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REPORT

OF THE

SUPERINTENDENT

OF THE

U. S. COAST AND GEODETIC SURVEY

FOR THE

FISCAL YEAR ENDING JUNE 30, 1892,

IN TWO PARTS.

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1892
pt. 2

PART II.

APPENDICES RELATING TO THE METHODS, DISCUSSIONS, AND RESULTS OF THE COAST AND GEODETIC SURVEY.

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National Oceanic and Atmospheric Administration

Annual Report of the Superintendent of the Coast Survey

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PREFATORY NOTE.

The text of this Report for the fiscal year 1892 has been arranged for publication in two parts, like that for the fiscal year preceding.

Part I, in quarto form, contains the historical portion, presenting reports of progress in the field and office operations of the Survey; estimates for future progress, and statements of expenditures during the fiscal year. Maps of general progress, and sketches showing localities of field work, exhibit graphically the advance of the Survey to June 30, 1892.

Part II, it will be observed, is in octavo, and includes the professional papers relating to the methods, discussions, and results of the Survey which have been approved for publication during the year. Such illustrations as are needed accompany them.

The octavo form is more convenient and suitable for the scientific and professional papers, while the quarto form appears to be demanded for the statistical matter and the progress sketches. Since the latter are of less general interest than the former, in the future distribution of the Report, Part II only will be sent, as it is believed that this will include all that is generally desired, and in a much more compact and convenient form than that of the old quarto.

In special cases, where both parts are desirable, they will be sent.

T. C. MENDENHALL,
Superintendent.

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ON THE VARIATION OF LATITUDE AT ROCKVILLE, MD., AS DETERMINED FROM OBSERVATIONS MADE IN 1891 AND 1892 IN COÖPERATION WITH THE INTERNATIONAL GEODETIC ASSOCIATION.

Submitted for publication October 14, 1892.

PREFATORY NOTE.

The Coast and Geodetic Survey having undertaken to coöperate with the International Geodetic Association in the observations for variation of latitude at Waikiki, near Honolulu, it seemed important to have determinations of latitude at points intermediate between that point and Berlin.

Washington, being nearly six hours west of Berlin in longitude, and in latitude nearly midway between Berlin and Waikiki, was geographically well placed for observations which would check those regularly made at these two stations. Still further light would be thrown upon the suspected progressive change in the position of the axis of rotation of the earth by a series of observations at San Francisco, nearly midway between Washington and Waikiki. The verification of the movement by these observations would remove all doubt as to its reality, and it was accordingly determined to do the best that could be done under the circumstances in the way of making observations for latitude as frequently as possible throughout a year at both of these points.

Other demands of the service forbade the detailing of one or more assistants to this duty alone. It was necessary to do it, if it was to be done at all, in addition to other regular work, and it therefore became, in a large degree, a voluntary service. Assistant George Davidson, in charge of the Sub-office at San Francisco, in carrying on the work there was much interrupted by necessary attention to office and field duties. In Washington the task was voluntarily assumed by Assistant Edwin Smith in addition to his regular duties as chief of the Instrument Division. The latter required his presence in the office during the day and necessitated a journey of 16 miles in the morning and again in the evening, to his home in Rockville, where the temporary observatory was located.

In considering the results herewith submitted, the above facts must not be lost sight of, and it must be remembered that the work at the

stations in the United States was undertaken primarily as a check upon the more careful and systematic investigation in progress at Berlin, and by Assistant Preston and Dr. Marcuse at Waikiki.

This paper contains the observations made at Rockville, and the reduction and computation of the results. An abstract of it was published in November, 1892, as Bulletin No. 25.

The instrumental methods and scheme of observing have been described by Mr. Smith, and the description of the method of reduction and the discussion of the results was prepared by Mr. Charles A. Schott, Assistant in charge of the Computing Division of the office, under whose direction the computations were made.

Mr. Preston's report of his observations and results at Waikiki is published as Appendix No. 2, 1892.

T. C. MENDENHALL,
Superintendent.

I.—DESCRIPTION OF THE STATION, INSTRUMENTS AND METHODS OF OBSERVING BY EDWIN SMITH, ASSISTANT U. S. COAST AND GEODETIC SURVEY, AND OBSERVER AT ROCKVILLE.

In conformity with the instructions of the Superintendent, dated March 25, 1891, the scheme adopted for the Rockville station was as follows: Eighty-eight pairs of stars were selected from the Coast and Geodetic Survey Catalogue of Stars for Observations of Latitude, Appendix No. 7, Report for 1876, extending over the twenty-four hours of right ascension and arranged in fifteen groups, which were observed as follows:

Group.	Nos. of pairs.	Dates of observations.
1	1 to 16	June 13, 15, 1891, and June 2, 6, 11, 12, 13, 15, 16, 17, 20, 23, 25, 26, 29, 1892.
2	5 to 19	June 23, 24, 25, 27, July 5, 6, 1891, and June 11, 12, 20, 23, 29, July 7, 8, 9, 1892.
3	9 to 24	July 9, 10, 12, 21, 22, 27, 29, 1891.
4	14 to 28	July 31, August 7, 9, 10, 11, 14, 15, 22, 1891.
5	20 to 35	August 15, 17, September 1, 3, 7, 9, 10, 12, 13, 14, 15, 17, 1891.
6	26 to 40	September 10, 12, 14, 17, 18, 19, 21, 23, 24, 25, 26, 28, 30, October 3, 5, 9, 14, 15, 16, 17, 1891.
7	34 to 48	October 22, 23, 24, 28, 29, 30, November 1, 2, 6, 7, 9, 1891.
8	41 to 55	November 17, 18, 20, 23, 24, 29, 30, December 1, 4, 5, 1891.
9	47 to 61	December 8, 9, 10, 13, 18, 28, 31, 1891.
10	54 to 65	January 7, 8, 16, 20, 21, 24, 25, 28, 1892.
11	59 to 70	January 31, February 10, 13, 15, 17, 1892.
12	66 to 77	March 10, 12, 14, 16, 19, 20, 21, 23, 24, 1892.
13	71 to 82	March 28, 29, April 4, 9, 11, 12, 13, 1892.
14	77 to 88 and 1	April 16, 19, 23, 26, 27, 29, 30, May 1, 3, 4, 5, 7, 8, 9, 1892.
15	83 to 88 and 1 to 8	May 16, 17, 19, 23, 24, 25, 28, 30, June 1, 1892.

On the variation of latitude at Rockville, Md.

The large number of groups is considered the weakest point in this series of observations. The conditions under which the work was done will readily explain why it would have been impossible to have limited the number of groups. It is to be regretted that the hours of observation were so limited, but to have extended them to two or three o'clock in the morning would have been fatal to the performance of the observer's duties at the office. The groups could have been limited to a small number only by so extending the hours of observation.

The observer has had little assistance in this work. With the aid of Mr. Frank A. Cook, the clerk of the Instrument Division, the stars were selected and the groups arranged only a few days before the observations were actually begun. Mr. Cook kindly volunteered to assist even after office hours, and with his help an effort was made to keep up the computations as the observations progressed; but this was soon found impracticable and the effort was abandoned. This part of the work was then referred by the Superintendent to the Computing Division of the Coast and Geodetic Survey Office, where the computations have been personally made by the chief of that Division, assisted by Messrs. Alexander Bonnot and Legh W. Reid, who were specially employed for the purpose.

DESCRIPTION OF STATION.

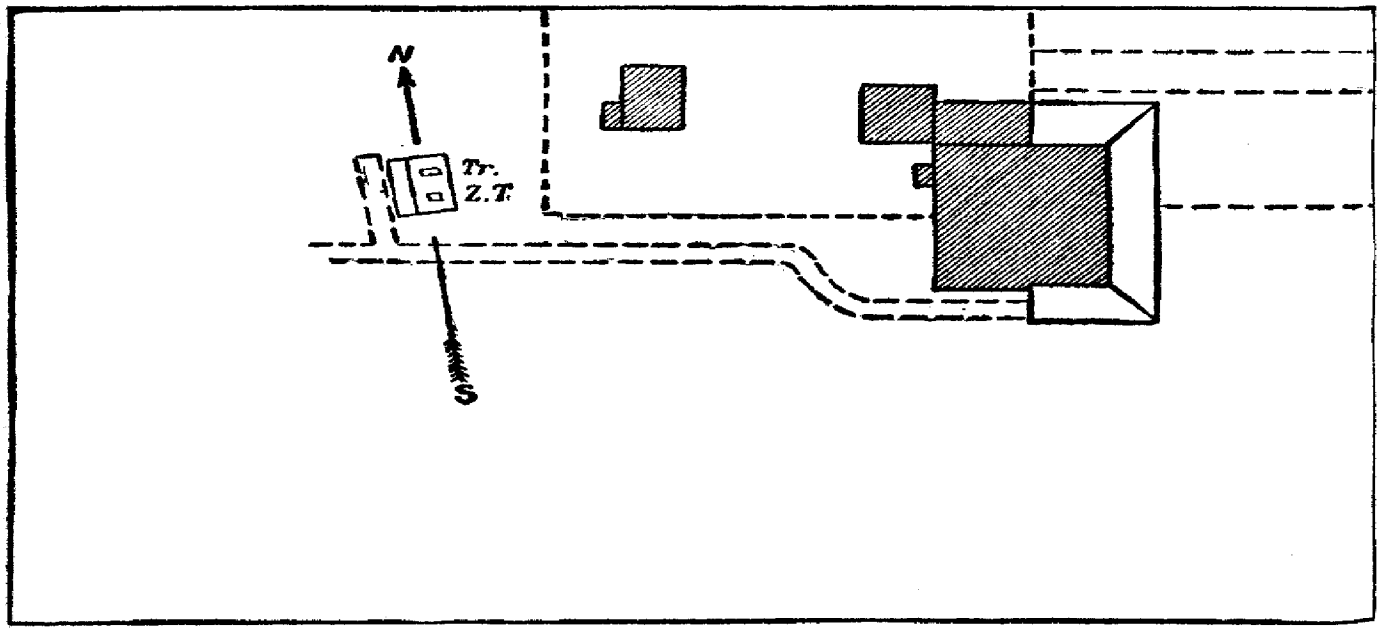
The town of Rockville is located on the Metropolitan Branch of the Baltimore & Ohio Railroad, about 16½ miles from Washington, D. C., in Montgomery County, Maryland.

The latitude station is located in the garden of the observer's residence on Forest avenue about 1 mile by the direct road from the railroad depot, but somewhat less than half a mile from the nearest point of the railroad. The station is about 500 feet above tide water, on one of the highest points within a radius of about 2 miles. The immediate vicinity is free from trees and other disturbing atmospheric influences, and altogether can be considered a very favorable location for such observations. The sketch shows the location of the observatory in the garden.

The pier upon which the zenith telescope is mounted is built of rough sandstone and cement, and its foundation is solid quartz rock. A ledge of this rock runs through the garden in a direction SW. to NE., and comes above the surface of the ground at several places. At the point where the pier is built the rock is about 2 feet below the surface. The pier has proved a very steady one. Throughout the thirteen months of observation the instrument required no adjustment in azimuth except when intentionally disturbed. The changes of level were slight and due more to temperature than to any other cause. The pier is 0·61 metre square, and the top 0·99 metre above the ground. At a distance of 1·98 metres north of this pier is another pier of the same

material, only one-half of which rests upon the solid rock. Upon this pier is mounted the transit instrument. As might be expected this pier is not as stable as the other, but serves its purpose very well.

Over the piers is built a wooden observatory 3·66 metres by 4·26 metres, divided into two rooms 4·26 metres by 2·44 metres and 4·26 metres by 1·22 metres, respectively, the smaller room being used as a recording and waiting room and the larger the observing room. By a system of ventilation under the eaves and at the peaks of the gables of the roof, an excessive rise of temperature is avoided. The width of the slit in the roof is 38 centimetres. The exterior of the building is painted a very light color, almost white. The zenith telescope pier is covered



Sketch of Garden, showing location of Observatory.

by a wooden case resting upon the floor of the observatory free from any contact with the pier.

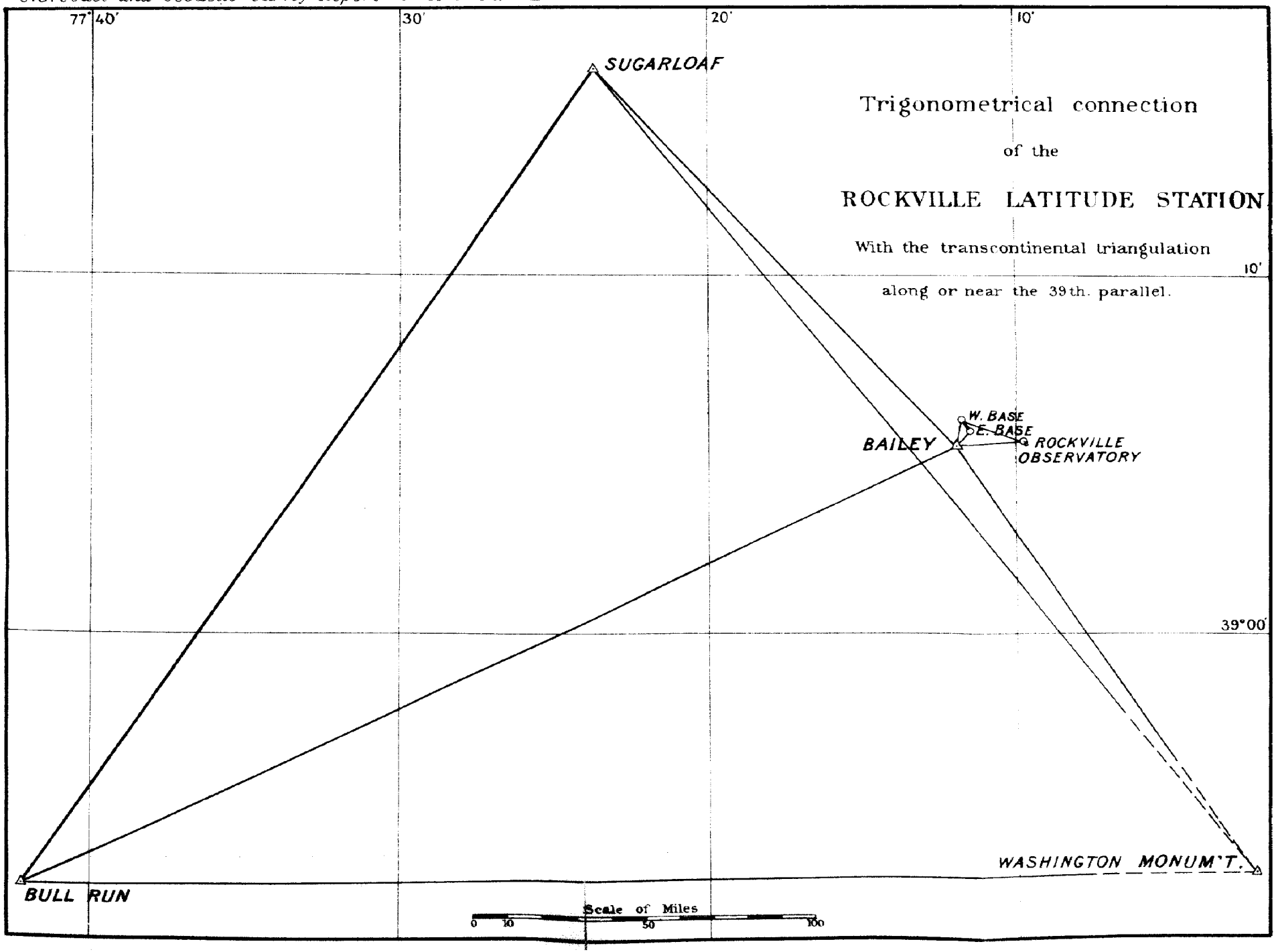
In the spring of 1892 this station was connected with the main triangulation of the Coast and Geodetic Survey, as shown in illustration No. 1 and the geodetic position of the zenith telescope pier was found to be as follows:

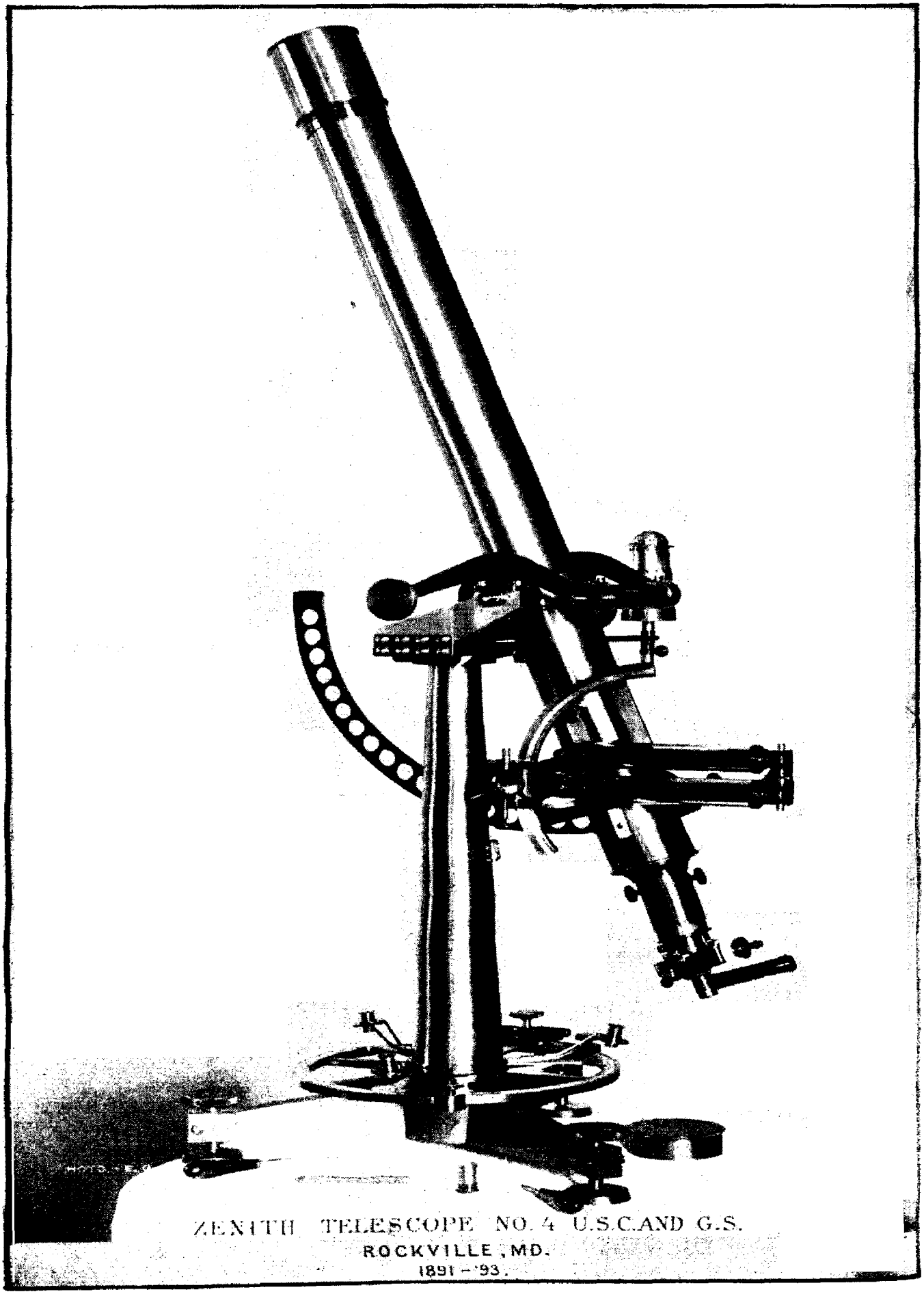
Latitude =39° 05′ 11″·21
Longitude =77° 09′ 36″·65

INSTRUMENTS.

Zenith telescope No. 4; transit No. 5; sidereal break circuit chronometers Negus No. 1589, Hutton No. 220, and Hutton No. 202; cylinder chronograph; electric lamps, batteries, etc.; Centigrade thermometers Nos. 6287, 6288, and 6289; mercurial barometer No. 1735.

Zenith telescope No. 4 (see illustrations Nos. 2 and 3) was originally made by Troughton & Simms of London in 1849. The Coast and Geodetic Survey possesses four of these instruments, known as Nos. 1, 2, 3,


77° 40'
30'
20'
10'
SUGARLOAF
Trigonometrical connection
of the
ROCKVILLE LATITUDE STATION
With the transcontinental triangulation
along or near the 39th. parallel.
10'
W. BASE
E. BASE
BAILEY
ROCKVILLE
OBSERVATORY
39°00'
WASHINGTON MONUM'T.
BULL RUN
Scale of Miles
0
10
50
100



ZENITH TELESCOPE NO. 4 U.S.C.AND G.S.
ROCKVILLE, MD.
1891 – '93.

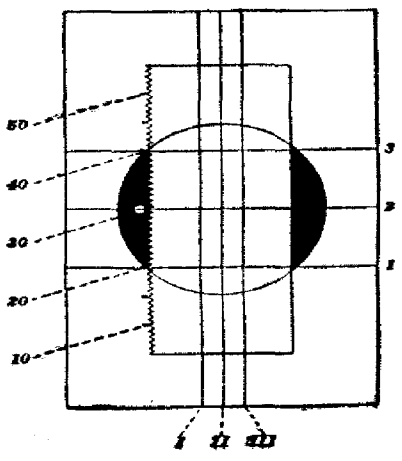
and 4, all made at about the same time. In their original form they had many defects and gave unsatisfactory results, and for some years past had been unused. When it was proposed that the Coast and Geodetic Survey should take part in these researches on the variations of latitude, in coöperation with the International Geodetic Association, no other instruments were available, and it was decided to have two of these remodeled at the Survey Office, one for the party going to the Hawaiian Islands and the other for the party to be located near Washington, D. C. No. 2 was selected for the former and No. 4 for the latter.

The remodeling of these instruments was done in the shops of the Instrument Division of the Coast and Geodetic Survey Office, and every precaution was taken to make them as perfect as possible under the circumstances. The new features are as follows for each instrument:

New base and leveling screws; new vertical axis; new wyes for horizontal axis of telescope, with adjustment for level; new and larger horizontal axis for telescope; new micrometer screw and reconstruction of micrometer box and slide; new and improved clamp to telescope; two new fine levels attached to telescope; new stride level for telescope axis; the whole instrument repolished, bronzed, etc.; electric lamps, batteries, etc., for illumination of telescope field, the reading of levels, etc.

Owing to want of time, No. 2 did not have the new base and vertical axis. This is very much to be regretted, as the old form is too weak. In every other respect the instruments are equal. Though these instruments are by no means such as would have been selected had there been time to purchase or construct new ones, it was believed they would answer the purpose.

The optical parts are those originally furnished with the instrument. The objective has a clear aperture of 7·6 centimetres and a focal length of 116·6 centimetres. The glass has a slight green color, but the images of the stars observed were bright and well defined. The diagonal eye-piece gives a power of 100. It gives good definition only very near the middle of the field, and having no parallactic motion in right ascension, only three good pointings could be made on a southern star while crossing the field. The micrometer is of the usual form. The diagram shows the field when the screw and eye-piece are set centrally. The three vertical lines are fixed spider webs about eight and a half equatorial seconds apart. The three horizontal lines are spider webs fixed to the slide moved by the micrometer screw at distances of ten revolutions of the screws. The micrometer is so arranged that if taken



apart the head and rack can be replaced so as to have the same relation to each other. Increasing readings of the screw indicate increasing zenith distances.

