .00 | .00 | .00 | .00 | .00 | 6.5 | .11 | .11 | .12 | .13 | .14 | .16 |
| 0.5 | .01 | .01 | .01 | .01 | .01 | .01 | 7 | .12 | .12 | .13 | .14 | .15 | .18 |
| 1 | .02 | .02 | .02 | .02 | .02 | .02 | 7.5 | .13 | .13 | .14 | .15 | .16 | .19 |
| 1.5 | .02 | .03 | .03 | .03 | .03 | .03 | 8 | .13 | .14 | .15 | .16 | .18 | .21 |
| 2 | .03 | .03 | .04 | .04 | .04 | .05 | 8.5 | .14 | .15 | .16 | .17 | .19 | .22 |
| 2.5 | .04 | .04 | .05 | .05 | .05 | .06 | 9 | .15 | .16 | .17 | .18 | .20 | .23 |
| 3 | .05 | .05 | .06 | .06 | .07 | .08 | 9.5 | .16 | .17 | .18 | .20 | .21 | .24 |
| 3.5 | .06 | .06 | .07 | .07 | .08 | .09 | 10 | .17 | .18 | .19 | .21 | .23 | .26 |
| 4 | .07 | .07 | .08 | .08 | .09 | .10 | 10.5 | .18 | .19 | .20 | .22 | .24 | .27 |
| 4.5 | .08 | .08 | .09 | .09 | .10 | .11 | 11 | .18 | .19 | .21 | .23 | .25 | .28 |
| 5 | .08 | .09 | .10 | .10 | .11 | .13 | 11.5 | .19 | .20 | .22 | .24 | .26 | .30 |
| 5.5 | .09 | .10 | .10 | .11 | .12 | .14 | 12 | .20 | .21 | .23 | .25 | .27 | .31 |
| 6 | .10 | .10 | .11 | .12 | .13 | .15 | | | | | | | |

difference of micrometer readings of the south and north stars of the several pairs, $\phi_1\ \phi_2\ \phi_3$ . . . the results for latitude by the several pairs; we then have the conditional equations—

$$M_1 dR - d\phi = \phi - \phi_1$$
$$M_2 dR - d\phi = \phi - \phi_2$$
&c.,

which gives the normal equations for finding $dR$ and $d\phi$—

$$\Sigma M dR - \Sigma d\phi = \Sigma(\phi - \phi_0)$$
$$\Sigma M^2 dR - \Sigma M d\phi = \Sigma M(\phi - \phi_0)$$

If weights are given to the several pairs depending upon the probable error of declination of stars and upon the number of observations on a pair, they may readily be introduced in the above normal equations. To find a reliable value, however, by this method, it is essential that the errors in the catalogue places of stars should be as small as possible.

(12.) *Reduction to the meridian.*—First, when the line of collimation of the telescope is off the meridian, the instrument having been revolved in azimuth, and the star observed at the hour angle $\tau$, near the middle thread, then

$$m = \frac{2\sin^2 \frac{1}{2}\tau}{\sin 1''} \cdot \frac{\cos\varphi\cos\delta}{\sin\zeta}$$

and the correction to the latitude, if the two stars are observed off the meridian, $= \frac{1}{2}(m' - m)$ as given in Art. (8.) The value of

$$\frac{2\sin^2 \frac{1}{2}\tau}{\sin 1''}$$

for every second of time up to two minutes (a star being rarely observed at a greater distance than this from the meridian in zenith telescope observations) is given in the following table:

| $\tau$. | Term. | $\tau$. | Term. | $\tau$. | Term. | $\tau$. | Term. | $\tau$. | Term. | $\tau$. | Term. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *s.* | $''$ | *s.* | $''$ | *s.* | $''$ | *s.* | $''$ | *s.* | $''$ | *s.* | $''$ |
| 0 | 0.00 | 20 | 0.22 | 40 | 0.87 | 60 | 1.96 | 80 | 3.49 | 100 | 5.45 |
| 1 | 0.00 | 21 | 0.24 | 41 | 0.91 | 61 | 2.03 | 81 | 3.58 | 101 | 5.56 |
| 2 | 0.00 | 22 | 0.26 | 42 | 0.96 | 62 | 2.10 | 82 | 3.67 | 102 | 5.67 |
| 3 | 0.00 | 23 | 0.28 | 43 | 1.01 | 63 | 2.16 | 83 | 3.76 | 103 | 5.78 |
| 4 | 0.01 | 24 | 0.31 | 44 | 1.06 | 64 | 2.23 | 84 | 3.85 | 104 | 5.90 |
| 5 | 0.01 | 25 | 0.34 | 45 | 1.10 | 65 | 2.31 | 85 | 3.94 | 105 | 6.01 |
| 6 | 0.02 | 26 | 0.37 | 46 | 1.15 | 66 | 2.38 | 86 | 4.03 | 106 | 6.13 |
| 7 | 0.02 | 27 | 0.40 | 47 | 1.20 | 67 | 2.45 | 87 | 4.12 | 107 | 6.24 |
| 8 | 0.03 | 28 | 0.43 | 48 | 1.26 | 68 | 2.52 | 88 | 4.22 | 108 | 6.36 |
| 9 | 0.04 | 29 | 0.46 | 49 | 1.31 | 69 | 2.60 | 89 | 4.32 | 109 | 6.48 |
| 10 | 0.05 | 30 | 0.49 | 50 | 1.36 | 70 | 2.67 | 90 | 4.42 | 110 | 6.60 |
| 11 | 0.06 | 31 | 0.52 | 51 | 1.42 | 71 | 2.75 | 91 | 4.52 | 111 | 6.72 |
| 12 | 0.08 | 32 | 0.56 | 52 | 1.48 | 72 | 2.83 | 92 | 4.62 | 112 | 6.84 |
| 13 | 0.09 | 33 | 0.59 | 53 | 1.53 | 73 | 2.91 | 93 | 4.72 | 113 | 6.96 |
| 14 | 0.11 | 34 | 0.63 | 54 | 1.59 | 74 | 2.99 | 94 | 4.82 | 114 | 7.09 |
| 15 | 0.12 | 35 | 0.67 | 55 | 1.65 | 75 | 3.07 | 95 | 4.92 | 115 | 7.21 |
| 16 | 0.14 | 36 | 0.71 | 56 | 1.71 | 76 | 3.15 | 96 | 5.03 | 116 | 7.34 |
| 17 | 0.16 | 37 | 0.75 | 57 | 1.77 | 77 | 3.23 | 97 | 5.13 | 117 | 7.46 |
| 18 | 0.18 | 38 | 0.80 | 58 | 1.83 | 78 | 3.32 | 98 | 5.24 | 118 | 7.60 |
| 19 | 0.20 | 39 | 0.83 | 59 | 1.89 | 79 | 3.40 | 99 | 5.34 | 119 | 7.72 |

Secondly, when the star is observed off the line of collimation, the instrument remaining in the plane of the meridian, then

$$m = \frac{2\sin^2 \frac{1}{2}\tau}{\sin 1''}\sin\delta\cos\delta, \text{ or } m = \frac{2\sin^2 \frac{1}{2}\tau}{\sin 1''} \cdot \frac{1}{2}\sin 2\delta$$

and the correction to the latitude is $\frac{1}{2}$ of this quantity, whether the star be north or south; and if the two stars forming a pair are observed off the line of collimation, two such corrections, separately computed, must be added to the latitude. If the stars should be south of the equator the essential sign of the correction is negative. The value of $m$ for every 5° of declination is given in the following table:

| $\delta$. | 10 *s.* | 15 *s.* | 20 *s.* | 25 *s.* | 30 *s.* | 35 *s.* | 40 *s.* | 45 *s.* | 50 *s.* | 55 *s.* | 60 *s.* | $\delta$. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ° | $''$ | $''$ | $''$ | $''$ | $''$ | $''$ | $''$ | $''$ | $''$ | $''$ | $''$ | ° |
| 5 | .00 | .01 | .02 | .03 | .04 | .06 | .08 | .10 | .12 | .14 | .17 | 85 |
| 10 | .01 | .02 | .04 | .06 | .08 | .11 | .15 | .19 | .23 | .28 | .34 | 80 |
| 15 | .01 | .03 | .05 | .09 | .12 | .17 | .22 | .28 | .34 | .41 | .49 | 75 |
| 20 | .02 | .04 | .07 | .11 | .16 | .22 | .28 | .36 | .44 | .53 | .63 | 70 |
| 25 | .02 | .05 | .08 | .13 | .19 | .26 | .34 | .42 | .52 | .63 | .75 | 65 |
| 30 | .02 | .05 | .09 | .15 | .21 | .29 | .38 | .48 | .59 | .71 | .85 | 60 |
| 35 | .03 | .06 | .10 | .16 | .23 | .31 | .41 | .52 | .64 | .77 | .92 | 55 |
| 40 | .03 | .06 | .11 | .17 | .24 | .33 | .43 | .54 | .67 | .81 | .97 | 50 |
| 45 | .03 | .06 | .11 | .17 | .25 | .33 | .44 | .55 | .68 | .82 | .98 | 45 |

(13.) *Record of the observations.*—The observations for latitude are recorded in a note book, ruled to suit the convenience of the observer, and of the following form, or nearly so:

(Record Z. T. latitudes.)

Station ———. Date ——— ———.
Instrument ———. Observer ———.

| No. | Star number. | Catalogue. | North or south. | Micrometer. | | Level. | | Chronometer time of observation. | Remarks. |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Turns. | Divisions. | North. | South. | | |
| | | | | | | | | | |

The first column shows the number of times each pair has been observed; the other columns explain themselves. (See example appended.)

(14.) *Reduction of the observations.*—The reduction will be facilitated by the use of any convenient printed form, of which the following may be taken as a specimen:

Reduction of Z. T. latitudes.

Station ———. Pair ———. ——— of zenith. ———,
———. ——— " " Computer.

| Date. | Micrometer. Difference in reading. | Level. Sum north. Sum south. | Difference of sums. | Mer. distance. | Declination. | Sum and half-sum. | Corrections. | | | | Latitude. | Remarks. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | Micr. | Level. | Refr. | Mer. | | |
| | | | | s. | ° ′ ″ | ° ′ ″ | | | | | | |

(15.) *Discussion of the results.*—The weights of the results by each pair differ with the number of observations upon each pair, with the number of times a star enters into any such combinations, and with the accuracy of the star's position as assigned by the catalogue.

To find the weight due to the number of observations or the probable error of observation, let $n$ = number of observations upon a pair, $p$ = number of pairs, $\Delta$ = difference of latitude between each observed result and the mean result deduced for the respective pair, (see example to Art. 14,) $[\Delta^2]$ = sum of all the $\Delta^2$ for any one pair, and $\Sigma[\Delta^2]$ the sum total from all the pairs; then, if $e$ = probable error of observation,

$$e^2 = \frac{0.455\ \Sigma[\Delta^2]}{n-p}$$

On the average, the value of $e$ from many determinations is rather less than $\pm 0''.50$, the corresponding probable error of an observed zenith distance equals $e\sqrt{2}$ or $\pm 0''.71$.

To find the probable error $e_\phi$ of a resulting latitude by any one pair and the probable error $e_\delta$ of the mean of two declinations, or of $\frac{1}{2}(\delta+\delta')$, we must consider separately the errors of observation $\varepsilon$ and the errors of declination; then

$$e_\delta^2 = e_\phi^2 - \varepsilon^2$$

The value of $e_\phi^2$ is obtained by means of the difference $\Delta\varphi$ of the result for latitude by any one pair and by the mean of all the pairs; then

$$e_\phi^2 = \frac{0.455\ \Sigma\Delta\varphi^2}{p-1}$$

and the value of $\varepsilon^2$ is found by

$$\varepsilon^2 = \frac{e^2}{p-1}\left[\frac{1}{n}\right]$$

The value of $e_\delta$ being thus known, the probable error of a single declination becomes $\varepsilon_\delta = e_\delta \sqrt{2}$ and our weights will be found to differ for different catalogues. If but one catalogue is used its declinations may be considered as affected with the same probable error, provided the authorities and number of observations from which the declinations are derived are the same, but if different catalogues are employed the probable error and weight of any declination may be taken as found for the same authority by our previous experience. The average probable error of any single declination from the Greenwich catalogues may be taken as less than $\pm 0''.5$.

(16.) *Combination of the results by weights.*—If $\varepsilon_\delta$ and $\varepsilon_{\delta 1}$ = probable errors of the declinations of the stars of a pair, and $n$ the number of observations on the pair, then

$$e_\delta = \tfrac{1}{2}\sqrt{(\varepsilon_\delta^2 + \varepsilon_{\delta 1}^2)} \text{ and } e_\phi = \sqrt{e_\delta^2 + \frac{e^2}{n}}$$

hence the weight of the result by a pair

$$\frac{n}{\frac{n}{4}(\varepsilon_\delta^2 + \varepsilon_{\delta 1}^2) + e^2}$$

and for an equal probable error of the declinations the weight

$$\frac{n}{\frac{n}{2}\varepsilon_\delta^2 + e^2}$$

and since the weights need only be proportional numbers we may divide by 4 and use the expressions

$$w = \frac{n}{n(\varepsilon_\delta^2 + \varepsilon_{\delta 1}^2) + 4e^2} \text{ and for equal errors of delination } \frac{n}{2n\varepsilon_\delta^2 + 4e^2}$$

In order to show how little is gained in weight after a pair has been observed on five or six nights, the following table has been computed:

Supposing $e = \pm 0''.50$ and $\varepsilon_\delta = \pm 0''.4$, also $w = \frac{n}{1 + 0.32n}$, then

| for $n$ = | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $w$ = | 0.75 | 1.22 | 1.53 | 1.76 | 1.92 | 2.05 | 2.16 | 2.25 | 2.32 | 2.38 |

To obtain the latitude $\varphi_0$ from the separate results $\varphi_1\ \varphi_2\ \varphi_3$ . . . by each pair, with their weights $w_1\ w_2\ w_3$ . . . , we have

$$\varphi_0 = \frac{[w\varphi]}{[w]}, \text{ with the probable error } \varepsilon_{\varphi_0} = \sqrt{\frac{0.455\,[w\Delta\varphi^2]}{(p-1)\,[w]}}.$$

It sometimes happens that, instead of the ordinary combination, one or more south stars are combined with one or more north stars. In this case the mean of the declinations of the south stars may be combined with the mean of the declinations of the north stars, the corresponding mean of the micrometer and level readings being also used; or else we may combine each south star, for instance, with each of the north stars, and the mean of these separate results of the latter method must equal the result by the entire combination. In general, the formation of a number of pairs is preferable, as errors of observation or of catalogue place are more readily detected. Suppose the more simple case of one star entering into combination with two others, forming doublets, or, entering in combination with three others, forming triplets, &c., and supposing the weight of an ordinary pair = 1, that of each doublet will be $\frac{2}{3}$, and of each triplet = $\frac{1}{2}$, &c., according to the form $\frac{2}{c+1}$, and our former weights $w$ must first be multiplied by this fraction before using them in the combination for $\varphi_0$. Supposing N. (north) combined with S. (south) stars, then the weight (that of an ordinary pair being 1) of the single mean of this combination becomes $\frac{2NS}{N+S}$. In case there are many stars in combination the computation by separate pairs may become troublesome, since the number of such combinations is NS. If all combinations are formed, the weight of any single one is $\frac{2}{N+S}$. If the declinations of stars entering in combination have different weights, the weights of the separate combination to pairs can readily be determined according to the above rules, and their sum will determine the weight of the entire combination, in case the latter form of computation is preferred.

The method of observing several north with several south stars is not now practiced, as observation by pairs is far preferable, and doublets or triplets generally enter now only in such cases where a S. (or N.) star is combined with a certain N. (or S.) star for a number of nights, and with another N. (or S.) star for a number of nights following, and perhaps still another star at a later period.

To Art. (9.)—*Example of observation and reduction of the value of one turn of the micrometer.*

Station Harris, *August* 24, 1855.—Observations on Polaris, near eastern elongation, for value of micrometer of zenith telescope No. 2. Elongation by chronometer, $19^h\ 15^m\ 02^s$. One division of level $= 1''.16$. Daily rate of chronometer, $6^s$, gaining. Temp., 64°.0 Fah. Observer, G. W. D.

| No. | Reading of micrometer turns. | Time by sid. chronometer. | Level readings. North end. | Level readings. South end. | Time from elongation. | Correction for *t*. | Reduction to mean state of level. | Correction for level. | Reduced time. |
|---|---|---|---|---|---|---|---|---|---|
| | | *h. m. s.* | *d.* | *d.* | *m.* | *s.* | *d.* | *s.* | *h. m. s.* |
| 1 | 5 | 18 36 11.5 | ........ | ........ | 38.8 | +11.0 | ........ | +3.0 | 18 36 25.5 |
| 2 | 5.5 | 37 09.7 | 24.9 | 12.2 | 37.8 | 10.2 | +1.00 | +3.0 | 37 22.9 |
| 3 | 6 | 38 10.0 | ........ | ........ | 36.8 | 9.4 | ........ | +2.3 | 38 21.7 |
| 4 | 6.5 | 39 09.5 | 25.2 | 11.6 | 35.8 | 8.7 | +0.55 | +1.7 | 39 19.9 |
| 5 | 7 | 40 10.0 | ........ | ........ | 34.8 | 8.0 | ........ | +2.0 | 40 20.0 |
| 6 | 7.5 | 41 10.0 | 25.1 | 11.7 | 33.8 | 7.3 | +0.65 | +2.3 | 41 19.6 |
| 7 | 8 | 42 11.0 | ........ | ........ | 32.8 | 6.7 | ........ | +2.0 | 42 19.7 |
| 8 | 8.5 | 43 10.0 | 25.2 | 11.6 | 31.8 | 6.1 | +0.55 | +1.7 | 43 17.8 |
| 9 | 9 | 44 08.4 | ........ | ........ | 30.9 | 5.5 | ........ | +1.5 | 44 15.4 |
| 10 | 9.5 | 45 07.2 | 25.3 | 11.5 | 29.9 | 5.0 | +0.45 | +1.2 | 45 12.4 |
| 11 | 10 | 46 08.0 | ........ | ........ | 28.9 | 4.5 | ........ | +1.4 | 46 13.9 |
| 12 | 10.5 | 47 03.0 | 25.2 | 11.5 | 27.9 | 4.1 | +0.50 | +1.5 | 47 08.6 |
| 13 | 11 | 48 04.7 | ........ | ........ | 26.9 | 3.7 | ........ | +1.1 | 48 09.5 |
| 14 | 11.5 | 49 03.0 | 25.4 | 11.2 | 25.9 | 3.3 | +0.25 | +0.8 | 49 07.1 |
| 15 | 12 | 50 02.4 | ........ | ........ | 25.0 | 2.9 | ........ | +0.5 | 50 05.8 |
| 16 | 12.5 | 51 60.0 | 25.6 | 11.1 | 24.0 | 2.6 | +0.10 | +0.3 | 51 02.9 |
| 17 | 13 | 51 56.0 | ........ | ........ | 23.1 | 2.3 | ........ | +0.1 | 51 58.4 |
| 18 | 13.5 | 52 57.2 | 25.7 | 11.0 | 22.1 | 2.0 | 0.00 | 0.0 | 52 59.2 |
| 19 | 14 | 53 52.4 | ........ | ........ | 21.1 | 1.8 | ........ | 0.0 | 53 54.2 |
| 20 | 14.5 | 54 53.0 | 25.7 | 11.0 | 20.1 | 1.6 | 0.00 | 0.0 | 54 54.6 |
| 21 | 15 | 55 54.1 | ........ | ........ | 19.1 | 1.3 | ........ | 0.0 | 55 55.4 |
| 22 | 15.5 | 56 50.9 | 25.7 | 11.0 | 18.2 | 1.1 | 0.00 | 0.0 | 56 52.0 |
| 23 | 16 | 57 48.0 | ........ | ........ | 17.2 | 1.0 | ........ | 0.0 | 57 49.0 |
| 24 | 16.5 | 58 48.4 | 25.7 | 11.0 | 16.2 | 0.8 | 0.00 | 0.0 | 58 49.2 |
| 25 | 17 | 59 47.0 | ........ | ........ | 15.2 | 0.7 | ........ | 0.0 | 59 47.7 |
| 26 | 17.5 | 19 00 44.4 | 25.7 | 11.0 | 14.3 | 0.6 | 0.00 | 0.0 | 19 00 45.0 |
| 27 | 18 | 01 44.4 | ........ | ........ | 13.3 | 0.5 | ........ | −0.1 | 01 44.8 |
| 28 | 18.5 | 02 42.0 | 25.8 | 10.9 | 12.3 | 0.4 | −0.10 | −0.3 | 02 42.1 |
| 29 | 19 | 03 43.4 | ........ | ........ | 11.3 | 0.3 | ........ | −0.2 | 03 43.5 |
| 30 | 19.5 | 04 39.9 | 25.8 | 11.0 | 10.4 | 0.2 | −0.05 | −0.1 | 04 40.0 |
| 31 | 20 | 05 39.8 | ........ | ........ | 9.4 | 0.2 | ........ | −0.3 | 05 39.7 |
| 32 | 20.5 | 06 36.9 | 25.8 | 10.8 | 8.4 | 0.1 | −0.15 | −0.4 | 06 36.6 |
| 33 | 21 | 07 39.0 | ........ | ........ | 7.4 | 0.1 | ........ | −0.4 | 07 38.7 |
| 34 | 21.5 | 08 34.5 | 25.9 | 10.9 | 6.4 | 0.1 | −0.15 | −0.4 | 08 34.2 |
| 35 | 22 | 09 33.0 | ........ | ........ | 5.4 | 0.0 | ........ | −0.5 | 09 32.5 |
| 36 | 22.5 | 10 31.0 | 25.9 | 10.8 | 4.5 | 0.0 | −0.20 | −0.6 | 10 30.4 |
| 37 | 23 | 11 30.7 | ........ | ........ | 3.5 | 0.0 | ........ | −0.6 | 11 30.1 |
| 38 | 23.5 | 12 25.2 | 25.9 | 10.8 | 2.6 | 0.0 | −0.20 | −0.6 | 12 24.6 |
| 39 | 24 | 13 27.0 | ........ | ........ | 1.6 | 0.0 | ........ | −0.7 | 13 26.3 |
| 40 | 24.5 | 14 23.9 | 25.9 | 10.7 | 0.6 | 0.0 | −0.25 | −0.8 | 14 23.1 |
| 41 | 25 | 15 23.0 | ........ | ........ | −0.6 | 0.0 | ........ | −0.9 | 15 22.1 |

| No. | Time for 10 turns. | Δ | $\Delta^2$ | No. | Time for 10 turns. | Δ | $\Delta^2$ |
|---|---|---|---|---|---|---|---|
| | *m. s.* | *s.* | | | *m. s.* | *s.* | |
| 1 to 21 | 19 29.9 | 2.2 | 4.8 | 11 to 31 | 19 25.8 | 1.9 | 3.6 |
| 2 22 | 29.1 | 1.4 | 2.0 | 12 32 | 28.0 | 0.3 | 0.1 |
| 3 23 | 27.3 | 0.4 | 0.2 | 13 33 | 29.2 | 1.5 | 2.2 |
| 4 24 | 29.3 | 1.6 | 2.6 | 14 34 | 27.1 | 0.6 | 0.4 |
| 5 25 | 27.7 | 0.0 | 0.0 | 15 35 | 26.7 | 1.0 | 1.0 |
| 6 26 | 25.4 | 2.3 | 5.3 | 16 36 | 27.5 | 0.2 | 0.0 |
| 7 27 | 25.1 | 2.6 | 6.8 | 17 37 | 31.7 | 4.0 | 16.0 |
| 8 28 | 24.3 | 3.4 | 11.6 | 18 38 | 25.4 | 2.3 | 5.3 |
| 9 29 | 28.1 | 0.4 | 0.2 | 19 39 | 32.1 | 4.4 | 19.4 |
| 10 30 | 26.6 | 1.1 | 1.2 | 20 40 | 28.5 | 0.8 | 0.6 |

| | *s.* | *log's.* |
|---|---|---|
| Mean time for 1 turn - - | 116.774 | 2.06735 |
| | cos $\delta$ | 8.40750 |
| | 15 | 1.17609 |
| | | 1.65094 |
| | $''$ | |
| One turn - - - - - - | =44.765 | |
| Correction for refraction - | — .025 | |
| Correction for rate - - - | — .003 | |
| Resulting value - - | 44.737 | |

Probable error of 10 turns

$$=\sqrt{\frac{0.455\times 83.3}{20\times 19}}=\pm 0^s.31.$$

Probable error of 1 turn $=\pm 0''.012$.

N. B.—Another set of observations immediately follows the above.

For the application of the method of least squares to the above set we prefer to take the 41 original measures of half turns, and find the mean reduced time $T_0=18^h\,55^m\,54^s.6$; and assuming $M_1=15$ and $\frac{1}{2}R_1=22''.4$, the first conditional equation results as follows: $M_1-M=+20$; corresponding difference in reduced time taken from last column of example above, and converted in seconds, $1169^s.1$; which multiplied by $15\cos\delta$ corresponds to $z-z_0=+448''.17$; also, $(M-M)R_1=448''.00$; hence, $n=+0.17$, and the conditional equation becomes $+0.17=22.4\mu+20\rho$. Forming the 41 equations, we find the normal equations, on account of the symmetry of the observations, to become much simplified and to give the unknown quantities directly. The result in the present case is $R=44''.768$, irrespective of refraction and rate. To find the probable error of the determination we must substitute the resulting values $\mu$ and $\rho$ into the conditional equations and proceed by the usual method.

## TO ART. (10.)—*Example of the determination of the value of one division of the level.*

STATION HARRIS, *August* 23, 1855.—Observations for value of a division of level B of zenith telescope No. 2. Collimator eight feet from object glass. Value of one division of micrometer screw (mean of 4 sets) =0″.448. Observer, G. W. D.

NOTE.—Only a part of the observations taken are here given.

| No. | Temp. | Micr. turns. | Level reading. North. | Level reading. South. | Difference of reading. In micr. | Difference of reading. In level. | Value of 1 div. of level. | Δ | $\Delta^2$ |
|---|---|---|---|---|---|---|---|---|---|
| | ° *Fah.* | | *d.* | *d.* | *d.* | *d.* | *d.* | | |
| 1 | 66.2 | 18.94 | 34.2 | 1.8 | 64 | 25.2 | 2.54 | 0.01 | .000 |
| | | 18.30 | 9.0 | 27.0 | | | | | |
| 2 | ........ | 18.26 | 34.7 | 1.4 | 61 | 24.55 | 2.48 | 0.07 | .005 |
| | | 17.65 | 10.0 | 25.8 | | | | | |
| 3 | ........ | 17.61 | 36.0 | 0.0 | 66 | 27.1 | 2.44 | 0.11 | .012 |
| | | 16.98 | 8.8 | 27.0 | | | | | |
| 4 | ........ | 16.95 | 35.5 | .0.0 | 73 | 30.0 | 2.43 | 0.12 | .014 |
| | | 16.22 | 5.5 | 30.0 | | | | | |
| 5 | 66.5 | 16.22 | 34.0 | 1.5 | 74 | 28.45 | 2.60 | 0.05 | .002 |
| | | 15.48 | 5.6 | 30.0 | | | | | |
| 6 | ........ | 15.43 | 34.3 | 1.2 | 76 | 30.85 | 2.46 | 0.09 | .008 |
| | | 14.67 | 3.5 | 32.1 | | | | | |
| 7 | ........ | 14.62 | 31.0 | 4.8 | 81 | 32.35 | 2.50 | 0.05 | .003 |
| | | 13.81 | − 1.5 | 37.0 | | | | | |
| 8 | ........ | 13.77 | 33.4 | 2.2 | 67 | 25.0 | 2.68 | 0.13 | .017 |
| | | 13.10 | 8.2 | 27.0 | | | | | |
| 9 | ........ | 13.07 | 35.0 | 0.2 | 71 | 27.55 | 2.58 | 0.03 | .001 |
| | | 12.36 | 7.5 | 27.8 | | | | | |
| 10 | 67.0 | 12.33 | 35.0 | 0.6 | 67 | 25.3 | 2.65 | 0.10 | .010 |
| | | 11.66 | 9.5 | 25.7 | | | | | |
| 11 | ........ | 11.65 | 30.5 | 4.8 | 60 | 22.35 | 2.68 | 0.13 | .017 |
| | | 11.05 | 8.0 | 27.0 | | | | | |
| 12 | ........ | 11.00 | 33.0 | 1.9 | 68 | 26.25 | 2.59 | 0.04 | .002 |
| | | 10.32 | 6.8 | 28.2 | | | | | |
| Mean...... | | | | | | | 2.55 | Sum.. | 0.091 |

One division of level $B = 2.55 \times 0''.448 = 1''.14$, at temperature 66°.6 Fahr., with a probable error of $\sqrt{\frac{0.455 \times 0.091}{12 \times 11}} = \pm 0^d.018 = \pm 0''.01$.