The two levels attached to the telescope were selected from a lot purchased from the Keuffel & Esser Company of New York in 1889. They were made by Adolph Pessler, mechanician, of Freiburg, in Saxony. These levels have proved very satisfactory, and it is doubtful if superior levels can be obtained. The glass tubes are 150 millimetres long and 18 millimetres in diameter. They have a chamber at one end, by which the length of the bubble can be controlled. The length of a division is 2 millimetres. As viewed upon the instrument, the graduations are from left to right, the upper level scale being numbered from 0 to 50 and the lower end from 60 to 110.

Each glass tube is mounted in a brass tube, in which it rests upon two points near each end, and is held in place by a light spring above these points at each end. Each brass tube is covered by a glass tube. The two levels thus mounted are attached to a brass arm by ball and socket joints at one end and opposing screws at the other end, by which the two levels may be made parallel to each other. The levels are mounted upon the left-hand side of the telescope and when pointing north the scales read from north to south.

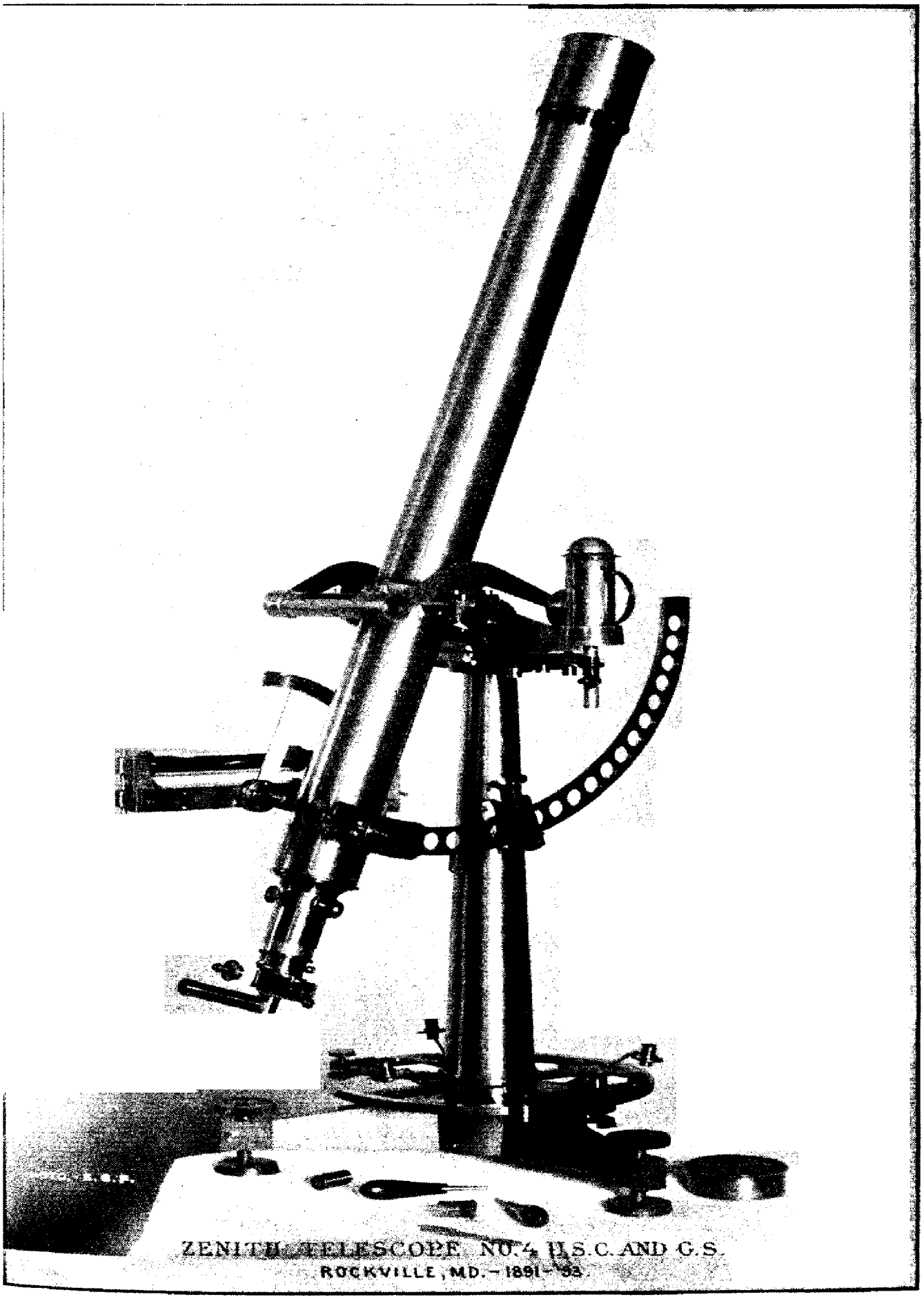
The illumination of the field is by a small mirror, mounted centrally in the horizontal axis, which reflects light received from a small electric lamp through one end of the axis.

Transit No. 5 is also an instrument made by Troughton & Simms in 1849, and remodeled at the Coast and Geodetic Survey Office in 1890. The dimensions and power of the telescope are about the same as those of zenith telescope No. 4. The pivots are so nearly equal that there is no appreciable correction. The value of one division of the stride level is 0·96 second. A similar instrument is illustrated in the Coast and Geodetic Survey Report for 1880, Appendix 14. This instrument was used for the determination of the corrections to the chronometers and also as a collimator for the adjustments of zenith telescope No. 4.

CONSTANTS OF ZENITH TELESCOPE NO. 4.

Levels.—The are values and errors of the level scales were determined and investigated at the Coast and Geodetic Survey Office by Mr. E. G. Fischer, the chief instrument maker of the Coast and Geodetic Survey instrument shop, in May, 1891, and again in January, 1892, at high and low temperatures. The details will appear with the report of the chief of the Computing Division on the computations and results. There was found to be no appreciable difference of value for change of temperature, and the errors of different parts of the scales are so small that mean values are used. These are as follows:

Upper level, 1 div. =1·600 seconds.
Lower level, 1 div. =1·364 seconds.



ZENITH TELESCOPE NO. 4 U.S.C. AND G.S.
ROCKVILLE, MD. - 1891 - '93.

On the variation of latitude at Rockville, Md.

Micrometer.—The true distances between the threads of the micrometer were determined by pointings made with the three threads upon a small speck on the glass diaphragm of the transit. The distances were found to be as follows:

Previous to June 13, 1892,

1 to 2 = 9·9418 revolutions;
1 to 3 = 19·9763 revolutions;
2 to 3 = 10·0345 revolutions;

After June 13, 1892,

1 to 2 = 9·8900 revolutions.

The arc value of the screw was determined and its errors investigated by observing transits of α Ursæ Minoris and λ Ursæ Minoris at eastern and western elongations, only that part of the screw between 20 and 40 revolutions being used. The head of the screw was set successively at 0, 25, 50, and 75 divisions in each of the 20 revolutions. The observations were recorded on the chronograph, three breaks being made at each transit, the mean of which was taken. Thirteen such sets of transits were observed as follows:

June 8, 9, and 10, 1891, on λ Ursæ Minoris at eastern elongation. September 18, 19, 21, 23, and 24, 1891, on α Ursæ Minoris at eastern elongation. January 27, February 22 and 23, and March 3 and 9, 1892, on α Ursæ Minoris at western elongation.

These observations show no appreciable periodic errors in a revolution, but there is an appreciable difference in whole revolutions and also an appreciable correction for temperature. The detailed results will appear in the report of the chief of the Computing Division on the computations and results. The mean arc value of one revolution is $44''·6606 - 0·001159\ t$, t being degrees centigrade.

The equatorial intervals between the vertical threads were determined August 6, by observations of transits of ω Draconis, φ Draconis, η^2 Draconis, and γ Draconis at upper culmination to be

I to II = 8·56 seconds;
II to III = 8·84 seconds.

METHOD OF OBSERVATION.

The collimation of the zenith telescope was frequently examined by pointings on the middle thread of the transit, the zenith telescope being reversed in its wyes. No re-adjustment of the collimation was required up to November 18, 1891. On this date the micrometer box had to be removed and was replaced the same day, and no re-adjustment has been required since.

The azimuth of the zenith telescope when pointing north was frequently examined by pointings upon the micrometer thread of the transit set in the meridian, and when pointing south by transits of

standard stars, the correction to the chronometer being accurately known. The stops required no re-adjustment throughout the whole series except when intentionally removed for observing for value of micrometer. The error of collimation and the deviation of the line of collimation from the meridian have not exceeded one second of time throughout the series.

It was intended to use only that part of the micrometer screw between 20 and 40 revolutions, 30 revolutions being the middle of the field. All pointings between 20 and 40 revolutions were to be made with thread 2, all below 20 revolutions with thread 1, and all above 40 revolutions with thread 3, the extreme limits being 10 to 50 revolutions, so the screw would have been moved only over those revolutions between 20 and 40. This scheme was carried out till November 18, when thread 3 was found to be slack. It was on this date that the micrometer box was removed with the intention of putting in a new thread. On examination at the office a new thread was thought unnecessary, and the micrometer box was replaced as it was. Observations the same night proved thread 3 to be useless and it was abandoned, thread 2 being used in its place.

In the beginning, the method of observation and record usually adopted in the Survey was followed—that is, a single pointing was made on the star when on the meridian, and a single reading of the levels was made after pointing on the star. Beginning on July 27, 1891, the method used at Berlin 1889–'90 was followed as nearly as practicable—that is, the levels were read, three pointings were made on the star near the three vertical threads, and the levels were again read.

Specimen of record.

[Rockville, Md., June 13, 1891. Observer, Edwin Smith.]

No. of pair.	Star No. U.S.C. and G.S. Catalogue.	N. or S.	Micrometer.		Level.		Microm. thread.	Time of culm. by chro. Hutton 202.	Remarks.
			Revs.	Divs.	North.	South.			
								h. m. s.	
1	1148	S.	40	49·0	*39·3	11·7	3	14 05 55	Thermometers :
	1188	N.	19	36·6	11·6	40·3	1	10 06	6287 outside 20°·0 C.
					†98·2	70·0			6288 north 21°·8.
					71·1	99·3			6289 south 22°·0.
2	1198	N.	26	61·8	11·1	40·0	2	15 48	
	1206	S.	33	87·0	39·8	10·7	2	28 11	
					70·9	99·1			
					98·9	70·2			
3	1215	S.	12	57·5	40·8	11·6	1	36 05	
	1220	N.	48	11·7	12·2	41·5	3	39 49	
					99·8	71·1			
					72·1	100·8			

* Upper level. † Lower level.

On the variation of latitude at Rockville, Md.

Specimen of record.

[Rockville, Md., July 27, 1891. Observer, Edwin Smith.]

No. of pair	Star No. U. S. C. and G. S. Catalogue.	N. or S.	Micrometer. Revs.	Micrometer. Divs.	Level. Left.	Level. Right.	Microm. thread.	Time of culm. by chro. Hutton 220.	Remarks.
								h. m. s.	
				86·5					
9	1336	N.	34	86·5	*11·6	42·6			Thermometers:
				84·8	†11·6	42·6	2	15 59 54	6287 outside 16°.6 C.
				0·0	*10·1	41·4			6288 north 19°.1.
	1354	S.	26	1.0	†10·1	41·6	2	16 17 11	6289 south 19°.5.
				0·0					
					*71·5	101·8			Barometers:
					†71·5	101·8			29·645 in. At. Th. 66° F.
					*70·0	100·7			
					†70·0	100·7			
				28·5					
10	1360	N.	32	27·5					
				29·5	12·5	44·0	2	20 46	
				54·0	11·1	42·6			
	1367	S.	27	54·8	11·2	42·8	2	25 29	
				54·1					
					72·7	103·4			
					71·0	101·8			
					71·1	101·9			
				62·5					
11	1382	N	31	61·0	10·2	41·9			
				61·0	10·2	41·9	2	33 44	
				5·0	9·3	40·9			
	1386	S.	28	5·8	9·2	40.9	2	36 33	
				6·2					
					70·2	101·1			
					70·2	101.2			
					69·1	100·0			
					69·1	100.0			

* Before bisecting star. † After bisecting star.

Thermometer No. 6288 was hung in the north end and No. 6289 in the south end of the observatory, and No. 6287 in the open air on the east side of the observatory. The barometer was hung in the observatory near the transit. The temperature and pressure of atmosphere, as indicated by these instruments, were noted at the beginning, the middle, and the end of each night's work.

Record of temperature and pressure of atmosphere at Rockville, Md., during observations for latitude.

Date.	Time.	Thermometers.			Barom. reduced to 0° C.
		Outside.	Inside N.	Inside S.	
1891.	*h.* *m.*	*C.*	*C.*	*C.*	*mm.*
June 13	14 00	+20·2	+21.8	+22·0	
13	17 32	18·0	18·8	19·0	
15	14 00	25·5	25·0	26·5	
15	17 32	23·1	24·1	24·0	
23	14 58	21·0	22·5	23·5	
23	18 20	19·5	21·5	21·5	
24	14 58	19·7	23·0	22·6	
24	18 00	19·6	20·5	21·0	
25	15 56	20·5	23·5	23·2	
25	18 00	19·0	21·5	21·3	
27	14 58	17·8	19·2	19·5	
27	18 20	15·5	16·4	16·5	
July 5	14 58	16·8	19·0	19·0	
5	18 20	13·7	16·0	16·0	
6	14 58	19·5	23·1	23·0	
6	18 15	17·5	19·5	19·5	
9	15 55	16·7	18·4	18·5	
10	15 55	17·5	20·0	20·0	
10	18 25	16·4	17·5	17·6	
10	19 56	15·6	17·0	17·0	
12	15 55	19·6	22·3	22·3	
12	19 56	16·8	18·6	18·6	
21	15 55	21·3	23·8	24.0	
21	19 55	19·5	21·0	21·0	
22	15 55	21·5	22·8	23·0	
22	19 20	20·5	21·0	21·3	
27	15 55	16·6	19·1	19·5	750.0
27	18 00	16·6	17·7	18·0	750·5
27	19 55	16·5	17·5	17·5	750·6
29	15 55	21·0	23·3	23·4	746·2
29	18 00	19·5	21·0	21·2	746·4
29	19 55	18·5	19·8	20·0	746·3
Aug. 7	17 00	23 0	24·8	24·8	750·8
7	20 35	20·0	21·6	21·8	751·2
9	17 40	25·5	26·6	26·6	749·2
9	20 49	22·5	24·5	24·5	749·0
10	17 00	26·2	27·8	27·6	750·0
10	20 49	24·5	25·5	25·5	750·2
11	17 00	24·5	26·5	26·5	748·6
14	17 28	21·2	22·3	22·5	749·5
15	17 28	20·0	22·5	22·5	749·5
15	21 05	16·1	18·5	18·5	750·7
15	22 33	15·5	17·6	18·0	750·9
17	18 37	20·0	21·0	21·0	750·3
22	18 00	23·0	24·5	24·7	745·7
22	20 12	22.4	23·1	23·2	745·7
Sept. 1	18 37	18·3	19·6	20·0	752.3
1	21 05	17·5	18·5	19·0	752·5
1	22 33	16·0	17·5	17·5	752·8
3	18 37	20·1	21·3	21·5	750·8
3	21 05	18·4	19·4	19·6	750·8
3	22 33	17·8	18·5	19·0	750·9
7	19 00	16·6	18·0	18·0	749·5
7	21 05	14·5	16·6	17·0	749·9
9	18 37	13·5	15·5	15·9	757·3
9	20 35	11·5	13·1	13·5	757·9

Record of temperature and pressure of atmosphere at Rockville, Md., during observations for latitude.

Date.	Time.	Thermometers.			Barom. reduced to 0° C.
		Outside.	Inside N.	Inside S.	
1891.	*h.* *m.*	*C.*	*C.*	*C.*	*mm.*
Sept. 9	22 19	+9·5	+12·8	+12·6	758·0
10	19 00	15·2	16·5	17·0	758·4
10	20 35	13·7	15·2	15·5	758·6
10	23 19	12·0	14·0	14·0	758·4
12	18 37	18·6	19·9	20·2	752·1
12	20 33	16·4	18·0	18·1	752·0
12	23 19	14·4	16·0	16·3	751·5
13	18 37	19·5	21·0	21·4	749·0
14	18 37	15·1	17·0	17·0	750·1
14	20 48	12·8	15·2	15·5	750·5
14	23 27	10·4	13·0	13·0	751·1
15	18 37	19·0	20·6	20·9	749·6
15	21 05	18·0	19·4	19·5	750·7
15	22 33	17·2	18·4	18·5	750·8
17	18 37	22·5	24·0	24.0	756·4
17	20 48	20·2	21·5	21·6	756·6
17	23 36	19·0	20·0	20·1	756·7
18	20 35	20·5	23·5	23·5	752·3
18	22 00	20·0	22·5	22·5	752·0
18	23 36	19·0	21·4	21·5	752·1
19	20 12	19·5	21·6	21·8	752·2
19	22 00	18·5	20·4	20·5	752·7
19	23 20	17·0	19·5	19·5	753·1
21	20 12	20·1	22·0	22·1	751·8
21	22 00	19.1	20·6	20·6	751·9
21	23 20	17·8	20·0	20·0	751·8
23	20 12	22·4	23·8	24·0	751·3
23	22 00	21·0	23·0	23·0	751·5
23	23 20	19·8	21·6	21·7	751·8
24	20 35	19·1	21·0	21·3	754·7
24	22 00	19·0	20·0	20·1	755·1
24	23 20	18·0	19·0	19·5	755·1
25	20 12	20·6	21·6	22·0	752·8
25	22 19	19·5	20·5	20·6	753·0
25	23 20	18·6	20·0	20·3	753·0
26	20 12	22·0	23·5	23·7	752·6
26	22 20	20·6	22·0	22·2	753·0
26	33 20	20·0	21·4	21·5	753·1
28	20 12	19·6	20·5	20·8	753·9
28	22 20	19·3	20·1	20·3	753·8
28	23 20	19·4	20·0	20·0	753·2
30	20 13	11·9	14·1	14·5	757·6
30	22 20	11·3	12·5	13·0	758·1
30	23 20	10·7	12·0	12·3	757·9
Oct. 3	20 13	20·5	22·5	22·6	749·8
3	22 02	18·1	20·5	20·6	749·6
3	23 20	16·5	19·5	19·5	749·0
5	20 13	13·8	15·3	15·5	748·3
5	22 20	13·5	14·6	14·7	749·3
5	23 20	11·9	13·2	13·5	749·7
9	20 13	9·5	10·6	11·0	751·4
9	22 10	8·4	9·5	9·8	751·4
9	23 20	8·4	9·0	9·2	751·4
14	20 13	11·1	12·3	12·5	757·5
14	22 20	9·6	11·0	11·0	747·8
14	23 20	9·6	10·5	10·7	747·7

Record of temperature and pressure of atmosphere at Rockville, Md., during observations for latitude.

Date.	Time.	Thermometers.			Barom. reduced to 0° C.
		Outside.	Inside N.	Inside S.	
1891.	*h.* *m.*	*C.*	*C.*	*C.*	*mm.*
Oct. 14	00 05	+10·6	+10·5	+10·5	747·7
15	20 13	9·0	11·0	11·0	749·2
15	22 10	8·0	9·5	9·5	750·0
15	23 20	6·6	8·5	8·6	750·8
16	20 13	8·1	9·4	9·6	755·6
16	22 20	6·5	7·6	8·0	756·8
16	23 20	6·1	7·1	7·5	756·9
17	20 13	7·5	9·0	9·5	756·4
17	22 10	6·3	7·8	8·0	756·5
17	23 20	5·5	7·0	7·0	756·6
22	22 02	+ 4·6	+ 5·4	+ 5·5	747·0
22	23 20	5·0	5·2	5·4	747·0
23	22 02	5·1	7·0	7·0	748·3
23	23 50	6·0	6·6	6·8	748·4
23	1 30	5·0	5·7	5·8	748·4
24	22 02	7·2	9·0	9·0	746·9
24	23 50	6·5	8·4	8·6	747·7
24	1 40	7·2	8·9	9·0	747·8
28	22 02	4·0	4·5	5·1	757·7
28	23 50	1·1	2·5	3·0	758·4
28	1 30	— 1·1	1·1	1·0	758·9
29	22 02	+ 8·0	8·5	8·3	759·7
29	23 50	6·6	7·4	7·3	759·7
29	1 30	6·0	6·5	6·5	759·6
30	22 02	11·6	12·7	12·5	756·4
30	23 50	11·2	12·0	11·9	756·1
30	1 40	10·1	10·8	10·7	755·6
31	22 02	15·7	16·5	16·2	747·6
Nov. 1	22 02	7·1	7·9	8·0	753·2
1	23 50	5·1	6·1	6·3	754·6
1	1 40	3·3	4·7	5·0	755·3
2	22 02	2·5	3·5	4·0	757·6
2	23 50	— 0·6	1·6	1·6	758·3
2	1 40	0·5	1·1	1·2	758·8
6	22 02	+ 1·0	2·6	2·7	752·5
6	23 50	— 0·2	1·6	1·8	752·4
6	1 40	+ 0·3	1·2	1·5	752·1
7	22 02	4·5	6·1	6·4	750·2
7	23 50	3·1	5·7	5·6	750·1
7	1 40	1·7	4·1	4·3	749·9
9	22 02	9·4	10·1	10·3	752·6
9	23 50	7·5	8·3	8·5	753·1
9	1 30	5·9	6·9	7·0	752·8
17	23 36	— 2·2	— 1·5	— 1·5	752·5
17	1 10	3·1	2·5	2·4	754·3
18	23 36	5·1	4·1	3·9	766·1
18	1 30	6·5	5·3	5·0	766·6
18	3 20	8·1	6·4	6·2	766·9
20	23 36	+ 3·2	+ 4·2	+ 4·5	773·5
20	1 30	3·0	3·9	4·0	768·5
20	3 20	1·9	2·8	3·0	767·3
23	23 36	6·3	7·2	7·2	735·1
23	1 30	5·0	6·0	6·0	736·5
23	3 20	3·6	5·0	5·0	737·2
24	23 36	3·9	4·9	5·0	746·0
24	1 30	2·4	3·6	3·5	747·0

Record of temperature and pressure of atmosphere at Rockville, Md., during observations for latitude.