## TO ART. (13.)—*Example of record.*

Station, Mount Desert. Date, September 4, 1856. Instrument, zenith telescope No. 5. Observer, S. H.

| No. | Star number. | Catalogue. | N. or S. | Micrometer. Turns. | Micrometer. Divisions. | Level. North. | Level. South. | Chronometer time of observation. | Remarks. |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | *h. m. s.* | |
| 4 | 7220 | B. A. C. | N. | 30 | 85.0 | 31.3 | 34.8 | 20 41 51 | Weather fair; clouds flying; |
| | 7256 | " | S. | 39 | 22.0 | 34.9 | 30.5 | 47 56 | wind moderately fresh from SW. |
| | 7721 | " | S. | 22 | 25.0 | 38.7 | 30.0 | 22 02 22 | |
| 6 | 7731 | " | S. | 22 | 52.0 | 38.5 | 30.0 | 03 06 | Bar. 28.65 in. |
| 7 | 7754 | " | N. | 31 | 16.0 | 28.8 | 39.8 | 06 07 | Ther., 65°.2 Fahr. |
| | 7778 | " | N. | 13 | 54.5 | 29.0 | 39.6 | 09 14 | |
| | 7800 | " | N. | 14 | 73.0 | 25.4 | 43.8 | 22 14 36 | |
| | 7803 | " | S. | ........ | ........ | ........ | ........ | ........ | Too faint to observe. |
| 5 | 7855 | " | N. | 16 | 62.0 | 37.1 | 33.4 | 22 24 52 | |
| | 7858 | " | S. | 6 | 75.0 | 28.2 | 42.0 | 25 35 | |
| 7 | 7882 | " | N. | 35 | 39.5 | 43.3 | 27.2 | 29 26 | |
| 5 | 8141 | " | S. | 23 | 13.0 | 20.5 | 52.0 | 23 14 23 | Observed off line of collimation. |
| | 8188 | " | N. | 15 | 88.0 | 51.9 | 21.3 | 22 55 | |
| | &c. | | | | | | | | |

N. B.—Value of one turn of micro. screws, 41″.42; and value of 1 div. of level, 0″.731. Chronometer slow, $30^s.2$.

## TO ART. (14)—*Example of reduction.*

STATION, MOUNT DESERT, MAINE.
(Reduction of Z. T. lat's.)

Pair { 8141 B. A. C., or 5161 Armagh catalogue, (Dublin, 1859.) S. of zenith.
8188 B. A. C., or 1968 G. 7-year catalogue. N. "

| Date. | Microm'r. Diff. in reading. | Level. Sum N. Sum S. | Diff. of sums. | Mer. dist. | Declination. | Sum and half sum. | Corrections. Micr. | Level. | Retr. | Mer. | Latitude. | Δ | Δ² | Remarks. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1856 | t. d. | | | s. | ° ′ ″ | ° ′ ″ | ′ ″ | ″ | ″ | ″ | ° ′ ″ | ″ | | |
| Aug. 26 | −7 19.0 | 68.8 | ...... | ...... | 31 01 41.13 | 88 47 08.41 | | | | | | | | |
| | | | +6.6 | | | | | | | | | | | |
| | | 62.2 | ...... | ...... | 57 45 27.28 | 44 23 34.21 | −2 29.00 | +1.21 | −0.05 | ...... | 44 21 06.37 | 0.43 | 0.185 | |
| 27 | −7 04.0 | 92.7 | ...... | ...... | 41.40 | 09.03 | | | | | | | | |
| | | | −9.0 | | | | | | | | | | | |
| | | 101.7 | ...... | ...... | 27.63 | 34.51 | −2 25.80 | −1.64 | −0.05 | ...... | 07.02 | 0.22 | 0.048 | |
| Sept. 2 | −7 17.0 | 87.9 | ...... | ...... | 42.97 | 12.73 | | | | | | | | |
| | | | −4.9 | | | | | | | | | | | |
| | | 92.8 | ...... | ...... | 29.76 | 36.36 | −2 28.49 | −0.90 | −0.05 | ...... | 06.92 | 0.12 | 0.012 | |
| 3 | −7 26.5 | 92.0 | ...... | ...... | 43.22 | 13.34 | | | | | | | | |
| | | | +4.4 | | | | | | | | | | | |
| | | 87.6 | ...... | ...... | 30.12 | 36.67 | −2 30.46 | +0.80 | −0.05 | ...... | 06.96 | 0.16 | 0.026 | |
| 4 | −7 25.0 | 72.4 | ...... | 33 | 43.47 | 13.95 | ...... | ...... | ...... | ...... | ...... | ...... | ...... | 8141 observed off middle thread. |
| | | | −0.9 | | | | | | | | | | | |
| | | 73.3 | ...... | ...... | 30.48 | 36.97 | −2 30.15 | −0.16 | −0.05 | +0.13 | 06.74 | 0.06 | 0.004 | |
| Mean. | | | | | | | | | | | 44 21 06.80 | Sum. | 0.277 | |

# APPENDIX No. 11.

## DETERMINATION OF THE ASTRONOMICAL AZIMUTH OF A DIRECTION.

[Prepared for the Coast Survey Manual by C. A. Schott, Assistant.]

(1.) It is intended to give in this paper a concise statement of the methods principally employed in the operations of the Coast Survey for the determination of the astronomical azimuth of a triangle side or of a direction, illustrated by specimens of record and examples of computation.

(2.) The astronomical azimuth, or the angle which the plane of the meridian makes with the vertical plane passing through the object whose direction is to be determined, is generally reckoned from the south, and in the direction from south to west; when circumpolar stars are observed it is more convenient to reckon from the north meridian.

(3.) The geodetic azimuth differs from the astronomical azimuth; the former is supposed free from local deviation from the vertical, it being the mean of several astronomical azimuths referred to one station, and in such a case the various deflections may be supposed to neutralize each other; the latter is subject to a displacement of the zenith from local attraction.

(4.) We may distinguish primary and secondary azimuths; the one giving the direction, with respect to the meridian, of sides of the primary triangulation, the other giving the same for sides of secondary or tertiary triangles and for directions in connection with the determination of the magnetic declination.

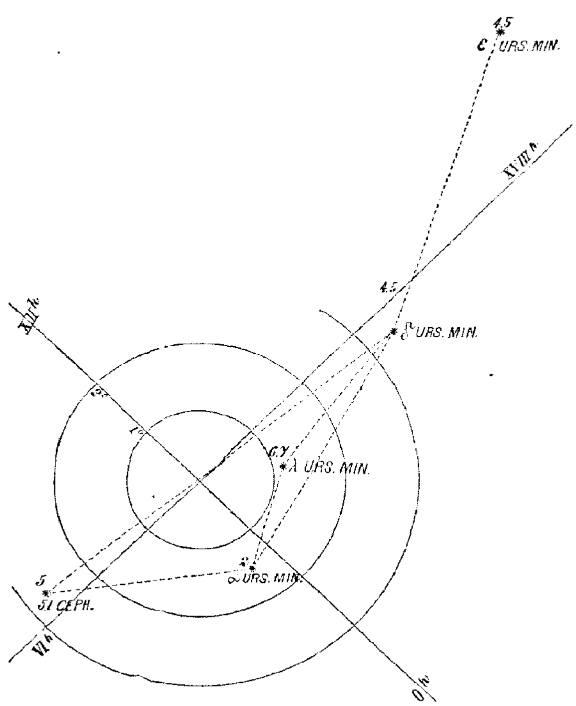
(5.) The determination of a primary azimuth supposes the local time to be known; for secondary azimuths observations for time and azimuth are sometimes made together. The local time for first class azimuths is generally determined by means of a transit instrument, (see preceding article on the determination of time;) for second class azimuths vertical circles or sextants are occasionally employed; with the latter instrument observations of equal altitudes give the most reliable results for time.

(6.) Primary azimuths are generally observed with an astronomical theodolite (altitude and azimuth instrument) of the largest portable size; in the Coast Survey practice theodolites of 30 and 24 inches diameter, also repeating circles of 18 and 12 inches (and others of smaller size) are employed, and if no greater accuracy than the nearest quarter of a minute is required, as in magnetic work, a five-inch theodolite suffices. Transits, either in the plane of the meridian or in that of the elongation of a circumpolar star, are also used exceptionally. The instrument is, of course, carefully adjusted in all its parts before use. A solid stone, firmly imbedded in the ground, gives the greatest stability to the instrument; if mounted on a brick pier or on a heavy block of wood, a coat of oil paint will prevent the action of moisture. Heavy tripods of seasoned wood may also be used. If it becomes necessary to elevate the instrument considerably above the ground, a pier of solid masonry must be constructed.

(7.) For the purpose of referring primary azimuths, observed at night, to the direction of any geodetic signal, a mark is set up, consisting of a perforated box, (about $\frac{3}{4}$ foot cube,) through the front face of which the light of a bull's-eye lantern is shown, appearing of about the size and brilliancy of the star observed upon. The distance of this mark from the station is generally determined by local circumstances, but should, if possible, not be nearer than about a statute mile, in order that the siderial focus of the telescope may not require changing. For day observations a vertical black stripe is painted on the white wand, centrally above and below the aperture, and of the same width; if the diameter of the aperture is a quarter of an inch it will subtend, at the distance of a statute mile, an angle of a little more than $0''.8$. Collimators were also tried; the preference has, however, been given to azimuth marks. The horizontal angle between the mark and any trigonometrical station is measured in connection with the triangulation; in the method of observing with the 30-inch theodolite the direction to the mark is combined with all other directions radiating from the station.

(8.) Let the time, $(t,)$ declination, $(\delta,)$ and latitude $(\varphi)$ be slightly in error by the quantities $dt$, $d\delta$, and $d\varphi$, and let $dA$ equal their effect upon the azimuth, (A;) then, in general, it will be seen that, all other circumstances being equal, $dA$ increases as the zenith distance $(\zeta)$ decreases; for a star near the pole and for a latitude not too high a small error in time and in latitude has but a slight effect upon the azimuth, and in the case of a circumpolar star at the elongation (when the parallactic angle is 90°) a small error in time, $dt$, will not affect the azimuth; but small errors in declination, $d\delta$, and in latitude, $d\varphi$, then attain nearly their maximum effect upon the azimuth. If observations are made upon a circumpolar star ($\delta > \varphi$) at the eastern and at the western elongation, effects of $d\delta$ and $d\varphi$ will disappear in the combination of the two results; this, therefore, is the most favorable condition for observing. In general, effects of $d\delta$ and $d\varphi$ disappear in mean results of observations of equal and opposite azimuths. In observations of a circumpolar star in the meridian, the effect of a small error in time and in right ascension may be eliminated by a combination of results from upper and lower culminations; for a star in the meridian the quantities $d\delta$ and $d\varphi$ do not enter in the azimuth. If the object to be observed, star or sun, is of great polar distance, (also $\delta < \varphi$,) and if $\delta$ is positive, the best time for observing is before the eastern transit, or after the western transit, over the prime vertical, when the change in azimuth with respect to time is a maximum; but the altitude of the star or sun should not be too near the zenith nor so low as to be affected by changes of refraction; if $\delta$ is negative the star or sun should be observed some distance from the meridian.

(9.) The circumpolar stars $\alpha$, $\delta$, $\lambda$, Ursæ Minoris, and 51 Cephei, are those almost exclusively used; their position is most accurately given in the second edition of Dr. Gould's Standard Places of Fundamental Stars, (Washington, 1866,) specially prepared for the use of the Survey. The annexed diagram will assist in readily finding the two fainter stars, which only become visible to




the naked eye under the most favorable circumstances; it also shows that when $\delta$ Ursæ Minoris and 51 Cephei culminate on either side of the pole, Polaris is not yet far from its elongation; and, on the contrary, if the pole-star culminates, the other two are on either side of the meridian, not far from their elongations. A similar approximate relation exists between $\alpha$ and $\lambda$ Ursæ Minoris. Polaris offers the advantage of being observable with portable instruments at eastern and western elongations, or at upper and lower culminations, provided the sun be not too high; $\lambda$, from its greater proximity to the pole and its smaller size, presents to the larger instruments a finer and steadier object for bisection than Polaris; 51 Cephei is also advantageously used on account of its small size. The sun is only employed in connection with inferior azimuths.

(10.) For a satisfactory determination of the azimuth from a high star, it is essential that the horizontal axis of the instrument be long and its inclination be carefully measurable by a delicate level, including, also, a determination of the inequality of the pivots. This inclination of the transit axis should be measured in the position when the telescope is pointed to the star, and again when pointed to the mark, unless the latter be in the horizon; the best results, however, are obtained by observing the star direct, and also its image reflected from the surface of still mercury; the mean result is free from the effect of any inclination of the transit axis. The collimation error is eliminated from the mean result by combining observations with telescope direct and with telescope reversed, the horizontal axis having been turned 180° in azimuth and the telescope again pointed to the star, (during this process the pivots of the transit axis remain undisturbed in their Y's.) Some of the smaller instruments, having their telescopes eccentrically mounted, as in some kind of declinometers, do not admit of reversing; in this case the pivots of the telescope may be inverted in their Y's. Errors of graduation are sought to be eliminated by observing in different positions of the instrument, the circle being shifted after each set of observations an equal amount of angular space, depending upon the number of positions intended to cover 360°, and upon the number of equidistant microscopes or verniers, so that no one shall occupy a position previously occupied by another. With the large theodolite, supplied with three reading microscopes, the number of positions generally adopted is either five or seven.

(11.) Observations for azimuth are generally made in sets, commencing, after the instrument is levelled, with a number of readings on the mark, (about six for primary and from three to one for secondary azimuths,) followed by about an equal number of readings on the star, preceded and followed by level readings, (unless reflections are intended, when no level readings in connection with the star are necessary.) The instrument is then reversed and the preceding operations are repeated in the inverse order, the number of observations upon the star and the mark being as before. Some observers reverse the instrument also upon the mark, before and after the reversal upon the star, but as every reversal renders the instrument liable to disturbance, their number might be reduced to three, or even to a single one, in each set; the number of pointings on mark and star varies with different observers and instruments. If the mark is not in the horizon, its zenith distance must be measured and level readings must be given also when pointing to it. Precautions should be taken to prevent the pivot or level being heated by the lamp or hand of the observer. The level value may be ascertained by a level trier, or by means of a vertical circle, or by a micrometer of known value. With smaller instruments the principle of repetition has been tried with very satisfactory results; whether repetitions should be employed or not depends upon the relative value of definition by telescope and of accuracy of graduation; the clamping apparatus, however, must have no tendency to disturb the relative position of the circles, and the motion of the instrument must be free.

(12.) The method of recording and reducing the different kinds of azimuth observations will next be stated in detail, and specimens of record and of computation will be given at the end of this paper. The formulæ and method of reduction in each case are as follows:

(13.) ***Observations of a close circumpolar star near its elongation.***—A table of chronometer corrections and rates, covering the period during which azimuthal observations are made, is prepared; the readings of the horizontal circle on the mark and star are corrected for over or under-run of micrometer of reading microscopes, if required by the instrument. The mean places of stars and their constants are taken from the Coast Survey Standard Places of Fundamental Stars, the apparent right ascensions ($\alpha$) and declinations ($\delta$) are computed by either of the two methods given in the American Ephemeris and Nautical Almanac, and the results are tabulated.

$$\text{Apparent} \begin{cases} \alpha \\ \delta \end{cases} \text{at time and place of observation} = \text{apparent} \begin{cases} \alpha \\ \delta \end{cases} \text{at upper culmination at Washington}$$

$$(\text{or Greenwich}) + (\text{difference of longitude [in hours]} \pm \text{hour angle of} \ast) \frac{\text{daily difference}}{24}$$

$$+ \text{correction for terms of nutation involving } 2\,☾.$$

The hour angle, $t_e$, and the azimuth, $A_e$, at elongation, for the latitude, $\varphi$, are computed by the formulæ

$$\cos t_e = \tan \varphi \cot \delta \quad \text{and} \quad \sin A_e = \sec \varphi \cos \delta.$$

Also, sidereal time $\left\{\begin{matrix}\text{W.}\\\text{E.}\end{matrix}\right.$ elongation $=\alpha\pm t_e$ and chronometer time of $\left\{\begin{matrix}\text{W.}\\\text{E.}\end{matrix}\right.$ elongation $=\alpha\pm t_e+$ correction of chronometer, $\left\{\begin{matrix}+\\-\end{matrix}\right.$ when chronometer is $\left\{\begin{matrix}\text{fast}\\\text{slow}\end{matrix}\right.$ of sidereal time.

Let $\tau=$ interval of times of elongation by chronometer and by observation, then reduction to azimuth, for $\alpha$ and $\lambda$ Ursæ Minoris, within 25 minutes of the time of elongation, with sufficient accuracy,

$$112.5\,\tau^2 \sin 1'' \tan A_e, \text{ or } \frac{2\sin^2\frac{1}{2}\tau}{\sin 1''}\tan A_e;$$

in which formulæ $\tan A_e$ may be exchanged for $\sin A_e$. Supposing the circle to read in the direction N., E., S., W., the reduction to elongation is applied to the reading of the star with the sign $\left\{\begin{matrix}+\\-\end{matrix}\right.$ when ✳ is $\left\{\begin{matrix}\text{E.}\\\text{W.}\end{matrix}\right.$ of the meridian. The means of all the readings of the star, reduced to elongation, for telescope "D" and for telescope "R," are corrected for error of inclination of axis by the formula

$$\frac{d}{4}\left\{(w+w')-(e+e')\right\}\frac{\sin h}{\cos\varphi},$$

when $d=$ value of one division of level scale in seconds of arc, $w$ $e$ and $w'$ $e'$ the west and east readings of the level before and after reversal, and $h$ the ✳'s altitude. For $\frac{\sin h}{\cos\varphi}$, $\tan\varphi$ may be substituted. The mean of the corrected readings for telescopes D and R is then taken for the reading of the star at elongation; hence, reading of meridian = reading of ✳ at elongation $\pm A_e$, where $\left\{\begin{matrix}+\\-\end{matrix}\right.$ for $\left\{\begin{matrix}\text{W.}\\\text{E.}\end{matrix}\right.$ elongation. The mean of D and R readings of the mark,* before and after the observations upon the star, is taken for the reading of the mark; and finally, azimuth of mark = difference of readings of meridian and mark. This result is yet to be corrected for effect of diurnal aberration.

Let $\zeta=$ zenith distance, and $A=$ azimuth of star; then $dA=\frac{0''.308\cos A\cos\varphi}{\sin\zeta}$, where $\sin\zeta=\frac{\sin p\sin t}{\sin A}$; and $p$ the polar distance, $t$ the hour angle. For elongation the formula becomes simply $dA=0''.31\cos A$ with sufficient accuracy. This correction is $\left\{\begin{matrix}-\\+\end{matrix}\right.$ when mark is $\left\{\begin{matrix}\text{W.}\\\text{E.}\end{matrix}\right.$ of north, and is always positive when applied directly to the azimuth, which, in the survey and geodetically, is counted from south to west to 360°. The final result for azimuth and its probable error is obtained by the combination of the separate results by each star, with application of the method of least squares.

(14.) *Observations of a close circumpolar star at any hour angle.*—The chronometer correction and rate are tabulated, the corrections for run of microscopes of azimuth circle is applied, and the right ascension and declination of the star are computed for the various dates, as in the preceding case. We may employ three different methods for the computation of the azimuth, viz: by the use of the fundamental trigonometrical formula, of Napier's analogies, and of a development in series.

(15.) By means of the fundamental formula, and counting the azimuth from the north,

$$\tan A=\frac{\sin t}{\cos\varphi\tan\delta-\sin\varphi\cos t};$$

the first term of the denominator may be tabulated for slightly different values of $\delta$ during the period of observation; the second term, for a close circumpolar star, may be computed by five-figure logarithms. The formula may be separately applied to each observation, if we desire individual results; but this work may be much shortened by computing only the azimuth corresponding to the mean hour angle and applying to it the correction to mean azimuth. Let $n$ be the number of observations on the star, A the azimuth corresponding to the mean hour angle, and, consequently, $\frac{\Sigma A}{n}$ the mean azimuth; let also $\tau=$ the difference between the time of any observation and the mean of the times; then for a circumpolar star—

$$\frac{\Sigma A}{n}=A-\tan A\cdot\frac{1}{n}\Sigma\frac{2\sin^2\frac{1}{2}\tau}{\sin 1''}.$$

* If the mark is not in the horizon its readings must be corrected for inclination of axis.

The reading of the meridian = reading of $* \pm \frac{\Sigma A}{n}$, where $\left\{\begin{matrix}+\\-\end{matrix}\right.$ for star $\left\{\begin{matrix}\text{W.}\\\text{E.}\end{matrix}\right.$ of meridian, and the circle is supposed to read as stated above.

The correction for level for a circumpolar star may be applied as in the preceding method, or by means of the general formula, which also includes the collimation, $\pm b \cot \zeta \pm c \operatorname{cosec} \zeta$, where $b$ the inclination of the transit axis and $c$ the collimation, both expressed in arc; the two signs refer to the position of the axis. The sign of the level correction in any case can readily be found from a special consideration. The application of the correction for diurnal aberration and the manner of obtaining the resulting azimuth have already been explained. In the prime vertical the diurnal aberration vanishes.

(16.) By Napier's analogies. Let $q$ = parallactic angle, or the angle at the star; then—

$$\tan \tfrac{1}{2}(q+A) = \frac{\cos \frac{1}{2}(\delta-\varphi)}{\sin \frac{1}{2}(\delta+\varphi)} \cot \tfrac{1}{2}t = m \cot \tfrac{1}{2}t;$$
$$\tan \tfrac{1}{2}(q-A) = \frac{\sin \frac{1}{2}(\delta-\varphi)}{\cos \frac{1}{2}(\delta+\varphi)} \cot \tfrac{1}{2}t = m' \cot \tfrac{1}{2}t;$$

hence, $A = \frac{1}{2}(q+A) - \frac{1}{2}(q-A)$; where A counts from the north. $m$ and $m'$ vary but slowly with a change in $\delta$. If the hour angle is reckoned from the lower culmination, we must employ the formulæ

$$\tan \tfrac{1}{2}(q+A) = m \tan \tfrac{1}{2}t$$
$$\tan \tfrac{1}{2}(q-A) = m' \tan \tfrac{1}{2}t.$$

The successive azimuths of the star at the times of observation are applied to the corresponding readings of the star, thus giving, after being corrected for level, as in the preceding case, a series of readings of the meridian, the mean of which is combined with the mean reading of the mark in order to obtain the azimuth of the mark. The latter is then to be corrected for diurnal aberration, unless the star be in the prime vertical.

(17.) By means of a development in series. We have—

$$A = \frac{\sin t}{\cos \varphi}\left\{p + p^2 \sin 1'' \tan \varphi \cos t + \tfrac{1}{3}p^3 \sin^2 1'' \left[(1+4\tan^2\varphi)\cos^2 t - \tan^2 \varphi\right]\right\};$$

where the azimuth may be reckoned either way from the north, and is expressed in seconds of arc; if the hour angle be reckoned from the lower culmination, the term $p^2 \sin 1'' \tan \varphi \cos t$ must be taken with the opposite sign. The third term, $\frac{1}{3}p^3 \sin^2 1'' [(1+4\tan^2\varphi)\cos^2 t - \tan^2\varphi]$, may be tabulated for each polar star for every $10^m$ of hour angle, and for every degree of latitude, within a certain range. Since $p$ varies slightly (for a given star) in time, the tabular quantities must be corrected accordingly; thus, in the case of Polaris, an increase or diminution of $1'$ in $p$ demands an increase or diminution of the tabular value nearly of its $\frac{1}{29}$th part. The remaining reduction is as above.

For the case of a close circumpolar star observed *near the culmination* the general formula becomes

$$A = \frac{\sin t}{\cos \varphi}\left\{p + p^2 \sin 1'' \tan \varphi \cos t + \tfrac{1}{3}^3 \sin^2 1'' (1 + 3\tan^2 \varphi)\right\}.$$

If the hour angle is counted from the lower culmination change the sign of the second term; we may use this formula for Polaris to within one hour of culmination. For a given star, time, and latitude the expression reduces to

$$A = [c] \sin t \left\{p + c' - [c''] \cos t\right\},$$

where $c$, $c'$, $c''$ are constants; the rectangular brackets include logarithms. For a very small hour angle the expression becomes $[C] \sin t$, where C may be taken as constant. The mean azimuth of the polar star is obtained from its azimuth computed from the mean hour angle by the formula

$$A_m = A - \tan A \,.\, \frac{1}{n}\Sigma \frac{2 \sin^2 \frac{1}{2}\tau}{\sin 1''}.$$

(18.) *Observations of a close circumpolar star at equal intervals before and after culmination.*—For chronometer correction and rate, and correction of run of reading microscopes, see first method; apparent $\alpha$ at time and place = apparent $\alpha$ at upper culmination at Washington (or Greenwich) + [difference of longitude (in hours)] $\frac{\text{daily difference}}{24}$ + correction for nutation involving 2☾.

Chronometer time of $\left\{\begin{matrix}\text{upper}\\\text{lower}\end{matrix}\right.$ culmination = $\left\{\begin{matrix}\alpha\\\alpha + 12^h\end{matrix}\right.$ + correction for chronometer error.

Reading of approximate meridian = mean of corresponding readings on the star before and after culmination. The means of these readings for telescopes D and R are separately taken, the instruments having been reversed at culmination.

$$\text{Correction for inclination} = \frac{d}{4}\left\{(w+w')-(e+e')\right\}\tan h.$$

The means require a further correction for error of assumed time of culmination by chronometer. Let $\tau$ = correct chronometer time of culmination, $\tau'$ = assumed chronometer time for observations, $dA$ = motion of the star in azimuth in one second of time, which quantity is readily found from the observations themselves, then correction $= \mp(\tau-\tau')\,dA$ for $\left\{\begin{matrix}\text{upper}\\ \text{lower}\end{matrix}\right.$ culmination; the circle being supposed to read in the direction from N. to E.

$$\text{Correction for diurnal aberration} = \frac{0''.31\cos A\cos\varphi}{\sin\zeta},$$

where $\sin\zeta = \sin\left\{\begin{matrix}\delta-\varphi\\ \delta+\varphi\end{matrix}\right.$ for $\left\{\begin{matrix}\text{upper}\\ \text{lower}\end{matrix}\right.$ culmination; the sign of this correction to the azimuth is as explained above. For interpolation, in case of accidental omissions, or a non-correspondence in time before and after culmination, or where the star is observed only *on one side* of the meridian, the reading may be referred to the meridian by means of any of the three methods given for the case of observations at various hour angles. The same formulæ apply in the case of one star observed on one side of the meridian and another star on the other side, and when the results of the two are proposed for combination.

The effect upon the azimuth for a small difference in time, near lower culmination, may be computed by the formula $dA = \frac{1}{2}(m-m')\cos t\,dt$, where $m$ and $m'$ are the factors developed by the use of Napier's analogies; or it may be derived from the observations themselves. Observations of a polar star within about $20^m$ of culmination may be reduced by the formulæ

$$A = \frac{\cos\delta\sin t}{\sin(\delta\mp\varphi)\sin 1''}, \text{ or } A = \frac{\sin p\sin t}{\cos(\varphi\pm p)\sin 1''},$$

where the sign $\left\{\begin{matrix}+\\ -\end{matrix}\right.$ refers to $\left\{\begin{matrix}\text{upper}\\ \text{lower}\end{matrix}\right.$ culmination, in the latter formula. We have also $dA = \frac{\sin p}{\cos(\varphi\pm p)}\cos t\,dt$, where $\cos t$ may be omitted.

(19.) Azimuths for tertiary triangulation, or in connection with the magnetic declination, where an accuracy of a fraction of a minute suffices, may be obtained with a small altazimuth instrument, (say of five inches diameter.) Supposing the latitude given, but the time only approximately known, the sun's zenith distance and azimuth may be observed as follows: reading of mark, three readings, noting the chronometer time at contacts of the sun's upper and first limb; instrument reversed, three readings of the sun's lower and second limb, reading of mark.

Let $h$ = altitude, corrected for refraction, parallax, (semi-diameter and dip, if necessary,) and $p$ = the sun's or star's polar distance, then—

$$\tan^2\tfrac{1}{2}A = \frac{\sin(s-\varphi)\sin(s-h)}{\cos s\cos(s-p)},$$

in which expression $s = \frac{1}{2}(\varphi+h+p)$. If the time should also be desired, it may be computed by

$$\tan^2\tfrac{1}{2}t = \frac{\cos s\sin(s-h)}{\sin(s-\varphi)\cos(s-p)}, \text{ or by } \tan\tfrac{1}{2}t = \cot\tfrac{1}{2}A\frac{\sin(s-h)}{\cos(s-p)}.$$

If the sun's limb is observed, the correction to the azimuth for reduction to centre is $\pm\frac{r}{\sin\zeta}$, where $r$ = sun's radius; whether + or — is to be used can readily be found in each particular case.

(20.) Examples of record and reduction for the various methods employed in determining astronomical azimuths are herewith appended:

## To Art. (13.)—*Example of record.*

STATION, AGAMENTICUS, YORK COUNTY, ME.

Polaris near western elongation. October 17, 1847.