Date.	Time.	Thermometers.			Barom. reduced to 0° C.
		Outside.	Inside N.	Inside S.	
1891.	*h.* *m.*	*C.*	*C.*	*C.*	*mm.*
Nov. 24	3 20	+1·9	+3·0	+3·4	747·8
29	23 36	— 6.5	— 6·2	— 5·9	756·8
29	1 40	7·5	7·1	6·7	757·1
29	3 20	8·0	7·5	7·4	757·0
30	23 36	4·1	3·0	2·6	758·5
Dec. 1	23 36	+ 0·5	+ 1·2	+ 1·1	757·7
1	1 40	— 1·4	— 0·4	— 0·4	757·6
1	3 20	1·5	0·9	0·6	757·2
4	23 36	+ 6·4	+ 6·9	+ 6·7	746·1
4	1 30	6·3	6·5	6·6	747·8
5	23 36	8·0	8·5	8·2	752·0
5	1 30	6·5	7·3	7·1	751·8
5	3 20	6·0	6·8	6·5	751·8
8	1 21	— 0·1	0·5	0·4	758·9
8	3 25	0·7	0·1	0·0	758·7
8	4 40	1·0	— 0·1	— 0·2	758·4
9	1 21	+ 4·6	+ 6·0	+ 6·0	756·5
9	3 25	3·0	5·1	5·0	756·4
9	4 40	1·8	3·9	4·0	756·4
10	1 21	3·5	6·5	6·5	755·2
10	3 25	3·2	6·5	6·5	755·1
10	4 40	2·4	4·8	5·1	755·1
13	1 21	4·9	6·5	6·2	755·3
13	3 25	2·2	5·0	5·0	755·1
13	4 40	2·5	4·3	4·5	754·6
18	1 21	— 6·9	— 5·5	— 5·3	762·8
18	3 25	5·9	5·5	4·9	762·2
18	4 40	8·0	6·4	6·0	759·5
28	1 21	0·5	+ 0·2	+ 0·1	756·2
28	3 25	1·5	— 0·8	— 0·6	755·9
28	4 40	0·9	0·5	0·5	755·8
30	1 21	+ 1·1	+ 3·0	+ 2·7	757·8
30	3 25	0·4	1·7	1·8	758·8
30	4 40	— 0·2	1·1	1·4	759·2
1892.					
Jan. 7	2 58	7·2	— 6·2	— 6·0	747·8
7	4 40	9·2	7·8	7·4	748·3
7	5 45	11·5	9·0	8·9	748·6
8	2 58	3·2	2·5	2·7	750·7
8	4 40	4·5	3·1	3·0	751·1
16	2 58	8·8	7·4	7·4	762·7
16	4 40	10·5	8·7	8·5	763·0
16	5 45	9·0	8·5	8·3	762·9
20	3 05	10·9	10·2	9·9	756·5
20	4 38	11·7	11·0	10·7	756·3
20	5 45	13·4	12·2	12·0	756·5
21	3 05	2·0	1·0	1·0	756·8
21	4 38	4·5	2·6	2·5	757·1
21	5 45	4·9	3·4	3·4	757·6
24	3 05	+ 0·5	+ 1·8	+ 2·0	743·9
24	4 38	— 0·4	1·0	0·8	743·4
24	5 45	1·0	0·0	0·0	743·3
25	3 05	+ 5·6	7·5	7·8	736·6
25	4 38	5·0	6·0	6·5	736·8
25	5 45	4·0	5·4	5·5	737·0
28	3 05	1·0	3·5	3·5	747·3

Record of temperature and pressure of atmosphere at Rockville., Md. during observations for latitude.

Date.	Time.	Thermometers.			Barom. reduced to 0° C.
		Outside.	Inside N.	Inside S.	
1892.	*h.* *m.*	*C.*	*C.*	*C.*	*mm.*
Jan. 28	4 00	+ 4·7	+ 4·8	+ 4·5	747·3
31	4 17	— 1·8	— 0·5	— 0·5	752·9
31	6 07	1·7	0·8	0·5	753·2
31	8 20	2·0	1·0	1·0	753·1
Feb. 10	4 17	0·6	0·0	0·0	743·5
10	5 17	0·5	0·0	0·0	742·7
13	4 17	3·6	2·7	2·5	746·6
13	6 05	1·6	0·4	0·2	758·0
13	8 17	3·9	1·8	1·6	758·4
17	4 17	5·8	3·8	3·5	763·4
17	7 05	6·7	5·8	5·7	762·7
17	8 17	7·2	5·7	5·7	762·1
Mar. 10	7 05	3·0	2·5	2·0	742·6
10	8 12	3·3	3·0	3·3	742·2
12	7 05	+ 8·3	+ 9·0	+ 9·5	745·4
12	8 35	7·0	8·0	8·3	746·4
14	7 05	— 3·5	— 3·0	— 2·8	755·6
14	8 45	4·5	4·0	3·7	756·5
14	10 28	6·0	5·0	4·9	756·8
16	7 05	7·5	3·8	3·6	756·0
16	8 46	7·8	5·2	5·9	756·2
16	9 25	7·4	5·5	5·4	757.6
19	7 05	0·5	+ 1·0	+ 1·0	745·7
19	8 40	1·3	0·0	0·2	746·8
19	10 25	2·6	— 0·8	— 0·8	746·9
20	7 05	1·4	0·4	0·3	755·4
20	8 40	3·5	2·0	2·0	756·4
20	10 30	3·3	2·5	2·2	756·9
21	7 05	4·8	1·7	1·5	760·7
21	8 53	5·1	3·6	3·5	761·1
21	10 30	6·0	4·5	4·5	760·9
23	7 05	+ 9·0	+10·0	+10·5	747·9
23	8 25	7·2	8·0	8·6	746·9
24	7 05	4·0	5·6	6·0	752·1
24	8 40	3·1	4·5	4·8	752·3
24	9 30	3·0	4·0	4·0	752·1
28	8 25	3·7	4·7	5·0	751·8
28	10 20	3·1	4·0	4·0	752·6
28	12 00	2·0	3·1	3·5	753·3
29	8 25	6·7	7·2	7·6	755·1
29	10 12	5·7	6·1	6·6	755·3
Apr. 4	8 25	20·7	21·6	21·5	748·1
4	10 20	19·0	19·6	19·6	748·1
4	12 12	17·0	18·0	17·8	747·8
9	8 25	0·0	0·6	0·6	745·2
9	10 20	— 0·9	— 0·3	— 0·2	746·3
11	8 25	+ 3·0	+ 4·3	+ 4·5	752·3
11	10 20	1·5	2·5	2·6	752·7
11	12 00	0·0	1·1	1·5	753·0
12	8 25	5·0	6·0	6·1	753·5
12	10 20	2·5	4·0	4·0	754·5
12	12 00	1·6	3·0	3·0	754·6
13	8 25	8·8	11·0	11·0	752·8
13	10 20	7·8	9·9	10·2	752·4
13	10 47	7·5	9·0	9·0	752·3
16	10 17	5·1	6·5	6·9	748·8

Record of temperature and pressure of atmosphere at Rockville, Md., during observations for latitude.

Date.	Time.	Thermometers.			Barom. reduced to 0° C.
		Outside.	Inside N.	Inside S.	
1892.	*h.* *m.*	*C.*	*C.*	*C.*	*mm.*
April 16	12 22	+ 4·4	+ 5·1	+ 5·6	748·8
19	10 17	6·8	8·3	8·5	757·3
19	11 00	7·2	8·1	8·3	757·7
23	10 17	10·6	12·0	11·7	749·9
23	12 22	8·9	10·2	10·1	750·7
23	14 00	7·4	8·6	8·6	751·3
26	10 17	10·0	11·3	11·4	758·7
26	12 17	7·1	9·0	9·5	759·2
26	14 00	5·4	7·8	8·1	759·4
27	10 17	10·6	11·8	12·0	757·5
27	12 22	10·5	10·9	11·0	757·0
29	10 17	9·5	10·9	11·0	752·3
30	10 17	13·9	15·1	15·0	756·9
30	12 22	10·7	11·8	12·0	757·3
30	14 00	10·1	10·9	10·6	757·1
May 1	10 17	20·5	21·0	21·0	748·6
3	12 22	21·6	22·5	22·4	747·5
3	13 53	21·0	21·8	21·6	747·0
4	10 40	23·2	25·4	25·2	751·2
4	12 22	21·0	23·1	23·0	752·1
4	13 40	21·1	22·5	22·4	752·2
5	10 17	17·7	17·8	18·0	751·3
5	12 22	15·1	15·6	15·9	750·9
5	14 12	13·8	14·3	14·4	750·1
7	10 17	13·5	14·5	14·6	748·3
7	12 22	11·0	11·9	12·0	749·8
7	14 12	9·1	10·0	10·3	750·4
8	10 17	12·1	13·9	13·6	754·6
8	14 00	9·9	10·3	10·7	755·9
9	10 17	15·5	18·9	18·5	753·5
9	12 22	14·8	16·4	16·5	753·9
9	14 12	12·5	14·0	14·0	753·8
16	12 10	17·5	18·5	19·0	752·4
16	14 00	16·4	17·2	17·5	753·3
16	15 50	13·6	15·1	15·5	753·9
17	12 10	17·1	18·0	18·1	752·5
17	13 55	15·0	15·9	16·0	752·3
17	15 25	14·1	15·0	15·0	752·0
19	12 10	13·1	14·6	14·8	740·2
23	12 10	8·1	10·0	10·0	742·5
23	13 55	6·6	8·5	8·7	743·5
23	15 48	5·8	7·2	7·4	744·2
24	12 10	13·6	15·9	16·0	748·3
24	15 58	12·1	13·0	13·0	748·6
25	12 10	20·0	20·9	20·9	744·8
25	15 58	17·2	18·0	18·0	744·8
28	12 10	14·4	15·6	16·0	748·5
28	14 30	12·8	13·5	13·5	749·0
28	15 58	11·9	12·5	12·5	749·1
30	12 10	21·6	23·2	23·0	750·8
30	14 00	20·0	21·0	21·1	751·4
30	15 48	18·6	19·5	19·7	751·3
June 1	12 10	24·0	24·8	24·7	752·5
1	14 00	22·0	22·6	22·7	752·9
1	15 48	20·5	21·5	21·5	752·6
2	14 00	23·2	23·7	23·5	749·3

Record of temperature and pressure of atmosphere at Rockville, Md., during observations for latitude.

Date.	Time.	Thermometers.			Barom. reduced to 0° C.
		Outside.	Inside N.	Inside S.	
1892.	*h.* *m.*	*C.*	*C.*	*C.*	*mm.*
June 2	15 48	+21·5	+22·2	+22·3	749·3
2	17 30	21·0	21·8	21·9	749·4
6	14 00	21·0	23·0	23·0	749·6
6	15 48	19·8	21·5	21·5	749·5
6	17 30	18·8	20·6	20·8	749·2
11	14 00	17·3	19·1	19·4	750·2
11	15 48	15·4	17·1	17·3	750·5
11	18 08	14·0	16·0	16·0	750·5
12	14 00	21·2	23·1	23·0	750·4
12	16 02	20·5	22·0	22·0	750·8
12	18 18	19·5	20·8	20·9	750·3
13	14 00	25·1	26·2	26·1	750·2
13	16 02	22·8	23·9	23·9	750·1
13	17 30	21·6	22·5	22·5	749·7
15	14 00	23·0	24·4	24·5	750·0
15	16 02	21·1	22·4	22·5	750·8
15	17 12	20·3	21·5	20·7	750·9
16	14 00	24·6	25·2	25·2	751·3
17	14 00	21·8	23·0	23·0	751·4
17	16 20	20·3	21·6	21·8	751·8
20	14 00	23·5	24·5	24·7	744·7
20	16 02	22·3	23·5	23·6	745·0
20	18 18	21·4	22·5	22·5	744·9
21	14 00	25·5	27·1	27·0	744·1
23	14 00	25·6	27·8	27·5	744·2
23	16 20	24·2	25·5	25·5	743·9
23	18 18	24·0	25·0	25·0	744·9
25	14 00	21·1	23·5	23·5	748·2
25	17 02	17·6	20·0	20·1	749·7
25	17 30	17·8	19·1	19·4	749·8
26	14 00	22·1	23·5	23·6	747·9
26	15 24	21·0	22·0	22·2	748·1
29	14 30	23·1	23·9	23·9	747·8
29	16 02	22·0	22·6	22·6	748·3
29	18 17	21·2	22·0	22·1	748·5
July 7	14 58	18·0	20·6	20·6	759·4
7	16 48	16·6	18·3	18·5	759·8
7	18 00	16·1	17·0	17·2	759·9
8	14 58	20·1	22·1	22·1	755·1
8	16 48	18·1	19·8	19·9	754·9
8	18 00	17·0	18·6	18·7	744·8
9	14 58	21·1	22·6	22·7	752·7
9	16 48	19·7	20·7	20·6	753·2
9	18 00	18·6	20·0	20·0	753·1

No lights were used in the observing room other than the small electric lamps for illuminating the field of the telescope, and the small electric hand lamps used for reading the micrometer, levels, setting, etc. These lamps were arranged with spring contacts, by pressing which the lamps were illuminated only for the time actually necessary. A dim illumination of the observing room was obtained from a lamp in the recording room, showing through a small window in a partition

separating the two rooms. Every precaution was taken to avoid any sudden change of temperature in the zenith telescope.

The first observations for latitude were made June 13, 1891, and the last July 9, 1892. In the whole series 1808 observations were made on 146 nights. On many of these nights the observations were very much broken on account of clouds, there being only 89 nights upon which complete groups were observed. No serious difficulties were encountered during these observations up to April 30, on which date the level readings were very wild. On May 4 it was discovered that the observatory had settled to such an extent that the casing about the pier rested against it. This difficulty was immediately remedied. The twelve observations of April 30, the four of May 1, and the first three of May 4 have to be rejected.

On June 13, 1892, it was necessary to strengthen the springs of the micrometer, to do which the slide carrying the spider webs had to be removed. The computation of observations subsequent to that date developed a discrepancy in the results from pairs in which one star was observed with thread 1 and the other star with thread 2. An investigation of the distance between threads 1 and 2 on August 24 and 26 developed the fact that the relation of these threads must have been disturbed on June 13. The distances between these threads before and after June 13, 1892, have already been given.

II.—REDUCTION OF THE OBSERVATIONS AND DISCUSSION OF THE RESULTS BY CHARLES A. SCHOTT, ASSISTANT U. S. COAST AND GEODETIC SURVEY.

COMPUTING DIVISION, COAST AND GEODETIC SURVEY,
September 28, 1892.

The results of the astronomical observations, specially made for the last three years, at several places in the northern hemisphere, for the purpose of inquiring into the supposed variation of the geographical latitude, and for investigating its laws of change, leave no further doubt of the fact that the latitude of a place is slightly variable. The phenomenon, as deduced from observations in Europe, in North America, and the Hawaiian Islands, indicates a periodic variation such as would be produced by the revolution of the axis of rotation about that of the earth's figure.

The length of the period, which was found to disagree with the Eulerian period of about 306 days, was first worked out by Dr. Chandler,* who made it variable, and at present increasing, and assigned to it 427 days, nearly, for the epoch 1875; on the other hand Prof. Newcomb's researches† point to a uniform rotation with a constant length of 430 days, nearly, as deduced from observations between 1865 and

* Gould's Astronomical Journal No. 267.

† Astronomische Nachrichten No. 3097.

1891. Further, according to the former investigator, the half range of the variation is 0″·22, whereas the latter suspects the radius of the circle described by the instantaneous pole of rotation to be on the increase, the Berlin observations for 1890 indicating 0″·30.

The direction of motion is necessarily from west to east, and the lengthening out of the period from 306 to 430 days is ascribed to the earth's elastic yielding, the Eulerian period demanding an absolutely rigid body.

The assistance afforded by the Survey for the elucidation of the phenomenon consists in the occupation of three stations at which latitude observations were to be made by means of the zenith telescope (Talcott's method) on every favorable or desirable night, and continued over a space of at least one year. These stations are Rockville, Md. (near Washington, D. C.), San Francisco, Cal., and Waikiki (near Honolulu), Hawaiian Islands. The observations at Rockville, which are here discussed, cover the period between June, 1891, and July, 1892.

The position of the latitude station at Rockville was connected, in the summer of 1892, by Mr. J. B. Boutelle, by triangulation with the transcontinental arc of the parallel, and its geodetic coördinates are as follows: Latitude 39° 05′ 11″·21; longitude 77° 09′ 36″·65 W. from Greenwich; hence it is about 15½ statute miles or about 25 kilometres N. 29° W. (true) from Washington, D. C. We have longitude of Rockville $5^h\ 08^m\ 38^s$ W., and longitude of Berlin $0^h\ 53^m\ 35^s$ E.; hence Rockville west of Berlin $6^h\ 02^m\ 13^s$. The astronomical latitude is 39° 05′ 10″·47, nearly, indicating a local deflection in the meridian of $A. - G. = -0''.74$.

The Rockville station was placed in charge of Edwin Smith, Assistant Coast and Geodetic Survey, by whom all observations were made. Between June 13, 1891, and July 9, 1892, nearly 1800 individual measures for latitude were secured on 146 nights. Early in January, 1892, these records were placed in my hands for reduction and report, and the temporary services of two computers, Mr. A. Bonnot and Mr. L. W. Reid, were specially provided for the computation of the apparent places of the 88 pairs of stars involved. Their mean places are due to Mr. H. Farquhar of the Computing Division.

The mean north polar distances of the stars depend on the places assigned to them in the principal and other available catalogues, corrected for their known systematic errors, following Boss's system, and are specially discussed with respect to the value of the catalogue itself as well as to the number of observations involved. The mean places are thus closely connected with Boss's system; the proper motions were independently discussed by the method of least squares after each star's N. P. D., found in the several catalogues, had been referred to the epoch 1891·0; the proper motions depend on special investigation, except in the case of Bradley-Auwers stars, of which the given

Results of observations for the variation of latitude at Rockville, Md.

proper motion was adopted unless found capable of improvement. Respecting weights Mr. Farquhar makes the following statement:*

"Combination weights used for adopted polar distances in Rockville latitude list: Twenty catalogues used in computing latitudes on this Survey were tested and their probable errors reported in June, 1890. These results were made the basis of the following series of weights; the unit of weight corresponding to a probable error of $\pm \sqrt{0.1}''$ and the ratio of observation error to systematic error being taken (for all catalogues alike) $= \sqrt{5}$, so that if w_∞ be the weight of an infinite number of observations and w_1 of one, $w = \frac{n}{n+5} w_\infty = \frac{6n}{n+5} w_1$.

"The weight of no polar distance therefore can exceed six times that of a single observation, given in the table below:

CATALOGUE.	w_1
Lalande δ, Weisse-Bessel ε,	0·015
d'Agelet α, Piazzi β,	·02
Rümker ε,	·025
Taylor β,	·04
Groombridge β,	·06
Armagh '75,	·07
Armagh '40, Jacob β, Smyth β,	·08
Auwers-Bradley α, Paris '45, Main β, Glasgow, Cape '40 ε,	·09
Radcliffe '60,	·10
Radcliffe '45,	·12
Pond β, Cambridge '30 β, Greenwich 6 y.	·14
Washington,	·16
Cape '50 ε, Bonn β, Paris '60, Rome,	·18
Paris '75,	·20
Henderson, Greenwich 7y1, Melb. β, Cape '80 ε, Ann Arb. ε,	·25
Struve Pos. M. β Greenwich 12y (1 + 2), Brussels, Becker ε,	·3
Pulkowa Merid. Circle, Greenwich 7y2, Cordoba γ,	·35
Abo β, Harv. '85 ε,	·4
Greenwich 9y., Harv. '75,	·5
Leiden β,	·6
Romberg α,	·7
Pulkowa Vert. Circle β, Greenwich 10y,	·8

"In this table weights were deduced for catalogues marked:

"α..... from probable errors of observation given in the prefaces to the catalogues.

"β..... from Boss's investigations, $\frac{2}{3}$ of his weights being taken—*i. e.*, his unit being supposed to correspond to a probable error of $\pm \sqrt{0.15}''$.

"γ..... from a determination of systematic error by myself, using Boss's method.

"δ..... from determination of observation error by myself and formula above as in (α).

"ε..... from simple estimate, the places being too few for better methods.

* Under date of September 12, 1892.

"Others from the results obtained in 1890.

"Piazzi, Taylor, Jacob, Main, and a few others used by Safford, I have to take at second hand, being without the originals. For stars of the Berliner Jahrbuch, Dr. Auwers's combination of the authorities used by him was usually accepted, $\frac{5}{6}$ of his total weights being allowed them (*i. e.*, his probable error taken $= \pm \sqrt{0.16}''$ for $w = 1$) and the weights of this table used for the remaining authorities."