Observer: A. D. B. Instrument: 30-inch theodolite, C. S., No. 1. 0 upon 140°.
Weather: Light fog. Wind: S. W., moderate. Temp.: 48° Fahr.

| No. | Object. | Appear. | Tel. | Time by sid. chro'r. | Azimuth circle. | | | | | | | Level. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | A | | B | | C | | | | |
| | | | | h. m. s. | ° ′ | | | | | | | | | |
| 1 | Mark. | m-d. St. | R | 6 30 | 63 55 | 39.7 | 39.0 | 27.5 | 27.0 | 27.7 | 26.5 | Correction for run +0.1. | 1 div. = 0″.97. | |
| 2 | | | | 33 | 63 55 | 41.0 | 39.7 | 27.0 | 28.0 | 26.0 | 24.3 | | | |
| 3 | | | | 34 | 63 55 | 41.0 | 41.0 | 29.8 | 29.0 | 26.4 | 26.3 | | | |
| 4 | | | D | 37 | 243 55 | 26.2 | 28.2 | 16.8 | 17.0 | 16.8 | 13.3 | | | |
| 5 | | | | 39 | 243 55 | 25.5 | 28.0 | 17.0 | 17.0 | 16.4 | 15.2 | | | |
| 6 | | | | 42 | 243 55 | 27.0 | 29.0 | 19.0 | 19.0 | 16.2 | 14.0 | | | |
| | | | | | | | | | | | | | Level C. | |
| 1 | Star. | m-l. m-st. | D | 6 47 12 | 127 42 | 68.0 | 67.0 | 61.5 | 63.0 | 64.5 | 64.3 | Correction for run 0.0. | E. | W. |
| 2 | | | | 49 06 | 127 42 | 65.0 | 65.0 | 63.5 | 63.2 | 63.1 | 60.5 | | 44 | 62 |
| 3 | | | | 51 38 | 127 42 | 62.8 | 62.8 | 57.0 | 59.8 | 60.0 | 58.2 | | 63 | 44 |
| 4 | | | | 52 12.5 | 127 42 | 58.0 | 58.0 | 54.0 | 52.5 | 55.3 | 53.5 | | 43 | 63 |
| 5 | | | | 55 55.5 | 127 42 | 56.0 | 57.0 | 51.0 | 52.0 | 53.0 | 52.0 | | 64 | 43 |
| 6 | | | R | 7 00 54 | 307 42 | 48.2 | 48.7 | 45.2 | 45.0 | 47.7 | 45.8 | | 46 | 62 |
| 7 | | | | 2 25.5 | 307 42 | 48.0 | 49.2 | 43.2 | 44.2 | 45.0 | 44.8 | | 62 | 46 |
| 8 | | | | 4 01.5 | 307 42 | 48.0 | 48.7 | 43.0 | 44.7 | 46.8 | 45.0 | | | |
| 9 | | | | 5 51 | 307 42 | 49.0 | 49.0 | 44.7 | 45.0 | 47.9 | 46.9 | | 43 | 63 |
| 10 | | | | 7 14.5 | 307 42 | 49.2 | 50.5 | 44.8 | 44.8 | 47.2 | 46.2 | | 63 | 43 |
| 7 | Mark. | m-d. m-st. | R | 7 16 | 63 55 | 40.0 | 40.0 | 23.0 | 25.0 | 26.8 | 25.2 | Correction for run +0.1. | | |
| 8 | | | | 17 | 63 55 | 39.7 | 39.7 | 23.0 | 23.0 | 25.7 | 24.8 | | | |
| 9 | | | | 18 | 63 55 | 38.0 | 39.0 | 21.5 | 22.7 | 25.0 | 23.8 | | | |
| 10 | | | D | 23 | 243 55 | 26.0 | 26.5 | 13.7 | 14.0 | 15.0 | 14.6 | | | |
| 11 | | | | 24 | 243 55 | 26.8 | 26.8 | 14.5 | 14.8 | 15.2 | 14.0 | | | |
| 12 | | | | 26 | 243 55 | 26.7 | 27.3 | 14.0 | 13.0 | 14.5 | 13.9 | | | |

## To Art. (13.)—*Example of reduction.*

STATION, AGAMENTICUS, 1847.

$\phi = 43° 13' 25''.0$ ; $\lambda = 4h\ 42m\ 44.8s$, west of Greenwich.

Specimen of ephemeris and of time and azimuth at elongation.

| Date. | Elongation. | $a$ | $\delta$ | $A_e$ | $t_e$ | Sid. time of elongation $a \pm t_e$ | Chro'r fast | Chro'r time of elong'n. |
|---|---|---|---|---|---|---|---|---|
| | | h. m. s. | ° ′ ″ | ° ′ ″ | h. m. s. | h. m. s. | m. s. | h. m. s. |
| Sept. 17, 1847 | E | 1 05 28.14 | 88 29 42.80 | 2 03 54.95 | 5 54 20.5 | 19 11 07.6 | 12.2 | 19 11 19.8 |
| Sept. 21, 1847 | W | 29.30 | 44.54 | 52.56 | 20.6 | 6 59 49.9 | 31.4 | 7 00 21.3 |
| Sept. 22, 1847 | E | 29.40 | 44.73 | 52.31 | 20.6 | 19 11 08.8 | 33.3 | 19 11 42.1 |
| Sept. 22, 1847 | W | 29.50 | 44.91 | 52.06 | 20.6 | 6 59 50.1 | 35.0 | 7 00 25.1 |
| Oct. 17, 1847 | W | 1 05 32.96 | 88 29 54.27 | 2 03 39.21 | 5 54 21.2 | 6 59 54.2 | 1 51.8 | 7 01 46.0 |

Polaris near western elongation, October 17.

| No. | Tel. | Time from elong'n. | Red'n to elong'n. | Corr'd mean reading. | Reading reduced to elong'n. |
|---|---|---|---|---|---|
| | | m. s. | ″ | ° ′ ″ | ° ′ ″ |
| 1 | R | | | 63 55 31.3 | |
| 2 | | | | 31.3 | |
| 3 | | | | 32.3 | |
| 4 | D | | | 243 55 19.8 | |
| 5 | | | | 19.9 | |
| 6 | | | — | 20.8 | |
| 1 | D | 14 34.0 | 15.0 | 127 42 64.7 | 127 42 49.7 |
| 2 | | 12 40.0 | 11.3 | 63.2 | 51.9 |
| 3 | | 10 08.0 | 7.3 | 60.1 | 52.8 |
| 4 | | 9 33.5 | 6.5 | 55.2 | 48.7 |
| 5 | | 7 50.5 | 4.3 | 53.5 | 49.2 |
| 6 | R | 52.0 | 0.1 | 307 42 46.7 | 307 42 46.6 |
| 7 | | 39.5 | 0.0 | 45.7 | 45.7 |
| 8 | | 2 15.5 | 0.3 | 46.0 | 45.7 |
| 9 | | 4 05.0 | 1.2 | 47.1 | 45.9 |
| 10 | | 5 28.5 | 2.1 | 47.1 | 45.0 |
| 7 | R | | | 63 55 30.1 | |
| 8 | | | | 29.4 | |
| 9 | | | | 28.5 | |
| 10 | D | | | 243 55 18.4 | |
| 11 | | | | 18.8 | |
| 12 | | | | 18.3 | |

Chronometer time of elongation $7^h\ 1^m\ 46^s.0$.

Reading of star reduced to elongation.

| Tel. | Mean reading. | Level corr'n. | Corr'd mean. |
|---|---|---|---|
| | ° ′ ″ | ″ | ° ′ ″ |
| D | 127 42 50.46 | −0.23 | 127 42 50.23 |
| R | 307 42 45.78 | 0.00 | 307 42 45.78 |

| | |
|---|---|
| Reading of ✱ at elongation. | 127° 42′ 48″.00 |
| Azimuth of ✱ ....... ...... | 2° 03′ 39″.21 |
| Reading of meridian........ | 129° 46′ 27″.21 |
| Reading of mark............ | 243° 55′ 24″.89 |
| Mark east of north.......... | 114° 08′ 57″.68 |
| Corr'n for diur'l aber'n...... | +0″.31 |
| Mark E. of N............... | 114° 08′ 57″.99 |

## To Arts. (11 and 13.)—*Example of record and reduction.*

Station Point Avisadera, San Francisco Bay, Cal.

Polaris near eastern elongation, Sept. 9, 1851.

Observer: R. D. C. Instrument: 10-inch Gambey theodolite, C. S., No. 20; (graduation from right to left.)

| No. | Tel. | Star. | Chro'r time, Dent, 1838. | Reading of horzl. circle, mark, and ✱. |
|---|---|---|---|---|
| | | | h. m. s. | ° ′ ″ |
| | D | | | 254 45 50.0 |
| | | | | 45.0 |
| | | | | 50.0 |
| | | | | 62.5 |
| 1 | | Dir. | 3 42 30.5 | |
| 2 | | Ref. | 44 08.0 | |
| 3 | | Dir. | 45 52.0 | |
| 4 | | Ref. | 47 15.0 | |
| 5 | R | Dir. | 48 59.0 | |
| 6 | | Ref. | 50 34.0 | |
| 7 | | Dir. | 52 28.0 | |
| 8 | | Ref. | 53 51.5 | |
| | | | | 149 15 25.0 |
| | | | | 15.0 |
| | | | | 25.0 |
| | | | | 22.5 |

N. B. Several such sets are taken in succession.
Dent, 1838, is a mean-time chronometer.

$\phi = 37° 43' 31''$; $\lambda = 8^h\ 9^m\ 23^s$, west of Greenwich,
Sept. 9, at time of east elongation.

Polaris $a = 1^h\ 6^m\ 15^s.33$.

Polaris $\delta = 88° 30' 51''.26$.

$t_e = 5^h\ 55^m\ 24^s.10$.

$A_e = 1° 52' 42''.84$.

| | h. m. s. |
|---|---|
| Sid. time, mean noon...................... | 11 13 05.49 |
| Sid. time, east elongation................. | 19 10 51.23 |
| Mean time, east elongation................ | 7 56 27.47 |
| Chronometer, Dent, 1838, correction....... | +3 50 06.93 |
| Chronometer time, east elongation......... | 4 06 20.50 |
| Mean chronometer time of observation..... | 3 48 12.30 |
| Time from elongation...................... | 18 08.20 |
| Time from elongation, sid. int............. | 18 11.20 |

| $\tau$ | Tab. quan. | | |
|---|---|---|---|
| m. s. | ″ | | |
| 5 41.8 | 63.7 | | |
| 4 04.3 | 32.5 | | |
| 2 20.3 | 10.7 | Log 27.4 ........ | 1.4380 |
| 57.3 | 1.8 | Log (ratio²) ..... | 0.0024 |
| 46.7 | 1.2 | Log tan A....... | 8.5140 |
| 2 21.7 | 11.0 | | |
| 4 15.7 | 35.7 | | 9.954 |
| 5 39.2 | 62.8 | Corr'n........... | −0″.90 |
| $\frac{1}{n}\Sigma$ Tab. quan. | 27.4 | | |

| | ° ′ ″ |
|---|---|
| Mean angle between ✱ and mark........ | 13 11 18.7 |
| Reduction to elongation.................. | −21.3 |
| Reduction to mean azimuth .............. | −0.9 |
| Corrected angle, elong'n, and mark....... | 13 10 56.5 |
| Azimuth of ✱ at elongation ............. | 1 52 42.8 |
| Mark east of north ....................... | 15 03 39.3 |

To which result the correction for diurnal aberratio is yet to be applied.

## To Arts. (14) and (15.)—*Example of record.*

STATION DOLLAR POINT, GALVESTON BAY, TEXAS.

Polaris at various hour angles. April 5, 1848.

Observer: J. E. H. Instrument: 18-inch Troughton theodolite, C. S., No. 4.

Pos. II, set 2.

| Object. | Tel. | Time by chronometer. | Level. | | Azimuth circle. | | | |
|---|---|---|---|---|---|---|---|---|
| | | Hardy, 50. | E | W | A | B | C | |
| | | h. m. s. | | | ° ′ ″ | ″ | ″ | |
| Mark. | D | 8 56 | | | 158 50 55 | 65 | 50 | |
| | R | | | | 51 20 | 20 | 00 | |
| | | | 129 | 71.5 | | | | |
| | | | 81 | 119 | | | | |
| | | | 126 | 74 | | | | 1 div. of level = 0″.82. |
| | | | 83 | 117 | | | | |
| Star. | D | 9 03 33.5 | | | 337 18 40 | 35 | 20 | |
| | | 4 47.5 | | | 18 55 | 55 | 35 | |
| | | 6 07.0 | | | 18 75 | 70 | 55 | |
| " | R | 9 08 06 5 | | | 19 45 | 55 | 40 | |
| | | 9 24.0 | | | 19 65 | 75 | 55 | |
| | | 10 23.5 | | | 20 20 | 30 | 10 | |
| | | | 121.5 | 79 | | | | |
| | | | 80 | 120 | | | | |
| | | | 121.5 | 78 | | | | |
| | | | 77.5 | 122 | | | | |
| Mark. | D | | | | 158 50 55 | 65 | 50 | |
| | R | 9 19 | | | 51 20 | 15 | 00 | |

## To Arts. (14) and (15.)—*Example of reduction.*

STATION DOLLAR POINT, 1848.

Specimen of ephemeris, Polaris at Dollar Point mean midnight, and table of chronometer correction and rate.

$\phi = 29^\circ\ 26'\ 02''.6$. $\lambda = 6^h\ 19^m\ 32^s.0$ west of Greenwich.

| Date. | $\alpha$ | $\delta$ | At sidereal time | Chronometer fast. | Daily rate, gaining. |
|---|---|---|---|---|---|
| 1848. | h. m. s. | ° ′ ″ | h. | m. s. | |
| March 23 | 1 04 05.51 | 88 30 01.88 | | | |
| &c. | | | | | |
| April 2 | 1 04 04.46 | 88 29 58.80 | 12.0 | 0 11.4 | Stopped. |
| 5 | 04.70 | 57.77 | 10.5 | 02.0 | $2^s.8$ |
| 6 | 04.85 | 57.46 | 10.4 | 04.8 | |
| &c. | | | | | |

*Polaris at various hour angles. April* 5.

| | | |
|---|---|---|
| | h. m. s. | |
| Mean of times | 9 07 03.7 | |
| Correction to chronometer | — 01.8 | |
| Sidereal time of observation | 9 07 01.9 | |
| $\alpha$ of ✱ | 1 04 04.7 | |
| Hour angle | 8 02 57.2 | 120° 44′ 18″.0 |

| Time from mean. | Tabular quantity. |
|---|---|
| m. s. | " |
| 3 30.2 | 24.1 |
| 2 16.2 | 10.1 |
| 0 56.7 | 1.7 |
| 1 02.8 | 2.2 |
| 2 20.3 | 10.7 |
| 3 19.8 | 21.8 |
| Mean.... | 11.8 |
| Reduct'n. | —0".30 |

| | ° ′ ″ |
|---|---|
| Mean reading of ✳ | 337 19 26 40 |
| Level correction | —01.12 |
| Reduction to mean azimuth | —00.30 |
| Corrected reading of ✳ | 337 19 24.98 |
| Azimuth of ✳ | 1 28 11.45 |
| Reading of meridian | 338 47 36.43 |
| Reading of mark | 158 51 04.60 |
| Mark west of south | 0 03 28.17 |

(To which result the correction for diurnal aberration is yet to be applied.)

## To Art. (17.)—*Example of record and reduction.*

### Station Santa Cruz, California.

Polaris before upper culmination. October 30, 1854. Observer, R. D. C. Instrument, twelve-inch Gambey theodolite, C. S. No. 30, (graduation from right to left.)

| Number. | Telescope. | Star. | Chronometer time. | Reading of horizontal circle, mark, & star. |
|---|---|---|---|---|
| | | | h. m. s. | ° ′ ″ |
| | D | | | 37 21 21 |
| | | | | 33 |
| | | | | 27 |
| | | | | 21 |
| 1 | | Dir. | 9 51 08.0 | |
| 2 | | Ref. | 52 19.5 | |
| 3 | | Dir. | 53 35.0 | |
| 4 | | Ref. | 54 44.0 | |
| 5 | | Dir. | 56 55.0 | |
| 6 | | Ref. | 58 44.0 | |
| 7 | R | Dir. | 10 00 35.5 | |
| 8 | | Ref. | 01 5'.5 | |
| 9 | | Dir. | 03 07.0 | |
| 10 | | Ref. | 04 21.5 | |
| 11 | | Dir. | 05 29.0 | |
| 12 | | Ref. | 06 46.5 | |
| | | | | 216 16 15 |
| | | | | 12 |
| | | | | 24 |
| | | | | 12 |

N. B —Several such sets are taken in succession. The chronometer used is regulated to mean time.

$\phi = 36° 58' 32''$. $\lambda = 8^h 08^m 09^s$ west of Greenwich.
October 30. Polaris $\alpha = 1^h 06^m 54^s.38$
" $p = 1° 27' 48''.35$.

| | h. m. s. |
|---|---|
| Santa Cruz, sidereal time at mean noon | 14 35 14.31 |
| Sidereal interval of upper culmination after mean noon | 10 31 40.07 |
| Chronometer time of observation | 9 59 08.29 |
| Chronometer slow | 12 46.11 |
| Mean time of observation | 10 11 54.40 |
| Sidereal interval of observation after mean noon | 10 13 34.92 |
| Sidereal interval of upper culmination after mean noon | 10 31 40.07 |
| Hour angle, + east, — west | — 18 05.15 |

We have—

$p = 5268''.4$

second term $= 101''.0$

third term $= 3''.1$

hence—

$t = 4° 31' 17''$

$A = 530''.14$.

| τ | In sidereal time. | Tabular quantity. |
|---|---|---|
| *m. s.* | *m s.* | ″ |
| 8 00 | 8 01 | 126 |
| 6 49 | 6 50 | 92 |
| 5 33 | 5 34 | 61 |
| 4 24 | 4 25 | 38 |
| 2 13 | 2 13 | 10 |
| 0 24 | 0 24 | 0 |
| 1 27 | 1 27 | 4 |
| 2 46 | 2 46 | 15 |
| 3 59 | 4 00 | 31 |
| 5 13 | 5 14 | 54 |
| 6 21 | 6 22 | 80 |
| 7 38 | 7 39 | 115 |
| Mean ............ | | 52.2 |
| Reduction to mean azimuth ........ | | —0″.13 |

| | ° ′ ″ |
|---|---|
| Angle, mark, and star.......................... | 14 54 34.19 |
| Azimuth of star, corrected...................... | —8 50.01 |
| Mark west of north............................ | 14 45 44.18 |

(To which a correction for diurnal aberration is yet to be applied.)

## TO ART. (18.) *Example of record.*

### STATION SEBATTIS, KENNEBEC COUNTY, MAINE.

Polaris near upper culmination. July 13, 1853. Observer, A. D. B. Instrument, 30-inch theodolite, C. S. No. 1. Position, V. Weather clear. Wind northeast, light. Temperature 59° Fahrenheit. Assumed time of culmination, $1^h\ 05^m\ 57^s$. Assumed chronometer error, $+17^s$. Approximate chronometer time of culmination, $1^h\ 06^m\ 14^s$.

| Number. | Object. | Appearance. | Telescope. | Time by sidereal chronometer. | Azimuth circle. | A | | B | | C | | Level. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | *h. m. s.* | ° ′ | | | | | | | |
| 1 | Mark. | m-d. st. | R | 0 5 | 202 04 | 32 | 30 | 30 | 28.5 | 34.5 | 32 | 1 divis'n of level = 1″.53. |
| 2 | | | | 0 8 | 202 04 | 32.5 | 34.5 | 31 | 31 | 35 | 34.5 | |
| 3 | | | | 0 11 | 202 04 | 32 | 34 | 31.5 | 30.5 | 35.5 | 34 | |
| 4 | | | D | 0 16 | 22 04 | 26 | 28 | 31.5 | 31 | 31 | 31 | |
| 5 | | | | 0 18 30 | 22 04 | 26 | 27.5 | 29 | 27 | 30 | 28 | |
| 6 | | | | 0 22 | 22 04 | 27 | 27.5 | 25.5 | 25.5 | 28 | 27 | E. W. |
| 1 | Star. | m-d. s. r. | D | 0 46 14 | 22 09 | 04.5 | 03 | 02 | 03.5 | 06 | 06 | ×72 75 |
| 2 | | | | 0 49 14 | 22 07 | 27 | 26 | 28 | 30 | 28.5 | 27 | 75 72× |
| 3 | | | | 0 52 14 | 22 05 | 44 | 42 | 45 | 46.5 | 48 | 49 | 71 72.5× |
| 4 | | | | 0 55 14 | 22 04 | 05 | 05 | 06 | 07 | 08.5 | 07 | ×69 74<br>×72 70 |
| 5 | | | | 0 58 14 | 22 02 | 25.5 | 24.5 | 26.5 | 27 | 29 | 29 | 67 74× |
| 6 | | tr. | R | 1 14 14 | 201 53 | 47.5 | 47.5 | 49.5 | 49.5 | 51.5 | 50 | ×69 68 |
| 7 | | | | 1 17 14 | 201 52 | 07 | 08 | 09 | 09 | 10 | 10.5 | 74 62× |
| 8 | | | | 1 20 14 | 201 50 | 33 | 33 | 34 | 34 | 33.5 | 35 | 74 61 |
| 9 | | | | 1 23 14 | 201 48 | 49 | 49 | 51 | 49 | 52 | 51 | ×72 64<br>×72 63 |
| 10 | | m-b. st. | | 1 26 14 | 201 47 | 10 | 12.5 | 10.5 | 12 | 12.5 | 12 | 73 61× |
| 7 | Mark. | ft. m-st. | R | 1 30 | 202 04 | 31.5 | 31 | 32.5 | 32.5 | 36 | 34 | |
| 8 | | | | 1 37 | 202 04 | 30 | 29 | 31 | 30 | 34.5 | 33 | |
| 9 | | | | 1 39 | 202 04 | 31 | 30 | 32 | 30 | 34 | 33 | |
| 10 | | | D | 1 41 | 22 04 | 26.5 | 27 | 30.5 | 30.5 | 32 | 31 | |
| 11 | | | | 1 43 | 22 04 | 26 | 27 | 30.5 | 30 | 32 | 31 | No correction for run of |
| 12 | | | | 1 45 | 22 04 | 28.5 | 29.5 | 31 | 31 | 32.5 | 33 | microscopes. |

TO ART. (18.) *Example of reduction.*

STATION SEBATTIS, 1853.

$\phi = 44^\circ\ 08'\ 37''.7.$ $\lambda = 4^h\ 40^m\ 17^s.5$ west of Greenwich.

*Specimen of table of sidereal time and of chronometer time of culminations.*

| Date. | Culminat'n. | Sidereal time. | Chron'r fast. | Chron'r time of culmina'n. |
|---|---|---|---|---|
| 1853. | | h. m. s. | s. | h. m. s. |
| July 13 | Lower.... | 13 05 57.3 | 15.7 | 13 06 13.0 |
| July 13 | Upper .... | 1 05 57.7 | 17.9 | 1 06 15.6 |
| July 14 | Lower.... | 13 05 58.2 | 20.2 | 13 06 18.2 |
| July 14 | Upper .... | 1 05 58.7 | 22.5 | 1 06 21.2 |
| &c. | | | | |

| Before upper culmination of Polaris. | | | | After upper culmination of Polaris. | | | | |
|---|---|---|---|---|---|---|---|---|
| Number. | Telescope. | Time from upper culmination. | Corrected mean reading. | Number. | Telescope. | Time from upper culmination. | Corrected mean reading. | Reading of meridian. |
| | | | ° ′ ″ | | | | ° ′ ″ | |
| 1 | R | ............ | 202 04 31.2 | 12 | D | ............ | 22 04 30.9 | |
| 2 | " | ............ | 33.1 | 11 | " | ............ | 29.4 | |
| 3 | " | ............ | 32.9 | 10 | " | ............ | 29.5 | |
| 4 | D | ............ | 22 04 29.7 | 9 | R | ............ | 202 04 31.6 | |
| 5 | " | ............ | 27.9 | 8 | " | ............ | 31.3 | |
| 6 | " | ............ | 26.8 | 7 | " | ............ | 32.9 | |
| | | m | | | | m | | ° ′ ″ |
| 1 | D | 20 | 22 09 04.2 | 10 | R | 20 | 201 47 11.6 | 21 58 07.9 |
| 2 | " | 17 | 07 27.7 | 9 | " | 17 | 48 50.2 | 08.9 |
| 3 | " | 14 | 05 45.7 | 8 | " | 14 | 50 33.7 | 09.7 |
| 4 | " | 11 | 04 06.4 | 7 | " | 11 | 52 08.9 | 07.7 |
| 5 | " | 8 | 02 26.9 | 6 | " | 8 | 53 49.2 | 08.0 |

Chronometer time of upper culmination.. $1^h\ 06^m\ 15^s.6$
Assumed time of upper culmination...... $14^s.0$

| Mean reading of meridian. | Level correction. | Reduct'n to meridian. | Corrected mean. |
|---|---|---|---|
| ° ′ ″ | ″ | ″ | ° ′ ″ |
| 21 58 08.44 | −2.82 | −0.85 | 21 58 04.77 |
| Reading of mark | | | 22 04 30.60 |
| Mark east of north | | | 0 06 25.83 |

(To which result the correction for diurnal aberration is yet to be applied.)

TO ART. (19.) *Example of record and reduction.*

STATION WASHINGTON, D. C., CAPITOL GARDEN.

Sun near prime vertical. August 15, a. m., 1856. Observer, C. A. S. Instrument, five-inch magnetic theodolite. Sidereal chronometer.

| Chronometer time. | Horizontal circle. | | Vertical circle. | | Temperature. |
|---|---|---|---|---|---|
| | A. | B. | A. | B. | |
| SET I. | ⊙'s upper and first limb. Telescope D. | | | | 73° Fahr. |
| h. m. s. | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | |
| 5 02 53.0 | 25 24 30 | 205 24 30 | 61 56 00 | 61 56 00 | (Bar. 30 in., assumed.) |
| 05 34.0 | 25 50 45 | 205 51 30 | 61 24 30 | 61 25 00 | |
| 06 55.5 | 26 04 30 | 206 05 15 | 61 08 45 | 61 09 30 | |
| | ⊙'s lower and second limb. Telescope R. | | | | |
| 5 09 12.0 | 205 54 15 | 25 54 00 | 61 19 30 | 61 18 30 | |
| 10 32.0 | 206 07 15 | 26 06 45 | 61 04 00 | 61 03 00 | |
| 11 42.0 | 206 18 30 | 26 18 15 | 60 50 00 | 60 49 45 | |
| SET II. | ⊙'s lower and second limb. Telescope R. | | | | |
| 5 13 22.0 | 206 35 30 | 26 35 30 | 60 30 45 | 60 30 15 | |
| 14 32.0 | 206 47 30 | 26 47 30 | 60 17 30 | 60 17 00 | |
| 15 36.5 | 206 58 30 | 26 58 00 | 60 05 15 | 60 04 30 | |
| | ⊙'s upper and first limb. Telescope D. | | | | |
| 5 17 07.0 | 27 47 30 | 207 48 15 | 59 11 45 | 59 12 00 | |
| 18 16.5 | 28 00 00 | 208 00 30 | 58 57 45 | 58 58 00 | |
| 19 19.0 | 28 10 15 | 208 10 30 | 58 45 30 | 58 45 15 | |
| SET III. | ⊙'s upper and first limb. Telescope D. | | | | |
| 5 20 44.0 | 28 25 00 | 208 25 00 | 58 29 00 | 58 29 30 | |
| 22 01.5 | 28 37 45 | 208 38 15 | 58 14 45 | 58 14 30 | |
| 25 26.5 | 29 13 30 | 209 14 00 | 57 36 00 | 57 35 45 | |
| | ⊙'s lower and second limb. Telescope R. | | | | |
| 5 27 32.5 | 209 01 30 | 29 00 30 | 57 48 00 | 57 47 30 | |
| 28 39.5 | 209 12 45 | 29 12 15 | 57 34 30 | 57 34 15 | |
| 30 01.0 | 209 27 00 | 29 26 30 | 57 19 15 | 57 18 30 | 78° Fahr. |

$\phi = 38° 53' 18''$.  $\lambda = 5^h 08^m 01^s.0$ west of Greenwich.

| | Mean chronometer time. | Mean reading horizontal circle. | Mean reading vertical circle. | Correct'n for parallax in altitude and refraction. | Corrected ζ. |
|---|---|---|---|---|---|
| | *h. m. s.* | ° ′ ″ | ° ′ ″ | ′ ″ | ° ′ ″ |
| Set I ..... | 5 07 48.1 | 25 56 40 | 61 17 02 | +1 34 | 61 18 36 |
| Set II .... | 5 16 22.2 | 27 23 17 | 59 38 00 | +1 27 | 59 39 27 |
| Set III ... | 5 25 44.1 | 28 59 30 | 57 50 07 | +1 21 | 57 51 28 |

| | Set I. | Set II. | Set III. |
|---|---|---|---|
| | ° ′ ″ | ° ′ ″ | ° ′ ″ |
| $\phi$ .................. | 38 53 18 | 38 53 18 | 38 53 18 |
| $h$ .................. | 28 41 24 | 30 20 33 | 32 08 32 |
| $p$ .................. | 76 04 27 | 76 04 37 | 76 04 44 |
| A, (from north) ...... | 95 06 06 | 96 32 34 | 98 08 54 |
| Circle reads ......... | 25 56 40 | 27 23 17 | 28 59 30 |
| South meridian reads. | 110 50 34 | 110 50 43 | 110 50 36 |

# APPENDIX No. 12.

[From Coast Survey Report for 1846.]