The resulting mean N. P. D.'s of the stars composing the 88 pairs observed for latitude of Rockville are given below. The first column gives the star's number in the Coast Survey Field Catalogue (Appendix No. 7, Report for 1876), the second column its number in the British Association Catalogue (London, 1845); column 4 contains the square of the probable error (e^2), and the last column the proper motion (μ'):

No. in—		Mean N. P. D. for 1891·0.				e^2	μ'
C. and G. S. Catalogue.	B. A. Catalogue.	°	′	″			″
1184	4706	64	23	30·66		·02	+0·076
1188	4726	37	42	0·85		·02	+0·016
1198	4758	50	42	18·25		·07	+0·030
1206	4812	51	12	53·21		·01	—0·149
1215	4847	73	6	50·99		·01	—0·005
1220	4874	28	16	24·45		·02	+0·036
1231	4906	52	16	50·06		·04	—0·090
1241	4958	49	10	45·54		·01	+0·043
1247	4980	41	25	39·94		·03	—0·023
1254	5031	60	25	51·42		·04	—0·019
1262	5071	37	38	55·92		·08	+0·004
1275	------	64	31	7·10		·06	+0·021
1278	5122	48	47	42·79		·01	+0·014
1280	5130	48	43	50·00		·02	+0·014
1294	5178	53	0	{32·81}{36·06}	35·08	·02	—0·007
1316	------	30	5	49·84		·05	+0·032
1330	5315	71	52	48·38		·04	—0·146
1336	5348	31	8	36·71		·02	—0·345
1354	5466	70	35	26·17		·02	—0·036
1360	5511	13	59	37·42		·02	—0·258
1367	5520	87	46	37·98		·01	+0·093
1381	5574	36	52	50·85		·03	—0·024
1382	5575	36	51	{22·88}{24·44}	23·30	·04	—0·024
1386	5597	64	55	50·59		·07	+0·003
1392	5628	25	12	15·39		·03	+0·012
1397	5647	76	32	52·10		·05	—0·001
1407	5740	24	41	55·28		·03	—0·047
1418	5765	77	6	32·80		·02	+0·018
1424	------	49	5	10·64		·05	—0·007
1432	5834	53	4	4·40		·02	+0·009

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No. in—		Mean N. P. D. for 1891·0.				e^2	μ'
C. and G. S. Catalogue.	B. A. Catalogue.	°	′	″			″
1442	5860	65	23	30·05		·03	+0·004
1448	------	36	28	32·50		·12	—0·02
1456	5918	31	15	24·79		·07	—0·021
1462	------	70	39	50·59		·06	+0·096
1473	5978	28	2	30·80		·03	+0·500
1476	5991	73	59	48·64		·05	—0·099
1502	6079	33	6	36·59		·01	—0·070
1514	6106	68	24	14·01 13·09	13·55	·04	—0·015
1539	6203	47	52	39·70		·03	+0·008
1547	6235	53	59	4·06		·05	—0·020
1589	6404	48	40	31·69		·04	+0·025
------	6456	53	9	51·90		·03	+0·010
1608	6466	53	14	22·67		·04	—0·006
1628	6520	43	13	10·60		·04	+0·092
1641	6571	58	53	53·65		·03	+0·009
1648	6582	68	57	29·31		·09	—0·005
1666	6640	32	33	39·71		·05	0·000
1676	6667	53	54	1·94		·03	—0·007
1690	6718	47	49	34·00		·04	+0·035
1718	6794	71	7	51·56		·04	—0·021
1741	6852	30	34	46·23		·04	—0·045
1756	6895	40	11	54·87		·02	+0·005
1779	6957	61	38	7·21		·04	+0·045
1801	7022	50	5	31·34		·01	+0·003
1809	7061	51	55	2·74		·04	+0·052
1819	----	38	3	40·80		·04	—0·067
1825	7126	63	55	2·22		·03	+0·024
1852	7211	23	44	20·15		·03	—0·023
1859	7223	77	51	43·69		·04	—0·086
1885	7301	42	54	16·44		·03	+0·014
1896	----	59	15	8·85		·08	—0·02
1917	----	57	51	0·80		·05	0·00
1926	7455	43	45	27·45		·12	—0·049
1948	7542	28	24	35·33		·02	+0·016
1959	7567	73	8	59·42		·04	+0·017
1968	7598	41	11	41·19		·01	+0·016
1971	7607	60	19	59·67		·04	+0·039
1974	7627	64	35	15·46		·02	+0·014
1984	7676	37	38	35·92		·05	+0·007
2004	7749	32	20	9·75		·03	+0·007
2020	7807	69	42	9·09		·03	+0·031
2026	----	50	44	43·80		·05	+0·018
----	7879	50	56	9·58		·10	+0·026
2043	7880	50	55	47·40		·08	+0·026
2047	7901	51	31	1·27		·02	+0·012

No. in—		Mean N. P. D. for 1891·0.				ϵ^2		μ'
C. and G. S. Catalogue.	B. A. Catalogue.	°	′	″				″
2055	7915	50	20	37·77		·04		+0·008
2059	7932	48	45	10·10		·03		—0·002
2063	----	53	9	23·88		·40		+0·053
2070	7972	47	16	1·36		·03		—0·009
2078	----	54	13	48·04		·06		—0·008
2093	8039	23	22	42·09		·02		—0·010
2121	8149	78	17	1·32		·03		+0·032
2131	----	61	56	5·34		·14		+0·020
2142	8231	40	7	54·98		·03		+0·012
2154	----	33	9	14·73		·14		+0·05
2159	8296	68	56	6·77		·08		+0·030
2178	8366	29	17	34·57		·07		—0·003
4	8	72	23	38·40		·04		+0·035
10	28	49	33	59·11		·04		+0·142
22	67	52	38	6·84		·02		+0·038
27	102	74	9	27·84		·07		+0·023
32	126	27	40	11·72		·01		+0·009
45	166	59	44	8·13		·01		+0·093
55	198	42	18	44·41		·01		+0·014
66	227	49	30	53·36		·03		+0·021
77	259	52	5	31·38		·01		—0·020
93	314	35	36	52·88		·02		+1·561
111	365	65	59	37·17		·05		+0·030
134	456	31	19	39·54		·03		+0·026
150	523	70	15	41·89		·03		+0·678
176	614	36	2	23·22		·03		+0·010
196	693	65	27	45·30		·09		+0·097
202	710	61	51	39·23		·03		+0·018
208	735	40	12	54·52		·02		+0·025
220	786	55	47	17·27		·03		+0·051
232	829	46	10	0·47		·04		+0·017
242	870	72	59	21·78		·04		+0·012
250	----	28	55	27·56		·05		—0·04
259	918	38	4	56·43		·03		+0·033
263	941	63	58	10·56		·04		+0·017
					[For 1892·0]			
					″			
275	974	61	20	23·65	9·70	·03		+0·038
287	1007	40	10	39·86	26·40	·04		+0·031
292	1025	61	20	50·62	37·36	·05		+0·039
297	1043	40	31	38·65	25·57	·02		+0·037
306	1062	31	29	59·40	46·59	·04	·05	—0·007
331	1135	70	38	57·43	45·67	·03		+0·019
370	1253	66	11	40·98	30·81	·05		+0·022
378	----	35	27	37·24	27·40	·14	·16	+0·13
392	1286	28	25	29·14	19·69	·03		+0·013
417	1350	73	28	39·99	31·35	·07		+0·036
430	----	59	52	52·47	44·20	·08		+0·04
456	----	41	54	42·94	35·58	·05		+0·043

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No. in— C. and G. S. Catalogue.	No. in— B. A. Catalogue.	Mean N. P. D. for 1891–1892.				e^2		μ
		°	′	″	″			″
463	1449	67	15	9·96	2·77	·02		+0·021
479	1494	34	55	18·22	11·88	·05		+0·011
483	1504	36	25	23·44	17·28	·03		—0·009
491	1528	65	7	6·91	1·07	·03		+0·061
512	1591	74	32		27·84		·05	+0·005
520	----	27	27		45·89		·12	-------
531	----	49	4		39·24		1·20	—0·015
533	1663	52	42		58·36		·04	+0·012
562	1751	24	21		43·20		·05	+0·039
577	1846	77	23		0·56		·09	+0·024
606	----	60	28		47·07		·12	0·000
611	1963	41	16		4·66		·07	+0·068
700	2381	48	55		32·27		·01	+0·001
705	2416	53	2		12·80		·04	+0·023
718	2464	58	0		4·50		·02	—0·180
729	2488	43	34		56·49		·07	+6·040
745	2558	71	13		36·87		·04	+0·056
757	----	30	39		36·86		·03	—0·012
769	2707	21	12		31·62		·03	—0·003
776	2778	80	28		55·31		·02	+0·055
781	2792	36	25		58·91		·09	+0·103
792	2850	65	33		18·70		·03	+0·071
796	2876	24	36		23·38		·05	—0·097
804	2942	76	55		56·04		·04	+0·004
813	2982	27	38		3·45		·01	—0·017
825	3047	74	15		48·68		·04	—0·020
835	3087	22	41		37·39		·03	+0·043
842	3111	78	53		50·88		·03	+0·005
849	3140	35	31		57·07		·04	—0·066
865	3246	66	33		21·79		·02	+0·040
873	3273	58	21		16·88		·20	+0·05
888	3341	43	28		34·31		·02	+0·098
911	3505	46	32		47·68		·01	+0·047
921	3542	55	32		48·82		·09	+0·020
926	3572	52	44		22·43		·01	+0·104
932	3607	49	1		8·42		·05	+0·016
936	3639	35	46		5·50		·10	+0·082
945	3671	66	14		47·00		·03	—0·014
952	----	36	51		39·76		·04	+0·034
			55		19·29			+0·064
958	3742	64	40		27·70		·02	+0·015
966	2767	33	2		19·71		·01	—0·030
976	3834	68	53		4·78		·01	+0·137
1003	3964	68	2		50·83		·04	+0·061
1010	3985	33	46		15·23		·04	+0·025
1022	4052	82	47		0·54		·03	+0·031
1035	4122	19	11		55·77		·04	+0·022

No. in—		Mean N. P. D. for 1892·0.			ϵ^2	μ'
C. and G. S. Catalogue.	B. A. Catalogue.	°	′	″		″
1044	4148	40	24	58·75	.07	—0·004
1057	4195	61	7	52·42	.01	+0·085
1063	4216	31	0	0·27	·02	—0·087
1070	{4241 / 4242}	71	1	{41·42 / 42·08}	·03	—0·019
1094	4335	33	27	14·38	·01	+0·016
1109	4387	68	16	1·65	·03	+0·056
1118	4433	49	16	30·69	·03	—0·008
1128	4479	52	24	7·21	·05	—0·001
1137	4526	65	5	31·95	·30	+0·19
1149	4564	36	31	57·68	·07	—0·059
1162	4607	40	8	51·39	·02	+0·018
1175	4656	61	58	41·65	·02	+0·057

For star [1819] C. and G. S. Catalogue, the catalogues contained no late determinations, but observations of it were found at six Coast Survey stations, which furnished the means of deducing for it a satisfactory place.

To secure accuracy in the apparent places two independent computations were made, the computers using the independent star (or day) numbers contained in the American Ephemeris and Nautical Almanac.

The instrumental constants are as follows:

Equatorial interval of vertical threads of diaphragm, I to II = $8^s{\cdot}56$ and of II to III = $8^s{\cdot}84$, thread II representing the line of collimation. The value of $I = i \sec. \delta$, where $i = 8^s{\cdot}7$, was tabulated for different values of δ to be used in connection with the usual table for the reduction to meridian.

Level values, recapitulation of results from the several sets of observations.

Level No. 7, the upper level, as attached to the telescope is graduated from 0 to 50 divisions. Two sets of observations on each date were made with a level trier, the bubble being made to traverse with forward and backward motion. The results are:

	″	°
January 11, 1892	1 division = 1·632 at	— 0·4 C.
12, 1892	1·640	+ 9·6
May 22, 1891	1·547	+ 23·9
January 13, 1892	1·589	+ 37·9
Mean	1·600	+ 17·8
	± 14	

N. B.—Two sets made May 19, 1892, at 38°·0 C. gave 1 division = 1″·632, but the tube was thought not sufficiently protected.

Results of observations for the variation of latitude at Rockville, Md.

Similarly we have for level No. 13, the lower level, graduated from 60 to 110 divisions:

		1 division =	″		°
January	11, 1892		1·393	at —	0·4 C.
	12, 13, 1892		1·374	+	9·6
May	18, 1891		1·342	+	19·2
January	13, 1892		1·348	+	37·9
Mean			1·361	+	16·6
			± 8		

N. B.—A series of observations made on the same day as above (May 19) was rejected for like reason.

In the latitude reduction the average value or 1″·482 ± 0″·010 was used irrespective of the temperature, changes of which apparently do not sensibly affect the value. As each level was read twice before and after reversal of telescope, any error of reading could readily be detected.

The investigation of the value of the micrometer screw.—A large number of observations were made to determine the value and irregularities of the screw. This was the more demanded since the sums of the micrometer corrections to the latitude do not exactly balance in the several groups. Observations were made about the time of elongation of the star, and for that part of the screw contained between 20 and 40 turns, also for $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ turns. Corrections were applied for deviations from the mean state of level during a series of observations, also for curvature*, but not for rate of chronometer, which was too small to have any effect. The correction for differential refraction was applied.

Date.	Star.	Elongation.	Whole turns and fractions.	Temperature.	Value of one turn.	Probable error.	Relative weight p.	No. of set.
				°	″	″		
1891, June 8	λ Urs. Min.	E.	$\frac{1}{4}$ to $\frac{1}{4}$	+16·5 C.	44·641	±0·008	2	1
9	λ Urs. Min.	E.	$\frac{1}{4}$ to $\frac{1}{4}$	+20·5	·660	8	2	2
10	λ Urs. Min.	E.	$\frac{1}{4}$ to $\frac{1}{4}$	+21·0	·616	4	2	3
Sept. 18	a Urs. Min.	E.	$\frac{1}{2}$ to $\frac{1}{2}$	+24·5	·637	9	1	4
19	a Urs. Min.	E.	$\frac{1}{2}$ to $\frac{1}{2}$	+21·7	·632	9	1	5
21	a Urs. Min.	E.	$\frac{1}{4}$ to $\frac{1}{4}$	+23·3	·613	5	2	6
23	a Urs. Min.	E.	$\frac{1}{2}$ to $\frac{1}{2}$	+25·0	·660	11	1	7
24	a Urs. Min.	E.	$\frac{1}{2}$ to $\frac{1}{2}$	+22·0	·653	8	1	8
1892, Jan. 27	a Urs. Min.	W.	$\frac{1}{2}$ to $\frac{1}{2}$	— 7·0	·740	10	0	
Feb. 22	a Urs. Min.	W.	$\frac{1}{2}$ to $\frac{1}{2}$	+ 2·9	·683	10	1	9
23	a Urs. Min.	W.	$\frac{1}{4}$ to $\frac{1}{4}$	+ 0·4	·666	6	2	10
Mar. 3	a Urs. Min.	W.	$\frac{1}{4}$ to $\frac{1}{4}$	+ 1·9	·653	4	2	11
9	a Urs. Min.	W.	$\frac{1}{4}$ to $\frac{1}{4}$	+ 7·6	44·634	±0·005	2	12

* A table was used for the various values of $\frac{1}{6}(15 \sin 1'')^2 \tau^3$

When these values for one turn at the several temperatures were plotted they indicated a dependence on t and t^2. They were treated

accordingly, and the expression for value of one turn or revolution of the screw or $R = 44''{\cdot}6773 - 0{\cdot}00865\ t + 0{\cdot}000311\ t^2$ was found to represent the above tabular values, but when applied for the reduction of the latitude observations made at *low* temperature the formula failed to be satisfactory. It became evident that the micrometer observations made at the temperature $-7^{\circ}{\cdot}0$ C. had to be rejected, though not suspected at the time. The 12 conditional and 2 normal equations were then formed anew, whence the final value $R = 44''{\cdot}6606 - 0{\cdot}001150\ t$. It represents the observations as follows:

Set.	R observed.	R computed.	C.—O.
	″	″	″
1	44·641	44·642	+·001
2	·660	·637	—·023
3	·616	·636	+·020
4	·637	·632	—·005
5	·632	·636	+·004
6	·613	·634	+·021
7	·660	·632	—·028
8	·653	·635	—·018
9	·683	·657	—·026
10	·666	·660	—·006
11	·653	·658	+·005
12	·634	·652	+·018

The probable error of an observation of unit weight is

$$\cdot 675\sqrt{\frac{\cdot 005337}{12-1}} = \pm 0''{\cdot}015$$

and the probable error of resulting value of R,

$$\frac{0{\cdot}015}{\sqrt{19}} = \pm 0''{\cdot}0034.$$

It may be remarked that the screw has 100 threads to the inch, or to 2.540 cm.; hence one turn moves the thread $\frac{1}{4}$ mm. and one division corresponds to a linear move of 0·0025 mm.

Investigation of the progressive and periodic inequalities of the screw.— The value of the screw for the successive intervals $40 - 30$, $39\frac{3}{4} - 29\frac{3}{4}$, $39\frac{1}{2} - 29\frac{1}{2}$, $39\frac{1}{4} - 29\frac{1}{4}$, $39 - 29$ turns, etc., gave the above average value of R for the whole screw from turn 20 to turn 40; hence, by comparing this value with the individual values at the different parts of the screw, reckoned from its middle, and taking means of corresponding values for all the sets available, we get the following corrections:

Results of observations for the variation of latitude at Rockville, Md.

Turns.	Divisions.	Adopted correction.	Turns.	Divisions.	Adopted correction.
		″			″
At 40	+0·2	+.09	At 30	+·05	+·13
39	+0·1	.00	29	+·01	+·18
38	—0·3	—.04	28	+0·7	+·18
37	—0·1	.00	27	+0·3	+·13
36	0·0	+·04	26	—0.2	+·13
35	+0·3	+·04	25	+0·4	+·09
34	0·0	·00	24	+0·1	+·04
33	—0·3	—·04	23	+0·0	—·04
32	—0·1	—·04	22	—0·8	—·13
31	—0·2	+·04	21	—0·6	—·22
--	-----	-----	20	—0·8	—·31

The adopted corrections are simply the smoothed out values of the preceding column converted to seconds of arc. Thus between 40 and 30 turns the screw is quite perfect, but less so towards the other end.

The micrometer reduction to the latitude includes this correction.

Periodic inequality of the screw.—A discussion of the measures of quarter, half, and three-quarter turns, indicated that any periodic inequality of the screw was too small to be detected with certainty from the material on hand; the mean correction from 13 determinations at any of the above fractional turns would not amount to 0″·02.

Intervals of the micrometer threads 1, 2, *and* 3.—The part of the screw utilized in the latitude work generally falls between turns 20 and 40, but for larger micrometric differences the fixed threads 1 or 3 were employed instead of 2, the middle thread. By means of a collimator a series of measures of the intervals was made with the following results:

September 21, 1891.	Turns and divisions.	″
Space between threads 1 and 2	9 94·18	Diff. from 10 whole turns 2·60
Space between threads 1 and 3	19 97·63	Diff. from 20 whole turns 1·06
Space between threads 2 and 3	10 03·45	Diff. from 10 whole turns 1·54

Thread 3 was found unsatisfactory, it being slack and dragging on the vertical threads, and was soon abandoned. About June 12, 1892, the spring holding the micrometer slide became weak; the slide was therefore removed. The space 1 to 2 was found to have changed in consequence, and was redetermined August 24 and 26, 1892. The weighted mean of six sets of measures gave the value 4″·91 for the difference from ten turns, which quantity was used after June 12, 1892.

The reduction of the observations for latitude was made in the usual way, checked, scrutinized, and results tabulated. A few obvious misreadings of the whole turns or other similar errors were corrected and a few observations with results too wide from the truth for a possible error of measure were rejected with the consent of the observer. In all, 1789 individual results for latitude were secured. These are given in the following table:

Results for latitude of

[39° 05′ + tab

Pair No ...		1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
C. and G. S. Cat.		1184 1188	1198 1206	1215 1220	1231 1241	1247 1254	1262 1275	1280 1294	1316 1330	1336 1354	1360 1367	1382 1386
1891.												
June	13	9·57	10·22	10·71	10·77	9·84	9·96	10·91	10·23	10·50	10·85	10·85
	15	9·46	10·50	10·36	10·65	-- --	10·32	11·13	10·28	10·64	10·85	10·76
	23					10·33	10·54	10·90	10·24	11·04	10·74	-- --
	24					9·83	10·32	11·50	10·78	11·30	10·95	10·94
	25					10·48	10·31	11·27	10·49	11·02	10·67	10·99
	27					11·13	11·30	11·36	10·66	10·93	10·88	10·87
July	5					10·56	11·13	11·17	10·46	10·60	10·51	10·44
	6					10·66	10·36	11·13	10·43	10·72	10·45	10·90
	10									10·51	10·52	10·79
	12									11·22	9·99	10·10
	21									11·19	10·47	10·32
	22									10·35	10·21	10·63
	27									9·69	10·61	10·21
	29									10·62	10·56	10·57
	31											
Aug.	7											
	9											
	10											
	11											
	14											
	15											
	17											
	22											
Sept.	1											
	3											
	7											
	9											
	10											
	12											
	13											
	14											
	15											
	17											

Rockville, Md., 1891–'92.

ular seconds.]

12.	13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.
1392	1407	1424	1442	1456	1473	1502	1539	1589	1628	1648	1676	1718
1397	1418	1432	1448	1462	1476	1514	1547	1608	1641	1666	1690	1741
11·60	10·54	9·95	10·96	10·64								
11·97	11·25	9·45	11·11	11·42								
11·61	11·05	10·83	10·71	11·25	10·93	9·77	10·41					
11·81	10·99	11·46	10·95	11·14	11·46	9·61	10·55					
11·90	11·09	10·50	11·22	11·35	11·20	9·50	10·62					
12·06	10·97	10·96	11·66	11·81	11·57	9·72	10·78					
11·57	10·93	10·88	10·93	11·05	11·55	9·56	10·83					
11·79	10·64	9·87	11·07	-- --	11·45	9·95	-- --					
11·20	10·30	10·43	11·19	10·43	10·96	9·53	9·73	10·80	10·37	10·22	10·24	10·33
11·54	10·98	9.83	9·74	10·58	10·76	9·35	10·08	10·08	10·59	10·15	10·51	10·61
11·42	10·50	9·96	10·65	10·68	11·06	9·78	9·58	10·32	10·20	10·09	10·69	10·48
11·50	10·55	9·86	10·87	10·71	10·89	9·91	10·29	10·28	9·68	-- --	-- --	-- --
11·58	10·83	10·40	10·63	10·93	-- --	9·89	10·38	10·70	10·39	10·57	10·65	10·84
11·45	10·79	10·16	10·57	10·34	10·53	9·74	10·23	10·16	10·28	10·24	10·71	10·23
		10·40	10·87	10·64	11·27	9·85	10·29	-- --	-- --	-- --	-- --	-- --
		10·02	-- --	11·00	11·17	9·98	9·93	10·29	10·28	10·10	10·40	10·89
		9·74	10·63	11·13	11·11	9·82	10·17	10·09	-- --	10·39	10·61	10·42
		10·04	10·82	10·93	11·22	9·75	9·98	10·36	10·30	10·55	10·62	10·36
		9·56	10·18	10·90	10·70	-- --	-- --	-- --	-- --	-- --	-- --	-- --
					10·89	10·01	-- --	-- --	-- --	-- --	-- --	-- --
					11·11	10·02	10·17	10·31	10·58	9·97	10·25	10·16
					-- --	-- --	-- --	10·17	-- --	-- --	-- --	-- --
					11·10	9·48	10·08	10·12	10·34	9·87	10·55	10·59
								9·80	10·21	10·41	10·72	10·28
								10·13	10·20	10·33	10·68	10·09
								10·93	10·34	9·30	10·94	10·06
								10·45	10·58	10·07	10·39	10·19
								10·10	10·26	9·83	10·30	10·28
								9·55	9·84	9·89	10·34	10·50
								9·71	10·20	-- --	-- --	-- --
								10·18	10·61	10·20	10·23	10·56
								10·09	10·32	9·90	10·19	10·85
								10·08	10·00	9·87	10·61	10·54