LETTER OF S. C. WALKER, ESQ., TO THE SUPERINTENDENT OF THE COAST SURVEY, IN RELATION TO THE DIFFERENCES OF LONGITUDE OF PHILADELPHIA AND GREENWICH, BY REDUCTION OF OBSERVATIONS MADE AT CAMBRIDGE, MASSACHUSETTS.

WASHINGTON, D. C., *January* 13, 1846.

DEAR SIR: I beg to acknowledge the receipt of copies of the report of Mr. Bond, relative to the longitude of the new Cambridge Observatory, New England, from which it appears that the most recent determinations of this longitude are, (west of Greenwich,)

| | *h. m. s.* |
|---|---|
| By moon culminations by Mr. Bond in 1839, 1840, and 1841, reduced by Prof. Peirce... | 4 44 31.7 |
| By occultations observed by Mr. Bond in the years 1831 to 1839, inclusive, reduced by Prof. Peirce........ | 4 44 32.4 |
| By Mr. Bond's report of direct comparisons by chronometers, transported in 1844 and 1845, between Cambridge, New England, and Liverpool, England........ | 4 44 31.7 |

I take occasion to remark that a discussion of all the available sources of information in 1842 relative to the difference of longitude between the High School Observatory, in Philadelphia, and Mr. Bond's old observatory in Cambridge, New England, gave me the value of $16^m 12^s.2$ in time.

Prof. Peirce places the new Cambridge Observatory $3^s.1$ west of the old observatory. This gives, between the present Cambridge, New England, and the Philadelphia observatories, the difference of $16^m 9^s.1$. Using this value, the reports of Mr. Bond furnish new results for the longitude of the High School Observatory, which now stands thus:

| | *h. m. s.* |
|---|---|
| By S. C. Walker's report in 1844........ | 5 0 40.6 |
| By S. C. Walker's report in 1845........ | 5 0 40.6 |
| By Prof. Peirce's calculations by moon culminations........ | 5 0 40.5 |
| By Prof. Peirce's calculations by occultations........ | 5 0 41.2 |
| By Mr. Bond's report by chronometers........ | 5 0 40.5 |

This coincidence is quite gratifying, and furnishes a strong motive for testing, with greater precision, the difference of longitude between Cambridge and Philadelphia.

Yours, respectfully,

SEARS C. WALKER.

ALEXANDER D. BACHE, LL. D.,
*Superintendent United States Coast Survey.*

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# APPENDIX No. 13.

[From Coast Survey Report for 1846.]

## REPORT OF S. C. WALKER, ESQ., TO THE SUPERINTENDENT OF THE COAST SURVEY, RELATING TO DETERMINATIONS OF DIFFERENCES OF LONGITUDE BY TELEGRAPH, &C.

WASHINGTON, D. C., *December* 4, 1846.

DEAR SIR: Since my last annual report I have been chiefly engaged, in the time that could be spared from the pressing labors for the Observatory, in preparing for determination of difference of longitude between the stations of the Coast Survey connected by the magnetic telegraph line. The Washington Observatory has been united to the line from the post office northward. The observatory of the central high-school of Philadelphia has been connected with the same continued line. Prof. Loomis's station at Jersey City has been connected with the northern terminus of the Philadelphia and Jersey City line.

The requisite apparatus for giving and receiving signals was prepared by Mr. Saxton, consisting of five magnet stands, of small size and easy transportation. For a minute description of the magnet stand and the mode of using it I beg to refer you to the lithographed circular and accompanying lithographed forms for registering and reducing the astronomical observations.

The right of constructing and using the line from the post office to the Washington Observatory has been purchased of the patentees, and is now the property of the Coast Survey. The delay that occurred in the negotiations with the telegraph company, and the time required to complete the main line and the astronomical stations, prevented the trial of the method till the 1st of October. The first night in which signals were successfully passed between the Philadelphia and Washington observatories was the 10th of October. For the result of that night's work—not as yet, however, corrected for personal equations—I beg to refer you to my partial report, dated October 22. The Washington Observatory, according to that night's work, is found to be $7^m\ 34^s.306$ in time west of the Philadelphia Observatory. In my partial report of June 16, last, I have given the longitude of Captain Wilkes's observatory, on Capitol Hill, $5^h\ 8^m\ 4^s.60$; hence, Captain Wilkes's observatory is $7^m\ 24^s$ west of Philadelphia. In the interim, previous to the reduction of the recent triangulation which connects Capitol Hill and the Washington Observatory, I have taken from Ellicott's original survey of Washington the westing of the Washington Observatory $= 10^s.05$ in time. This added to $7^m\ 24^s$ makes $7^m\ 34^s.05$ for the west longitude of the Washington Observatory. This result differs only $0^s.256$ from that of the telegraphic comparison of October 10.

I submit the correction for personal equations as far as now known. I have often, in past years, compared personal equations with Prof. Kendall, and never found any sensible difference. For the Washington observers the equations of Messrs. Almy, Keith, and myself are as follows, for the clock correction by transits of stars:

| | | *m.* | *s.* |
|---|---|---|---|
| Observed September 29, 1846, (Almy — Keith) | = + | | 0.307 |
| Observed October 21, 1846, (Keith — Walker) | = + | | 0.014 |
| Concluded, (Almy — Walker) | = + | | 0.321 |
| Concluded, (Almy — Kendall) | = + | | 0.321 |
| Uncorrected longitude, (+ east) | = — | 7 | 34.306 |
| Uncorrected longitude by telegraph, October 10, 1846 | = — | 7 | 33.985 |
| Reported longitude June 16, by Gilliss's observations, 1838 to 1842 | = — | 7 | 34.050 |
| Discrepancy | = | | 0.065 |

The longitude reported June 16 is effected with the personal equations of Lieutenant Gilliss, and the numerous observers at the more eastern stations. If we suppose the latter to compensate each other, that of Lieutenant Gilliss, compared with Professor Kendall and myself, from observations of October 22, (at which time Lieutenant Gilliss visited the Washington Observatory,) is—

| | | s. |
|---|---|---|
| Observed October 22, 1846, (Almy — Gilliss) | = | + 0.271 |
| Concluded, (Gilliss, Walker = Gilliss, Kendall) | = | — 0.050 |

From this comparison it appears that Lieutenant Gilliss's observations correspond well with those of Professor Kendall and myself, and that there is no reasonable ground to suppose that any correction is required, in my report of June 16, for Lieutenant Gilliss's personal equation. The coincidence between the two results, within $0^s.065$, is too close not to be partly ascribed to accident. If affords, nevertheless, the highest encouragement for the prosecution of telegraph operations.

The violent storm of October 13 rendered the line nearly useless during the remainder of the month. Signals were, however, passed, on the 22d, from Philadelphia to Washington. The repairs of the line between Baltimore and New York, in November, and the putting up of new wires, rendered it inexpedient to continue the comparisons.

The registering apparatus made by Mr. Pike for the Coast Survey, (three complete sets, with portable local batteries,) under the directions of Dr. Morse, have been just received. It does not appear that the operators at the Washington and Jersey City telegraph offices were at any time in October in conversation with each other. This accounts for our failure to pass clock signals between Jersey City and Philadelphia and Washington, and for the loss, in this respect, of the valuable observations of Professor Loomis. This want of success is to be attributed to the imperfect insulation of the old telegraph line from Baltimore to New York. Such imperfection is not necessarily incident to the method. The wires from Baltimore city to the Washington Observatory never failed, in the most violent storms, (even of the 13th of October,) to transmit signals, night and day, when desired. The line from the post office to the Washington Observatory, erected by Dr. A. C. Goell, is the best specimen yet constructed, and should serve as a model for further progress. Great hopes are entertained that the new lines now being erected, with better materials and more perfect insulation than those of the old ones between Baltimore and Jersey City, will afford proper facilities for the successful application of the telegraph method to the determination of longitudes, which gave so satisfactory a result on the 10th of October last.

The subject of the reduction of the astronomical observations now in the collection of the Coast Survey has received all the attention which my necessary labor for the Observatory would allow, and the preparation of the telegraph operations would spare. No new individuals of the first two classes are received. Those which have been received of classes second and third, viz., transits of Mercury and eclipses of the sun, are only partially reduced. I do not expect, on the final reduction of them, much additional weight to be given to the longitudes already obtained. The classes of occultations from the fourth to the eighth, inclusive, present a great number of observations not yet reduced. As these have formed the basis of the longitudes of my former reports, I have been desirous to put the ninth class, that of moon culminations, in progress. For distinction's sake, the telegraph operations may be called the tenth class.

The Coast Survey has in its collection more than a thousand American observations of the moon culminations. These commence with the year 1838, and form an uninterrupted series up to the present time; some at one point, some at another, of the survey. I include those of Prof. Loomis, of Western Reserve, on account of their coincidence in date, as well as of the valuable papers of Prof. Loomis on the longitude of the Hudson Observatory, in the transactions of the American Philosophical Society. I have compared the American lists with the European lists from 1838 to 1845. The number of coincidences of one American with another, or with one European moon culmination, is near two thousand. I have prepared lithographed forms for computing the most probable longitude from one coincidence, and the most probable value for the result of an entire series. Previous to the filling of these blanks the series of observations must be freed from instrumental errors, if this has not been previously done by the observers. Since a large portion of the American series requires this application of the instrumental correction, and since, after these corrections are applied, the series on hand accumulates faster than I can, as yet, find time to reduce them, I have deemed it necessary to ask for aid in this labor. Since Lieutenant Gilliss has been applied for and ordered on duty in the Coast Survey he has corrected the American series for 1838, and is engaged with those of 1839. The series of 1838 being thus ready for the filling of the lithographic blanks, I have filled them to number of 162, for the year 1838. These 162 coincidences are between Hudson, Washington, and Dorchester observatories, and between some one of them and a European observatory. The number of results is not sufficient to warrant any modification of the values furnished in my previous reports.

Yours, respectfully,

SEARS C. WALKER.

A. D. BACHE, LL. D.,
*Superintendent United States Coast Survey.*

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# APPENDIX No. 14.

[From Coast Survey Report for 1848.]

ANNUAL REPORT TO THE SUPERINTENDENT ON LONGITUDE COMPUTATIONS, BY S. C. WALKER, ASSISTANT UNITED STATES COAST SURVEY.

CAMBRIDGE, MASS., *September* 16, 1848.

SIR: Since my last annual report of September 20, 1848, I have presented fifteen reports, numbered from x to xxv, inclusive, in compliance with special instructions, and in furtherance of the general duty of making computations for astronomical longitudes of stations of the Coast Survey. The telegraph operations of last year were handed, in February last, to the check computer, Mr. Ruth, for final revision. That labor was temporarily interrupted by his early resumption of field operations, and remains to be completed this autumn.

I inclose sections 1st, 2d, and part of section 3d, of my full report on the telegraph operations between Cambridge and New York. This will embrace 41 articles, by number, in the first three sections, and the remainder in section 4. Of these I send you 29 articles of the work, before furnishing the remaining 12 articles of section 3d, and the whole of section 4th. I send, however, the heads of the 12 articles partially completed for section 3d. With regard to the telegraph operations for 1848, I can only remark that the preliminary computations give an approximate result differing only a small fraction of a second of time from the values already reported in 1846, January 13, (iii,) and March 11, 1848, (xii,) based on the observations of lunar culminations and the transportations of chronometers.

It becomes my duty again to call your attention to a subject of grave importance in the determination of the longitude of the cardinal point (New York) from those of the trigonometrical surveys of other nations alluded to in my special report of the 3d and 8th of May last, (xvi.)

In my first report of November 16, 1844, in which I stated the most probable longitude of the Philadelphia Observatory, viz., $5^h\ 00^m\ 40^s.52 \pm 0^s.35$, I made a reserve in respect to any constant errors of the methods. I there remarked, "it is to be hoped that those of the latter kind (constant errors) do not much exceed a second of time."

I have only to add, after an interval of four years, that all the additional light thrown on this subject by referring Washington and Cambridge to Philadelphia by the telegraph operations, and combining the results of reports i, ii, iv, and xiv, still confirms the conclusion of my first report in 1844, that the probable accidental error of the method of eclipses and occultations, subject to parallax and semi-diameter, was at that time not greater than $\pm 0^s.35$. None of the subsequent results by the same method have varied the longitude of the Philadelphia observatory from Greenwich by more than $\pm 0^s.35$ from the value of $5^h\ 0^m\ 40^s.52$ then reported. This result was based on the use of Burckhardt's constant or mean value of the moon's semi-diameter and horizontal equatorial parallax. Any error in the assumption of the correctness of the mean semi-diameter of the moon may be supposed to vanish in the mean result of all these groups of eclipses and occultations.

Such is not, however, the case with the error of the assumed value of the moon's mean horizontal equatorial parallax. It does not disappear in the aggregate of these groups, but remains a constant residual error, in the final result, of nearly the same order in longitude in seconds of time as that of the error itself in seconds of arc. This constant source of error was more clearly anticipated in 1845, in my report ii, in which I remarked as follows:

"There is, however, a source of constant error common to classes iv to vii, inclusive, viz., the uncertainty concerning the true value of the moon's horizontal parallax. This element is not, like the other lunar elements, susceptible of immediate observation in particular instances, but must be derived theoretically from the tables. The fact that Prof. Airy, in the Greenwich observations for 1840, has adopted Mr. Henderson's determination, and increased the value of Burckhardt's constant of the moon's horizontal parallax by $\frac{1}{2600}$, shows the uncertainty of this element. For the sake of uniformity I have not yet applied this correction, but have used Burckhardt's elements throughout as the basis for reducing both the occultations and corresponding meridian observations. If I had uniformly employed Mr. Henderson's value instead of Burckhardt's, I should have placed Philadelphia about $2^s$ in time further east than at present reported. The only instance in which the error from this source has been completely eliminated is in (class iv) the eclipse of May 14, 1836. Here the full discussion of all the equations of condition, computed by Rumker, gave me, for the correction of Burckhardt's constant, a value of 0''.2 greater than Henderson's. I hope, in the course of another year, to complete the discussion of all the observations of classes vi and vii within the limits of the survey, using Burckhardt's elements with equations of condition, by means of which the effect of any correction of these that may hereafter be indicated on the longitude of Philadelphia, may be applied at once." It is useless to attempt to determine the amount and consequent correction of this error of the moon's mean parallax by the ordinary process of accumulating results of solar eclipses and lunar occultations, compared with similar phenomena and with meridian observations in Europe. In all the history of the solar eclipses of the past, the illustrious Bessel, in 1841, could find none that afforded the necessary data for making this computation.

It is proper to remark, however, that he was probably not aware at that time that the eclipse of May 14 and 15, 1836, had been so extensively observed in America. I beg to call your attention to the report of the committee on that eclipse, appointed by the American Philosophical Society, which contains my computation of the error of Burckhardt's mean value of the moon's horizontal parallax, from the group of conditional equations computed by Rumker from the European and American observations.

If we adopt the value of the correction of Burckhardt's mean parallax of the moon, from that eclipse, (viz., +1''.52,) and apply it to the series of eclipses and occultations that form the basis of my several reports above mentioned, it will probably be necessary to diminish the longitude of all the stations of the United States Coast Survey by about two seconds of time, or half a minute of arc. There are other grounds which strengthen the presumption of the existence of this error in Burckhardt's constant value of the moon's parallax.

Olufsen has deduced a similar correction of +2''.22 from Lacaille's meridian altitudes of the moon, observed in the last century at the Cape of Good Hope, and compared with corresponding European observations. Henderson finds, from his observations with the Cape mural circle, as compared with the European, a similar correction of +1''.3.

Mason, Burg, and Damoiseau, in their lunar tables, used a greater constant value of the moon's parallax than Burckhardt's.

Plana's theory of the moon gives a similar correction. In my report of April last I gave the following proposed supplementary equations to be added to the ordinary result from the tables of Burckhardt's, to wit:

$$\begin{aligned} d\pi' = &+0''.9 \times \cos(2☽ - 3☉ + 2 \text{ perigee } ☽ - 2 \text{ perigee } ☉.) \\ &+1''.1 \times \cos(\ ☽ - 3☉ + 2 \text{ perigee } ☽ - 2 \text{ perigee } ☉.) \\ &-0''.9 \times \cos(2☽ - \ ☉ + 2 \text{ perigee } ☽ - 2 \text{ perigee } ☉.) \\ &-1''.1 \times \cos(\ ☽ - \ ☉ + 2 \text{ perigee } ☽ - 2 \text{ perigee } ☉.) \\ ds' = &+0.0022 \times \pi' + 0.27272 \times d\pi'. \end{aligned}$$

I there mentioned that these equations, which rest upon theoretic grounds, would very nearly explain the excess of the moon's semi-diameter over Burckhardt's value, as deduced from several hundred observations of both limbs of the moon.

All these concurrent sources of information would have warranted a new computation of the American longitudes with the corrected semi-diameter and parallax of the moon. Such a labor I had proposed to undertake, as soon as leisure could be found. It might be fairly inferred that the result would be a diminution of the difference of longitude between the European and all the American stations of one or more seconds.

It is proper to add, that on consultation with Prof. Peirce, by the instruction of the Superintendent, about the first of July, it was concluded to recommend the following course, viz: to compute the correction of Burckhardt's semi-diameter by the formula above mentioned, and compare the computed and observed semi-diameter in all cases of direct measurement, and thence deduce the most plausible value for the co-efficient $\frac{s'}{\pi}$, to be used in the Coast Survey computations of longitudes.

There was further information in the archives of the Coast Survey, confirmatory of the propriety of this step.

The longitude of the Cambridge Observatory from Greenwich has been determined by Mr. Bond, from the transportation of 116 chronometers in 34 voyages of the Cunard steamers from Liverpool to Boston.

This result is stated in my report xii to be $4^h\ 44^m\ 30^s.492 \pm 0^s.754$. It is to be regretted that the data are yet wanting for computation of the longitude by the return voyages from Boston to Liverpool. The possession of them would greatly increase the value of the chronometric result by removing the constant source of error in the acceleration or retardation of the sea rates.

In my report xiv, on the longitude of Cambridge from Greenwich, as derived from lunar occultations and solar eclipses, I find for the result $4^h\ 44^m\ 31^s.95$.

An increase of Burckhardt's parallax of about $1''.5$ would reconcile these discordant results. It will appear from my report xxiii, on the American longitudes by moon culminations, derived chiefly from the elaborate and very accurate computations of Lieutenant J. M. Gilliss, Assistant United States Coast Survey, that this class of phenomena place the four stations of Washington, Philadelphia, Cambridge, and Hudson, Ohio, more than two seconds of time nearer to Europe than the eclipses and occultations with Burckhardt's parallax and semi-diameter of the moon. I deem it my duty, therefore, to state my belief, from all the information now before me in the archives of the Coast Survey, that all the astronomical stations of the United States Coast Survey, including the cardinal point, New York, must be set down one or two seconds of time further east than the places hitherto assigned them by American and European astronomers.

If my previous report, based on the authority of the lunar tables, has fallen short of the precision that more correct tables would have furnished, I have the consolation to reflect that other computers have shared a similar fate. I might mention in this connection the names of Rittenhouse, Bowditch, Paine, and Peirce, among the American astronomers, and of Friesnecker, Zach, Lalande, De Ferrer, and Wurm, among the European, all of whom have more or less confirmed by their computations the results of my reports. De Ferrer had, in fact, noticed this constant source of error in the moon's mean parallax, but had not sufficient data to guard against its effect.

Such was the state of our information from the archives of the Coast Survey up to the 1st of July last, and such were the conclusions to which I had then arrived. Since that time, however, a new era has occurred in our knowledge of the lunar theory. I allude to the recent publication of the report of the astronomer royal of England, (Professor Airy,) on the reduction of the Greenwich observations of the moon. An abstract of this report has just been received in this country, in the proceedings of the Royal Astronomical Society of London for the 9th of June last. In it I find full confirmation of the view contained in my report (xvi) of 3d and 8th May last. The importance of the subject, and its direct bearing on the longitudes of the United States Coast Survey, will justify the quotation from the article:

"The astronomer royal, after proposing the best hypothesis of error which he can suggest, and bringing the later Cambridge and Greenwich observations to bear on the subject, concludes by saying, that probably Plana's mean value of parallax should be a little increased, and the moon's mass be correspondingly diminished. He proposes in the future reductions at Greenwich to increase Burckhardt's parallax by $\frac{1}{1200}$ part." Again he remarks: "The corrected co-efficient of the parallactic equation is 122″.37, but that is uncertain. An empirical equation would render the observations more accordant, but this has no probable physical foundation. The more likely cause for the observed irregularities is a change in the moon's semi-diameter, depending on changes in the telescope or in the observer. From the law of the inequality this co-efficient will always be somewhat uncertain."

I will merely remark that the increase of Burckhardt's mean value of the moon's parallax of 3″, (the $\frac{1}{1200}$ part,) if adopted, will fully confirm the report of April last, and will justify the opinion above stated, that all our American longitudes from Europe must be diminished one or more seconds of time. As it is of great importance to obtain definite results before venturing on a change that might have to be retracted, I beg to suggest the propriety of the following course of investigation:

1. To reduce promptly the telegraph operations so as to refer all the American astronomical observations to the cardinal point, (New York.)

2. To complete the geodetic connection of the points interrupted in the line of telegraph operations.

3. To employ a greater force of computers, and urge forward, by every possible means, the reduction of the American observations of moon culminations from 1842 to 1846, using the blank forms of the Coast Survey.

4. To employ additional force in forming the conditional equations for the eclipses and occultations, with corresponding meridian observations in Europe, to the end of 1846, also according to the Coast Survey blank forms.

5. From the report of Professor Airy, and from any other available sources, and subsidiary computations, to find the most plausible theoretical elements for the moon's place on the nights of the American observations.

6. To apply these values to the conditional equations in articles 3d and 4th, so as to render the residual error of theory the least possible.

7. To complete the discussion of the value 122″.37 of the co-efficient of the parallactic equation given by Professor Airy. For this purpose it will be necessary to extend the discussion which Professor Airy has limited to the instruments of Greenwich and Cambridge, so as to include all those with which the transits and altitudes of both limits of the moon have been observed at the same time in any country, so as to decide whether the theoretical co-efficient of 122″.37, or a more plausible empirical one, is to be adopted in the longitude computations of the Coast Survey.

8. To institute a full discussion of all the observations of occultations of the stars in the group Pleiades extant in any country. This discussion is recommended by Bessel as certain to afford more perfect data than those from any other source, for the determination of longitudes and for the correction of the moon's parallax and semi-diameter.

9. To resolve by the method of least squares, or otherwise, conditional equations so obtained, after applying all the theoretical and empirical corrections to the lunar constants and co-efficients, so as to obtain the most plausible value of the correction of the longitude of our cardinal point from the average of the European meridians.

I regret that this outline is so copious that my personal efforts, after making and reducing the telegraph operations, and after making out the numerous special reports that are needed in the department of longitude computations, are hardly sufficient to make a sensible impression upon the accumulating mass of longitude observations in the Coast Survey collection.

I cannot, therefore, too strongly urge upon your consideration the importance of taking proper steps to increase the number of computers in the longitude party, especially in the winter time, whenever the opportunity shall offer.

Yours, truly and respectfully,

SEARS C. WALKER,
*Assistant United States Coast Survey.*

Professor A. D. BACHE, LL. D.,
*Superintendent United States Coast Survey.*

# APPENDIX No. 15.

[From Coast Survey Report for 1850.]

EXTRACT FROM THE REPORT OF S. C. WALKER, ESQ., ASSISTANT UNITED STATES COAST SURVEY, TO THE SUPERINTENDENT, ON THE TELEGRAPHIC OPERATIONS AND THE COMPUTATIONS IN HIS CHARGE.

1.—EXPERIMENTS FOR GALVANIC WAVE TIME BETWEEN WASHINGTON AND ST. LOUIS.

Standing orders were left in January, February, March, and April, to work, with the aid of the several telegraph companies, by junction at their respective termini, through as long circuits as possible by the chemical and mechanical methods.

Although the Seaton station was in readiness during this period, yet there were only three nights in which the instrumental and meteorological circumstances and the personal arrangements admitted of experiments on extensive lines. Among these the work of the 4th of February holds a prominent place, from the favorable concurrence of all these particulars.

Owing to the kindness of Mr. D. Brooks, the chief operator on the Pittsburg and Louisville, and of Mr. Stager, of Cincinnati, and of Mr. E. Calton, on the Washington and Pittsburg line, we were able on that night to effect a junction directly between Seaton station and St. Louis, through a distance of 1,045 miles of iron wire, and of 742 miles of ground between these termini. The temperature was 0° Fahrenheit from Pittsburg to St. Louis, and 8° at Washington. The sky was clear and the wind northeast. The snow, on the average more than twelve inches deep, afforded so perfect an insulation that Washington, Pittsburg, Cincinnati, Louisville, and St. Louis could each during the same second receive the writing of all without change of adjustment. The presence of Mr. K. Culmann, of the Bavarian engineers, of Dr. B. A. Gould, of Professors Hubbard and Coffin, added interest to the experiment. The operations were divided into stages of ten minutes each, during which the Saxton clock at the Seaton station graduated the time scales on the Morse registering fillets at all the stations, and arbitrary dots or signals were given at one station and received at all the others. Thus Pittsburg, Cincinnati, Louisville, and St. Louis were alternately made the stations for these ten-minute terms of arbitrary signals, which were printed on all the registers every three seconds. In one ten-minute term between Washington and Pittsburg, the Seaton battery of fifty Grove's pint cups was between the stations on the short junction of 300 miles through the ground. In the other term the battery was on the long junction, or zinc pole, through the ground to St. Louis. It can hardly be expected that the Coast Survey will be able, for some time to come, to meet with another combination of circumstances so favorable as this.

2.—ATTEMPTED EXPERIMENTS ON WAVE TIME THROUGH DIFFERENT CONDUCTORS.

An arrangement was made with Major B. B. French, president of the Morse line to New York, to use the four wires of that line for an experiment on galvanic wave time, in which two registers, placed side by side, should in reality be separated by a circuit of iron wire 700 miles long on each pole. During the period from January to June no single occasion presented itself in which all four lines were in good working condition, so that our hopes in this respect were not realized.

3.—EXPERIMENTS WITH THE CHEMICAL TELEGRAPH LINE.

An arrangement has also been made with Henry J. Rogers, esq., superintendent of the Bain chemical line, (the North American Telegraph Company,) to work by the chemical method, but without success, owing to difficulties of insulation over the Hudson river; accordingly we did not succeed in our experiment with chemical imprints till the 8th of July, when, by the courtesy of Marshall Lefferts, esq., president of the Merchants' line of chemical telegraphs, the experiment was made between Boston and New York, on a circuit of 225 miles of wire and and 187 miles of ground. We were frustrated in our efforts to work from New York to Buffalo by the impossibility, in the actual state of the art, of making the double record by the chemical process at the two termini. The battery of sixty Grove's cups, required to work a chemical line of a thousand miles circuit, causes a burning of the paper at the battery station where the short duplicate circuit is used and the writing is made.

An ingenious experiment has recently been performed by Mr. Bain, which obviates the difficulty just mentioned. This consists in dipping the two poles of the short circuit into a plate of water, and gradually bringing them nearer to each other till the resistance on the short and long circuit, both of which pass through the battery, is the same. The discoloration of the paper by the galvanic current from the present or from the remote battery is then the same. It is proper, also, in this connection, to invite your attention to an ingenuous contrivance of Mr. Westbrook, chief operator on the North American Telegraph Company's line, by which the residual current not destroyed by the remote electrotome from imperfect intervening insulations is carried onward through a branch circuit without affecting the chemical registering disk, a slight waning of the galvanic current causing a perfect electrotome at the receiving station and forcing the current into the short branch or multiple circuit, so as not to interfere with a still remoter receiving station. The two contrivances of Messrs. Bain and Westbrook—the one for tapping the circuit at the writing station, the other for diverting it through a supernumerary circuit at the receiving station —give to the chemical method the same universality of application as that of the mechanical method. The chemical lines of any length may now write at any one station and receive at all the others; moreover, the batteries may now be equalized along the line instead of expending their whole force at the writing terminus. It is also proper to mention an important invention of Mr. Westbrook, of the electro-metallic mode of recording, which surpasses all others in distinctness and legibility. As the obstacles in the way of the telegraph operations for longitude by the chemical method are now removed by the ingenuity of Messrs. Bain and Westbrook, and as the lines, mechanical and chemical, in all directions from Washington, have been generously placed at the disposal of the Coast Survey without charge, after commercial business hours, the prospect of success in our very remote telegraph connections is much increased.

4.—PROGRESS OF THE RESEARCHES ON THE VELOCITY OF THE GALVANIC CURRENT.

The physical researches on the velocity of the galvanic current, made in 1849 and 1850, have been concluded for the present. All the experiments of the Coast Survey, on this subject, concur in showing a velocity of the propagation of the galvanic waves of about 15,400 miles per second in the iron wires of the American telegraph lines. These experiments have been made on lines extending from Seaton station north to Cambridge, Massachusetts, on a circuit of 1,021 miles; west to St. Louis on a circuit of 1,787 miles; and south to Charleston, South Carolina, on a circuit of 1,157 miles. They have been made in all varieties of temperature and in all degrees of excellence of insulation of the lines. They have been made with the chemical and mechanical registers. The results of the electrotome comparisons on the Morse registers, and of both kinds on the chemical registers, are so uniform in their indications of this velocity, and the number of single comparisons made and measured is so numerous, (exceeding ten thousand single results,) that it will require a strong accumulation of counter evidence, of which none has yet appeared, to impair confidence in the general character of our conclusions.