Pair No.		25.	26.	27.	28.	29.	30.	31.	32.	33.	34.	35.
C. & G. S. Cat.		1756 1779	1801 1809	1819 1825	1852 1859	1885 1896	1917 1926	1948 1959	1968 1971	1974 1984	2004 2020	2026 2043
Aug.	7	10·64	10·97	10·80	10·21							
	9	10·67	10·86	10·45	10·54							
	10	10·61	11·04	10·67	10·34							
	11	-- --	-- --	-- --	-- --							
	14	-- --	-- --	-- --	-- --							
	15	10·40	10·62	10·76	10·32	10·70	10·72	10·21	10·40	9·54	10·58	10·91
	17	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --
	22	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --
Sept.	1	10·26	10·88	10·25	10·07	9·89	-- --	10·74	10·68	10·60	10·72	10·71
	3	10·15	10·86	10·42	10·17	9·95	9·90	10·54	10·46	10·35	9·96	10·03
	7	10·39	10·86	10·20	10·14	-- --	-- --	-- --	-- --	-- --	-- --	-- --
	9	10·27	10·76	10·36	10·40	-- --	10·50	10·78	10·50	10·79	10·84	10·27
	10	10·75	10·97	10·65	10·38	9·95	10·38	10·63	10·62	10·30	10·87	10·10
	12	10·35	10·78	10·28	10·00	9·89	10·07	10·06	9·98	10·12	10·68	9·60
	13	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --
	14	10·42	10·99	10·84	10·26	9·99	10·68	10·64	10·67	10·30	10·64	10·13
	15	10·20	10·78	10·92	-- --	9·99	10·27	10·72	10·44	10·48	10·36	10·13
	17	10·39	10·96	10·09	10·17	9·69	10·36	10·28	10·17	10·37	10·68	9·95
	18		11·27	10·57	9·82	10·21	10·64	10·45	10·08	10·57	10·68	9·71
	19		11·30	11·11	10·45	10·01	10·62	10·41	10·23	10·84	10·62	10·38
	21		10·66	10·39	10·46	9·97	10·34	10·07	10·32	10·44	10·78	10·26
	23		10·84	10·72	10·40	10·05	10·74	10·37	10·62	10·96	10·50	10·87
	24		10·66	10·43	9·95	9·53	9·41	9·93	10·11	10·28	10·35	9·87
	25		11·14	10·72	9·76	9·94	10·48	10·20	10·15	10·49	10·47	9·90
	26		11·14	10·77	10·38	9·93	9·94	10·15	10·37	10·15	10·40	9·99
	28		10·90	10·64	10·10	10·25	10·26	10·19	10·42	10·37	10·77	10·02
	30		10·96	10·66	10·50	-- --	9·91	10·03	10·34	10·45	10·33	9·78
Oct.	3		11·57	10·59	10·33	10·47	9·97	10·82	10·13	10·44	10·73	10·23
	5		10·86	10·33	10·48	9·85	10·08	10·22	9·92	10·59	10·75	10·07
	9		11·24	10·79	10·11	10·13	10·67	10·06	10·24	11·04	10·65	10·10
	14		10·85	10·57	10·38	10·05	10·29	10·07	10·39	10·38	10·59	10·30
	15		10·95	10·42	10·16	10·17	10·46	10·06	9·96	10·78	10·62	9·91
	16		11·04	10·81	10·24	10·18	10·12	10·17	10·10	10·06	10·22	9·88
	17		10·73	10·13	10·06	9·79	9·75	9·99	10·13	10·69	10·05	10·01
	22										10·33	9·93
	23										10·89	10·43
	24										10·88	10·10
	28										10·28	9·79
	29										10·76	9·77
	30										10·85	10·59
	31										10·28	-- --
Nov.	1										10·54	9·53
	2										10·35	9·52
	6										10·05	9·37
	7										10·52	9·58
	9										10·30	10·07
	17											
	18											
	20											
	23											
	24											
	29											
	30											
Dec.	1											
	4											
	5											

36.	37.	38.	39.	40.	41.	42.	43.	44.	45.	46.	47.
2047	2059	2070	2093	2131	2154	2178	10	27	45	66	93
2055	2063	2078	2121	2142	2159	4	22	32	55	77	111
10·56	10·59	9·83	10·13	-- --							
10·63	10·85	10·14	10·46	11·08							
-- --	-- --	-- --	-- --	-- --							
10·53	10·75	9·95	10·02	10·97							
-- --	-- --	-- --	-- --	-- --							
10·42	10·39	9·81	10·27	10·62							
10·88	10·82	10·33	10·01	10·52							
10·30	10·85	9·98	10·19	10·67							
10·45	10·76	10·36	10·60	10·90							
10·62	10·79	10·13	10·78	10·91							
10·49	10·56	9·59	10·07	-- --							
10·40	10·40	10·15	10·06	10·47							
10·23	10·35	9·82	10·27	10·73							
9·98	10·73	10·18	10·29	10·06							
10·36	10·87	9·80	10·59	10·61							
10·77	10·84	9·86	10·49	10·84							
10·50	10·58	9·45	10·13	10·71							
10·13	10·42	9·68	10·41	10·56							
10·45	11·01	10·04	9·94	10·28	10·30						
10·42	10·65	9·95	10·19	10·36	-- --						
10·33	10·13	9·61	10·24	10·30	-- --						
10·42	10·59	10·05	10·53	10·77	-- --						
9·88	10·25	9·72	-- --	-- --	-- --						
10·00	10·57	9·92	9·94	10·59	10·29	10·52	10·85	10·47	10·22	10·15	10·57
10·36	10·38	9·92	9·68	10·91	10·25	9·46	10·13	10·50	9·85	9·27	10·10
10·16	9·75	10·09	10·11	10·11	10·21	10·39	10·83	9·92	10·94	10·37	10·46
10·58	10·46	9·73	10·25	10·87	10·52	10·60	11·26	10·70	10·60	10·62	10·31
10·37	10·51	10·15	10·75	10·77	10·50	10·69	11·09	10·65	10·99	10·67	10·41
-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --
10·58	10·30	9·67	10·06	10·48	10·13	10·57	11·30	10·96	10·92	10·79	10·56
10·03	10·67	9·54	9·88	10·41	10·81	10·40	10·60	10·11	10·45	10·19	10·38
10·36	10·42	9·38	9·93	10·80	9·98	10·19	10·83	10·37	10·56	10·02	10·09
9·86	10·28	9·59	9·97	10·63	10·38	10·49	11·09	10·42	10·89	10·42	10·17
10·23	10·41	9·95	10·19	10·75	10·45	10·20	10·63	9·91	10·38	9·90	10·65
					10·91	10·29	-- --	10·55	11·19	10·65	10·80
					10·78	10·21	10·33	10·43	10·75	11·37	9·70
					10·69	10·75	10·48	10·07	10·98	10·04	10·57
					10·34	10·64	10·26	10·14	10·91	10·08	10·30
					10·18	10·34	10·26	10·59	10·45	10·24	10·19
					10·25	10·25	10·30	10·34	10·73	10·00	10·11
					10 30	10·74	10·67	11·17	-- --	-- --	-- --
					10·50	10·37	10·51	10·66	10·68	10·30	10·14
					10·43	10·73	10·56	10·33	10·55	10·41	-- --
					10·74	-- --	10·58	10·67	10·76	10·48	10·44

Pair No.		48.	49.	50.	51.	52.	53.	54.	55.	56.	57.
C. & G. S. Cat.		134 150	176 196	202 208	220 232	242 250	259 263	275 287	292 297	306 331	370 378
1891.											
Oct.	23	10·66									
	24	10·58									
	28	10·73									
	29	11·00									
	30	10·40									
	31	-- --									
Nov.	1	10·53									
	2	10·54									
	6	10·08									
	7	10·41									
	9	10·64									
	17	-- --									
	18	9·98	11·17	-- --	11·14	10·34	10·24	9·09	9·05		
	20	10·62	10·09	9·91	11·42	10·77	10·77	10·13	9·90		
	23	10·60	9·75	10·48	10·80	10·74	10·43	10·10	9·63		
	24	10·34	9·79	10·31	11·10	10·52	10·32	9·47	9·76		
	29	9·99	9·80	10·30	10·77	10·57	10·60	9·63	9·71		
	30	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --		
Dec.	1	9·70	9·74	10·58	10·92	11·15	10·87	10·06	9·98		
	4	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --		
	5	10·73	10·38	10·45	11·34	10·83	10·70	10·30	10·05		
	8	10·58	10·03	10·50	11·16	10·76	10·54	10·23	9·91	10·75	9·77
	9	11·07	10·74	10·87	11·62	10·68	10·53	10·47	9·97	11·02	9·86
	10	11·07	9·96	10·17	11·33	11·05	10·49	9·72	10·21	10·67	9·77
	13	10·94	9·89	10·60	11·03	10·82	10·69	10·89	9·80	10·24	9·56
	18	10·13	9·92	10·43	10·72	10·44	9·90	9·57	9·79	10·28	9·53
	28	11·32	10·24	10·37	10·78	10·50	10·35	9·99	9·68	10·84	9·49
	30	10·41	9·76	10·01	10·99	10·46	10·07	9·82	9·37	10·89	8·94
1892.											
Jan.	7							9·46	9·51	10·53	9·23
	8							10·01	9·83	-- --	9·64
	16							9·75	10·47	10·44	8·28
	20							9·84	10·16	10·61	9·44
	21							10·26	9·48	10·78	8·46
	24							9·62	9·46	10·48	8·88
	25							9·92	9·46	10·92	9·49
	28							10·20	9·68	10·78	-- --
	31										
Feb.	10										
	13										
	15										
	17										
Mar.	10										
	12										
	14										
	16										
	19										
	20										
	21										
	23										
	24										

58.	59.	60.	61.	62.	63.	64.	65.	66.	67.
392	430	463	483	512	531	562	606	700	718
417	456	479	491	520	533	577	611	705	729
10·75	11·04	10·28	10·53						
10·84	11·03	10·89	10·58						
10·46	10·77	10·57	-- --						
10·25	11·02	10·98	10·55						
9·91	10·95	10·57	10·89						
10·43	10·78	11·05	9·29						
10·15	10·85	10·55	10·82						
10·78	10·63	11·50	10·51	10·61	11·02	10·48	9·87		
10·59	10·74	-- --	-- --	-- --	-- --	-- --	-- --		
10·10	10·83	10·17	10·60	10·08	10·37	9·59	9·61		
10·21	10·44	9·51	10·13	10·63	10·35	10·12	-- --		
10·25	10·21	10·84	10·68	10·81	10·48	10·38	9·76		
10·02	9·96	10·09	10·39	10·60	11·00	10·50	9·82		
10·74	10·14	9·85	10·77	10·54	10·80	10·21	9·69		
-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --		
	10·42	10·21	9·91	10·45	10·52	10·16	9·64	10·42	9·63
	10·34	10·29	9·62	10·85	-- --	-- --	-- --	-- --	-- --
	10·66	10·71	10·52	10·73	10·97	10·49	9·62	10·70	10·55
	10·04	10·29	10·34	10·42	10·73	10·37	10·17	10·10	9·87
	10·68	10·06	10·47	10·57	10·88	-- --	10·90	10·78	10·27
								10·68	10·12
								10·41	10·33
								10·61	9·50
								10·38	10·01
								10·16	9·86
								10·23	10·17
								10·46	9·53
								10·36	10·60
								10·36	10·08

Pair No.		68.	69.	70.	71.	72.	73.	74.	75.	76.	77.
C. & G. S. Cat.		745 757	769 776	781 792	796 804	813 825	835 842	849 865	873 888	911 921	926 932
1892.											
Jan.	31	9·90	9·76	9·95							
Feb.	10	-- --	-- --	-- --							
	13	10·76	10·37	10·42							
	15	10·20	10·30	10·09							
	17	10·27	10·43	10·25							
Mar.	10	10·69	10·32	-- --							
	12	10·97	10·66	10·38	-- --	10·56	10·11	-- --	-- --	-- --	-- --
	14	10·12	10·18	10·38	10·10	10·19	10·01	10·77	8·99	10·61	10·34
	16	10·46	9·81	10·01	10·08	10·41	10·20	-- --	-- --	-- --	-- --
	19	10·24	10·32	10·31	10·49	9·87	10·27	10·68	9·38	10·44	-- --
	20	10·21	10·15	9·91	10·40	10·26	9·81	10·48	9·45	10.39	10·04
	21	10·56	10·27	10·00	11·29	9·98	10·20	-- --	9·76	10·60	9·94
	23	10·39	10·39	10·40	-- --	-- --	-- --	-- --	-- --	-- --	-- --
	24	10·49	10·75	10·43	10·77	10·13	10·10	10·78	-- --	-- --	-- --
	28				10·39	10·33	10·28	10·46	9.75	9·64	10·09
	29				10·71	10·53	10·30	10·74	-- --	-- --	-- --
Apr.	4				10·82	10·57	10·32	11·05	10·22	10·58	10·55
	9				10·32	10·30	10·41	10·83	9·83	10·70	10·15
	11				10·67	10·66	10·32	11·08	9·61	10·43	10·33
	12				10·25	10·17	10·13	10·65	9·68	10·62	10·57
	13				11·05	10·99	10·63	11·01	10·16	10·12	10·26
	16										10·18
	19										10·38
	23										10·41
	26										10·31
	27										10·11
	29										11·10
	30										10·38
May	3										-- --
	4										10·22
	5										10·26
	7										10·39
	8										10·89
	9										9·83
	16										
	17										
	19										
	23										
	24										
	25										
	28										
	30										
June	1										

78.	79.	80.	81.	82.	83.	84.	85.	86.	87.
936	952	966	1003	1022	1044	1063	1094	1118	1137
945	958	976	1010	1035	1057	1070	1109	1128	1149

10·60	9·64	10·25	10·21	10·60					
-- --	-- --	-- --	-- --	-- --					
10·54	9·64	10·42	-- --	10·75					
10·78	-- --	-- --	-- --	-- --					
10·61	9·84	10·29	10·19	10·50					
10·61	10·01	10·26	10·45	10·67					
10·55	-- --	-- --	-- --	-- --					
10·66	10·33	10·15	10·47	10·72					
10·63	-- --	-- --	-- --	-- --					
11·10	10·09	10·64	10·67	10·77	11·21	10·91	10·01	10·77	10·96
10·79	10·23	10·55	10·78	10·63	10·86	10·73	10·11	10·41	10·24
10·88	10·16	10·50	10·98	10·82	-- --	-- --	-- --	-- --	-- --
10·88	10·49	10·37	-- --	-- --	-- --	-- --	-- --	-- --	-- --
10·95	10·24	10·40	11·05	10·64	10·86	10·37	10·29	10·71	11·05
-- --	-- --	-- --	10·40	10·67	10·96	10·48	10·18	10·61	10·73
10·77	-- --	10·77	10·86	11·30	11·02	10·87	10·44	-- --	10·86
10·37	10·20	10·24	10·85	10·70	10·67	10·50	9·87	10·48	10·61
10·87	10·34	10·58	10·89	10·75	10·83	10·55	10·31	10·48	10·44
11·17	10·38	10·59	10·65	10·73	10·46	10·74	10·34	10·20	10·68
10·33	9·72	10·72	10·77	10·74	11·10	10·16	10·55	10·40	10·88
					10·91	10·62	10·16	10·51	10·41
					10·78	10·49	9·90	10·39	10·39
					10·70	10·71	-- --	-- --	-- --
					11·17	10·92	10·34	10·39	10·82
					11·07	10·39	10·03	10·32	10·44
					10·86	10·39	10·17	10·61	10·59
					10·85	10·47	9·77	10·59	10·46
					10·53	10·24	10·16	10·38	10·49
					10·67	10·54	10·08	10·50	10·27

Pair No.		88.	1.	2.	3.	4.	5.	6.	7.	8.	9.
C. & G. S. Cat.		1162 1175	1184 1188	1198 1206	1215 1220	1231 1241	1247 1254	1262 1275	1280 1294	1316 1330	1336 1354
1892.											
Apr.	23	10·19	10·70								
	26	10·39	10·34								
	27	-- --	-- --								
	29	-- --	-- --								
	30	11·06	10·70								
May	3	10·14	10·33								
	4	-- --	-- --								
	5	10·20	9·97								
	7	10·20	9·92								
	8	10·39	9·83								
	9	10·58	10·88								
	16	10·00	10·14	10·08	10·22	10·52	10·17	10·72	10·45	09·97	
	17	10·04	9·86	9·77	10·21	9 80	-- --	-- --	-- --	-- --	
	19	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	
	23	10·37	10·17	9·84	10·49	10·22	10·55	10·78	11·02	10·67	
	24	10·19	10·01	9·93	10·09	10·45	10·32	-- --	-- --	-- --	
	25	10·59	10·09	10·06	10·45	10·84	10·14	10·45	10·99	10·04	
	28	10·38	9·92	10·05	10·51	10·09	10·55	10·44	10·70	10·16	
	30	10·15	9·96	9·81	10·36	10·57	10·35	10·41	10·65	10·04	
June	1	10·28	9·88	10·11	-- --	-- --	10·56	10·38	10·86	10·08	
	2		9·96	9·91	10·41	10·35	10·34	10·36	10·69	10·14	10·62
	6		10·07	10·07	10·71	10·20	10·46	-- --	11·21	-- --	-- --
	11		9·63	9·65	10·66	10·18	10·14	10·45	10·46	10·14	10·25
	12		9·72	9·21	10·23	10·52	-- --	10·23	10·50	10·01	10·46
	13		9·95	9·91	10·29	10·51	10·01	9·96	10·73	9·94	10·51
	15		9·96	10·11	10·16	9·97	9·89	10·71	10·37	9·99	10·25
	16		9·72	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --
	17		9·71	9·82	10·31	10·83	-- --	10·80	10·63	-- --	10·73
	20		9·20	9·65	10·32	10·49	10·21	10·36	10·69	10·06	-- --
	21		9·34	9·64	10·85	10·63	-- --	-- --	-- --	-- --	-- --
	23		9·74	9·75	9·42	9·92	10·38	10·58	11·08	10·32	10·85
	25		9·52	10·01	10·26	11·19	-- --	9·89	10·12	10·89	10·62
	26		9·63	9·44	9·83	10·11	9·95	-- --	-- --	-- --	-- --
	29		9·92	10·11	10·38	10·59	10·44	10·38	10·68	10·16	10·48
July	7						10·36	10·38	10·68	10·26	10·45
	8						9·76	10·48	10·43	9·59	10·13
	9						9·90	10·41	10·65	10·02	10·39

10.	11.	12.	13.	14.	15.	16.	17.	18.	19.
1360	1382	1392	1407	1424	1442	1456	1473	1502	1539
1367	1386	1397	1418	1432	1448	1462	1476	1514	1547
10·81	-- --	11·71	10·81	11·44	11·02	10·91	-- --	-- --	-- --
10·36	10·55	11·91	10·59	11·07	10·83	10·69	-- --	-- --	-- --
10·52	10·39	11·72	10·83	10·94	10·84	10·67	10·94	9·57	9·93
10·42	10·37	11·50	10·66	11·11	10·66	10·54	11·37	-- --	10·34
10·52	10·24	11·65	10·70	9·87	10·76	10·93	-- --	-- --	-- --
10·47	10·40	11·35	-- --	-- --	-- --	-- --	-- --	-- --	-- --
-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --
-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --
10·49	10·49	11·61	10·81	10·05	10·74	10·73	11·23	9·46	10·35
-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --
10·90	10·63	11·69	11·25	10·20	11·04	11·07	11·52	10·30	10·28
10·33	10·37	11·65	10·37	9·08	10·78	-- --	-- --	-- --	-- --
-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --	-- --
10·47	10·59	11·55	10·93	10·36	10·88	11·10	11·54	9·87	10·15
-- --	10·33	11·63	10·85	9·42	-- --	11·03	11·50	10·14	10·24
10·54	10·27	11·32	10·24	9·90	10·75	10·55	10·83	9·59	10·15
10·46	10·42	11·61	10·98	9·89	10·73	10·91	11·23	9·65	10·07

COMBINATION OF RESULTS FOR VARIATION OF LATITUDE, AS OBSERVED AT ROCKVILLE, MD., IN 1891–'92.

In order to deduce any result for systematic variation from the individual results for latitude, as obtained directly from observations, it is essential that in the combination, the declinations of the stars, taken from various catalogues, should be referred to a uniform system, and at the same time that any change in latitude, whether periodic, progressive, irregular, or any absence of change, should not be interfered with. This was secured by the method of combination adopted in the reduction of the Berlin and other series of observations as carried out by Dr. Th. Albrecht* and is the same as followed for the Rockville series.

The first step of the reduction consists in dividing the whole series of results into groups, each of a sufficient number of pairs of stars to give a closely approximate value of the mean latitude and with the sums of the positive and negative micrometer corrections, as near as may be, balancing each other. The observer's arrangement respecting groups was altered with a view of strengthening the connections of the groups. At the close of a year's work the re-observation of the same pairs of a group would afford the means of improving the whole series by the introduction of the condition that the closing error of the sum of the corrections to the mean results of the groups at the end of the year be zero.

The second step is the reduction of the declinations of each pair of stars to the mean declination system of the group. Comparing the mean result for latitude of each pair in the group with the mean result by all the pairs—*i. e.*, with the mean latitude of the group—a correction was obtained to the result by each pair. In general, the smallness of these corrections (as given below) indicate that the tabular declinations were fairly accurate.

The pairs constituting each group, the number of days of observation for each, and the mean latitude for the same are tabulated below, together with the reduction of the result by each pair to the mean of its group.†

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*Provisorische Resultate der Beobachtungen in Berlin, Potsdam und Prag, betreffend die Veränderlichkeit der Polhöhe, etc. Von Th. Albrecht. Berlin, 1890.

† From these observations it has been found that the mean error of reduction to the mean declination system of a group for a single determination is $\pm 0''\cdot23$. Since the number of determinations of the various groups ranges from eight to twenty-eight, the mean errors of the reductions to the mean declination system in the above table are within $\pm 0''\cdot04$ and $\pm 0''\cdot08$.—*Edwin Smith.*—[Added November 9, 1892.]

					°	′	″
Group	I	Pairs, 9, 10, 11, 12, 13	Days, 21	φ_{i} =	39	05	10·83
	II	14, 15, 16, 17, 18, 19	19	φ_{ii}			10·53
	III	20, 21, 22, 23, 24, 25	12	φ_{iii}			10·30
	IV	26, 27, 28, 29, 30, 31, 32, 33	21	φ_{iv}			10·38
	V	34, 35, 36, 37, 38, 39, 40	28	φ_{v}			10·32
	VI	41, 42, 43, 44, 45, 46, 47, 48	16	φ_{vi}			10·43
	VII	49, 50, 51, 52, 53, 54, 55	13	φ_{vii}			10·36
	VIII	56, 57, 58, 59, 60, 61	12	φ_{viii}			10·32
	IX	62, 63, 64, 65	8	φ_{ix}			10·33
	X	66, 67, 68, 69, 70	12	φ_{x}			10·26
	XI	71, 72, 73, 74, 75, 76	9	φ_{xi}			10·33
	XII	77, 78, 79, 80, 81, 82	12	φ_{xii}			10·49
	XIII	83, 84, 85, 86, 87, 88	16	φ_{xiii}			10·49
	XIV	1, 2, 3, 4, 5, 6, 7, 8	13	φ_{xiv}			10·26

		″	″	″	″	″	″	″	″
Group	I	Red'n +·21	+ ·27	+·28	—·80	+·05			
	II	+·19	— ·30	—·38	—·62	+·83	+·29		
	III	+·11	·00	+·27	—·17	—·10	—·10		
	IV	—·60	— ·20	+·16	+·34	+·10	+·12	+·14	—·05
	V	—·24	+ ·33	—·05	—·23	+·42	+·10	—·31	
	VI	+·05	+ ·06	—·24	+·04	—·20	+·15	+·14	·00
	VII	+·35	— ·02	—·72	—·35	—·12	+·33	+·53	
	VIII	—·33	+1·03	—·05	—·34	—·20	—·16		
	IX	—·20	— ·41	+·06	+·56				
	X	—·15	+ ·23	—·12	—·02	+·05			
	XI	—·17	— ·05	+·09	—·46	+·66	—·07		
	XII	+·18	— ·26	+·39	+·06	—·17	—·20		
	XIII	—·37	— ·04	+·35	·00	—·10	+·17		
	XIV	+·40	+ ·33	—·08	—·13	+·03	—·17	—·46	+·12

The sum of the corrections for each group should be zero.