The entire experience of the Coast Survey up to this time cannot be reconciled with a velocity of galvanic waves in the ground greater than two-thirds of the velocity in the iron wires. Perhaps the proportion is even smaller. The subject is reserved for future investigation, in which the proportion of ground and wire circuit shall be changed at pleasure on the same evening. The work of February 4, 1850, between Washington and St. Louis, indicates that no change in the wave time between two stations is produced by the presence or absence of a powerful battery of fifty Grove's cups on the iron wire between them, in the shortest junction route.

In our experiments of February 4, 1850, a phenomenon was noticed, indicating an apparent crossing of the waves on the two poles of the telegraph circuit. The clock at Seaton station was on the platinum pole, and graduated the registers at all the stations with dots or pauses of the galvanic current of one-tenth of a second in length. The other nine-tenths of the second were, as usual, exhibited on the scale as a line of continuous action of the current.

Now, when the operator at a station distant some 800 miles made arbitrary signals in the following order, viz., dot, line, dot, all of the length of one-tenth of a second, and so timed that the line corresponded in absolute date to that of the Seaton clock pause, this apparent crossing took place. Thus: let us call A, B, and C, the operator's successive dot, line, and pause, and A′ the Seaton clock pause; then the Seaton station Morse register exhibits these four signals in the order A and A′ coincident, and forming a single pause, followed by B and C. The signal station register (Louisville, for instance) exhibits first A and B, then C and A′ coincident, and forming a single pause. This phenomenon was exhibited in more than 100 instances in the case of the Louisville and St. Louis signals, on the 4th of February, 1850. In other cases, where the middle line at the western stations did not correspond to the absolute date of the Seaton clock pause, since the mechanical register could not imprint both pause and signal at the same time, it appears to have followed the laws of mechanics and to have obeyed the influence of the resultant of the forces, caused by the simultaneous influence of the line or current on the zinc pole and the pause on the platinum pole, combined with the acquired armature momentum. This interference of the waves sometimes registered a line on the scale for the western signal, when the eastern would have given a pause. Sometimes the two effects neutralized each other, and the armature remained for a fifth part of a second motionless, exhibiting on the register either a continuous line or pause. This circumstance of the apparent crossing of the waves, and of their apparent interference, as indicated on the registers, affords ground for interesting physical researches on the interference of the galvanic waves which go out to meet each other on the two poles of the telegraphic circuits. This subject has already been alluded to in your brief communication to the Charleston meeting of the American Association. Some discussion has arisen, at the August meeting in New Haven, whether the analogy of the crossing and interference of waves of sound, light, heat, &c., applies to the galvanic waves in consequence of the existence of a re-entering circuit, while the other waves are propagated in a right line through space. But may we not conceive that for a short portion of the telegraphic circuit the action of the galvanic medium, whatever it may be, is similar to that of the medium of light and heat, and to that of the air, as the known medium of sound?

The progress of invention in regard to the mechanical registers, in the last year, has been very remarkable. The defect of all the registers in use heretofore has been the uncertainty of the time of a revolution of the registering apparatus, whether by cylinder, disk, or drawing rollers. Although the approximate portions of the graduated scale were very nearly equal, yet the accumulation even of the small discrepancies became manifest in the course of a few minutes. Professor O. M. Mitchel's revolving disk, with the Munich centrifugal fly, revolving in a conical box for its governor, had, in 1849, approached nearer to perfection than any of the registering machines on which our experiments had been made.

Mr. Saxton's fly, inclosed in a vessel of quicksilver, gave a very good performance in graduating the recording sheet, rolled round the revolving cylinder.

All these methods, however, were liable to the objection which I have mentioned, that they did not guard against the cumulative error. It was obvious that the pendulum alone afforded an effectual safeguard against this cumulation error; but the difficult point was to derive from it a uniform rotary or rectilinear motion.

This difficulty has been in a great degree obviated by a machine called the spring governor, which is the joint invention of Mr. William Cranch Bond and his sons, Messrs. George P. and Richard Bond. It consists in the application of a spring like the mainspring of a watch, (having, however, only one coil,) which takes motion from the primary train moved by a pendulum, and communicates it to a secondary train, controlled by a centrifugal fly. A cylinder on a delicate axis on friction rollers is made to rotate by this secondary train. The pen which graduates the sheet rolled round the cylinder is moved by an independent train and weight. The cylinder, controlled in the single seconds by the centrifugal fly, and in the long periods by the pendulum, performs its revolutions with all the accuracy of a clock in its measurement of time. The cylinder revolves once in a minute, so that the enveloping sheet has sixty seconds on a line. It has sixty lines on a sheet. By stamping the 0's, 5's, and 10's of seconds on the top, and of minutes on the left margin, the eye seizes instantly the correct reading of the minute and second. The fraction to the tenth of a second may be estimated by the eye, or it may be read off to the hundredth of a second by a glass or horn scale, graduated to suit the intervals. In a perfect register the scale of seconds should be straight and vertical from the top to the bottom of the sheet. In the sheet now before me the maximum deviations of this line from a straight vertical line are not more than one-tenth of a second, and the discrepancies of any single second's length from that of the average scale quite insensible. The Messrs. Bond deserve the highest commendation for this useful invention, which seems to have removed the only obstacle in the way of the practice of registering and of reading off the dates of observations from the printed scales.

# APPENDIX No. 16.

[From Coast Survey Report for 1851.]

## REPORT OF SEARS C. WALKER, ASSISTANT IN THE COAST SURVEY, COMMUNICATING THE MEASURRES OF WAVE-TIME MADE FROM 1849 TO 1851.

CAMBRIDGE, *September* 30, 1851.

DEAR SIR: I beg to submit a statement of the experience of the Coast Survey on the subject of galvanic wave-time since my last annual report of October 15, 1850.

The result of our experience was then stated, as follows:

1. That the average of all our experiments to that time indicates a velocity of propagation of the inducing waves of 15,400 miles per second in the iron wires of a telegraph line.

2. That the velocity of propagation through the ground appears to be less than two-thirds of the velocity in the iron wires.

These conclusions were in accordance with the independent results of the researches of Dr. B. A. Gould and Mr. Karl Culmann, previously read, and since published in the proceedings of the American Association for the Advancement of Science, at their meeting at New Haven, in August, 1850.

There have been three independent series of observations for the value of wave-time, made since October last, 1850.

The first experiment was repeated on several nights, between Seaton station and Portsmouth, Virginia. The distance on the iron wires is 268 miles, and the distance through the ground is 180 miles. The clock station excess, in the electrotonic readings, by a mean of 221 measures, was $+0^s.024$, while the computed excess for the assumed velocity of 15,400 miles per second, in the iron wires, was $+0^s.035$. The difference between theory and computation is, theory greater by $+0^s.011$.

The second experiment was made from Charleston, South Carolina, to Augusta, Georgia, in the winter of 1851. The distance on the iron wire from Columbia (where the Charleston end went to the ground) to Augusta was 301 miles, and from Augusta to Savannah 146 miles, making the total connection through the iron wire 447 miles, and the distance through the ground, from Columbia to Savannah, 135 miles. The clock was at Savannah. The arbitrary signals were given at Charleston. The observed clock excess was, by 59 measures, $+0^s.056$. The computed wave-time, for the above assumed velocity, was $+0^s.058$, leaving a difference of $+0^s.002$.

The clock excess of Augusta above Savannah was, by observation, (40 measures,) $+0^s.019$; by theory, $+0^s.019$; difference, $+0^s.000$.

The third experiment was made at Cincinnati, on the 9th of May last, on the occasion of the meeting of the American Association for the Advancement of Science. The telegraph line was composed of 840 miles of iron wire, without ground connection. The distances were as follows: From Cincinnati to Steubenville 295 miles, thence to Cincinnati the same, thence to Louisville 125 miles, thence to Cincinnati the same. The personal clock signals were given by Mr. Stager, chief operator at Cincinnati. In the first experiment the arbitrary signals were given by the operator at Steubenville, and recorded at Steubenville, and also on the two registers at Cincinnati, on opposite branches of the line. These registers I will call, respectively, Stager and Jones; Stager being the register for the clock station. The observed excesses were, for the Steubenville arbitrary signals, as follows:

| | |
|---|---|
| Stager—Steubenville | $+0^s.040$ by 31 measures. |
| Stager—Jones | $+0^s.039$ by 31 measures. |

Again, for the Jones arbitrary signals on the Stager clock scale, we found:

| | |
|---|---|
| Stager—Steubenville | $-0^s.004$ by 39 measures. |
| Stager—Jones | $+0^s.050$ by 226 measures. |

The direction of the current from the platinum to the zinc, through the junction wires, was from Stager to Steubenville, thence to Jones, thence round by Louisville to Stager.

This is the first experiment made by the Coast Survey on a telegraph line of iron wire exclusively, without ground connection.

The first conclusion to be drawn from this experiment is, that the excesses of the clock station readings, in the experiments heretofore made, have not been owing to the fact that a part of the galvanic circuit has been made through the ground, since they are here found to be as great for the dimensions of the line as in former experiments with the partial ground connections.

This experiment was made with a long circuit of iron wire, without ground connection. It confirms the general conclusion respecting the value of wave-time.

It gives a new field for the discussion of the physical question, whether the wave is propagated round in one direction, and only affects the magnets as it reaches them in succession in this direction, or whether the wave travels by the *shortest* direction from one magnet to another, without reference to the character of the poles.

Our experiments with lines composed partly of ground and partly of iron wire stretched on poles, led to the preference of the latter view of the subject.

The experiment at Cincinnati, in 1851, raises some doubt on this conclusion. It was made with a single battery at Cincinnati, and with 840 miles of wire, all in the air. The work of this night was not as complete as I could have desired. I must, therefore, wait till similar experiments are made under more favorable circumstances, before attempting a further examination of the question.

| Number. | Date. | Clock station. | Signal station. | Name of excess. | Miles of wave space. | Observed wave time. | No. of measures. | Obs'd—comp'd wave time. |
|---|---|---|---|---|---|---|---|---|
| | | | | | | s. | | s. |
| 1 | January 23, 1849 | Pa.. | Ca .. | Pa—Ca ... | 938 | +0.063 | 25 | +0.002 |
| 2 | ......do.......... | ..do.. | ..do.. | ..do—N .... | 418 | .024 | 16 | +0.003 |
| 3 | ......do.......... | ..do.. | ..do.. | Wn—Ca ... | 938 | .053 | 34 | +0.008 |
| 4 | ......do.......... | ..do.. | ..do.. | ..do—N .... | 418 | .011 | 23 | +0.016 |
| 5 | ......do.......... | ..do.. | ..do.. | N—Ca .... | 520 | .039 | 18 | —0.005 |
| 6 | ......do.......... | ..do.. | N ... | Pa—N .... | 418 | .036 | 7 | —0.009 |
| 7 | ......do.......... | ..do.. | ..do.. | ..do—Ca ... | 418 | .004 | 8 | +0.023 |
| 8 | ......do.......... | ..do.. | ..do.. | Wn—Ca... | 418 | .034 | 9 | —0.007 |
| 9 | ......do.......... | ..do.. | ..do.. | Wn—N.... | 418 | .020 | 14 | +0.007 |
| 10 | October 31, 1849 | Ci... | Wn . | Ci—Wn ... | 1,151 | .068 | 88 | —0.008 |
| 11 | ......do.......... | ..do.. | ..do.. | ..do—Cd ... | 651 | .043 | 12 | —0.001 |
| 12 | ......do.......... | Wn.. | Cd .. | Wn—Ci ... | 330 | .019 | 34 | —0.002 |
| 13 | ......do.......... | ..do.. | ..do.. | ..do—Ci ... | 330 | .023 | 29 | +0.002 |
| 14 | ......do.......... | ..do.. | ..do.. | H—Cd .... | 182 | .008 | 22 | +0.004 |
| 15 | ......do.......... | ..do.. | ..do.. | ..do—Ci ... | 182 | .009 | 25 | +0.003 |
| 16 | ......do.......... | ..do.. | Wg . | Wn—Cd .. | 330 | .017 | 17 | +0.004 |
| 17 | ......do.......... | ..do.. | ..do.. | ..do—Ci .. | 618 | .025 | 12 | +0.015 |
| 18 | ......do.......... | ..do.. | ..do.. | H—Cd .... | 182 | .038 | 5 | —0.026 |
| 19 | ......do.......... | ..do.. | ..do.. | ..do—Ci ... | 470 | .043 | 9 | —0.013 |
| 20 | ......do.......... | ..do.. | ..do.. | Cd—Ci.... | 288 | .004 | 12 | +0.015 |
| 21 | February 4, 1850 | ..do.. | Pg .. | Wn—Pg .. | 576 | .028 | 41 | +0.009 |
| 22 | ......do.......... | ..do.. | ..do.. | ..do—do.... | 576 | .030 | 44 | +0.007 |
| 23 | ......do.......... | ..do.. | ..do.. | .do—Ci.... | 576 | .034 | 33 | +0.003 |
| 24 | ......do.......... | ..do.. | ..do.. | ..do—do.... | 576 | .032 | 36 | +0.005 |
| 25 | ......do.......... | ..do.. | ..do.. | ..do—L .... | 576 | .035 | 57 | +0.002 |
| 26 | ......do.......... | ..do.. | ..do.. | ..do—S. L.. | 576 | .051 | 11 | —0.014 |
| 27 | ......do.......... | ..do.. | Ci... | ..do—Pg.. | 576 | .028 | 32 | +0.009 |
| 28 | ......do.......... | ..do.. | Le .. | ..do—do.... | 576 | .028 | 47 | +0.009 |
| 29 | ......do.......... | ..do.. | Ls .. | ..do—do.... | 576 | .045 | 56 | —0.008 |

| Number. | Date. | Clock station. | Signal station. | Name of excess. | Miles of wave space. | Observed wave time. | No. of measures. | Obs'd—comp'd wave time. |
|---|---|---|---|---|---|---|---|---|
| | | | | | | s. | | s. |
| 30 | February 4, 1850 | Wn.. | Ci... | Wn—Ci ... | 1,244 | +0.075 | 33 | +0.004 |
| 31 | ......do.......... | ..do.. | ..do.. | ..do—Le ... | 1,244 | .075 | 32 | +0.004 |
| 32 | ......do.......... | ..do.. | ..do.. | ..do—Ls ... | 1,244 | .070 | 26 | +0.014 |
| 33 | ......do.......... | ..do.. | Le .. | ..do—Ci ... | 1,244 | .081 | 58 | +0.001 |
| 34 | ......do.......... | ..do.. | Ls .. | ..do—Ci ... | 1,244 | .095 | 60 | —0.016 |
| 35 | ......do.......... | ..do.. | Le .. | ..do—Le ... | 1,494 | .109 | 56 | —0.013 |
| 36 | ......do.......... | ..do.. | ..do.. | ..do—Ls ... | 1,494 | .102 | 49 | —0.006 |
| 37 | ......do.......... | ..do.. | Ls .. | ..do—Le ... | 1,494 | .134 | 65 | —0.038 |
| 38 | ......do .......... | ..do.. | ..do.. | ..do—Ls ... | 2,090 | .145 | 61 | —0.049 |
| 39 | ......do.......... | ..do.. | Ci... | Pg—Ci .... | 668 | .047 | 32 | —0.004 |
| 40 | ......do.......... | ..do.. | ..do.. | ..do—Le ... | 668 | .047 | 32 | —0.004 |
| 41 | ......do.......... | ..do.. | ..do.. | ..do—Ls ... | 668 | .042 | 29 | +0.001 |
| 42 | ......do.......... | ..do.. | Le .. | ..do—Le ... | 918 | .081 | 48 | —0.023 |
| 43 | ......do.......... | ..do.. | ..do.. | ..do—Ls ... | 918 | .074 | 58 | —0.016 |
| 44 | ......do.......... | ..do.. | Ls .. | ..do—do ... | 1,514 | .089 | 60 | +0.008 |
| 45 | ......do.......... | ..do.. | ..do.. | Ci—do..... | 846 | .050 | 60 | —0.005 |
| 46 | ......do.......... | ..do.. | ..do.. | Le—do .... | 596 | .032 | 57 | +0.006 |
| 47 | February 5, 1850 | ..do.. | Cn .. | Wn—Cn... | 1,416 | .084 | 45 | —0.008 |
| 48 | July 8 .......... | Bn .. | N .. | Bn—N .... | 484 | .033 | 63 | —0.002 |
| 49 | December ........ | Sn .. | Ps .. | Sn—Ps .... | 536 | .024 | 221 | +0.011 |
| 50 | Feb. and March .. | Sa .. | Cn .. | Sa—Cn .... | 596 | .056 | 59 | +0.017 |
| 51 | ..do........do ... | ..do.. | Aa .. | Sa—Aa .... | 292 | .019 | 40 | +0.000 |
| 52 | May 9, 1851 | Ci... | Se ... | Ci—Se .... | 580 | .040 | 31 | +0.006 |
| | | | | | 38,294 | +2.496 | 2,051 | |

From which it appears that the time of traversing 15,342 miles is one second. The column marked (obs'd—comp'd) is based upon this value.

Yours, respectfully,

SEARS C. WALKER,
*Assistant Coast Survey.*

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# APPENDIX No. 17.

[From Coast Survey Report for 1851.]

## ABSTRACT OF REPORTS ON LONGITUDES, BY SEARS C. WALKER, ASSISTANT IN THE COAST SURVEY, TO THE SUPERINTENDENT.

CAMBRIDGE, *September* 30, 1851.

DEAR SIR: I beg to submit an abstract of all my reports on longitude hitherto made:

*Harvard Observatory, west of Greenwich.*

| | h. m. s. |
|---|---|
| (A) By moon culminations at Harvard, 1843–1845 .................. | 4 44 28.47 |
| (A) By moon culminations at Hudson, Ohio, 1838–1844 .................. | 28.62 |
| (A) By moon culminations at Wilkes's observatory, 1838–1842 .................. | 28.52 |
| (A) By moon culminations at Washington Observatory, 1845 .................. | 28.06 |
| (A) Mean by moon culminations .................. | 4 44 28.42 |

| | h. m. s. | Weight. |
|---|---|---|
| (B) By eclipses, transits, and occultations at Dorchester and Harvard, 1820–'40 | 4 44 32.16 | — 6.4 |
| (B) By eclipses, transits, and occultations at Brooklyn, New York | 31.22 | — 0.4 |
| (B) By eclipses, transits, and occultations at Philadelphia, 1769–1840 | 32.56 | — 2.5 |
| (B) By eclipses, transits, and occultations at Wilkes's observatory, 1838–1842 | 33.13 | — 1.0 |
| (B) Mean by eclipses, transits, and occultations | 4 44 32.27 | —10.3 |

These phenomena have been reduced by Burckhardt's tables, and include, on the average, the constant error of his parallax of the moon. Airy, in his reductions of the Greenwich observations of the moon, makes the correction of this parallax to be $\triangle\pi_{\odot} = +1''.78$. Professor Peirce and myself have computed the average value of the co-efficient $\left(\frac{\triangle d}{\triangle \pi}\right) = -1^s.5$, whence $\left(\frac{\triangle d}{\triangle \pi}\right) \times \triangle\pi_{\odot} = -2^s.67$; and $4^h\ 44^m\ 32^s.27 - 2^s.67$ are—

| | h. m. s. |
|---|---|
| (B') Corrected mean by eclipses, transits, and occultations | 4 44 29.60 |
| (C) By chronometers with Liverpool— | |
| Indiscriminate mean of 373 chronometers in all | 4 44 30.92 |
| Indiscriminate mean of 175 chronometers, (great special exp. of 1849) | 30.96 |
| Bond's indiscriminate mean of 175 chronometers, (great special exp. of 1849) | 30.10 |
| (C) Adopting the last value | 4 44 30.10 |

| | h. m. s. |
|---|---|
| (A) Longitude of Harvard Observatory | 4 44 28.42 |
| (B') Longitude of Harvard Observatory | 29.64 |
| (C) Longitude of Harvard Observatory | 30.10 |
| Adopted for the present, Harvard Observatory | 4 44 29.50 |

Then we have, by the telegraph operations of the Coast Survey, the following results from Greenwich, depending on this assumed longitude of Harvard Observatory:

| | h. m. s. |
|---|---|
| New York, (City Hall) | 4 56 00.150 |
| Philadelphia Observatory | 5 00 37.504 |
| Seaton station, (Washington, D. C.) | 5 07 58.564 |
| Capitol, Washington | 5 08 00.853 |
| Wilkes's observatory | 5 08 00.958 |
| Washington Observatory | 5 08 11.206 |
| Georgetown Observatory, (Georgetown, D. C.) | 5 08 17.206 |
| Charleston Observatory, S. C., (Sec. V) | 5 19 43.832 |
| Savannah Exchange, (Sec. V) | 5 24 20.572 |
| Hudson Observatory, Ohio | 5 25 43.205 |
| Cincinnati Observatory | 5 37 58.062 |

| The following results depend on moon culminations and occultations: | h. m. s. |
|---|---|
| Sand Key, Florida, (Sec. VI) | 5 27 31.641 |
| Moro Castle, (Havana) | 5 29 24.000 |
| Point Conception, (Sec. X) | 8 01 42.640 |

Respectfully submitted.

SEARS C. WALKER,
*Assistant Coast Survey.*

Prof. A. D. BACHE, LL. D.,
*Superintendent Coast Survey.*

# APPENDIX No. 18.

[From Coast Survey Report for 1851.]

NOTES OF A DISCUSSION OF TIDAL OBSERVATIONS, MADE IN CONNECTION WITH THE COAST SURVEY, AT CAT ISLAND, IN THE GULF OF MEXICO, BY PROFESSOR A. D. BACHE, SUPERINTENDENT OF THE COAST SURVEY. (SEE SKETCHES H, NOS. 2 TO 6, INCLUSIVE.)

In executing the hydrography of the entrance of Mobile bay and of Mississippi sound, connected tidal observations were made under the immediate direction of Lieutenant Commanding C. P. Patterson, United States navy, Assistant in the Coast Survey.

The observations at Cat island, at the entrance to Lake Borgne, Louisiana, and at Fort Morgan, at the entrance to Mobile bay, have undergone more than one discussion, the peculiarities of the tides giving great interest to the observations.

The results, as obtained from a year's hourly observations, day and night, at Cat island, will be given as far as obtained, the steps taken for further progress stated, and the information which has been obtained from other sources, bearing upon this most interesting problem of the tides in the Gulf of Mexico, will be briefly touched upon.

I hope, in the progress of the survey along this part of our coast, to develop the subject of these tides, full of importance to the navigator, and of interest to the man of science. These tides, with special exceptions, ebb and flow but once in twenty-four hours.

The tide-gauge was of the kind known as the box gauge, with a float and staff, graduated into feet and decimals of a foot. It was placed in the harbor of Cat island, near the light-house, at the extremity of a temporary wharf.

The harbor, as the Coast Survey chart which I now present to the meeting shows, turns its widest and deepest opening to the east.

Apparent time was given by a mark, and the observations were made at mean solar time by applying the equations. The time was of less consequence than ordinary in these observations, from the small rise and fall of the tide, which prevented small differences of time from being noticeable by differences of rise and fall. Slight inequalities, caused chiefly by wind, were also found to affect the observations so materially that it was not deemed advisable to observe oftener than once the hour; and after attempting to determine the epoch of high and low water by more frequent observations, it was decided that errors would probably be introduced by aiming at a degree of precision which the phenomena themselves did not present.

The observations were made day and night, hourly, for a year, with exceedingly rare omissions, and, as the discussion has shown, with a degree of faithfulness which merits very great praise. The observers were Messrs. Gustavus Würdeman and R. T. Bassett, attached to the Coast Survey.

The general opinion of nautical men on the subject of these tides is that they mainly depend upon the action of the wind; and the very regular effect which may be shown to result from a discussion of the tides, in reference to the local action by the wind, lends plausibility to this generalization, which, nevertheless, is unfounded.

The causes are of a much more general character, and such as usually influence the tides, so modified as to be difficult to bring out; phenomena which are only accessory in the ordinary discussions assuming here the chief and overruling part.

The regular tabulation of the observations was made by Lieutenant Commanding C. P. Patterson, who did not fail to perceive that the ordinary methods of discussion of the tides were inapplicable. His removal from the survey on other professional service has devolved upon me the labor of discussing the results.

Their importance, interest, and novelty, so far as our coast and their striking peculiarities are concerned, have justified me in giving much time to the discussion, which has been carried on, under my immediate direction, by Mr. G. W. Dean, sub-assistant in the Coast Survey, and by Messrs. R. M. Bache, A. S. Wadsworth, jr., and W. M. Johnson.

I am indebted, for the diagrams necessary to illustrate the conclusions already arrived at, to Messrs. Bache, Johnson, and Keyser.

I present a part only of the labors of these gentlemen. The whole of the hourly observations for the year have been thrown into the form of curves, and numerous tables for examining and verifying the different hypotheses have been made by them. Though the subject was reached inductively, I do not propose to present it strictly in that form.

The work, even now, is far from being complete; indeed, we have rather reached the true method of discussion than have completed the discussion, and we may yet have to modify our hypothesis, though I think not materially. I present it to the association as a work in *progress*. When the investigation for this station is made complete, the application of the methods to the other stations on the Gulf of Mexico will be in a degree mechanical.

It is curious that one among the earliest complete series of tidal observations on record, is of tides ebbing and flowing but once in twenty-four hours. The observations were made by Mr. Francis Davenport, at Batsha, of the tides on the bar of Tonquin, and communicated to Dr. Halley, who gave them, with a diagram connecting the phenomena with the moon's motion in the ecliptic, in the thirteenth volume of the Philosophical Transactions for the year 1683. Newton explained these tides by his lunar theory, but in a way, as appears to me, to leave it doubtful whether he supposed the interference of two ordinary or six-hour tides to produce the phenomena. These tides have been referred to since by almost every writer of note, who has given a general theory of the tides.

The subject of the diurnal inequality of the tides has been so completely and ingeniously discussed by Mr. Whewell, Master of Trinity, that it may be said emphatically to be his own. He first pointed out the empirical law of variation of this inequality. The first distinct attempt to trace the cause of apparent ebb and flow once in twenty-four hours to the influence of the diurnal irregularity, is also, so far as I know, his. In discussing (Phil. Trans. for 1837, Part I) the tides at Singapore, where the diurnal inequality is very large, he was led to the conclusion, if carried a little further, "at a certain stage of it the alternate tides would vanish." To this effect he attributed the "single-day tides of King George's sound, on the coast of New Holland, as observed by Captain Fitz Roy," and gives the curves for a week's observations on the diagram accompanying his papers. The progress of the diurnal inequality wave along the coast of Europe forms an interesting part of Mr. Whewell's labors, the conclusions of which are given in the same volume of the Philosophical Transactions.

In all these cases, however, there are two tides in the course of the day, so as to bring out the diurnal inequality by the comparison of the consecutive high or low water. The subject is followed up in the eleventh series of tidal researches by Mr. Whewell, and in the appendix, in which the diagram of the tides of Petropaulovski, in the bay of Avatcha, Kamstchatka, approaching very nearly, at certain parts of the lunar month, to the order of single-day tides, is given, to prove that the diurnal inequality may be so large "as to lead to the appearance of only one tide in twenty-four (lunar) hours." The equations of the diurnal and semi-diurnal tide-waves are given in this paper, and the wave produced by certain cases of their interference is discussed. (Phil. Trans. for 1840.)

I do not pretend to give such notice of these important papers as would be necessary in a formal communication. Unquestionably the observations now under examination would have furnished to Mr. Whewell only the means of trying ideas and consequences flowing from those which have been already discussed by him; yet the forms of discussion are original, and perhaps new, and the conclusions present so much of novelty that they remain to be fully put to the test by more elaborate discussion, and by bringing the results at other places to bear upon the same question. I am forced, by the necessity for brevity, to omit a reference to the learned, ingenious, and elaborate paper of Mr. Airy, in the Philosophical Transactions for 1848.

The small rise and fall of the tides, amounting on the average to but one foot, would seem to make it difficult to obtain the law of the phenomena, even with the aid of the most careful and truthful observations—the class to which those under discussion have proved to belong. In regard both to time and height, we may expect to be baffled by small irregularities, requiring long continuance of observations and comparisons of means to get rid of. Thus far few cases have occurred which do not exhibit more striking coincidences than differences.

1. To show the time of high or low water in such a way that the discussion might be readily

generalized, the diagrams, of which a specimen is before the association, were made, (Pl. 3, or H. No. 2.) The hours of the day are the ordinates, and the days of the month the abscissa. The signs H and L show in their proper place the hour of occurrence of high and low water for each day. The time of the moon's superior transit is marked, and the periods of greatest declination, and of crossing the equator. The result is easily generalized, that there is ordinarily but one high and one low water at Cat island in twenty-four (lunar) hours, and that when there are two tides they occur about the time of the moon's crossing the equator, and are usually most regular and strongly marked when in syzigies, with declination nearly zero. Following one set of high and low waters, it will be found that they occur later and later as the lunar day gains on the solar, with very remarkable differences, of which the explanation will be given towards the period of small declinations. The interval from high to low water is generally less by some hours than that from low to high. That as the moon approaches the equator, there are a few days of singular double tides, or of single tides, in which the times from low to high water are very much increased. That when the declination changes its name, a high tide takes nearly the place of a low in time, and *vice versa*, with an interval of irregularity; or, in other words, the tides are displaced by nearly twelve hours.