The preceding corrections were applied to the results of the respective pairs, forming a new table of results (marked A_1 B_1 C_1 D_1 E_1 F_1, in MS. and not reproduced here) preparatory to the next step, viz:

Comparison of the mean declination of the groups by means of the results obtained on the *same night* from pairs ranging over *two* or more groups. Such comparisons between results of the same date leave, of course, any changes in latitude absolutely untouched. This third step, then, consists in ascertaining the differences in the mean declination system of any two adjacent groups or overlapping groups. For this purpose all results of pairs belonging to the same group were made use of, and not only those results which form the full or complete set of all pairs for every night. For any date *common* to two or more groups, the mean latitude was taken for all pairs observed in each, and their number noted as the weight to the mean, and the same was done for the other groups. These comparisons being made for every day of observation, the final difference between any two groups will be that of their

weighted means. The following tables marked A_{II} B_{II} C_{II} contain these comparisons:

Comparison of mean values of groups.

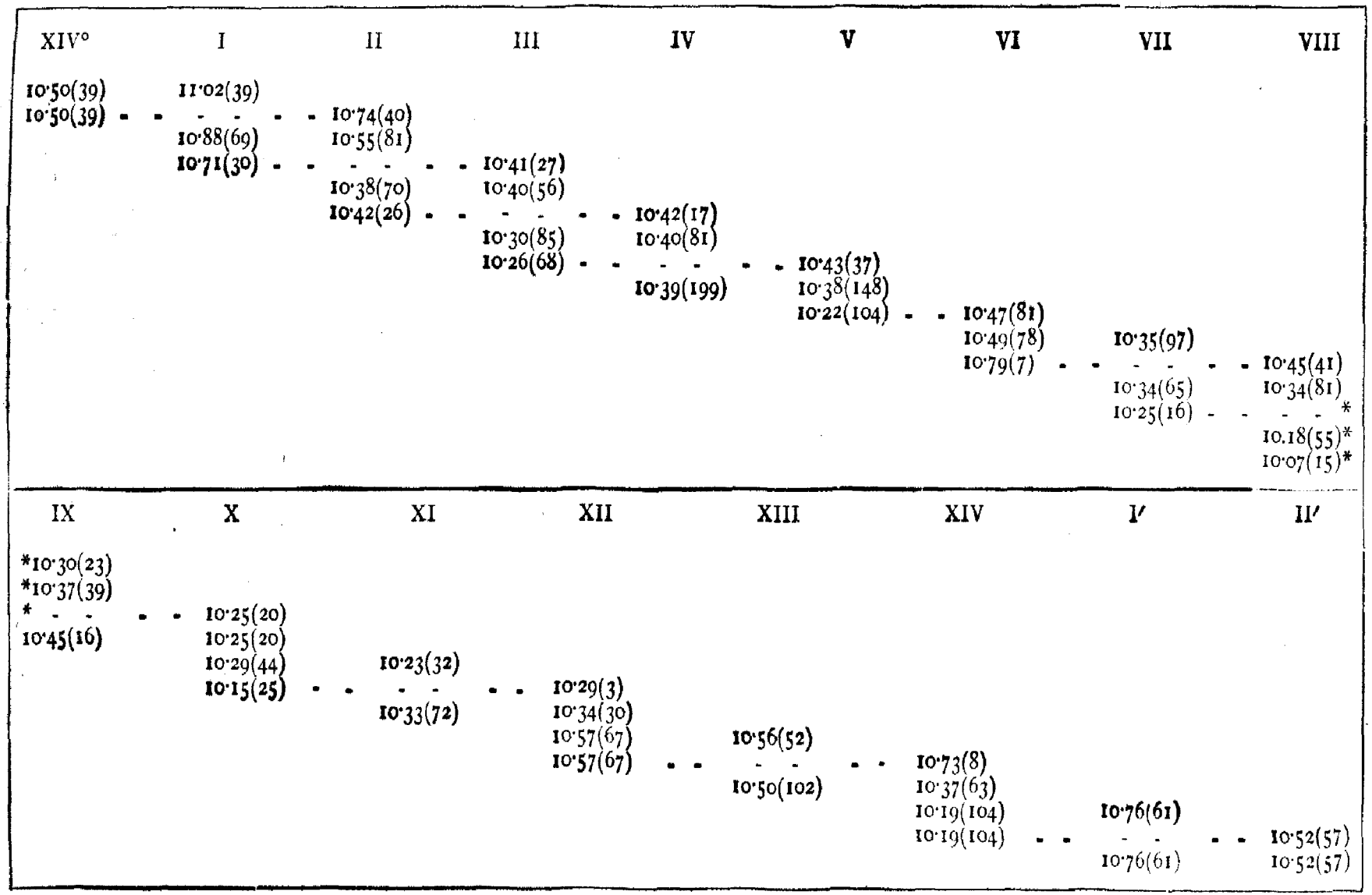
Date.	XIV.	I.	II.	III.	IV.	V.	VI.
1891.	"	"	"	"	"	"	"
June 13	10·28 8	10·87 5	10·35 3				
15	10·39 7	11·09 5	10·50 3				
23	10·38 4	11·04 4	10·65 6				
24	10·49 4	11·20 5	10·86 6				
25	10·52 4	11·13 5	10·73 6				
27	10·99 4	11·14 5	11·08 6				
July 5	10·71 4	10·81 5	10·80 6				
6	10·52 4	10·90 5	10·61 4				
10		10·66 5	10·38 6	10·41 5			
12		10·77 5	10·06 6	10·41 5			
21		10·78 5	10·28 6	10·38 5			
22		10·65 5	10·42 6	10·04 2			
27		10·58 5	10 57 5	10·65 5			
29		10·80 5	10·26 6	10·35 5			
31			10·55 6				
Aug. 7			10·48 5	10·43 6	10·45 3		
9			10·43 6	10·44 5	10·40 3		
10			10·46 6	10·47 6	10·47 3		
11			10·06 4				
14			10·56 2				
15			10·60 3	10·28 6	10·41 8	10·79 2	
17				10·28 1			
22			10·39 3	10·32 5			
Sept. 1				10·28 6	10·43 7	10·76 2	
3				10·26 6	10·33 8	10·04 2	
7				10·33 6	10·19 3		
9				10·33 6	10·54 7	10·60 2	
10				10·25 6	10·48 8	10·40 6	
12				10·08 6	10·15 8	10·50 7	
13				10·01 2			
14				10·37 6	10·55 8	10·43 7	
15				10·26 6	10·49 7	10·29 2	
17				10·25 6	10·26 8	10·31 7	
18					10·45 8	10·42 7	
19					10·62 8	10·43 7	
21					10·33 8	10·59 7	
23					10·60 8	10·66 7	
24					10·04 8	10·21 6	
25					10·36 8	10·27 7	
26					10·36 8	10·26 7	
28					10·39 8	10·29 7	
30					10·36 7	10·33 7	
Oct. 3					10·54 8	10·54 7	
5					10·29 8	10·31 7	
9					10·54 8	10·28 7	
14					10·37 8	10·37 7	10·35 1
15					10·37 8	10·30 7	
16					10·34 8	10·10 7	
17					10·16 8	10·35 7	

Comparison of mean values of groups.

Date.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.
1891.	"	"	"	"	"	"	"	"
Oct. 22	10·01 5							
23	10·33 7	10·47 8						
24	10·32 7	10·02 8						
28	10·04 7	10·48 8						
29	10·35 7	10·70 8						
30	10·57 7	10·68 8						
31	10·04 1							
Nov. 1	10·17 7	10·72 8						
2	10·06 7	10·43 8						
6	10·04 7	10·26 8						
7	10·06 7	10·53 8						
9	10·27 7	10·35 8						
17		10·77 6						
18		10·44 8	10·18 **6**					
20		10·52 8	10·43 **7**					
23		10·41 8	10·28 **7**					
24		10·32 8	10·18 **7**					
29		10·25 8	10·20 **7**					
30		10·70 4						
Dec. 1		10·36 8	10·47 **7**					
4		10·48 6						
5		10·62 7	10·58 7					
8		10·58 1	10·45 7	10·52 **6**				
9		11·07 1	10·70 7	10·70 **6**				
10		11·07 1	10·42 7	10·48 **5**				
13		10·94 1	10·53 7	10·43 **6**				
18		10·13 1	10·11 7	10·36 **6**				
28		11·32 1	10·27 7	10·31 **6**				
30		10·41 1	10·07 7	10·37 6				
1892.								
Jan. 7			9·92 2	10·53 6	10·50 **4**			
8			10·35 2	10·55 3				
16			10·54 2	10·07 6	9·91 **4**			
20			10·43 2	10·06 6	10·18 **3**			
21			10·30 2	10·20 6	10·36 4			
24			9·97 2	9·97 6	10·48 4			
25			10·12 2	10·32 6	10·31 4			
28			10·37 2	10·45 1				
31				9·95 3	10·19 4	9·93 **5**		
Feb. 10				9·85 3	10·65 1			
13				10·40 3	10·45 4	10·56 **5**		
15				9·99 3	10·42 4	10·11 5		
17				10·17 **3**	10·77 3	10·40 5		
Mar. 10						10·44 4		
12						10·55 5	10·36 **2**	
14						10·16 5	10·11 6	10·52 1
16						10·13 5	10·19 3	
19						10·18 5	10·19 6	
20						10·13 5	10·13 6	10·22 1
21						10·16 5	10·46 5	10·12 1
23						10·43 5		
24						10·42 5	10·30 4	

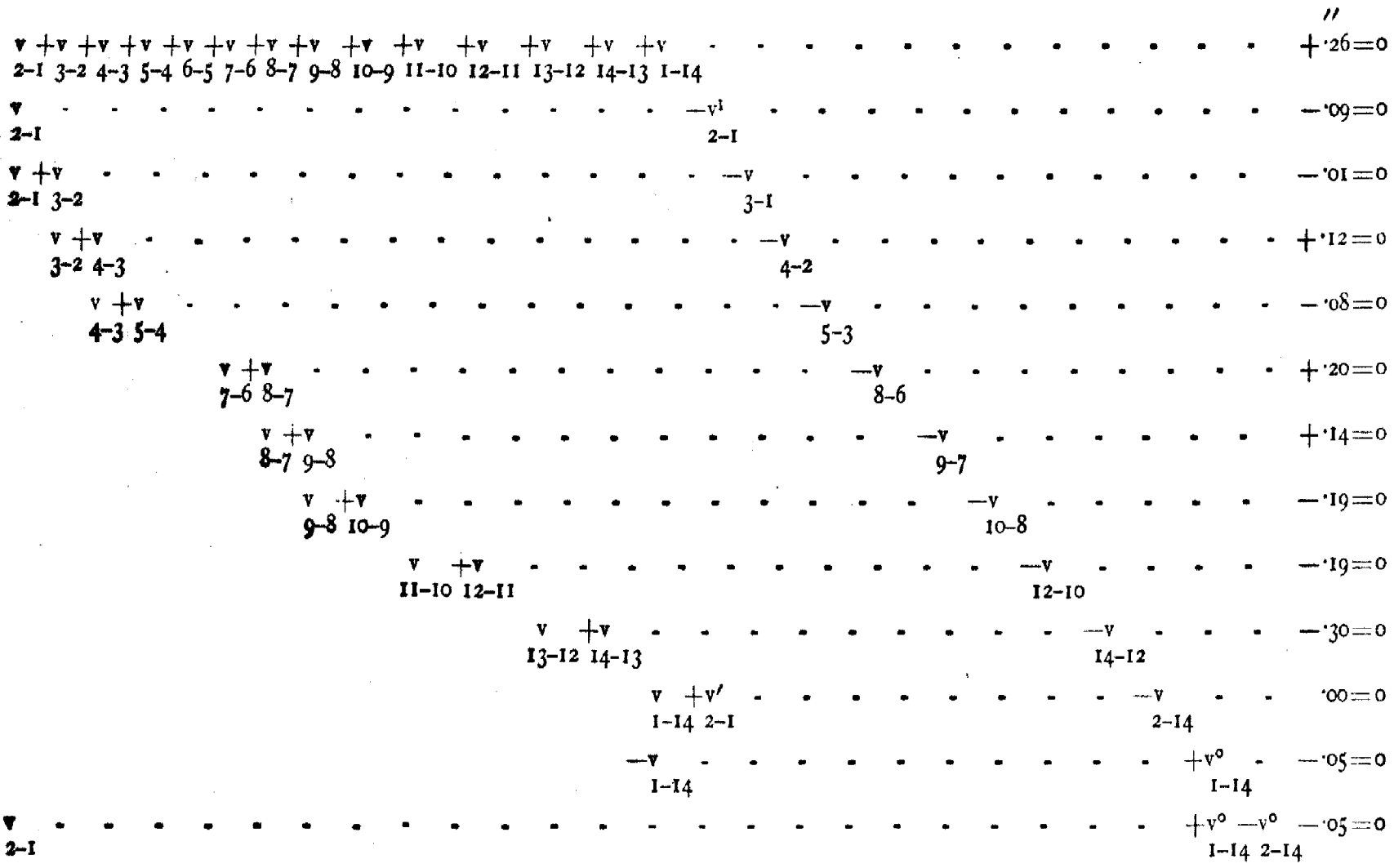
Comparison of mean values of groups.

Date.	XI.	XII.	XIII.	XIV.	I.	II.
1892.	″	″	″	″	″	″
Mar. 28	10·14 6	10·23 6				
29	10·42 4					
April 4	10·59 6	10·41 5				
9	10·40 6	10·42 2				
11	10·46 6	10·29 6				
12	10·25 6	10·43 6				
13	10·66 6	10·36 2				
16		10·42 6				
19		10·46 2				
23		10·61 6	10·68 6	11·10 1		
26		10·55 6	10·46 6	10·74 1		
27		10·58 6				
29		10·80 4				
30		10·61 6	10·72 6	11·10 1		
May 3		10·35 2	10·52 6	10·73 1		
4		10·71 5	10·76 4			
5		10·44 6	10·39 6	10·37 1		
7		10·64 6	10·47 6	10·32 1		
8		10·73 6	10·47 6	10·23 1		
9		10·35 6	10·61 6	11·28 1		
16			10·44 6	10·28 8		
17			10·33 6	10·04 4		
19			10·50 2			
23			10·67 6	10·47 8		
24			10·41 6	10·27 5		
25			10·54 6	10·38 8		
28			10·42 6	10·30 8		
30			10·32 6	10·27 8		
June 1			10·39 6	10·35 6		
2				10·27 8	10·92 4	10·96 3
6				10·47 6	10 80 4	10·70 3
11				10·16 8	10·74 5	10·48 6
12				10·06 7	10·68 5	10·64 5
13				10·16 8	10·72 5	10·36 3
15				10·14 8	10·61 4	
16				10·12 1		
17				10·33 6	10·94 1	
20				10·12 8	10·80 4	10·43 6
21				10·24 4		
23				10·15 8	11·06 5	10·73 6
25				10·27 7	10·67 5	9·88 2
26				9·90 5		
29				10·33 8	10·80 5	10·65 6
July 7				10·30 4	10·75 4	10·53 5
8				9·95 4	10·50 5	10·30 6
9				10·12 4	10·77 5	10·41 6



*Continued on same line.

The preceding differences demand 13 conditions to be satisfied; these conditions are expressed by the following conditional equations, to which are added the values of the reciprocal weight or $\frac{1}{p}$ for each difference depending on the number $n\,n_1$: for the observed values for latitude in each group, I take $\frac{1}{p} = \frac{10\,(n + n_1)}{nn_1}$



$\frac{1}{p}$ = ·268 ·321 ·241 ·118 ·220 ·231 ·277 ·438 1·126 ·540 ·472 ·342 ·257 ·260 ·339 ·704 ·973 ·417 1·672 1·060 1·167 3·734 1·399 ·272 ·513 ·506

Multiplying these equations with the correlates k_1 we form the normal equations as below.

NORMAL EQUATIONS.

k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	
+5·11	+0·27	+0·59	+0·56	+6·36	+0·51	+0·72	+1·56	+1·01	+0·60	+0·26	−0·26	+0·27	+0·26=0
	+0·61	+0·27								−0·34		+0·27	−0·09=0
		+1·29	0·32									+0·27	−0·01=0
			+1·54	+0·24									+0·12=0
				+0·78									−0·08=0
					+2·18	+0·28							+0·20=0
						+1·78	+0·44						+0·14=0
							+2·73						−0·19=0
								+4·75					−0·19=0
									+2·00				−0·30=0
										+0·87	−0·26		0·00=0
											+0·77	+0·51	−0·05=0
												+1·29	−0·05=0

Combination of results for the variation of latitude at Rockville, Md.

Solving, we get

$k_1 = -0.162$	$k_5 = +0.196$	$k_9 = +0.075$
$k_2 = +0.359$	$k_6 = -0.048$	$k_{10} = +0.198$
$k_3 = +0.031$	$k_7 = -0.049$	$k_{11} = +0.230$
$k_4 = -0.055$	$k_8 = +0.170$	$k_{12} = +0.130$
		$k_{13} = -0.060$

and after substituting in the equations of correlates

$v_{2-1} = +''.045$	$v_{9-8} = -''.018$	$v'_{2-1} = -''.044$	$v_{12-10} = -''.280$
$v_{3-2} = -''.060$	$v_{10-9} = +''.009$	$v_{3-1} = -''.022$	$v_{14-12} = -''.277$
$v_{4-3} = -''.005$	$v_{11-10} = -''.047$	$v_{4-2} = +''.054$	$v_{2-14} = -''.062$
$v_{5-4} = +''.004$	$v_{12-11} = -''.041$	$v_{5-3} = -''.082$	$v^\circ_{1-14} = +''.036$
$v_{6-5} = -''.036$	$v_{13-12} = +''.013$	$v_{8-6} = +''.080$	$v^\circ_{2-14} = +''.030$
$v_{7-6} = -''.048$	$v_{14-13} = +''.009$	$v_{9-7} = +''.052$	
$v_{8-7} = -''.072$	$v_{1-14} = -''.016$	$v_{10-8} = -''.198$	

The adjusted values of the differences between the groups are as follows:

		$''$
I—XIV° =	I′—XIV =	+0.555
II— I =	II′— I′ =	−0.285
III— II =		−0.040
IV— III =		+0.095
V— IV =		−0.006
VI— V =		+0.214
VII— VI =		−0.188
VIII— VII =		−0.072
IX— VIII =		+0.172
X— IX =		−0.191
XI— X =		−0.107
XII— XI =		−0.031
XIII— XII =		+0.003
XIV— XIII =		−0.121

Check: $\Sigma \begin{matrix} +1.039 \\ -1.041 \end{matrix}$ $''$

We also have the closing error:

	$''$
For group XIV	−.21
For group I	−.26
For group II	−.35

Taking group IV for reference group as having the greatest number of pairs and observations ($\varphi_{IV} = 39° 05' 10''·39$), and applying the above differences we get the final correction to each group, as below:

		″			″
Correction to group	IV	·000	and to group	III	+ ·095
	V	+ ·006		II	+ ·055
	VI	— ·208		I	— ·230
	VII	— ·020		XIV°	+ ·325
	VIII	+ ·052			
	IX	— ·120			
	X	+ ·071			
	XI	+ ·178			
	XII	+ ·209			
	XIII	+ ·206			
	IV	+ ·327			
	I′	— ·228			
	II′	+ ·057			

From these corrections, when applied to the individual results in each group respectively or more conveniently to the mean value of each group as tabulated on the tables marked $A_{,,}$ $B_{,,}$ $C_{,,}$, and combined according to weights, we get the final daily mean value for each day of observation as below. Column headed n gives the number of pairs observed each night; the next column of figures the seconds of the resulting latitude φ_1 or the combined result from two or more nights; in the last column the number of observations n_1 is added for convenience of plotting the results, as properly grouped, in order to equalize number of observations for suitable short intervals of days.

Date.	$\varphi = 39° 05'$	n	φ_1	n_1	Date.	$\varphi = 39° 05'$	n	φ_1	n_1
1891.	″		″		1891.	″		″	
June 13	+10·58	16	10·65	31	Aug. 22	+10·43	8	10·43	8
15	·73	15			Sept. 1	·46	15	10·38	31
23	·74	14			3	·31	16		
24	·91	15	10·90	59	7	·35	9		
25	·84	15			9	·51	15	10·39	65
27	11·11	15			10	·42	20		
July 5	10·81	15	10·77	28	12	·28	21		
6	·72	13			13	·11	2		
10	·46	16	10·42	32	14	·49	21	10·44	38
12	·37	16			15	·41	15		
21	·45	16	10·43	29	17	·31	21		
22	·40	13			18	·44	15	10·43	66
27	·58	15			19	·54	15		
29	·44	16	10·52	37	21	·46	15		
31	·61	6			23	·63	15		
Aug. 7	·52	14			24	·12	14	10·36	44
9	·49	14	10·51	43	25	·32	15		
10	·53	15			26	·32	15		
11	·12	4			28	·35	15	10·34	44
14	·62	2	10·43	26	30	·35	14		
15	·48	19			Oct. 3	·54	15	10·42	45
17	·38	1			5	·30	15		

Combination of results for the variation of latitude at Rockville, Md.

Date.	$\varphi = 39°05'$	n	φ_1	n_1	Date.	$\varphi = 39°05'$	n	φ_1	n_1
1891.	"		"		1892.	"		"	
Oct. 9	+10·42	15	10·42	45	Mar. 16	+10·26	8	10·38	31
14	·36	16			19	·32	11		
15	·34	15	10·30	61	20	·27	12	10·36	39
16	·23	15			21	·43	11		
17	·25	15			23	·50	5		
22	·02	5	10·23	20	24	·49	9		
23	·30	15			28	·38	12	10·45	25
24	·05	15			29	·60	4		
28	·17	15	10·22	45	April 4	·70	11	10·65	19
29	·43	15			9	·59	8		
30	·52	15			11	·57	12		
31	·05	1	10·34	46	12	·54	12	10·61	32
Nov. 1	·36	15			13	·77	8		
2	·15	15			16	·63	6		
6	·05	15			19	·67	2	10·80	21
7	·20	15	10·15	45	23	·90	13		
9	·21	15			26	·74	13		
17	·56	6	10·31	20	27	·79	6	10·84	36
18	·20	14			29	11·01	4		
20	·36	15			30	10 92	13		
23	·23	15	10·24	45	May 3	·73	9		
24	·13	15			4	·94	9	10·75	31
29	·11	15			5	·63	13		
30	·49	4	10·23	34	7	·76	13		
Dec. 1	·29	15			8	·79	13	10·77	39
4	·27	6	10·42	20	9	·76	13		
5	·48	14			16	·63	14		
8	·49	14			17	·47	10	10·57	26
9	·72	14	10·57	41	19	·71	2		
10	·49	13			23	·83	14	10·73	25
13	·51	14	10·36	28	24	·61	11		
18	·22	14			25	·73	14	10·68	28
28	·36	14	10·29	28	28	·63	14		
30	·22	14			30	·57	14		
1892.					June 1	·64	12	10·64	41
Jan. 7	·40	12			2	·71	15		
8	·49	5	10·28	29	6	·72	13	10·60	32
16	·08	12			11	·51	19		
20	·15	11			12	·50	17	10·49	33
21	·25	12	10·20	47	13	·48	16		
24	·12	12			15	·44	12		
25	·26	12			16	·45	1	10·52	20
28	·40	3	10·10	15	17	·67	7		
31	·02	12			20	·49	18		
Feb. 10	·06	4	10·38	16	21	·57	4	10·58	41
13	·48	12			23	·67	19		
15	·18	12	10·31	23	25	·45	14		
17	·45	11			26	·23	5	10·52	38
Mar. 10	·51	4			29	·65	19		
12	·60	7	10·38	31	July 7	·58	13		
14	·30	12			8	·31	15	10·45	43
					9	·49	15		

The columns headed φ_1 and n_1 were added as more convenient for graphical representation than the daily values; they contain 52 weighted means depending on 1789 observed values for latitude, and are represented on the accompanying diagram. (Illustration No. 4.)