2. There is, as Mr. Whewell has remarked, no proper establishment to be derived from such tides; yet, we may obtain a desirable datum by throwing the results into the form of tables, in which the luni-tidal intervals are arranged according to the days from the zero of declination and the corresponding superior and inferior transits, and for north and south declinations. This will be made more clear by subsequent explanation. These afford a test of the theory of these tides by showing the displacement of the ordinate of high and low water, and might be used for the inverse purpose of forming prediction tables. Such tables of luni-tidal intervals for three months I now submit. They show considerable steadiness and similarity of intervals towards the maximum of declinations and great variations near the zero, and greater discrepancies than is usual in ordinary tides. These are from a series of tables computed by Mr. R. M. Bache for the year, and containing the times of high and low water, deduced from the daily curves, the readings of the gauge, the rise and fall of the tides, the times of the moon's superior and inferior transit, and the moon's declination.

The intervals serve to show that the high water belongs alternately to the superior and inferior transits of the moon, according as the moon's declination is north or south, with a few cases only which admit of doubt. Two sets of luni-tidal intervals were computed (see tables) for three months, to ascertain the proper epoch of reduction, or age of the tide. In one case, the intervals were referred to the superior transit of one day before; and in the other, to the superior transit of two days before. The square of the discrepancy of the mean in the latter case was greater than in the former. An establishment deduced from these numbers for high water, without correction, would have a probable error, as tried by discrepancy from the mean, of nearly eighty-four minutes. I have little doubt of being able to reduce this error, by computation, much within the limits of observation, so as to give useful prediction tables. The foregoing results point distinctly to a ruling cause depending upon the moon's declination.

3. The hourly observations for the year were thrown into the form of curves—the abscissas representing the hours, and the ordinates the heights. Of these I present, as characteristic, the months of January and March, (Pl. 4, 5, or H. Nos. 3, 4.) In January the tides are single throughout the month, the rise and fall diminishing towards the zero of declination; and in March, two periods of marked double tides occur. The times of new and full moon coincide nearly with the zero of declination of March; in January the syzigies occur at times of greatest declination. A series of diagrams, prepared for periods of declination zero, show irregularities, or double tides, near these times. Before disappearing, the tide which is lost appears rather as an irregularity than as a real tide, puzzling to the observer, and a severe test of his faithfulness. A similar set of diagrams for the periods of greatest declination show uniformly single tides and the greatest comparative rise and fall at the same periods, whether coinciding with syzigies or with first and last quarters. In computing the height of spring and neap tides by the common methods, four months gave zero or negative differences.

To discuss the epochs of the phenomena, as compared with greatest and least declinations, I prepared two sets of tables, which require revision. They show sometimes an actual coincidence in the epoch of least tides and zero of declination—sometimes a precedence and sometimes a subsequence—which, when not caused by irregularity of winds, I believe will find a satisfactory explanation; at a mean, there was little advantage in the discussion found from displacing the epoch. The average rise and fall for the second day before the greatest declination was 1.68 feet; for the day next preceding the greatest declination 1.78; for the day of greatest declination 1.81; for the next day 1.86; and for the next 1.77. Tracing a curve from these would give the epoch of greatest rise and fall about 0.75 days after the greatest declination. The average rise and fall on the corresponding days, in reference to declination zero, were 0.96 feet, 0.75, 0.60, (dec. zero,) 0.63, 0.73, the curve giving the epoch about one-sixth of a day after the zero of declination. The numbers, as stated, require revision; and there are causes for apparent displacement, which require further examination.

4. This general examination tends to point to the diurnal irregularity, as Mr. Whewell has stated, as the cause of the occurrence of these single-day tides; a view which is confirmed by such examinations as I have been able to make of the hourly tidal observations at Fort Morgan, at the entrance of Mobile Bay. The interference in this case would be between the diurnal tide-wave, which represents the diurnal inequality, and the ordinary semi-diurnal wave; whether this wave has a regular progress along the coast, independently of the semi-diurnal wave, as was at first supposed by Mr. Whewell, or whether its phenomena are local, as he has since been led, from his investigations, to believe. If the observed wave is produced by its interference with a semi-diurnal wave, we can only study the phenomena to advantage after the observed wave has been separated into its components.

5. As a first approximation, I assumed the two waves to be governed by the law of sines, and then determined the curve which would result from the superposition of two such waves, having the same or different origins. The mean of the regular double tides, about the zero of declination, would present a first approximate value of the rise and fall of the semi-diurnal tides, and the mean of double and single tides, at the maximum of declination, would, especially when near the quadratures, give a first approximation to the height of the diurnal tide. The comparisons with the forms of curves already traced, addressing the eye, are easily made.

I present, herewith, diagrams (Pl. 6, or H. No. 5) for the case, in which the maximum of the diurnal tide coincides with that of the semi-diurnal, is three hours in advance, (or coincides with mean water falling,) six hours (or coincides with low water,) and nine hours, (or coincides with the second mean, or mean water rising,) using the approximate quantities referred to above for the greatest height of two component curves. It requires little examination to see that neither of the first three forms represents the case, and that the fourth does so remarkably, even in what appear to be small irregularities in the daily curves. This will be seen in the results for October, of which a diagram on a large scale is presented, giving the tidal curves near the zero, and thence up to the maximum of declination, for the first half of the month. In the single-day tides there was the same slow rise compared with fall; sharp rise and fall near high and low water, with the tendency to a stand during the rise; the same excess in the interval of time from low to high water, over that from high to low water. This hypothesis as to the position of the two waves may perhaps be slightly improved by further discussion. It is obvious, from the equation of the curve, (which I have already referred to, as given by Mr. Whewell,) that the form and position of remarkable points will vary with the constants in the component curves, as well as with the position of the origin of each in reference to that of the other.

To carry out the representation graphically, I have drawn the curves for four values of the constants of the diurnal and semi-diurnal, formed from the observations with the same displacement of nine hours in the time of high water of the diurnal curve, and corresponding to the epochs of the maximum declination, two, four, and six days before or after the maximum. These show the general features of the curve sufficiently, and the variations in the times and heights, the passage from single to double tides, and the reverse; and the coincidence with observations is such as to warrant a close numerical discussion.

6. The equation of the curve shows how much the time of high and low water depends on the constants in the diurnal and semi-diurnal curve.

The equivalent of the equation given by Mr. Whewell is—

$$C \cos 2t + D \cos (t - E) - y = O,$$

in which $t$ is the time in hours from the place of the maximum ordinate of the semi-diurnal curve as an origin; C is the constant of that curve of sines; E is the distance of the maximum ordinate of the diurnal curve for the former, and D the constant for the curve of sines; $y$ is the ordinate of the complex curve.

By an easy transformation this takes the form—

$$2\,C \cos^2 t + D \cos t \cos E + D \sin t \sin E - C = y.$$

$$\text{For } E = 9 \text{ hours.} \quad \text{Cos } E = -\sin E = -\sqrt{\tfrac{1}{2}},$$

$$\text{and } \; y = 2\,C \cos^2 t + D \sin E\,(\sin t - \cos t) - C.$$

The differential co-efficient of which for the case of the maximum or minimum is—

$$\frac{dy}{dt} = -4\,C \cos t \sin t + D \sin E\,(\sin t + \cos t) = O,$$

$$\frac{1}{\sin t} + \frac{1}{\cos t} = \frac{4\,C}{D \sin E} = \frac{4\,C}{D\sqrt{\frac{1}{2}}},$$

or, since the second term is negative when $t > 6$ hours,

$$\text{cosec } t - \sec t = \frac{4\,C}{D\sqrt{\frac{1}{2}}}.$$

Applying this to the four cases shown in the diagrams—

| | h. m. |
|---|---|
| E = 9 hours, C = 0.175, D = 0.700, we find maximum at | 10 25.4 |
| = 0.615, | 10 33.3 |
| = 0.400, | 10 51.1 |
| = 0.157, | 11 56.8 |

and for the intervals between high and low water, in lunar hours, $9^h\ 09^m.2$, $8^h\ 53^m.4$, $8^h\ 17^m.8$, and $6^h\ 06^m.4$.

We might apply this mode to test the hypothesis, using for the values of C the half difference of the ordinates of six and twelve hours from the mean, and of eighteen and twenty-four hours with the signs changed; and for D, the average of the ordinates of six and eighteen hours from the first mean. The means present the best criterion, because not displaced in this combination, as the equation shows. This mode of proceeding, however, throws the test too much on the weak part of the results—the times of occurrence of high and low water, or of mean water—and does not take in all the points of the curve; and I have, therefore, preferred a different form of discussion.

7. Placing the maximum of the semi-diurnal curve at O hours, in the hypothesis that the high water of the diurnal curve is nine hours in advance of the semi-diurnal curve, the two curves cross the line of mean water at three hours, the diurnal curve rising and the semi-diurnal falling; at six hours, the semi-diurnal curve has reached its maximum, and rises again at nine hours to its intersection with the mean water line, at which time the diurnal curve has reached its maximum; the semi-diurnal curve attains its greatest rise at twelve hours, and the mean level at fifteen; the diurnal curve also descending to the same point at that time.

Within these two intervals from mean level to mean level, the combination of the ordinates forming the actual tidal curve are exhausted, the part of the curve below the mean level being symmetrical with the above; from three to nine hours the ordinates of the semi-diurnal curve as subtractive; from nine to fifteen hours, additive. The mean is the average between high and low water. The tides of each day will give the forms of the component curves, beginning with the mean, and ending with it, considering as symmetrical the parts above and below the axis of X.

In tabulating, the branch above the axis should be referred to the mean of the preceding and succeeding low water $\left\{\frac{1+1}{4}+\frac{h}{2}\right\}$ and of the high water which it includes, and that below to the mean of the two high and of one low water. From three to nine hours, the difference of the ordinates giving the actual curve, and from fifteen to nine in the reverse order, the sum of the same ordinates, half the sum of the two series of ordinates gives the value of the ordinates of the semi-diurnal curve. The same being repeated with the second branch of the curve, the average will give two results for each day's observation.

The case given in the table on the board, for March 5, will serve to illustrate the simple nature of this method of proceeding.

The mean ordinate for the first and second branches of the curve having been obtained, and the hourly observation which coincides most nearly with it having been found before and after high water, the hourly observations are arranged from it forward for seven hours ($m$,) and backwards for seven ($n$.) The same is done for low water ($m'$ and $n'$.) The half sums and half differences are taken in each case, and then the means. The computation of the diurnal curve is made in the upper part of the table, and that of the semi-diurnal curve in the lower part. The number representing the mean level is eliminated by the mode of taking the means in each table, and the ordinates below the axis are treated as if having the same sign as those above. The semi-diurnal curve is turned over on its maximum ordinate, and the mean value of a single branch of it found. Then each curve is reduced to zero, in the mean level of the period. The last two columns of the upper and lower part of the table contain, respectively, the curves of sines corresponding to the diurnal and semi-diurnal curves.

In the case shown in the first diagram, the ordinates of the semi-diurnal curve from mean water to high water, and corresponding nearly to a minimum of declination, and new moon, are 0.00 foot, + 0.02, + 0.03, + 0.05, + 0.04, — 0.02, + 0.02. The moon's declination during the period being about from 2° 54′ S., to 1° 45′ S., this curve obviously contains a residual of the semi-diurnal curve not taken out; but supposing it to be deduced from a just mean, the corresponding ordinates of a semi-diurnal curve, calculated with 0.04 foot as the maximum, would be 0.00 foot, 0.01, 0.02, 0.03, 0.03, 0.04, 0.04, differing, at the most, 0.06 of a foot, or about three-quarters of an inch, and, in a single instance, the sum of all the six differences being 0.03 foot, and the average 0.004.

The ordinates of the semi-diurnal curve are 0.00 foot, 0.14, 0.28, 0.32. The curve of sines computed with the greatest ordinate has, in this case, for its corresponding ordinates, 0.00 foot, 0.16, 0.28, 0.32, differing but .02 foot at the greatest.

At the next period of declination, near zero and full moon in the month of March, the ordinates of the diurnal curve deduced are 0.00 foot, 0.05, 0.06, 0.06, 0.08, 0.06, 0.09, and the corresponding computed ordinates 0.00 foot, 0.02, 0.04, 0.06, 0.07, 0.09, 0.09, differing at the greatest 0.03 foot, and on the average 0.004 foot, the observed ordinate being this time in excess, as it was before in defect. The ordinates of the semi-diurnal curve are 0.00 foot, 0.12, 0.22, 0.26, and the computed ones 0.00 foot, 0.13, 0.24, 0.26, the greatest difference being 0.02 foot, and the average 0.007 foot in excess, as was the former.

For March 12, corresponding to the maximum of the diurnal curves, and to neap tides, (one day after last quarter,) the ordinates of the hourly diurnal curve from mean to high water are 0.00 foot, 0.21, 0.36, 0.51, 0.63, 0.69, 0.71, the corresponding ordinates of the curve of sines being 0.00 foot, 0.18, 0.35, 0.63, 0.69, 0.71, in which the greatest difference is 0.03 foot, and the mean + 0.007 in the curve computed from observation. The ordinates of the semi-diurnal curve are each zero. Two days afterwards, viz: March 13, gives for the diurnal curve, 0.00 foot, 0.18, 0.34, 0.47, 0.61, 0.68, 0.74, corresponding to which is the curve of sines, 0.00 foot, 0.18, 0.37, 0.51, 0.63, 0.72, 0.74, in which the greatest difference is 0.04 foot, and the mean — 0.02 foot, the curve of observation having the least ordinates. The semi-diurnal curve is 0.00 foot, 0.00, 0.03, 0.02.

The average of three months taken by weeks, gives, for the mean curve and curve of sines, the following table:

| | Diurnal curve. | | | Semi-diurnal curve. | |
|---|---|---|---|---|---|
| | From observation. | Of sines. | Difference. | From observation. | Of sines. |
| *Hours.* | *Feet.* | *Feet.* | | *Feet.* | |
| 0 | 0.00 | 0.00 | | 0.00 | 0.00 |
| 1 | 0.17 | 0.15 | 0.02 | 0.04 | 0.04 |
| 2 | 0.32 | 0.30 | 0.02 | 0.07 | 0.07 |
| 3 | 0.43 | 0.42 | 0.01 | 0.08 | 0.08 |
| 4 | 0.52 | 0.52 | 0.00 | | |
| 5 | 0.56 | 0.58 | 0.02 | | |
| 6 | 0.58 | 0.60 | 0.02 | | |
| Sum.. | ............ | ........ | 0.01 | | |

These results are shown by a curve in the diagram herewith presented (Pl. 7, or H. No. 6,) on the full scale, the greatest difference between the curve from the observation and the curve of the sines being less than a quarter of an inch in the mean, deduced from three months' observations. Whether this will disappear in the mean of more observations, or whether a modification of the hypothesis of displacement of nine hours must be made to meet it, further computations now in progress will show.

8. When this analysis has been made as complete as possible, and applied to the year's observations, it will remain to take up the two series into which we have divided the observations, and to discuss them numerically in detail, as we have heretofore done, generally, in regard to the known laws of the diurnal irregularity, and of the ordinary tides.

Each determination gives a corresponding value of the maximum, or of the ordinate of high water, and in the case of the mean of the curves for January, February, and March, these maxima are 0.66 foot, 0.65, 0.60, 0.60, 0.58, 0.58. Mean 0.61 foot, differing 0.03 of a foot from the maximum found directly from the observations, and if the discrepancies are accidental, give a mean probable error by the variations from the average of 0.02 foot (one-quarter of an inch) of any one of the determinations, and for the mean, 0.01 foot nearly.

9. By the kindness of Colonel Abert, of the topographical engineers, of Major Bache, of the same corps, and of Lieutenant Maury, superintendent of the National Observatory, I have been put in possession of tidal registers which have been kept during the progress of the local surveys made of harbors on the coast of the Gulf of Mexico. The tidal observations of Major Bache, United States topographical engineers, at Key West and the Tortugas, are the most complete of this series, and show, as a general phenomenon, the prevalence of the semi-diurnal wave at that point. I have not yet had the opportunity to examine fully these results, which are, however, under discussion.

# APPENDIX No. 19.

[From Coast Survey Report for 1851.]

## EXTRACTS FROM THE REPORT OF PROFESSOR AGASSIZ TO THE SUPERINTENDENT OF THE COAST SURVEY, ON THE EXAMINATION OF THE FLORIDA REEFS, KEYS, AND COAST.

CAMBRIDGE, *August*, 1851.

SIR: The following report of the examination made by me of the Florida reefs, keys, and coast, is prepared in compliance with your request:

### TOPOGRAPHY OF FLORIDA.

To form a correct idea of the Florida reefs, it is of paramount importance to keep in mind the topographical features of the whole country. The peninsula of Florida projects between the Gulf of Mexico and the Atlantic, from the 30th degree of northern latitude nearly to the 24th, as a broad, flat, low promontory, which has generally been considered a continuation of the low lands of the southern States. But, as we shall see hereafter, this is not the case, or, at least, not with respect to the southern extremity of the peninsula, which consists of the same formations as the reef itself. Again, in a physical point of view, Florida is not limited to those tracts of land, forming the peninsula, which rise above the level of the sea, for the extensive shoals along its southern extremity, between the main land and the keys and reefs, as well as those extending to the west as far as the Tortugas, whence they stretch along the western coast, in fact belong to it, and are intimately connected with it by their physical character. There is a similar tract of flats along the eastern shore, but it is not so extensive as on the southern and western shores, nor does it partake as largely of the peculiar character of the peninsula, being chiefly formed of the alluvial sand drifted ashore by the waters of the Atlantic.

We shall have occasion, however, to show hereafter that the narrow longitudinal islands, which extend close to the main land almost for the whole length of the eastern shore, are probably a direct continuation of the keys, covered with drifted sand.* This is certainly the case with the range of keys extending from the main land to Cape Florida, which limits to the east the bay of Miami, their formation being of coral rock, but covered by silicious drift-sand.

As to the southernmost extremity of the main land proper, it is very difficult to determine its outlines, as it consists of innumerable islands, sometimes separated by narrow channels, and sometimes assuming the character of real islands only at high water, being mostly connected with the main land by very shallow flats. This is especially the case along the southwestern extremity of the peninsula. The outline of the southern shore, however, between Cape Florida and Cape Sable, is better defined—presenting, in almost unbroken continuity, steep bluffs of the same coral limestone which forms the bottom of the everglades, and may be traced, without interruption, along the Miami from the seashore to the everglades.

South of the main land, between it and the range of keys, there are extensive flats, which, even at high water, are but slightly covered, and which the retreat of the tide lays bare, leaving only narrow and shallow channels between the dry flats, with occasional depressions of greater depth. These mud flats extend not only between the main land and the keys as far as Cape Sable, but may be traced to the north along the western shores of the continent, and to the west along the northern shores of the keys, not only as far as Key West and the Marquesas, but even to the Tortugas.

There is, however, this remark to be made—that to the west the mud flats become covered, by degrees, with deeper and deeper water; or, in other words, that these low grounds, extending between the main land and the main range of keys, dip slightly to the west, being gradually lost in the shoals extending north of the Marquesas and the Tortugas, along the western shore of the peninsula. These flats are interspersed with innumerable low islands, known in the country by the generic appellation of the Mangrove islands, respecting which we shall give further details hereafter.

*A direct investigation of this point, which did not come within the limits of my survey, would be of considerable practical importance, inasmuch as it may lead to the discovery of a basis of coral rock, affording a far more solid foundation for the construction of the light-houses wanted along that coast than the loose shore detritus.

The shoals between Cape Sable, Cape Florida, and the main range of keys, are literally studded with these Mangrove islands. Sometimes they are distributed without apparent regularity; sometimes, as to the north of Key Largo, they form a continuous range between the main land and the keys. They are also very numerous along the main keys, or at least along that side of them which is turned towards the most extensive mud flats. Sometimes these Mangrove islands form little archipelagos of innumerable small islets, so intimately interwoven, and separated by such narrow and shallow channels, as to be almost impenetrable. Such archipelagos occur chiefly to the north of Bahia Honda and the Pine islands, as well as to the northwest of Key West. The luxuriant vegetation which rises from these low islands, consisting chiefly of mangroves, gives them a very peculiar appearance. We shall have occasion to return to this subject, when we attempt to explain the formation of the different islands connected with the Florida reef and the main land. The whole tract between Cape Sable and the keys, east of Bahia Honda, as far as Cape Florida, or at least as far as Soldier key, is so shoal that it will forever remain inaccessible, except to very small vessels.

The keys consist of an extensive range of low islands, rising but a few feet, perhaps from six to eight or ten, or at the utmost to twelve or thirteen feet, above the level of the sea. They begin to the north of Cape Florida, where they converge towards the main land, extending in the form of a flat crescent in a southwesterly direction, gradually receding from the main land until, opposite Cape Sable, they have so far retreated as to be separated from it by a shallow sheet of water forty miles wide. Farther to the west they project in a more westerly course, with occasional interruptions, as far as the Tortugas, which form the most western group. They consist either of accumulated dead corals, of coral rocks, or of coral sand, cemented together with more or less compactness. Their form varies, but is usually elongated and narrow, their greatest longitudinal extent following the direction of the main range, except in the group of the Pine islands, where their course is almost at right angles with the main range—a circumstance which we shall attempt hereafter to explain.

Most of these islands are small, the largest of them, such as Key West and Key Largo, not exceeding ten or fifteen miles in length; others only two or three, and many scarcely a mile. Their width varies from a quarter to a third or half a mile, the largest barely measuring a mile across; but whatever the difference in their size, they all agree in one respect—that their steepest shore is turned towards the Gulf Stream, while their more gradual slope inclines towards the mud flats which they encircle.

This is a point which it is important to notice, as it will assist us in our comparison between the keys and the shore bluffs of the main land, as well as with the outer reef and the reefs of other seas, in all of which we find that the seaward shore is steeper than that turned towards the main land, or, in the case of circular reefs inclosing basins (atolls,) than that which borders the lagoon.

The reef proper extends parallel to the main range of keys, for a few miles south or southeast of it, following the same curve, and never receding many miles from it. The distance between the reef and the main range of keys varies usually from six to two or three miles, the widest separation being south of Key West and east of the Ragged keys, where the space is about seven miles. Between this reef, upon which a few small keys rise at distant intervals, and the main range of keys already described, there is a broad navigable channel, extending the whole length of the reef from the Marquesas to Cape Florida, varying in depth from three to six and seven fathoms, and, except off Looe key, where the passage is not more than fourteen feet deep at low water, averaging from three to four fathoms.

Farther east the average depth is again the same as at Looe key; but it becomes gradually more and more shoal towards the east, measuring usually about two fathoms, or even less, to the east of Long key and Key Largo, but deepening again somewhat towards Cape Florida, where the reef converges towards the main keys and the main land. Protected by the outer reef, this channel affords a very safe navigation to vessels of medium size, and would allow a secure anchorage almost everywhere throughout the whole length of the reef, were the numerous deep channels which intersect the outer reef well known to navigators and marked by a regular system of signals. As it is, however, the reef seems to present an unbroken range of most dangerous shoal grounds, upon which thousands of vessels, as well as millions of property, have already been wrecked. These facts have a stronger claim upon the attention of the government, since there are, as already remarked, numerous passages across the reef which might enable even the largest vessels to find shelter and safe anchorage behind this threatening shallow barrier. * * * * *

The reef proper, as we have remarked above, runs almost parallel to the main range of keys from Cape Florida to the western extremity of the Marquesas, where it is lost in the deep. It follows in its whole extent the same curve as the keys, encircling to the seaward the ship channel already mentioned. This is properly the region of living corals.

Throughout its whole range it does not reach the surface of the sea, except in a few points where it comes almost within the level of low-water mark, giving rise to heavy breakers, such as Carysfort, Alligator reef, Tennessee reef, and a few other shoals of less extent, but perhaps not less dangerous. In a few localities fragments of dead coral and coral sand begin to accumulate upon the edges of the reef, forming small keys, which vary in form and position according to the influence of gales blowing from different directions—sometimes in the direction of the Gulf Stream from southwest to northeast, but more frequently in the opposite direction, the prevailing winds blowing from the northeast. Such are Sombrero key, Looe key, the Sambos, and Sand key. Here and there are isolated coral boulders, which present projecting masses above water, such as the Dry Rocks, west of Sand key, Pelican reef, east of it, with many others, more isolated. Though continuous, the outer reef is, however, not so uniform as not to present many broad passages over its crest, dividing it, as it were, into many submarine elongated hillocks, similar in form to the main keys, but not rising above water, and in which the depressions alluded to correspond to the channels intersecting the keys. These broad passages leading into the ship channel, which may be available as entrances into the safe anchorage within the reef, are chiefly the inlet in front of Key Largo and to the west of Carysfort reef, with nine feet of water; a passage between French reef and Pickle reef, with ten feet; another between Conch reef and Crocus reef, also with ten feet; another between Crocus reef and Alligator reef, with two fathoms; another between Alligator reef and Tennessee reef, with two fathoms and a half; and a sixth to the west of Tennessee reef, varying in depth from two and a half to three fathoms.

The remark which has been made respecting the mud flats and their gradual deepening from east to west, applies equally to the general features of the main reef, as well as to the intervening channel. To the eastward the channel is shallower, the ground around the keys and reef becomes shoaler, and there is a gradual dip towards the west, which makes the connection less marked between the keys west of Key West, in the large groups of the so-called Mangrove islands and the Marquesas, beyond which there is even an extensive interruption in the succession of the keys before we reach the Tortugas. These last, however, as well as the bank west of these keys, belong none the less to the main range of keys, from which they are only separated by a more extensive and deeper depression. West of Sand key the reef itself becomes gradually less elevated, until it is finally lost where the ship channel, south of the Marquesas, expands into the broad depression, separating that group of keys and shoals from the Tortugas.

In order to understand fully not only the topography but also the mode of formation of all these keys and reefs, it must be remembered that the rising reefs, which form more or less continuous walls, reaching at unequal heights nearly to the surface, or above the level of the waters, are only a particular modification of those formations growing upon coral grounds under special circumstances. It has been ascertained, whenever similar investigations have been made, that living corals do not occur in depths exceeding twenty fathoms; that the reef-building species prosper from a depth of about twelve fathoms nearly to the surface, and that different species follow each other at successive heights. Now if we keep in mind these facts, we shall see that all the coral-bound islands of the West Indies, as well as of the main land of Central America, constitute an extensive coral field, divided by broad, deep channels, over which the coral reefs extend, with different features, according to the depths in which they occur and the changes which their own growth has gradually introduced upon the localities where they are found, influenced and modified to some extent also by the direction of the prevailing currents and the action of the tides.

The formation of the main range of keys in their primitive condition as a reef—for, as we shall see hereafter, they have been a sub-marine reef before they rose as islands above the level of the ocean—the formation of this range, we repeat, at gradually greater distances from the main land, as we follow their course from east to west, has been simply owing to the depth of the bottom from which the reef has risen. It has followed the line of ten or twelve fathoms depth ; and if there is so wide an interruption between the Marquesas and the Tortugas, it is because the ground is deeper over that space. Again, if the Pine islands have a northwesterly direction, while the main range runs more from east to west, it is no doubt because the body of water emptying from the northern part of the gulf, along the western shores of the peninsula, has, for a time, run chiefly over that field, while the tract of mud flats between the keys and the main land was filling prior to the formation of the outer reef, the rising of which, as an external barrier, must have modified greatly the course of the currents north of the keys at a later period, leaving between them only a few narrow but navigable channels, such as exist now between the Marquesas and the Mangrove islands, between these and Key West, and between the Pine islands and the group of Bahia Honda.

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We would only add that the absence of corals along the western shore of the peninsula, at present, is probably owing to the character which that shore has assumed in the progress of time, for the peninsula itself has once been a reef, at least as far as the 28th degree of north latitude, as is shown by the investigation of the everglades, and by the examination of the rocks at St. Augustine.

This latitude is the natural northern limit of the formation of coral reefs, as also of the extensive growth of stony corals; though on the southern shores of the North American continent, these formations seem to have extended far beyond their usual bounds, probably under the influence of the high temperature of the Gulf Stream, for not only do the narrow, longitudinal islands which extend along the eastern shore, and their direct connection with the small keys north of Cape Florida, indicate their coralline origin, but we have even under the 32d degree of north latitude extensive coral formations at the Bermudas still flourishing in the present day. If the growth of corals has been stopped along the eastern shore, it must be ascribed to the invasion of drift sand, which extends over the everglades, as well as along the eastern shores as far south as the Miami, Key Biscayne, and the bay of the Miami.

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### MODE OF FORMATION OF THE REEF.

The reefs of Florida as they have been described in the foregoing sketch of the topography of that State, and, indeed, the separate parts of each of these reefs, in their extensive range from northeast to southwest, present such varieties as will afford, when judiciously combined, a complete history of the whole process of their formation.

Here we have groups of living corals, beginning to expand at considerable depth, and forming isolated, disconnected patches, the first rudiments, as it were, of an extensive new reef. There we have a continuous range of similar corals in unbroken continuity for miles, or even hundreds of miles, rising at unequal heights nearly to the surface.

Here and there a few heads or large patches, or even extensive flats of corals, reach the level of low-water mark, and may occasionally be seen above the surface of the waters, when the sea is more agitated than by the simple action of the tides. In other places coral sands or loose fragments of corals, larger or smaller boulders, detached from lower parts of the living reef, are thrown upon its dying summits, and thus form the first accumulation of solid materials, rising permanently above low-water mark; collected sometimes in such quantities and at such heights as to remain dry, stretching their naked heads above high water.

In other places these accumulations of loose, dead materials have entirely covered the once living corals, as far as the eye can reach into the depth of the ocean; no sign of life is left, except perhaps here and there an isolated bunch of some of those species of corals which naturally grow scattered, or of those other organisms which congregate around or upon coral reefs; but the increase of the reef by the natural growth of the reef-building corals is at an end. Again, in other places, by the further accumulation of such loose materials, and the peculiar mode of aggregation which results from the action of the sea upon them, and which will be more fully explained hereafter, extensive islands are formed, ranging in the direction of the main land, which support them. Elsewhere we may find the whole extent of the reef thus covered, which, after a still more protracted accumulation, perhaps becomes united with some continental shore.

Now it must be obvious, that from a comparison of so many separate stages of the growth of a coral reef, a correct insight may be obtained into the process of its formation; and, indeed, in thus alluding to the different localities which came under our own observation, we have already given a general history of its progress, which we now proceed to illustrate more in detail.

We would, however, first remark, that the extraordinary varieties which exist in the natural condition of different parts of the same reef, or of different reefs, when compared with each other, fully explain the discrepancies between the reports which have been obtained, respecting the reefs of Florida, prior to our investigations.

It had been stated that the reefs consisted solely of living corals; and, indeed, this report is true of the outer reef, which is called by all the inhabitants of Florida *"the reef" par excellence*, and is unfounded only with regard to those few islands which rise above the surface of the sea at Sand key and the Sambos. Others, who had noticed only the larger accumulations of coral fragments which occur on the shores of some of the islands forming part of the Florida reef, had reported the islands to be formed of coral rocks; while some who had, perhaps, observed the extensive excavations made around Key West, have told us only of the existence of oolitic and compact rocks, almost destitute of corals or other remains of animal life; and from still other localities comes the opinion, that the rocks consist of nothing but more or less disintegrated shells, cemented together. * * * * * * * *

ANIMAL LIFE.

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This fullness and variety of animal life is particularly obvious within the boundaries of coral fields, the natural limits assigned to the growth of these animals being those in which animals of other classes range in greater profusion, and the coral reefs themselves also affording very favorable circumstances for the display of numerous living forms. Hence the extraordinary assemblage of all classes of animals upon the reef, where, besides those particular kinds of corals which contribute largely to its formation, we find upon it, or on the foundation from which it rises, a great variety of other corals, which, though too insignificant in size to take a conspicuous part in building up these extensive accumulations of organic lime-rock, add none the less their small share in the work, contributing especially to fill up the vacant spaces left by the more rapid and durable growth of the larger kinds. They are to the giants of the reef what the more slender parts are to the lords of the forest, adding the elegance and delicacy of slighter forms to the strength, power, and durability of their loftier companions.

But besides the stony corals, we find in the reef a great variety of soft polyps, either attached to the surface of dead corals, dead shells, or of the naked rock, or boring into the coral sand and mud.

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Such are different species of Arca, the date-fish among the Mollusca, and many worms, especially Serpula among articulates, the agency of which in the formation of the keys will be described hereafter. All these animals and plants contribute, more or less, to augment the mass of solid materials which is accumulating upon the reef, and increase its size. Not only are the hard parts of shells, echinoderms, worms, or their broken fragments, heaped among the detritus of the corals, but occasionally even the bones of fishes and turtles, which are very numerous along the reef, may be found in the coral formations.

The decaying soft parts of all these animals undoubtedly have their influence upon the chemical process, by which the limestone particles of their solid frame are cemented together, in the formation of compact rocks. Upon this point we may expect further information from Professor Horsford, who is now submitting to chemical analysis all the variety of rocks and the solid stems of the different corals obtained in Florida. Respecting the relations of the solid and soft parts of the living coral, and their mode of growth, we would refer to a paper of ours now in press, to appear in the next volume of the Smithsonian Contributions to Knowledge. * *

THE KEYS.

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We see everywhere that the larger boulders and the coarser fragments have been the first to find a resting-place upon the dead reef; the minuter particles and coral sand, which are periodically washed away from its crest during heavy gales, never accumulating upon it till large boulders and more solid materials have collected to such an extent as to form sufficient protection for the more movable, looser fragments. This fact is beautifully illustrated by an accurate survey of Sand key, where a wide field of large boulders is partially laid bare at low water, presenting the appearance of an extensive key, with a low hill of minute materials, the product of some heavy gale, heaped upon the summit, against which the sea plays without disturbing it materially, even at high water, when it leaves in sight only a nucleus, as it were, for a greater accumulation of such loose materials which may in time cover the whole surface of the larger boulders. We have here in reality the same phenomenon which is observed upon all beaches, where larger materials have first accumulated on a shoal shore, being followed, in the course of time, by more minute fragments which have found a resting-place upon levels where the sea was powerless to increase the collection of coarser matter. In attempting to understand these formations, it must be remembered that the accumulation of the larger materials, collected at a certain level, may modify the action of the water at a subsequent period, thus producing a combination of substances, heaped unconformably upon each other. This is, in reality, the case throughout the whole main range of keys, which have been raised to their present level by the action of the tides and gales for ages past, the fragments of which they are composed having been thrown up at different periods, and overlying each other in such a manner as to present the same irregularity which is found in all drift stratification. Layers upon layers are seen resting unconformably, dipping in different directions so as to present all the modifications which may be observed in torrential stratification, each layer following, with more or less regularity, the course of the flood under which it has been accumulated.

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By a process, not yet fully understood, but to which we shall return hereafter, these loose collections are gradually cemented into solid rock, presenting the most diversified appearance, according to the substances of which it is composed. Then we find a coarse breccia, consisting of larger fragments of corals and shells, inclosing sometimes coral boulders; and this is the sort of rock which generally overlies the immediate surface of that portion of the keys which has been formed by the progress of the reef, growing *in situ*. Such rock was seen among the foundations of the new light-house at Sand key, where the large boulders are very numerous, and seem almost as fresh as if they had been lying on the spot but for a few years. It may be, indeed, that during the hurricane of 1846, the whole cap of the reef was renewed at that spot.

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A careful survey of the character of the rocks in the keys affords satisfactory evidence that they have been formed at whatever height they may rise, by the same action which is now going on upon the reef—that is, by the accumulation of loose materials above the water-level. That part of the keys which rises above the level of the water is therefore a sub-aerial and not a submarine accumulation of floating matter, thrown above high-water mark by the tempestuous action of the water. We insist upon the fact, that the keys furnish in themselves, by the internal structure of their rock, the fullest evidence that they have been formed above high-water mark by the action of gales and hurricanes, instead of having grown as a reef up to the water-level, and been subsequently raised to their present height. The evidence of this statement rests upon certain facts obtained from observation of the reef itself, at Sand key and the Sambos.

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Let us now return from this digression to the consideration of the keys themselves, under the different aspects which they present. We find, then, that some have more abrupt shores, being, as it were, narrow shelves with ragged edges, rising without a beach from deep water; these are undoubtedly such as were formed upon the narrowest part of the old reef. Others spread more uniformly, having an extensive beach, and dip gradually under the sea, presenting a gentle, submarine slope, covered with coral sand and mud; these were, no doubt, formed upon the broader parts of the reef, where it descends gently on both sides. Again, we find those which, though resembling the last in general appearance, may have more abrupt shores, owing to the denudation of parts of their earliest deposits. Occasionally we see that more recent layers have filled again such worn places, thus presenting, on a miniature scale, among the latest formations among layers which belong altogether to the present geological age, all the diversity of unconformable deposits which occur in former geological periods.

CORAL REEFS.

After examining a growing coral reef, so full of life, so fresh in appearance, so free from heterogeneous materials, in which the corals adhere so firmly to the ground, or if they rise near the surface seem to defy the violence of the ocean, standing uninjured amid the heaviest breakers, an observer cannot but wonder why, in the next reef, the summit of which begins to rise above the level of the water, the scene is so completely changed. Huge fragments of corals, large stems, broken at their base, gigantic boulders, like hemispheres of Porites and Macandrina, lie scattered about in the greatest confusion—flung pell-mell among the fragments of more delicate forms, and heaped upon those vigorous madrepores which reach the surface of the sea.

The question at once arises, how is it that even the stoutest corals, resting with broad base upon the ground, and doubly secure from their spreading proportions, become so easily a prey to the action of the same sea which they met shortly before with such effectual resistance? The solution of this enigma is to be found in the mode of growth of the corals themselves. Living in communities, death begins first at the base or center of the group, while the surface or tips still continue to grow, so that it resembles a dying centennial tree, rotten at the heart, but still apparently green and flourishing without, till the first heavy gale of wind snaps the hollow trunk, and betrays its decay. Again, innumerable boring animals establish themselves in the lifeless stem, piercing holes in all directions into its interior, like so many augers, dissolving its solid connection with the ground, and even penetrating far into the living portion of these compact communities. The number of these boring animals is quite incredible, and they belong to different families of the animal kingdom: among the most active and powerful we would mention the date-fish, Lithodomus, several Saxicava, Petricola, Arca, and many worms, of which the Serpula is the largest and most destructive, inasmuch as it extends constantly through the living part of the coral stems, especially in Macandrina.

On the loose basis of a Macandrina, measuring less than two feet in diameter, we have counted not less than fifty holes of the date-fish—some large enough to admit a finger—besides hundreds of small holes made by worms.

But however efficient these boring animals may be in preparing the coral stems for decay, there is yet another agent, perhaps still more destructive. We allude to the minute boring-sponges, which penetrate them in all directions, until they appear at last completely rotten throughout.

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SHIP CHANNEL.

The broad channel extending the whole range of the reef, between the main keys and the outer reef, is rather uniform, having the same width throughout, with the exceptions of those few places where the reef widens, or the mud flats from the keys encroach upon it. Its narrowest passages are between Looe key and the Pine islands, between Pickle and French reefs, and between Key Rodriguez and Tavernier. It is also somewhat narrowed between Alligator reef and Indian key, and is widest off Key West. Its depth varies also slightly, being shoaler in its eastern range than to the west. The shallowest part is between Pickle reef and Key Rodriguez, and between Looe key and Pine islands.

But if we do not take into account those spots where the depth is reduced from local circumstances, we may say that as a whole the ship channel begins to the east, with a depth of about two fathoms between Fowey Rocks and Soldier key, increasing gradually thence, until it reaches three fathoms between Pacific reef and Old Rhodes, then becomes again slightly reduced between Carysfort reef and Key Largo; after which, with the exception of the shoals between Pickle reef and Key Rodriguez, it deepens again to three, four, five, or even six fathoms, until, between Looe key and Pine islands, it shoals once more to fourteen feet. Farther on it increases again to five, six and seven fathoms, the average depth between Key West and the reef being five or six fathoms; and still beyond, more toward the west, sinks to eight, nine, and ten fathoms between the western extremity of the Marquesas and the western end of the reef, where it spreads into the great depression separating the Tortugas from the Marquesas. The character of the bottom varies in different parts, as do also the living beings which it supports. Where it is the most shoal, as between Fowey Rocks, Triumph reef, and Long reef, on one side, and Soldier key and the Ragged keys on the other, the bottom consists of coral sand, overgrown with what is called the country *grass;* that is to say, a variety of the limestone algæ, mingled with Gorgonia, among which rise a number of coral heads.

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To the west of Long reef, especially between Carysfort and Key Largo, the coral sand rises here and there in the form of shoal sandbanks, intermixed with coral heads—an arrangement which is probably owing to the more rapid currents flowing in that part of the channel, which is precisely the turning point of the direction of the reef. Such heads occur again about a mile and a half off Vermont key, half way between Key Tavernier and Indian key, outside of which Gorgonia and sponges are very abundant, upon a hard white sand bottom. Similar heads are seen between Long key and Tennessee reef, and nearer the reef there are shoals of white coral sand, covered with Gorgonia; but farther west, off Duck key, the bottom becomes softer. Off Bahia Honda, again, it is rocky—that is, studded with large heads, surmounted with soft muddy sand. This change in the character of the bottom is more obvious westward, where the heads are fewer and the bottom more generally muddy, or covered with finer-grained sand. For instance, hard sand is observed between Loggerhead key and Saddle bluff; but nearer the reef, as far as the American shoals, we have soft mud, with shoals and coral heads. Off Boca Chica, the channel way has also a bottom of soft coral mud, while shoals, with coral heads, may be traced for three-fourths of a mile along the shores, as again towards the Sambos, in a depth of from three to two fathoms. The softness of the bottom in the vicinity of Key West, considered in connexion with the scarcity of coral heads in that region, shows that a soft mud formation is unfavorable for the growth of corals; and, indeed, this holds also good for the flats north of the keys.

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## THE MAINLAND.

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A careful survey of all the varieties of rock occurring at Key West, as well as their peculiar superposition, had prepared us for a minute comparison between the Keys and the mainland; but, nevertheless, we were no less surprised than delighted to find that the solid foundation of the mainland consisted of the same identical modifications of coral rocks which form the keys. Along all that part of the shore which was examined, as well as upon the shores of the Miami, we found everywhere the same coarse, oolitic rock, with cross stratification, consisting of thin beds, dipping at various angles in different directions, precisely as we find it at the western extremity of Key West, excepting, perhaps, that the cross stratification is here more prominent, the strata dipping more frequently in several directions within the same extent.

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## COAST SURVEY.

But it may be asked, what is the practical use of such detailed descriptions of the coral reefs for the coast survey? We need only allude to the universal impression of the dangers arising to navigation from the growth of such reefs, to satisfy the most skeptical that a minute knowledge of the extent and mode of formation of those belonging to our own shores must be of paramount importance, were it only with reference to the position of light-houses. But there is another subject connected with this investigation, which is not less momentous. It is well known that in the Pacific coral reefs have been raised above the levels at which they were formed by the agency of the living animals, and also that in other localities, sometimes in close connection with those just mentioned, the ground is subsiding. These changes have been so often observed, whenever coral reefs occur that the idea of subsidence and upheaval is naturally connected with the features of coral reefs, and the question at once arises, whether the reefs on our shores are thus undergoing variations of level, independently of their natural growth. We have seen how extensive are the changes produced merely by the normal growth of the corals, and the facts accompanying their increase. It now remains for us to ascertain whether this growth has taken place, or does at present take place, upon ground which has changed or is now changing its relative level in reference to the sea.

The facts already described afford a sufficient answer to the question. We are satisfied that as far as coral formations have been observed upon the mainland of Florida, and within the present extent of the coral reefs, no change of the relative level has taken place either by subsidence or upheaval of the coral ground, and that all the modifications which the reef has presented at successive periods have been the natural consequence of the growth of reef-building corals, with the subsequent accumulation of their products in the manner described above.

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There is in reality but one way of accounting for this equality of level in the successive reefs; which is to suppose that their loftiest ridges are the maximum height at which materials can be accumulated by the natural agency of gales, and we have sufficient evidence to justify the adoption of this view.

The fact that at present the highest tides during the most severe gales do not reach the level of the bluff summits along the shores of the main-land, or even that of the maximum height of Key Largo, or Key West, does not invalidate this supposition, for when the shore bluffs of the main-land were formed, the ocean had full sweep over the ground now occupied by the reef and mud flats, which did not then exist; and when Key Largo and Key West attained their maximum height, the outer reef did not yet form a barrier, checking the violence of the Gulf Stream in that direction. But, even with the present obstruction, we have evidence of the occasional rise of the water to heights which fully justify our assumption, that even the highest ridges on the shores of the main-land and on the reef have been formed by the action of severe gales. For, in the year 1846, the water rose eight and a half feet above high-water mark at Key Vacas. Key West was entirely inundated during the same gale; and though that island is somewhat protected by the reef, even at present the rushes, driven upon it by the flood, may be seen among the trees and bushes, at a height almost equal to its loftiest summit. In 1841 the water rose ten feet above high-water mark at Cape Romaine, on the western shore of the peninsula.

These facts suffice to show that the explanation we have given of the formation of the reef is in accordance with the powers of the agencies to which it is ascribed, and when taken in connection with the peculiar arrangement of the materials of which they consist, seems to us to prove the justness of this view.

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## PHYSICAL CHANGES IN THE GULF STREAM.

There are several questions of the deepest scientific interest, which may be advanced by a due consideration of the facts observed upon the reefs of Florida. There we have a peninsula—a narrow, flat strip of land, projecting for about five degrees from the main-land, between the Atlantic ocean and the Gulf of Mexico, and forming an effective barrier between the waters of the two seas, which otherwise, even by the change of a few feet in the relative level of the intervening peninsula, would communicate freely with one another; and this peninsula we now know to have been added to the continent, step by step, in a southerly direction.

We know that the time cannot be far behind us when the present reef, with its few keys, did not exist, and when the channel, therefore, was broader, and the Gulf Stream flowed directly along the main range of keys. We know, further, that at some earlier period the keys themselves were not yet formed, and that then the channel between Cuba and Florida was wider still, washing freely over the grounds now known as the mud flats, between the keys and the main-land, and that there was then nothing to impede a free communication between the Gulf of Mexico and the Atlantic ocean. The channel of the Gulf Stream was not only wider—it was also less shallow along its northern borders, for the whole extent of soundings south of the main-land of Florida was an uncovered coral ground, upon which the deep-water species were just beginning to spread. But we may trace the change further. There was a time when neither the southern bluffs of the continent, nor Long key within the everglades, nor even the everglades themselves, existed; when, therefore, the Gulf Stream had a broad communication with the Atlantic, and the southern shores of the United States extended in almost unbroken contiguity from west to east, from the shores of Texas and Louisiana to St. Augustine. At that time the Gulf channel was in reality a broad bay, as broad as the gulf itself, destitute of all those obstructions which now cause the tropical current to follow such a circuitous course between the West India islands, through the Caribbean seas, and around the peninsula of Florida. The influence which the Gulf Stream has upon the climate of the

Atlantic is so well known that its connection with the changes which the current itself has undergone within a comparatively recent period cannot be overlooked. It is true, as we have every reason to believe, that the temperature of the Gulf Stream, in connection with the temperature of the southwesterly winds blowing obliquely across the Atlantic, modifies that of the western coast of Europe. If it is true that the Gulf Stream and the southwest winds have an influence in determining the course of the isothermal lines upon the two sides of the Atlantic, and of raising beyond their normal altitude the mean annual temperatures of northwest Europe, then we may look to the physical changes which have occurred on the southeastern extremity of the North American continent for the cause, or at least a partial cause, of those changes of temperature which have taken place in the beginning of the present period, in those very northwestern portions of Europe which are now so much warmer than the corresponding latitudes on the American continent, and which, soon after the accumulation of the glacial drift, had as low mean annual temperatures as the coasts of Labrador, Nova Scotia, and New England in our day.

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### CHANGES IN AGES TO COME.

Among the questions contained in your instructions you ask whether the growth of coral reefs can be prevented or the results remedied, which are so unfavorable to the safety of navigation. I may say that here, as in most cases where the operations of nature interfere with the designs of man, it is not by a direct intervention on our part that we may remedy the difficulties, but rather by a precise knowledge of their causes, which may enable us if not to check, at least to avoid the evil consequences. I do not see the possibility of limiting in any way the extraordinary increase of corals beyond the bounds which nature itself has assigned to their growth. We have seen how successfully several reefs have been formed, more or less parallel, within the limits of the peninsula of Florida, as well as beyond the mainland. We have seen, also, how these parallel or concentric reefs have been gradually transformed into mainland by the accumulation of coral, sand and mud, with other loose materials, and also that the keys are now slowly annexed to the mainland by the same process. We may, therefore, safely infer, that, as far as the conditions exist for the formation of similar accumulations of loose materials, they will continue to occur, but they will never extend beyond the natural foundation from which a coral reef may rise; and as we now have sufficient evidence that this foundation is a sea-bottom, under from 12 to 20 fathoms, we may be satisfied that outside of the present outer reef, where the slope is steep, sinking rapidly to unfathomable depths, there is no opportunity for the growth of a new reef.

Here and there the reef may widen somewhat towards the Gulf Stream, within those limits at which the depth does not exceed twenty fathoms, and from the knowledge we already possess of the soundings outside the reef, we know positively that this is nowhere a broad stream; we may therefore rest assured that the changes which are going on will chiefly consist in bringing up the reef, for its whole extent, to the surface of the water, with occasional intervening channels kept open by the currents, such as exist now between the keys; that this reef once matured will be covered by coral debris, becoming transformed into a range of keys, similar to that which exists now inside of it; that the depth of the ship channel between the reef and the main range of keys will gradually lessen, and the channel itself be changed into mud-flats, similar to those stretching now between the keys and the mainland. In still more remote ages the present mud-flats may become swamps, elevated above the reach of the tide-waters, like the everglades; and this process may perhaps be extended to the present ship channel. But unless some great revolution in nature modifies the present relative level between land and sea, it may be safely maintained that the present outer reef is the final southern boundary of the North American continent, and that the sooner a system of light-houses and signals is established along the whole reef the better; for this is, after all, the shore which is to be lighted, and not the range of keys which is within the reef. In relation to the western range of keys, and the western extremity of the reef we may expect, in course of time, to see the depression between the Marquesas and Tortugas gradually lessened by the increase of the reef, so that the westernmost group of islands may finally stand in as close connection with the keys more to the west as they now bear to each other, the passage betwen them being reduced to as narrow a channel as Boca Grande, between the Marquesas and the Mangroves.

The shoals west of Cape Sable may, undoubtedly, also increase in extent westward; but how far the currents from the northwest may limit this accumulation, in connection with the changes which the currents themselves may undergo by the increase of the keys to the west, it is beyond the power of human foresight to determine.

These practical results—for so we venture to call the general conclusions last presented—although they are purely scientific deductions from general principles, may satisfy the most obstinate supporters of the matter-of-fact side of all questions, of the advantages of scientific illustrations in the daily walks of life, and also justify the course which has been followed with so much success by the Coast Survey, in combining the strictest scientific methods with its practical operation.

Respectfully submitted:

L. AGASSIZ.

Professor A. D. BACHE,
*Superintendent of the Coast Survey.*

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# APPENDIX No. 20.

[From Coast Survey Report for 1851.]

REPORT TO THE ASSISTANT IN CHARGE OF THE COAST SURVEY OFFICE ON THE ELECTROTYPING OPERATIONS OF THE COAST SURVEY, BY GEORGE MATHIOT, ELECTROTYPIST.

ELECTROTYPE LABORATORY, COAST SURVEY OFFICE,
*Washington, November* 29, 1851.

DEAR SIR: In compliance with your request, I present the following report of the electrotype art as now practiced in this office. Most of the apparatus and processes here used are entirely new.

To clearly exhibit the advantages derived from their introduction, it will be necessary to consider the scientific principles involved in their use, and also to take a cursory view of the history of the electrotyping art.

The art of working metals by electric currents is of very recent introduction; and, although it has advanced with great rapidity, it is yet, perhaps, but in a state of infancy in its applications, and of crudeness in the modes of conducting it.

The electro-deposition of metals was observed by most experimenters with the voltaic battery. As early as 1804 electro-gilding had been successfully practiced; but the idea of making castings by electric currents does not seem to have occurred to any one previous to the introduction of Daniel's battery, to which electro-casting is incidental.

After the introduction of Daniel's battery, it simultaneously occurred to several persons that electric currents might be used to make castings of a finer kind than were obtained by melting and pouring. Propositions to this effect are about all that can be attributed to the rival claimants for the invention of electro-metallurgy; for neither the English nor Russian philosopher revealed what had not been known before.

Yet to Jacobi and Spencer is due the merit of having called public attention to the subject; for in doing that, they have conferred benefits on the world greater, perhaps, than by making an original discovery.

After the publications of Jacobi and of Spencer had called the attention of the scientific world to the new art, the principles involved in it became the study of several eminent philosophers, who disclosed the methods to be followed for obtaining reguline metal. After this, several departments of electro-metallurgy rapidly advanced. Electroplating, and the multiplication of pages of letter-press work, as pages of type, and woodcuts, (electro-stereotyping,) were soon extensively practiced; but the copying of the delicate touches of the copperplate engraver (the electrotype proper) was beset with difficulties. On account of the great value of the engraved plate, together with the risk of its being destroyed in the attempt to copy it, and the uncertainty as to whether the duplicate would have good metallic properties, even if the operator should have the good fortune to obtain one, this department of the art (the first and most beautiful of Spencer's suggestions) was allowed to rest as an experiment or be confined to articles of small size and value.

ADHESION OF DEPOSIT TO MATRIX.

Electro-metallurgy requires that the deposited metal should have all its cohesive properties. If such a deposit of copper is made on a clean plate of copper, it is obvious that the deposited metal will cohere with the plate on which it is made, and an elaborately engraved plate would thus be converted into a mere mass of metal. The electrotype art, therefore, cannot exist before means are provided for preventing this destructive adhesion.

Various plans for overcoming this difficulty have been proposed. All these, however, have a common feature, which is to prevent the deposit and matrix from touching by means of an intervening film of heterogeneous matter.

Mr. Smee proposes to use that coating of air which adheres so firmly to polished metals, (so strikingly exhibited when the attempt is made to wet a polished knife-blade.) To obtain the air coating, he directs that, after every attachment has been made to the plate, it be placed in a cool and moist cellar for a few days before introducing it into the electrotype vat.

Smoke, black lead, oils, and powders, and wax, have also been proposed for covering the face of the plate.

The method used in the British ordnance survey is perhaps the best of all these. This is conducted as follows: The plate is first well oiled, and the oil well wiped away with soft bread. The plate is then heated to above the temperature of melting wax, and a cake of white wax pressed against the edge. The oil having removed the air from the plate, the wax will flash over it in an extremely thin sheet or film. All excess of wax is then to be wiped away with a fine linen cloth, free from lint. The plate must be left to cool before introducing it into the vat.

To smear the face of the finely engraved plate is in opposition to the fundamental idea of the electrotype, which is that of atomic casting. In the process of Mr. Smee, air bubbles will be retained in the fine lines of the graving, thus mutilating the copy; moreover, the face of the new plate is waved from the agitation of the stratum of air when receiving the first portion of copper.

In the waxing process it is almost impossible to free every line from excess of wax. Even days of tedious application do not insure perfection. In addition to the coarseness of these various methods, they are extremely uncertain as to whether they effect the purpose for which they are applied.

It was always observed that if the deposited metal was *not deficient* in mechanical properties, it stuck very hard to the original, and the plates had to be subjected to violent jarring, heating, or beating, to separate them. But if the deposited metal was of very fine quality, then most likely the deposit was inseparably united to it. From these circumstances attending the adhesion of the deposit, it occurred to me that when the cohesive force was but feebly developed in the deposited metal, then the force of cohesion or homogeneous attraction could not extend the distance presented by the thickness of the film of heterogeneous matter between the plates, but that when these forces were well developed the spheres of homogeneous attraction of each plate would extend through the wax or air film.

It may be proper here to remark that the above views of adhesion have been applied to another department of electro-metallurgy with the most gratifying success.

In electro-plating the difficulty of obtaining a firm adhesion of the film of precious metal is entirely obviated by making such arrangements as insure a rapid deposition of highly ductile metal at the moment the article to be plated is immersed in the electrolyte.

In considering the sticking of the plates, after homogeneous attraction or cohesion, heterogeneous attraction or adhesion demands attention; for two similar bodies may be separated by a film of heterogeneous matter which binds them more firmly together than their particles are held together by cohesion, as we see in the use of cements.

This force is very powerful between some bodies, while between others it is very slight. Air adheres very strongly to metals, as before referred to; hence a film of air may unite two copper plates, even though they are separated beyond the distance at which cohesive attraction takes place.

Wax is a common ingredient in cements; its adhesive properties have become proverbial; its use is evidently improper. Therefore a substance having a strong adhesive attraction for the plates must not be on the face, and the cohesive force of the surface particles must be suspended by other methods than making the deposited metal deficient in mechanical properties.

It was hoped that a substance could be found that would act uniformly and gently on the surface of the engraved plate, and, in destroying the homogeneous attraction of the surface particles, would, by chemical union with them, form an insoluble and friable compound, having but a slight adhesion to the plate. I was led to select iodine for the experiment on account of its sparing solubility in water, its high equivalent number, and innoxious qualities. A copper plate was well cleaned, exposed to the vapor of iodine, and electrotyped; the deposit separated from it readily. This was repeated some hundred times with invariable success.

It was found, in cleaning large plates for the application of the iodine vapor, that while one part of the plate was being cleaned, another part would tarnish, and hence a uniform action of the iodine could not be obtained. This led to silvering the plates before iodizing, which facilitated the cleaning and exhibited the action of the halogen. A silvered plate was washed with an alcoholic solution of iodine and electrotyped; the electrotype separated from the matrix yet more readily than before, the iodide of silver serving better to prevent adhesion than the iodide of copper.