The main feature of the representation is a decided minimum latitude towards the close of the year 1891 and apparently two epochs of high values of latitude, one of which, however—that about April, 1892—is probably due to some unknown disturbances affecting the observations. Besides this there are two large and systematic declines in the resulting latitudes which in magnitude greatly surpass the minor irregularities of the ordinary zigzag lines of variability. It cannot be supposed that these features are due to real changes in latitude; about this, however, corroborative evidence can be had as soon as the results for changes in the latitude of San Francisco shall have become known.

The probable error of observation for latitude for the period covering the best part of the series—*i. e.*, from July to November—is as follows:

	″
For group III,	± 0·17
For group IV,	± 0·17
For group V,	± 0·17

embracing together 544 observations, and for the period, unfavorable for observation—*i. e.*, from November (in part) to February, inclusive—the probable error of a single observation for latitude becomes

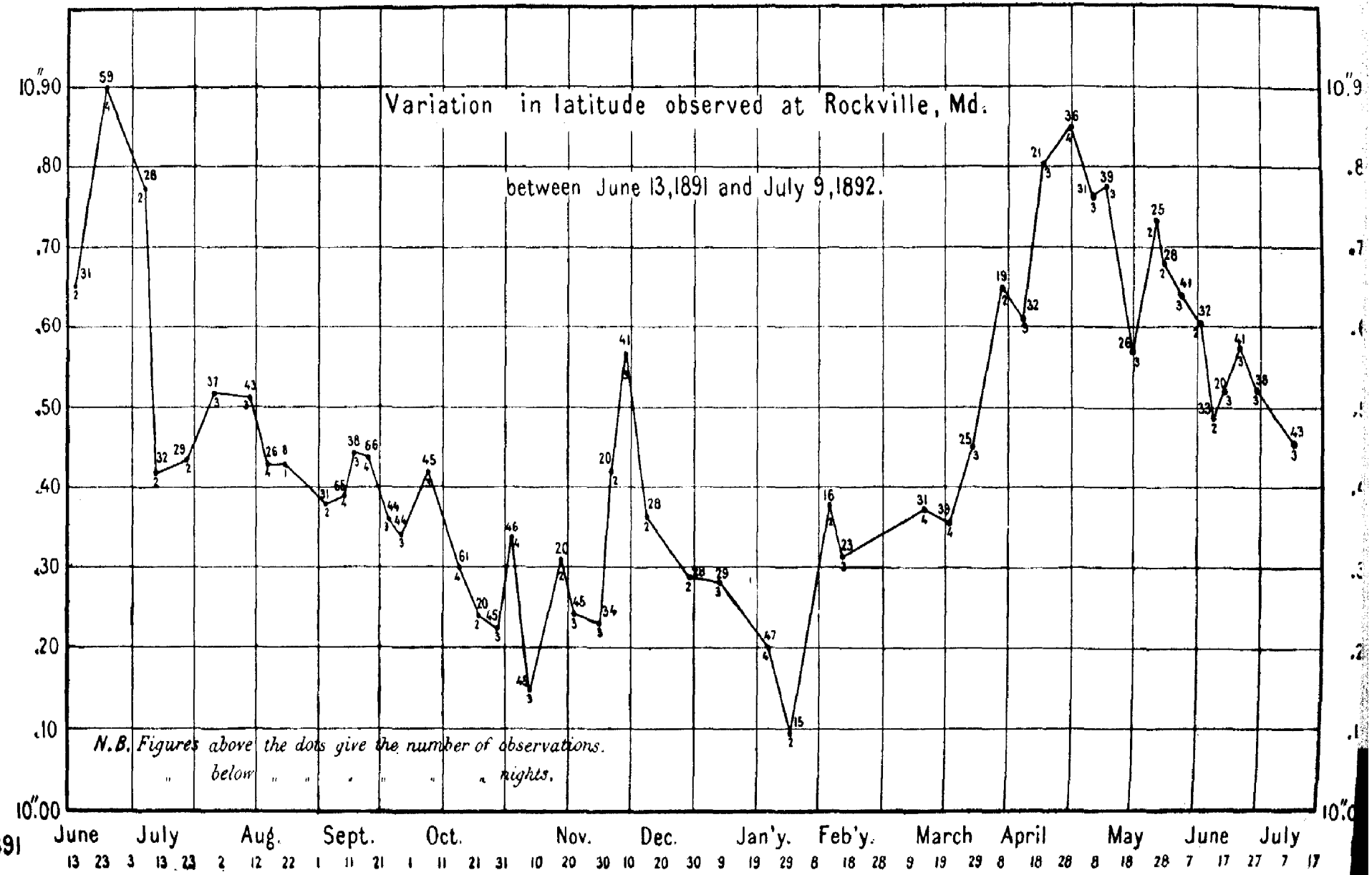
	″
For group VI,	± 0·21
For group VII,	± 0·22
For group VIII,	± 0·26
For group IX,	± 0·20

embracing together 407 observations.*

The average value for the whole series is ± 0″·20 nearly. Comparatively this may be regarded as rather a large value; it may be attributed mainly to the circumstance of the use of three in the place of one micrometer thread, since there was evidence to show that the distances between the threads did not remain invariable. There is also liability to error in recording the particular thread used. The nature of suspected systematic errors, notably those affecting the results of December and January, has so far eluded scrutiny, but is probably related to a feature of obscure origin which may produce a systematic difference in the results of all pairs observed on two consecutive nights, as on September 23–24, when each of 14 pairs gave a latitude in defect on the second night, with an average difference of 0″·51.

The unexpected rise of the latitude in April and the subsequent sharp decline was accentuated by the introduction of the adjusted

*From the 1789 observations on 146 nights it has been found that the mean error of a single determination for latitude is ± 0″·25. Since the results in column ϕ, depend upon eight to sixty-six single results, their mean errors are within ± 0″·03 and ± 0″·09.—*Edwin Smith.*—[Added Nov. 9, 1892.]


Variation in latitude observed at Rockville, Md.
between June 13, 1891 and July 9, 1892.
10".90
.80
.70
.60
.50
.40
.30
.20
.10
10".00
10".9
.8
.7
.6
.5
.4
.3
.2
.1
10".0
N.B. Figures above the dots give the number of observations.
" below " " " " " nights.
1891 June July Aug. Sept. Oct. Nov. Dec. Jan'y. Feb'y. March April May June July
13 23 3 13 23 2 12 22 1 11 21 1 11 21 31 10 20 30 10 20 30 9 19 29 8 18 28 9 19 29 8 18 28 8 18 28 7 17 27 7 17

Combination of results for the variation of latitude at Rockville, Md.

differences between the groups, and hence may be referred, at least in part, to a weak connection of several of the groups.

A further concentration of latitude values by months leads to the following normals:

Date.	φ	n	Date.	φ	n
1891.	″		1892.	″	
June 21	10·81	90	Jan. 20	10·21	91
July 17	10·53	126	Feb. 14	10·34	39
Aug. 12	10·47	77	Mar. 20	10·39	95
Sept. 17	10·39	288	April 18	10·73	108
Oct. 17	10·30	171	May 15	10·71	149
Nov. 14	10·25	190	June 14	10·56	205
Dec. 15	10·43	117	July 8	10·45	43

If we suppose the period of the latitude variation to be 430 days, hence the daily angular motion of the pole of rotation $n = 0°{\cdot}837$, and l = the longitude of this pole west of the place at an arbitrary epoch t_0, here January 1, 1891, then the variation in latitude $\delta\varphi$ may be expressed by $\delta\varphi = \varphi - \varphi_0 = a \cos(nt - l)$, where φ_0 = mean latitude $= 10''{\cdot}47$ and φ = latitude at the time t.

Using Cauchy's method we get the expression

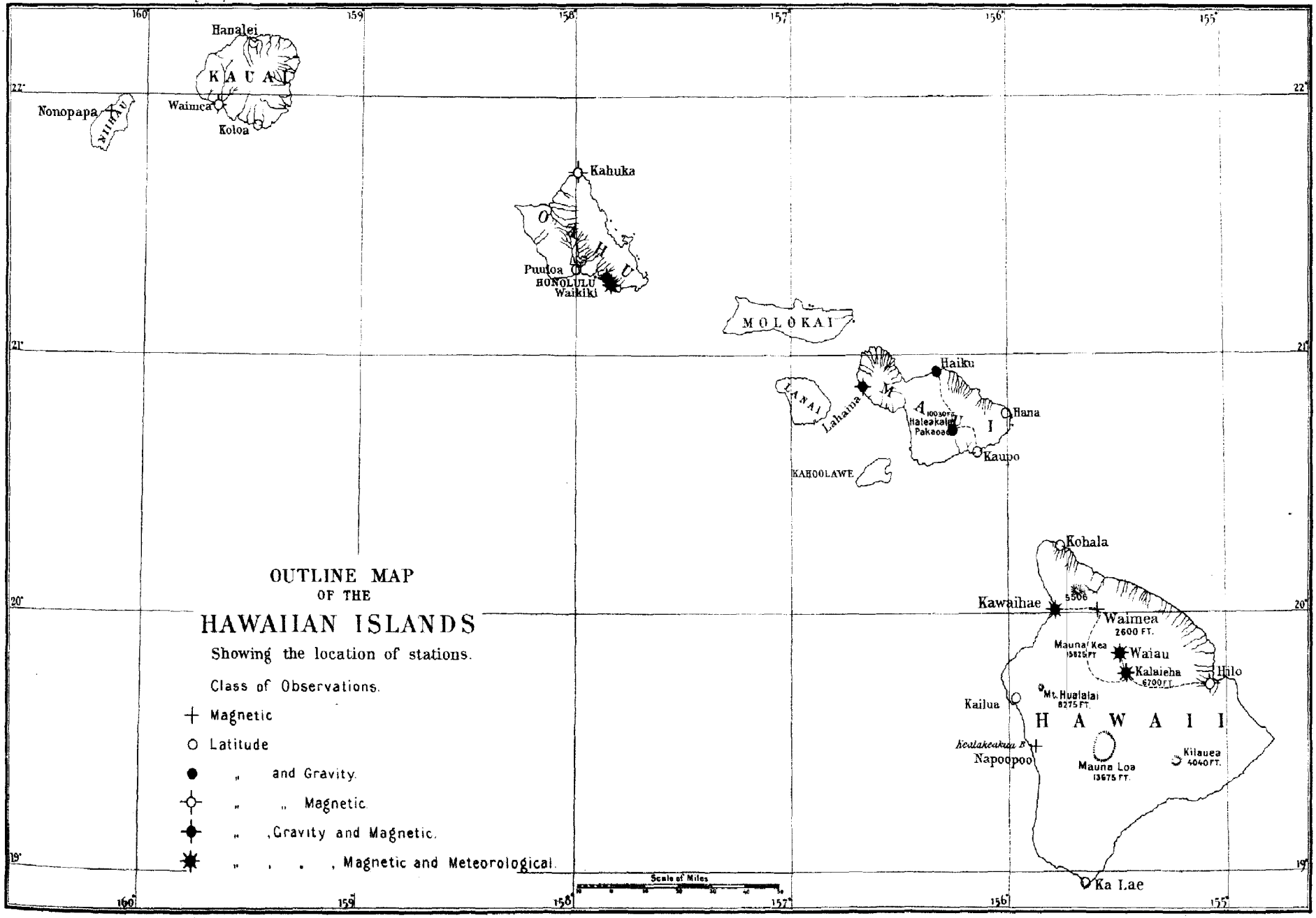
$$\delta\varphi = + 0''{\cdot}20 \sin(nt - 5°)$$

which places the minimum at November 24, 1891.*

Perhaps it will be best to defer further consideration of the observed variability of the Rockville latitude until the results of the other two stations observed during the same time shall have become known.

C. A. S.

*It seems likely that a second term, probably of an annual period, will be needed for the expression of the variation in latitude.—C. A. S.—[Added Nov. 9, 1892.]


160°
159°
158°
157°
156°
155°
22°
21°
20°
19°
Hanalei
KAUAI
Waimea
Koloa
Nonopapa
NIIHAU
Kahuka
OAHU
Puuloa
HONOLULU
Waikiki
MOLOKAI
LANAI
Lahaina
Haiku
MAUI
10030 FT.
Haleakala
Pakaoa
Hana
Kaupo
KAHOOLAWE
OUTLINE MAP
OF THE
HAWAIIAN ISLANDS
Showing the location of stations.
Class of Observations.
Magnetic
Latitude
" and Gravity.
" " Magnetic.
" ,Gravity and Magnetic.
" , , , Magnetic and Meteorological.
Scale of Miles
Kohala
5306
Kawaihae
Waimea
2600 FT.
Mauna Kea
13825 FT.
Waiau
Kalaieha
6700 FT.
Hilo
Mt. Hualalai
8275 FT.
Kailua
HAWAII
Keulakeakua B.
Napoopoo
Mauna Loa
13675 FT.
Kilauea
4040 FT.
Ka Lae

APPENDIX NO. 2—1892.

THE VARIATION OF LATITUDE AT WAIKIKI, NEAR HONOLULU, HAWAIIAN ISLANDS, AS DETERMINED FROM OBSERVATIONS MADE IN 1891 AND 1892 IN COÖPERATION WITH THE INTERNATIONAL GEODETIC ASSOCIATION.

A report by E. D. PRESTON, Assistant.
Submitted for publication, May 15, 1893.

In compliance with the instructions of the Superintendent, there were made in the Hawaiian Islands, between May 8, 1891, and September 14, 1892, the following observations:

I. Of the transit of Mercury on May 9, 1891.
II. Of the variation of latitude from June 6, 1891, to June 25, 1892.
III. Of the force of gravity on Oahu and Hawaii from June 9, 1891, to July 25, 1892.
IV. Of the magnetic elements at fourteen stations at intervals from August 11, 1891, to September 9, 1892.
V. For latitude on the island of Hawaii at the base, on the flank, and at the summit of Mauna Kea, from July 4, 1892, to July 25, 1892.
VI. Of meteorological phenomena, including the barometric determination of the height of Mauna Kea (nearly 14,000 feet), from July 1, 1892, to July 27, 1892.

Illustration No. 5 shows the location of these stations, as well as of those that had been previously occupied for scientific observations under the direction of Prof. W. D. Alexander, the Surveyor-general of the Hawaiian Government.

The following table gives a summary of the season's work:

Summary of observations in the Hawaiian Islands in 1891–'92.

Station.	Island.	Date of occupation.	Class of observations.	Nights or days of observations.	No. of determinations.	Remarks.
		1891. 1892.				
Waikiki.	Oahu.	June 6–June 25	Latitude.	220	2434	
		June 9–June 11	Gravity.	199	827	
		June 9–June 11	Time.	202	202	
		1891.				
Kahuku.		Aug. 11–Aug. 13	Magnetic.	3	3	
		Nov. 25–Nov. 27	do.	3	3	
		1892.				
Honolulu.		June 23–June 25	Gravity.	3	57	
		June 15–June 27	Time.	10	10	
		June 2–June 4	Magnetic.	3	3	
Kawaihae.	Hawaii.	July 4–July 6	Latitude.	3	23	
		July 3–July 6	Gravity.	4	91	
		July 3–July 6	Time.	4	4	
		July 1–July 3	Magnetic.	3	3	
		June 30–July 7	Meteorology.	8		
Waimea.		July 8	Magnetic.	1	1	West base.
		July 9–July 11	do.	2	2	Old station.
		July 7–July 11	Meteorology.	5		
Kalaieha.		July 14–July 15	Latitude.	2	3	
		July 14–July 16	Gravity.	3	67	
		July 14–July 18	Time.	6	6	
		July 14–July 16	Magnetic.	3	3	
		July 13–July 18	Meteorology.	6		
Waiau.		July 21–July 25	Latitude.	4	55	
		July 22–July 25	Gravity.	4	81	
		July 21–July 25	Time.	4	4	
		July 21–July 24	Magnetic.	3	3	
		July 21–July 26	Meteorology.	6		
Hilo.		July 30–Aug. 3	Magnetic.	5	3	
Napoopoo.		Aug. 18–Aug. 21	do.	4	3	
Lahaina.	Maui.	Aug. 23–Aug. 25	do.	3	3	
Waimea.	Kauai.	Sept. 2–Sept. 3	do.	2	2	Latitude station, 1887, and transit of Venus, 1874.
		Sept. 5–Sept. 6	do.	2	2	Thorny Croft.
Nonopapa.	Niihau.	Sept. 9	do.	1	1	

In the column "Number of determinations" the figures indicate: For latitude, the number of pairs of stars observed. For gravity, the number of intervals, each one giving a value for the period of oscillation of the pendulum. For time, the number of sets of stars, each one giving a correction to the chronometer. For terrestrial magnetism, the number of determinations of all three elements, declination, dip, and horizontal intensity, and the time and azimuth.

The meteorological observations were made many times during the day. The barometer was read at the times of maximum and minimum—at 9 a. m. and 3 p. m. On the summit of Mauna Kea it was read more frequently.

The National Academy of Sciences, the United States Coast and Geodetic Survey, and the Hawaiian Government have each contributed to the accomplishment of this work. The first named supplied the

On the variation of latitude at Waikiki, Hawaiian Islands.

funds for the prosecution of the international latitude observations, our own service furnished the instruments and observer, and the Hawaiian Government survey defrayed most of the expenses for the gravity and magnetic determinations. I left Washington on April 18, 1891, and returned on October 16, 1892.

The results of the observations of the transit of Mercury have already appeared as Appendix No. 12 of the Report of the Superintendent for 1891, Part II.

It is proposed in the present paper to deal with the international latitude observations only, reserving the last four heads for a subsequent report. This work was undertaken at the request of the International Geodetic Association. The following extract from a letter written by Prof. Helmert to the Superintendent of the Coast and Geodetic Survey, under date of October 18, 1890, refers to our coöperation, gives reasons for sending two observers, and specifies the conditions necessary for successful work:

> . . . Da die Geldmittel der permanenten Commission etwas beschränkt sind, so ist beschlossen die Expedition nur auf 1 Jahr auszudehnen, wovon 2 Monate auf die Reise und 10 auf Beobachtungen kommen. Auch kann 1 Beobachter (Dr. Marcuse bisher in Berlin Beobachter) und nur 1 Instrument ausgesandt werden. Die hiesigen Erfahrungen haben aber gezeigt, dass sehr leicht in einer Reihe durch Krankheit des Beobachters, Versagen des Instruments, etc., Unterbrechungen entstehen.
>
> Deshalb ist es dringend erwünscht, dass Ende Februar 1891 von der Coast and Geodetic Survey gleichzeitig ein Beobachter und ein Instrument nach Honolulu gesandt werden, so dass doppelte Reihen gewonnen werden wie seinerseit hier in Berlin und Potsdam. Natürlichweise muss das Instrument auch die Genauigkeit einer Breitenbestimmung aus einem Sternpaare an einem Abend von 0″·1 bis 0″·2 gewähren. Das wird sich ja wohl erreichen lassen.

The observations were treated according to the method adopted by Prof. Albrecht in his reduction of Dr. Marcuse's work as embodied in the "Resultate der Beobachtungsreihen in Honolulu, betreffend die Veränderlichkeit der Polhöhe, Berlin, 1890," with this difference, that the adjustment in the present case involved five conditional equations. This was made necessary from the fact that group II was again observed in 1892, and also that during the year's observations three separate combinations of three groups were made.

DESCRIPTION OF STATION.