But it was soon observed that a plate prepared on a dull day did not separate so readily as one prepared under a bright sky, and on experimenting it was found that a plate iodized and exposed to sunshine would separate with very great facility; while a plate iodized on a rainy day, and placed in a dark room for a few hours before introducing it into the vat, might stick so hard as to require some of the old resorts of heating and jarring to separate it from the matrix.

The process of iodizing and exposing to light has now been applied to a very great extent of finely engraved surface, and in no case has the least difficulty been found in lifting one plate off the other when the requisite thickness had been obtained.

I am aware that it may be thought that the iodine acts only by intervening between the plates; but the quantity of iodine applied to a plate must be thought insufficient to effect it by mere mechanical separation when we consider the large quantity of silex and carbon found in ordinary copper. If but one ounce of copper be dissolved from a square foot of ordinary plate, a very heavy deposit of impurities is left, (sometimes five per cent.,) and the quantity of wax which may be applied to a plate, and fail to prevent sticking, is ten thousand times more than the quantity of iodine which prevents it.

In preparing our largest plates, having ten square feet of face, I use a solution of one grain of iodine in 20,000 grains of strong alcohol. If one grain of the solution is required to wet a square foot, it will give but $\frac{1}{20000}$ of a grain of iodine on a square foot. But as the iodine evaporates rapidly with the alcohol, probably the actual quantity on a square foot does not exceed one-hundred-thousandth part of a grain.

Taking the weight of a cubic inch of iodine at 1,250 grains, and supposing that it remains on the silver surface in its elementary state, instead of forming iodide of silver, then we have $1{,}250 \times 144 \times 100{,}000 = 18{,}000{,}000{,}000$, one-eighteen-thousand-millionth part of an inch for the thickness of the coating of iodine. Even if we suppose that the solar rays decompose the iodide of silver, and leave the iodine in vapor on the plate, it will still be only one-forty-four-millionth part of an inch—a thickness to be taken as nothing in a mechanical view.

To test the effect of the chemical method of preventing adhesion on the sharpness of the engraved lines, an engraving was seven times successively transferred from plate to plate, when the closest inspection failed to show any inferiority of impressions from the last plate as compared with those from the first.

### TIME AND EXPENSE OF ELECTRO-CASTING.

Next in importance to securing a certain and easy separation of the matrix and casting is bringing the entire time and expense of electrotyping within the narrowest limits.

Mr. Smee and others have shown that the quality of electro-metal is determined by certain relations between the rapidity of forming the plate and the strength of the solution in which it is formed. Both the common operations of the electric-metallurgist, and the improvements he proposes, must conform to these relations.

As small quantities of electricity are easily set in motion, small-sized electro-castings are readily made in six or eight days. To make large castings in a short time requires a powerful current. To accomplish the corresponding augmentation in the effective electric action has proved a somewhat difficult matter.

At the date of the "Aide Mémoire to the Military Sciences," it is stated that in the ordnance survey one pound of copper was deposited in twenty-four hours on a plate of eight square feet, the plates being made ductile enough to bear hammering only by continued agitation of the electrolytic solutions.

At this rate, to make a plate one-eighth of an inch thick will require forty-five days. So far as I am informed, the above performance has not been excelled, as to quality and time, on large work anywhere prior to being attained as now to be described.

The first and most obvious suggestion for increasing the rate of deposition is to enlarge the battery; this, however, is incapable of producing the desired end.

To present this subject in a clear and satisfactory manner, I will make use of the celebrated formula of Professor Ohm, who deduced from mathematical reasoning, and established by experiment, that the effective force of the current from any battery was directly as the electromotive force, and inversely as the resistance offered to that current. To express this, he gave the equation $\frac{E}{R+r}=Q$, in which E represents the electromotive force, or affinity of acid for zinc, and $R+r$ the resistance to the current generated by that force; R representing the resistance offered to it from the liquid contained between the positive and negative elements of the battery, and $r$ the resistance offered by the object on which the battery is working, and Q the amount of work executed, or the quantity of the current obtained.

The resistance of conductors has been found to be directly as the length, and inversely as the section.

So far as concerns form of arrangement, E is constant, for the materials used, as it depends on their chemical relations; Q can therefore be favorably affected only by varying R or $r$. Now, as R represents the resistance of the liquid contained between the battery plates, to increase the size of the plates is only to increase the section of the liquid, or, in other words, to diminish the resistance represented by R. The expression, $\frac{E}{R+r}=Q$, shows that if the resistance in the battery is small, compared to the external resistance, the gain of effect from enlarging the battery plates is but small.

To determine the relative value of R, as compared with $r$, a battery was constructed so as to collect and measure the gas evolved by its action.

The plates were placed in contact with each other, and the gas evolved in thirty minutes taken as a unit of effect. As in this case the current did not pass through anything but the battery, there is no resistance to be represented by $r$, or $r$ in the formula will be equal to 0, and $Q=\frac{E}{R}=1$.

The battery was then attached to a pair of electrodes, in a certain solution of sulphate of copper and sulphuric acid, especially recommended by all the writers on electro-metallurgy, the arrangement being such as to produce good metal. The gas now evolved in thirty minutes was found only one-twentieth of the former amount; hence the introduction of the resistance, $r$, had diminished Q twenty times, and $\frac{E}{R+r}=Q=\frac{1}{20}\frac{E}{R}$, whence $r$ is equal to $19R$. To exhibit the effect of battery enlargement, we now have $Q=\frac{1}{\frac{1}{m}+19}$. If $m=1$, then $Q=.05$; if $m=2$, $Q=.0512$; if $m=3$, $Q=.0518$; if $m=4$, $Q=.0524$, &c., &c. This shows a gain of only a fortieth from doubling the size of the battery, &c.: an advantage too small to repay for the enlargement. These calculations are in accordance with experimental results from small batteries; but in large ones the necessity of further separating the plates, in increasing their size, makes the resistance increase instead of diminish, and there is consequently a loss from enlargement. It is not, therefore, by merely increasing the battery surface that the time for electrotyping can be shortened.

Mr. Smee, the distinguished writer on electro-metallurgy, by covering the negative plate of the battery with pulverulent platinum, produced a very energetic form of the instrument. When the plate is freshly platinized, it acts violently, and throws off the hydrogen in torrents. But this increased energy of the plate is gradually lost, from the electric current depositing upon it impurities from the zinc.

As this deposit has a strong attraction for the hydrogen, it is retained on the plate. The plate, being thus encased in air, is virtually excluded from the liquid of the battery. The ordinary solvents of the metals do not readily remove this coating of impurity. The plate can be renewed by replatinization; but, as this is both tedious and expensive, I was urged to find a menstruum which would restore the original platinum to its energy. This I attained, at length, by immersing the plate in a solution of per-chloride of iron, which almost immediately restores the action of the plate.

The plates are now daily immersed in the chloride of iron, by which the tone of the battery is constantly maintained.

By this last discovery, together with obtaining better solutions for the decomposing cell, the time for making a casting was reduced; but still the time required for making a plate was too long when only one electrical equivalent was employed.

The effective force of one battery may be added to another. This is increasing E in the formula, and this will sometimes increase Q.

We unite the effective force of many batteries by joining their dissimilar ends in consecutive order. As the current in such an arrangement has to traverse every battery in the chain, R will be multiplied as many times as we multiply E. The formula then becomes $Q = \frac{n\,E}{n\,R + r}$. When the values of $r$ and R are nearly equal, and we have batteries of definite construction to work with, it becomes a matter of some importance to determine whether we shall use the whole galvanic apparatus, as a single electrical equivalent, by connecting all the similar parts of all the battery cells, or whether we shall convert it into a battery of two pairs, in consecutive order, by joining dissimilar ends. As doubling the battery is doubling R, and to double the electrical equivalents is also to double R, we shall increase R fourfold by the double arrangement. Instead of $Q = \frac{E}{R + r}$ we have $Q = \frac{2\,E}{4\,R + r}$. Taking $R = r$, we have $Q = .50$ in the single arrangement, and $Q = .40$ in the double, showing that we may double the expense and yet make the casting more slowly than before. Conditions as above are of frequent occurrence, and a knowledge of them without experimenting is of very great importance.

For $R = 10r$, with a single equivalent of battery, $Q = \frac{1}{1 + 10} = 0.0909$. For two batteries in series $Q = \frac{2}{2 + 10} = 0.166$. The use of two batteries in consecutive order, as thus exhibited, doubles the expense, but does not double the effect. A regard for economy prohibits us from further increasing the series. To represent an effect double of $\frac{E}{R + r}$ we have $2\left(\frac{E}{R + r}\right) = \frac{2\,E}{\frac{2R}{2} + r}$.

As dividing R by 2 is doubling the battery surface, we may now make $Q = .183$. The gain per cent., now indicated by doubling the surface, makes it advantageous to make this increase when two consecutive batteries are used.

The difficulty of obtaining large flat plates of silver proved a serious obstacle in effecting an increase of battery surface, for the irregularity of the surface requires the plate to be placed at an increased distance from the zinc, thereby augmenting R, the very thing sought to be diminished.

Plates could be made flat by the use of the planishing hammer; but the operation being expensive, and the plates continually liable to accidents in use, economy prohibited this mode of forming flat plates. Though the plating of metallic bodies with silver had been well executed, it had not yet been determined that electro-casting of silver could be executed in a desirable manner, and at a moderate expense and trouble. At first every attempt to make plates weighing 2,500 grains to the square foot failed, on account of the difficulty of observing Mr. Smee's laws relative to the E for the time required.

But after modifying the solutions of silver, and using a register battery, a plate could be made in 30 hours, perfectly flat, and possessing the mechanical qualities of hardness, elasticity, and malleability, in an eminent degree, and not costing over 16 cents per ounce for the making.

The perfectly flat plates admit of a very close approximation to the zincs. Their size may therefore be increased to more than twice their former surface. As in the double arrangement, $r$ is relatively smaller to R.

Important changes have also been made in the modes of operating, and in the arrangement of the apparatus. It had early been noticed that changes of temperature influenced the rate of working; and every electro-metallurgist knows the importance of keeping the laboratory warm.

To determine where and how the effect of temperature took place, a battery, at 60 degrees of Fahrenheit, was connected with a wire 120 feet long, and enclosing a galvanometer. The deflection was 40 degrees; the battery was then cooled until the temperature was 48 degrees; the needle was still deflected nearly 40 degrees.

This experiment indicated that the batteries were not greatly affected by ordinary variations of temperature. Advantage was then taken of this development to secure a more perfect ventilation. Accordingly, a small room, to contain the battery, was partitioned off from the general apartment by a glass partition, and large outward openings made at the top and at the bottom of the room, to give a circulation of air for carrying off the battery fumes.

At the stage of improvement now described, one of our medium plates, having 8 square feet of surface, could be readily made in from 8 to 10 days. But wishing to still further quicken the process, or attain my first desire—to deposit one pound per day on the square foot, with a single equivalent of battery—improvements were again sought after. As the E of the formula has been increased to the greatest extent the cost would permit, and $r$ had been diminished, or the plates increased in size to the greatest useful extent, it was sought to increase Q by diminishing $r$, or the electrolytic resistance. It was sought to increase the conducting power of the electrolyte by adding easily decomposable salts to it; but with no success. The accelerating effect of temperature being found, as above stated, to be confined chiefly to the decomposition cell, it was evident that by using the electrolyte alone, at a high temperature, a considerable advantage might ensue.

To determine the most advantageous working temperature, and the resulting gain of effect, a voltameter battery was connected to a pair of electrodes, in the solution formerly described as being generally recommended. Each electrode had five square inches of face, and was coated on the back to prevent radiation. They were placed one inch apart, and had thin plates of wood bound against their edges, to prevent any lateral spread of the current in passing between them. The following was then obtained:

| | |
|---|---|
| Battery plate in contact gave 300 cubic inches gas per hour. | |
| Electrodes in contact gave 216 cubic inches gas per hour. | |
| Current through electrolyte, at 58°, gave 16 cubic inches gas per hour ....... | 23.15 |
| Current through electrolyte, at 60°, gave 20 cubic inches gas per hour ....... | 18.15 |
| Current through electrolyte, at 100°, gave 27 cubic inches gas per hour ....... | 13. |
| Current through electrolyte, at 175°, gave 37 cubic inches gas per hour ....... | 8.96 |

The last column of figures shows the value of the resistance of the solution, as compared with R of the formula. This column was obtained by first uniting the battery plates, and afterwards the electrodes.

From the above table it appears that heat may be made to diminish the resistance in the decomposition cell in the proportion of 2.58 to 1; and the whole resistance by 2.25. And as $\frac{2\,E}{R+r}=\frac{E}{\frac{R+r}{2}}$; therefore, by heating the electrolyte, we may with a single electrical equivalent make a plate as rapidly as by working at atmospheric temperatures with two batteries in consecutive order, with double surfaces, (four times the battery and twice the expense.)

But as Smee's laws require that, in forming a plate, certain mutual conditions of apparatus be maintained, it follows that alterations in one element or condition must be attended by corresponding changes in the others. Hence, if the temperature of the electrolyte be raised to a certain point, and the apparatus correspondingly adjusted, it is evident that, to avoid incessant adjustment, the original temperature must be maintained.

Thus, to avail ourselves of the advantages experimentally found from heating the solutions, an apparatus for steadily maintaining a high temperature in the electrolyte through several successive days becomes indispensable.

As the electrotype operations are not suspended at night, it is important that the heating apparatus should perform its office for at least 12 hours without supervision or replenishing its fuel; and its action should be sensibly uniform, during all the time, between successive replenishings.

Such an apparatus I have devised, and is now in use. A peck of charcoal furnishes fuel for 12 hours, and maintains 100 gallons of copper solutions steadily at any required point between 100° and 200°.

With the above arrangement in use, I have made a large reverse or alto, and returned the original to the engraving department, in 55 hours from its being placed in my hands. This time included trimming the edges and the preparations to prevent adhesion.

Again recurring to Ohm's formula, the relative value of R to $r$ was once more experimentally found. This gave $R : r :: 1 : 4$ or $Q=\frac{1}{1+4}=0.20$, a great improvement as compared with the first determination of $R : r :: 1 : 19$, or $Q\ \frac{1}{1+19}=0.05$. Having now made $r$ so small compared with R, the size of the battery can be profitably increased until the result is about 0.24. Moreover, using a double arrangement of cells with double surfaces, for a double effect, we now have $2\left(\frac{1}{1+4}\right)=\frac{2}{2+4}=0.40$. As the relative resistance of the electrolyte becomes now still smaller, we may yet more increase the battery surface until the result is nearly 0.5.

The electrotype has now ceased to be a mere experiment, uncertain, expensive, and slow. I have lately formed plates of most excellent quality, at the rate of 3 lbs. to the square foot, in 24 hours. This rate will require but two days to form one of our largest plates, having ten square feet of face, and one-eighth of an inch thick.

### ACTIONS IN THE ELECTROLYTIC SOLUTION.

The quality of the deposited metal is governed solely by the relations between the *quantity* of electricity passing through any solution and the amount of metal the solution contains. The usual supposition is, that the acid of the salt goes to one electrode and the metal to the other. It is now ascertained that no such mutual transfer takes place; for, while the acid is carried to the positive electrode, the metal is *not carried* to the negative electrode. Hence, however strong the solution on commencing the process, the negative electrode, by abstracting the metal in its vicinity, is soon surrounded with a weak solution. With a simple wire electrode, the exhausted solution surrounding the electrode is readily renewed by mere difference of specific gravity producing a flow. But, with large parallel plate electrodes, this rapid renewal of dense solution becomes impossible, and the electrode is soon surrounded with a weak solution. This state of things must be recognised in adjusting our battery arrangements. Electrotypists not aware of this fact find themselves much perplexed by failing to accomplish with large plates what is so easily done with medals or small plates.

It would, at first sight, appear that, by strengthening the solution of sulphate of copper, a more rapid supply of metal to the electrode would be obtained. Unfortunately, the effect of this is to diminish the solvent capacity of the water in the solution for the sulphate formed on the positive electrode by the action of the transferred acid. The grand essential in electrolysis is liquidity in the solution. Thus, if the quantity of free water surrounding the positive electrode be small, this electrode is soon enveloped in a saturated solution, and the newly-formed salt remains undissolved upon it. This salt, being a non-conductor, virtually excludes the electrode from the solution, and thus arrests the current, except when the efflux of saturated solution permits the salt to dissolve, and so reopens the passage for the current in irregular quantities. From this spasmodic action result plates of copper-sand, or sometimes copper as soft as lead.

By applying heat to the solution when this state of things exists, the solvent capacity of the water for the salt is increased, rapid diffusion takes place, the salt is carried to the negative electrode, and the exhausted water to the positive electrode; the dormant batteries rush into uninterrupted action, and in a short time a plate is deposited, having all the hardness and elasticity of hammered or rolled copper. Smee's conditions, then, seem to maintain themselves. The electrotypists' axiom of "work slowly," requires to be reversed into "the quicker the work, the better the quality."

### LABORATORY APPARATUS.

Figure I is a plan of the Coast Survey electrotype laboratory. The glazed partition, *b*, *b*, *b*, *b*, with a door, *d*, separates the battery room from the general laboratory, and permits an easy inspection of the batteries, without exposure to their fumes. The laboratory floor is about six feet above the ground, and slopes inward from the sides towards the scuttle holes, *h*, *h*, *h*, *h*, arranged for discharging the waste liquids spilled upon the floor. To obviate the deleterious effects of working on a floor saturated with chemical agents, when any solutions are spilled, the floor is well flooded and brushed, the water passing off through the scuttle holes. There are four battery cells, placed as indicated, B, B, B, B. A rectangular India-rubber bag, supported by a deep wooden box, contains the battery solutions. Each cell can contain nine silver and eight zinc plates. A metallic connection unites all the zinc plates of a cell, and another one all the silver plates. Each cell can be used as an independent battery, or two, three, or four cells can be connected in consecutive or simultaneous order, or all combined into two pairs of two in consecutive or simultaneous order, or into one group of three and one of one. The position of the vertical decomposing vat is shown at V, and that of the horizontal vat at H. S is a large tub for washing plates. The tub C contains the solution of chloride of iron. Q is the quicksilver tub, and W, W, are fresh-water tubs. F is the furnace, and *d*, *d*, *c*, *c*, are heating tubes connecting with the vat H. T is a flat iron table.

Fig. 2 exhibits a cell and its included plates, with their mode of suspension.

Fig. 3 represents the suspending frame of wood and the attached plate P, prepared for immersion in the vertical vat.

Fig. 4 shows the vertical vat and the plates suspended in it.

Fig. 5 represents the adjustable plate-supporting frame used in the horizontal vat.

Fig. 6 exhibits the interior arrangement of the horizontal vat, a blank plate and an engraved original being in position; also the connecting copper wires leading to the battery.

Fig. 7 represents the heating furnace. The door for admitting air is shown at *a*, and is so connected with an adjusting compound bar of iron and zinc that by an adjusting screw it can be arranged to regulate the draught, opening or closing the door, thus maintaining a uniform heat in the solution. After getting the fire started, this door is set so as to close when the solution reaches a heat of 150°. In principle this furnace is similar to a bath-heater. A tubular helix of lead is coiled within it like the worm of a still, and the terminating branches *c* and *d* lead to the horizontal vat, the branch *c* uniting the top of the vat just below the liquid surface with the bottom of the coil, and *d* the bottom of the vat with the top of the coil. Hence follows a circulation of the solution from the furnace at top and into it at bottom.

### MANIPULATION.

When a plate is to be electrotyped, it is placed on trestles above the open scuttle holes, *h*, *h*, *h*, *h*, and thoroughly cleaned by washing with alkalies and acids. It is then silvered, iodized, and placed before a window. A plate of rolled copper an inch larger than the engraved plate is then selected, placed on the flat iron table, and beaten with mallets until a steel straight edge shows it to be plane. It is then weighed and fixed in the vertical plate frame by two copper hooks. The engraved plate is then similarly fixed in a similar frame, when both are placed in a vertical vat and connected with the battery.

The process does not go on well when the plates are vertical, but it is necessary to start the castings in this position to prevent dust, motes, or specks of impurities from settling on the face. As the rolled plate dissolves, its impurities rapidly render the solution muddy, and endanger the face of the forming plate. For common electrotypes dust or mote specks are not detrimental; but the Coast Survey copper plates being not inferior in fineness of lines to fine steel plates, the effect of impurities settling on the face of their copies is to give the impressions a clouded appearance. On first immersing the plate, the solution should, therefore, be perfectly clean. Formerly, after each use of the vertical vat, it was emptied and washed out. When the solution had deposited its sediment it was drawn off and strained through very fine cotton. This whole operation was extremely disagreeable, and consumed a whole day of one man.

By a simple expedient I have saved the necessity of cleaning the vat oftener than once a month. To guard the new plate from specks and impurities, a bag of fine cotton is drawn over a slight wooden frame, which keeps it distended. An hour or more before the solution is wanted, the bag, with its included frame, is placed on top of the solution and loaded with the copper bars used to support the plate frames. The weight causes the bag to sink gradually, filtering the contained solution as it goes down. The impurities cannot wholly choke the meshes of the cloth, as a fresh portion is constantly brought into action during the sinking. I thus filter the solution without taking it from the vat or disturbing the sediment, saving much labor, time, and annoyance.

The plate remains in the vertical vat over night, and preparations are made in the morning to transfer it to the horizontal vat. The furnace is first brought into action. A new plate of blank copper, an inch larger than the matrix, is flattened on the iron table, and bolted to the edges of wooden bars by platinum bolts, for the purpose of preventing the plate from sagging downwards when supported horizontally. The plate so arranged is called the strapped plate. The coated matrix is then taken from the vertical vat, disengaged from its frame, and arranged in the horizontal frame. A wooden wall, an inch high, then surrounds the plate, and in this wall the strapped plate is laid, when the whole combination is placed in the horizontal vat and the connection with the battery established. The positive plate is then taken from the vertical vat and its loss of weight noted and recorded. From the known superficial area of the matrix, the quantity of copper required for the casting one-eighth of an inch thick is computed and recorded. The blank copper consumed in both vats must equal this amount before the required thickness is reached, allowance being made for impurities of rolled copper and roughness on the back of the electrotype. After a few hours of action the strapped plate becomes so loaded with impurities that they will begin to drop on the electrotype; this plate must, therefore, be removed from the vat and a new one immediately supplied. The dirty plate is then washed in the large water tub, and when cleaned its loss of weight is found and recorded. By the amount of loss the action of the batteries is tested, and it is found, if Smee's laws are being observed. Vigilance must now be exercised in watching the batteries and rate of work, and the power must be varied to suit circumstances.

The entire working battery generally requires renewal once a day, the process being conducted as follows: One zinc and one silver plate are taken from the battery; the silver placed in a solution of chloride of iron, and the zinc taken to the water tub outside the door of the battery room, where it is scrubbed clean with a hard brush. It is then re-amalgated at the quicksilver tub, and taken back to the battery. The silver plate is transferred from the chloride of iron solution to the adjacent fresh-water tub. Another silver plate is then transferred from the battery to the chloride solution, and another zinc cleaned, washed, and put back in the battery with the first silver. In this manner the whole battery can be renewed without sensibly interrupting its action.

When the loss of weight from the rolled copper in both vats indicates that the required thickness of the electrotype is gained, the plate is withdrawn from the battery, detached from its frame, its back smoothed and its edges filed until a separation can be made. By separation the original becomes liberated, and the alto or reversed relief is silvered and electrotyped exactly as an original.

The copy from it, or the electrotyped basso, will, if the process has been properly conducted, be a perfect fac-simile of the original, and in hardness, ductility, and elasticity, will equal the best rolled and hammered or planished copper plate.

Yours, respectfully,

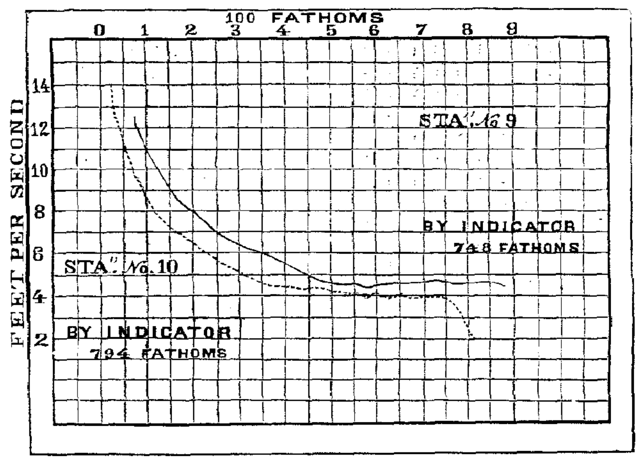
GEORGE MATHIOT.

Major I. I. STEVENS,
*Assistant in charge of office.*

# SUPPLEMENT TO APPENDIX No. 5.

## RATES OF OUTRUN OF LINE.

Berryman apparatus, with 96 pounds lead and Italian hemp line $\frac{3}{4}$ inch in circumference.




An indication of bottom appears in the case of station No. 10, sudden falling off of velocity between 750 and 800 fathoms; not so with station No. 9. Station No. 10 is in the northern edge of the stream, where the current is feeble; while station No. 9 is but little to the southward of the axis or thread of the current.

# SUPPLEMENT TO APPENDIX No. 8.

## LENGTH OF THE KENT ISLAND BASE LINE.

Kent island, on which this base line was measured, is situated on the eastern side of Chesapeake bay, nearly opposite Annapolis, (see Sketch No. 10.) The line is about five and a half miles in length, and was measured in May and June, 1844, by Assistant James Ferguson, with the same apparatus which had been previously used in the measurement of the Fire Island base line, (see Coast Survey Report for 1865, Appendix No. 21.)

The comparisons of the four double-meter bars with the committee meter, by means of Bessel's contact-level comparator, in 1844 and 1845, gave their combined length, or A + B + C + D, at 32° Fahrenheit, = 7.99987165 meters, as stated in the account above referred to.

The mean temperature of the bars during the measurement was 77°.79, and when corrected for graduation error, 77°.33 Fahrenheit; hence the excess of temperature above that of melting ice, 45°.33. The co-efficient of expansion for iron is adopted, as before, at 0.000006963535, and the temperature correction computed accordingly.

The correction to the length of the base for inclination of bars was calculated by means of the tabulated versed sines of the observed inclinations. The average elevation of the line above the half-tide level of the bay, which cannot sensibly differ from that of the ocean, is found by leveling to be 5.0 meters, inclusive of the height of apparatus.

The resulting length of the base is then found as follows:

| | | *Meters.* |
|---|---|---|
| 1086 boxes, at 8 meters | | 8688 |
| Defect of each on 8 meters, or 1086 × 0.00012835 | − | 0.1394 |
| Correction for expansion of bars, or for excess of temperature over 32° | + | 2.7424 |
| Correction for inclination | − | 1.0007 |
| Excess of box at south end, measured by bar D and scale | − | 2.0508 |
| Reduction to half-tide level of bay | − | 0.0069 |
| Resulting length | | 8687.5446 |

To estimate the accuracy of this value we first consider the probable error of the assigned length of the combined bars, derived from their comparison with the standard meter. This is found to be ±0.00000550. The probable error in 1086 boxes is consequently ±0.00597 meter.

If we arrange the boxes according to rising, stationary, and falling temperatures, we shall find 457 boxes laid during rising, 150 during falling, and the remainder during stationary temperatures. We have, therefore, 307 uncompensated cases; and supposing again an average of 2° difference between the temperature of the bars and that indicated by the thermometers, (as in the account of the Fire Island base,) the effect would amount to 0.0342 meter, which has been adopted as the probable error in the length arising from imperfect temperature indications; combining with this the uncertainty from the graduation error, or ±0.0151, we have the probable error ±0.0374 meter, arising from the temperature corrections.

The probable error arising from the instability of the microscopes, estimated as in the case of the Fire Island base, cannot exceed $\pm 0.000127\sqrt{1086}$, or ±0.0042 meter.

Putting together these three principal probable errors, that of the assigned length of the base becomes $\sqrt{(0.0374)^2 + (0.0060)^2 + (0.0042)^2} = \pm 0.0381$ meter, equal to ±1.50 inches, or to $\frac{1}{228000}$ of the length; the corresponding value in the logarithm is ±19 046 in units of the seventh place of decimals.

We have, therefore, the following value for the resulting length of the Kent Island base line:

8687.5446 ± 0.0381 meters,

and its logarithm,

3.9388970 472
± 19 046

# CORRIGENDA IN COAST SURVEY REPORTS OF PRECEDING YEARS.

**1859, page 336.** Co-ordinates of curvature; lat. 9°, col. $y$, long. 2°, for 400 read 600.

**1862, page 233.** To expression for $12\,b_2$, line 3 from top, add $+ S_3 - S_9 + S_{15} - S_{21}$.

**1864, page 143,** line 2 from bottom, for 40''.009 read 41''.009.

**1865, page 198.** In triangle 20, distance opposite Humpback, for 22161.352 read 26161.352.

# LIST OF CHARTS AND SKETCHES.

# National Oceanic and Atmospheric Administration

## Annual Report of the Superintendent of the Coast Survey

### Please Note:

This project currently includes the imaging of the full text of each volume up to the "List of Sketches" (maps) at the end. Future online links, by the National Ocean Service, located on the Historical Map and Chart Project webpage (http://historicals.ncd.noaa.gov/historicals/histmap.asp) will includes these images.