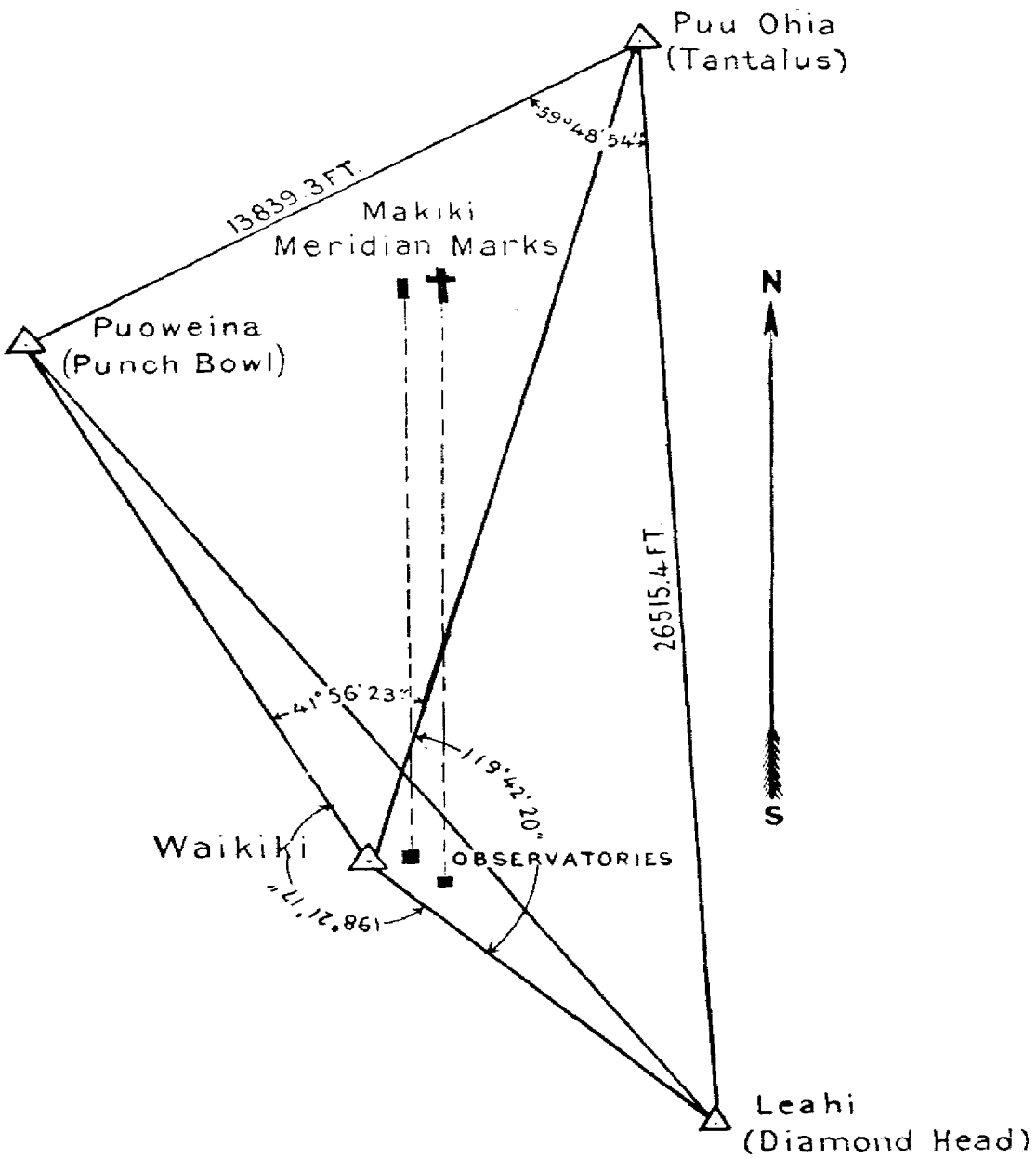
The station of observation was located at Waikiki, 2 miles SE. of Honolulu, on the property of Mr. J. F. Brown, who generously gave the use of the ground for the whole time of occupation. The position of the observatory is shown in illustration No. 10, which also includes the three triangulation points to which the Waikiki triangulation station was referred. The distance from the seashore to the zenith telescope pier is 400 feet. The geodetic difference as given by

Mr. Lyons between the positions of the Government Survey observatory in Honolulu and the U. S. Coast and Geodetic Survey observatory at Waikiki is as follows:

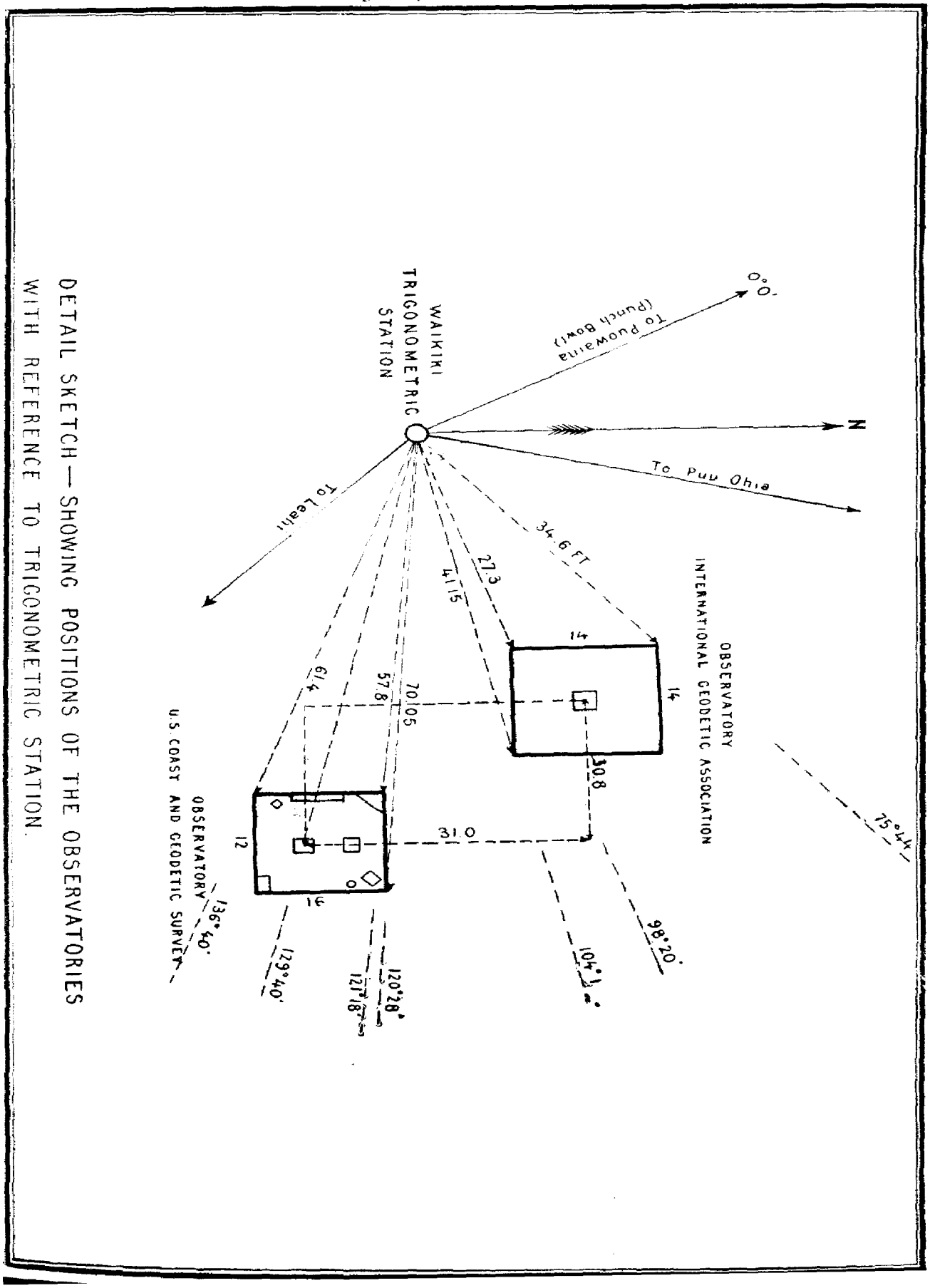
$$d\varphi = 9658 \text{ feet} = 1' \ 35''{\cdot}5$$
$$d\lambda = 9785 \ \text{“} \ = 1' \ 43''{\cdot}6 = 6^{s}{\cdot}9$$

The astronomical latitude of the former (Coast and Geodetic Survey Report, 1888, Appendix 14, p. 495) is 21° 18′ 2″·48. This determination was made about April 1. The latitude of Waikiki at this season of the year is not far from 21° 16′ 24″·70, so that we have a difference of astronomical latitudes of 1′ 37″·8, which would indicate a deflection of the plumb line between Honolulu and Waikiki of 2″·3 in latitude. The astronomical amplitude being greater than the geodetic one, would indicate that the plumb line at the northern station (Honolulu) was deflected farther south than the one at Waikiki. This result is not in accordance with what we might expect, since the great mass of the island lies to the northward, and the greater proximity of Waikiki to the seashore would tend to draw its vertical to the southward. The astronomical difference of longitude between the same two stations was determined by Mr. C. J. Lyons, assistant in charge of the Government Survey Office, and myself. The method employed was by exchanging signals over the telephone wire. The chronometers were rated at the two observatories, and the signals were sent in both directions by tapping on the transmitter box. A number of taps were made at different parts of the second-hand dial to eliminate any effect of eccentricity. The Waikiki chronometer had a rate of about 20^{s} daily, which enabled us to get a coincidence of beats nearly every day of the exchanges, so that there were no fractional parts of a second to be estimated. No high degree of precision was expected in the results, yet they agreed within one-fifth of a second for the six days. Stars were employed at Waikiki in getting the diurnal correction, whereas at Honolulu the sun was used. On one day the exchanges were made to depend entirely on the sun at both places as a test. This day's work agreed well with the others. The azimuth of the Honolulu instrument was not very accurately known; but as the sun only was observed at this place, and as moreover its declination at the time was about 21° 30′, and the latitude of the station was 21° 18′, it is evident that the azimuth was without any effect on the time observations. The mean result from several days' work was 7^{s}·4, which gives a difference between the astronomical and geodetic amplitude of 0^{s}·5, the astronomical being the greater. The connection of the two stations would seem to indicate a deflection of the plumb line at Honolulu towards the ESE., or in the opposite direction at Waikiki.

The connection of the Waikiki triangulation point with the Government Survey stations is shown in illustration No. 10. This connection was made independently by Mr. J. S. Emerson and myself. The two measures checked each other and the agreement was satisfactory.



CONNECTION OF OBSERVATORIES WITH OAHU TRIANGULATION.



DETAIL SKETCH — SHOWING POSITIONS OF THE OBSERVATORIES WITH REFERENCE TO TRIGONOMETRIC STATION.

On the variation of latitude at Waikiki, Hawaiian Islands.

With a 6-inch theodolite, kindly loaned by Prof. Alexander, I measured at Waikiki the three angles defined by Punch Bowl, Tantalus, and Diamond Head, thus closing the horizon and determining the position of Waikiki by the three-point problem.

The result of my own measures gave the following geodetic position for the zenith-telescope pier in the observatory:

	°	′	″
$\varphi =$	21	16	26·7
$\lambda =$	157	50	1·2

As the observatories and the piers above ground were removed after the completion of the year's work, it was necessary to connect their positions with the permanent mark at the Waikiki trigonometric station. This was done, and the measurements are given in illustration No. 11.

The observatory was built by Harrison Bros., of Honolulu, and was essentially similar in construction to that built by Dr. Marcuse for the International Association, the Coast and Geodetic Survey building being somewhat different in shape to accommodate a greater number of instruments. Illustration No. 6 gives an exterior view and shows the method of ventilation, the large observing slits, and the means of opening these by a system of pulleys. An interior view appears in illustration No. 7, and shows the disposition of the instruments and indicates the particular class of work to which each one was devoted.

A plan (illustration No. 12) is added, drawn to scale, giving exact dimensions.

INSTRUMENTS.

The entire instrumental outfit taken on the expedition was as follows: Zenith telescope No. 2; meridian telescope No. 2; reconnoitering telescope No. 30; pendulum apparatus B; chronograph, Fauth, No. 6; chronograph, Breguet, No. 5; magnetometer No. 11; dip circle No. 21; mercurial mountain barometer No. 3388; aneroid mountain barometer No. 211; Baudin thermometers 9252, 11316, 11319; sidereal break-circuit chronometer Negus, 1825; mean time break-circuit chronometer, Bond, 177; sidereal watch 1335020; photographic outfit.

The following miscellaneous objects were also taken: Two tents, 8 by 8; wall tent and fly, 10 by 12; tarpaulins; 8 dry-cell batteries; 8 Fuller compound batteries; chronometer cases; electric lamps; switches; oil stove; metre scale; plane-table compass; steel tape, and insulated wire.

THE ZENITH TELESCOPE.

This instrument was used in all the latitude observations at Waikiki. It has a focal length of 112·1 cm., with a clear aperture of 8·2 cm. It is shown in illustration No. 8. A power of 100 was used in the observations.

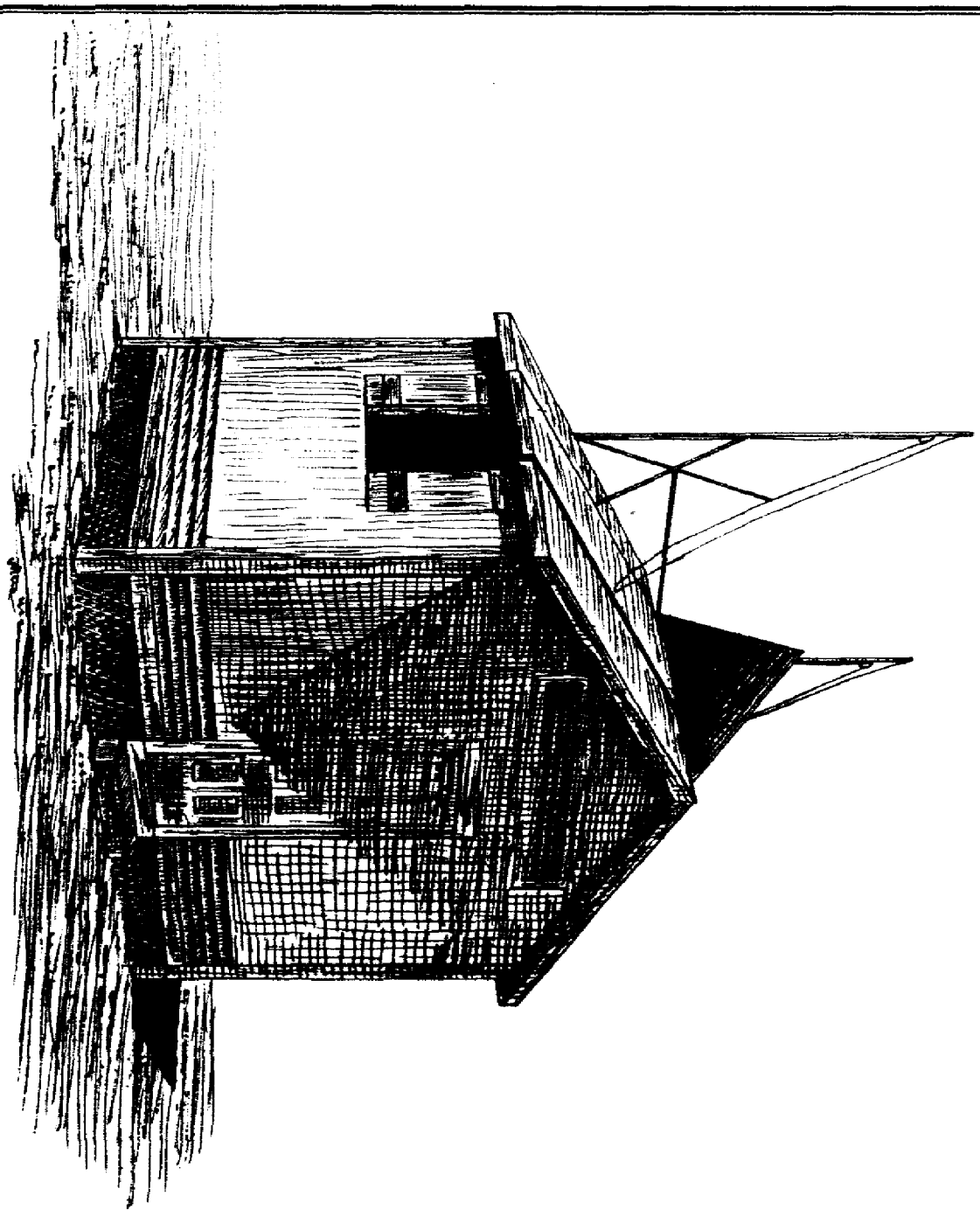
Although very easy of manipulation it has serious defects, a few of which will be briefly indicated. The instrument was much too light. A moderate wind affected the levels, and frequently it was necessary to reject observations on account of the uncertainty of the level reading. On one or two occasions of strong wind it was necessary to give up work altogether.

The fact that the time was very limited before leaving Washington will sufficiently explain the haste necessary in the preparation. The zenith telescope was an adaptation of an old form of instrument. The tube is placed centrally, and the remodeling involved the construction of a new projecting platform at the top of the vertical axis. The horizontal axis was made longer than in the old form. The supporting tripod at the base was confessedly much too light, and the double foot-screws, although they gave a fine means of adjustment, possessed very little stability. This defect, combined with the somewhat difficult motion of the micrometer screw, produced considerable discrepancy between the level readings before and after the bisections. This effect was nearly always in the same direction. It varied from 1/10 of a division to 8/10 of a division, and indicated that the whole instrument was pushed bodily by the manipulation of the micrometer and revolved about a line passing through the lower part of its base. It is evident that the clamps holding the perforated arm and the fitting of the pivots in the horizontal axis resisted the pressure at the ocular much better than the total weight of the instrument.

No provision was made for reading the micrometer while sitting, because it had been recommended to make only one bisection. When for various reasons it was deemed advisable to point three times on the star, a mirror was attached at an angle of 45° to the line of sight, which enabled the observer to make the readings of the micrometer by reflection without changing position.

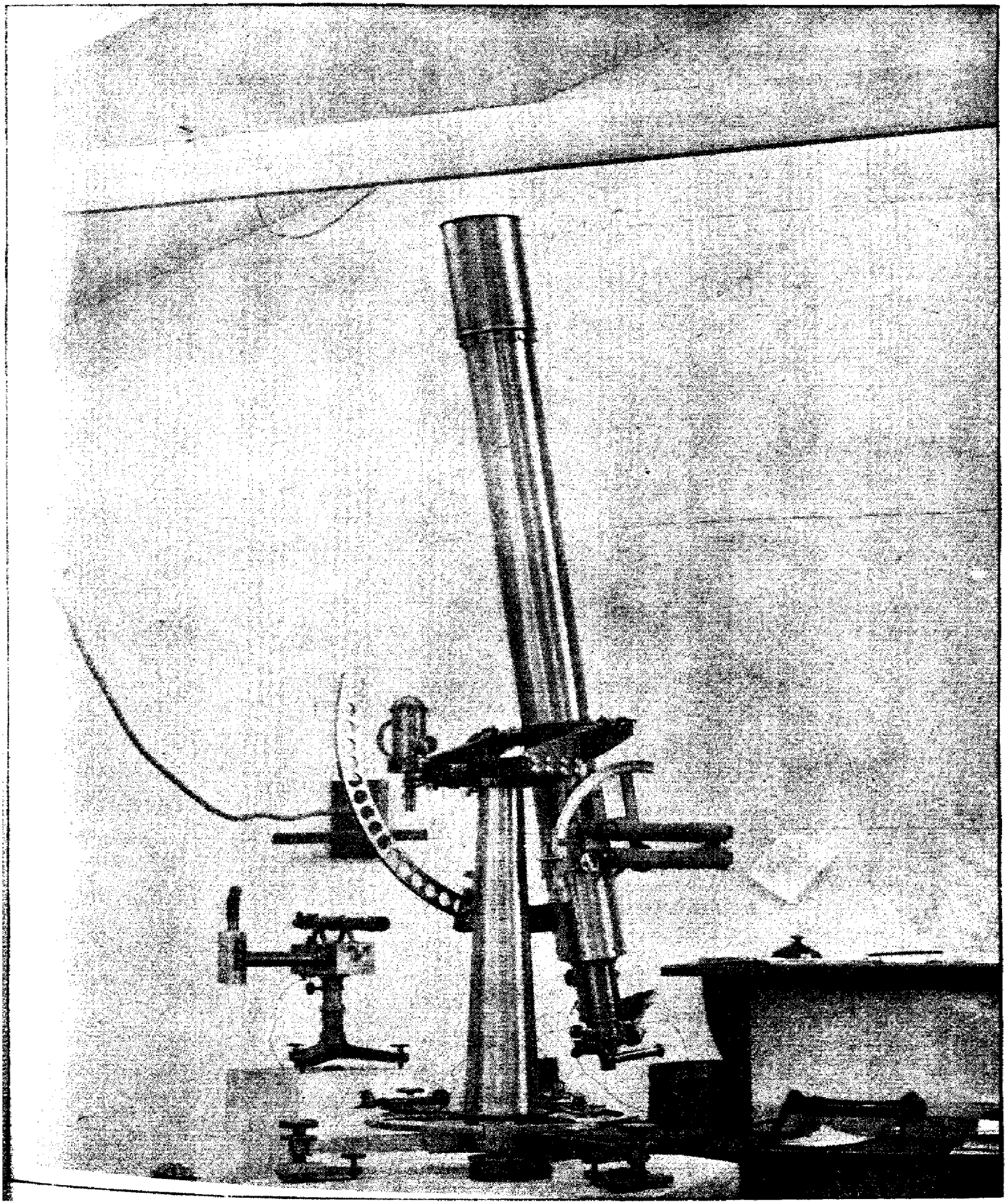
By reference to illustrations Nos. 7 and 12, it will be seen that the meridian telescope was mounted north of the zenith telescope. This made it possible to use the former as a collimator for the latter, and as the meridian telescope was kept accurately in the meridian from daily star observations for use in the gravity work, a fine meridian mark was always available inside the observatory. In addition to this a mark was placed on Makiki Ridge 13,000 feet away exactly north of the two piers. This was done by Mr. F. S. Dodge, and its position is shown in illustration No. 9. The deviation in azimuth of the meridian telescope was seldom more than a few tenths of a second of time. The zenith telescope was examined frequently, and was never found out of adjustment beyond the limits prescribed for this kind of work.

The stops on the horizontal circle used in defining the plane of the meridian were only moved when it was necessary to observe circumpolar stars for micrometer value, and only on one or two occasions were latitude and micrometer both observed on the same night. When this

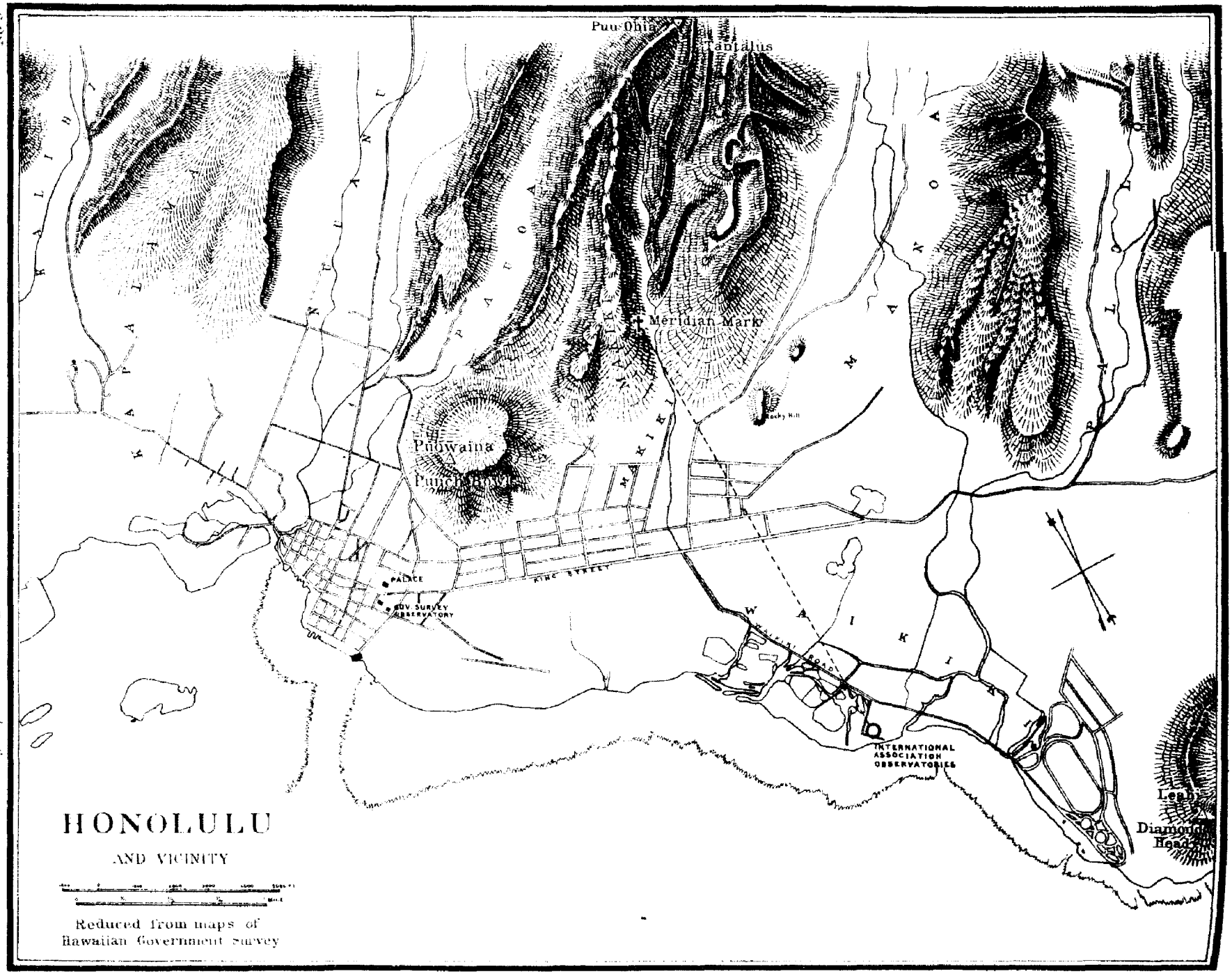


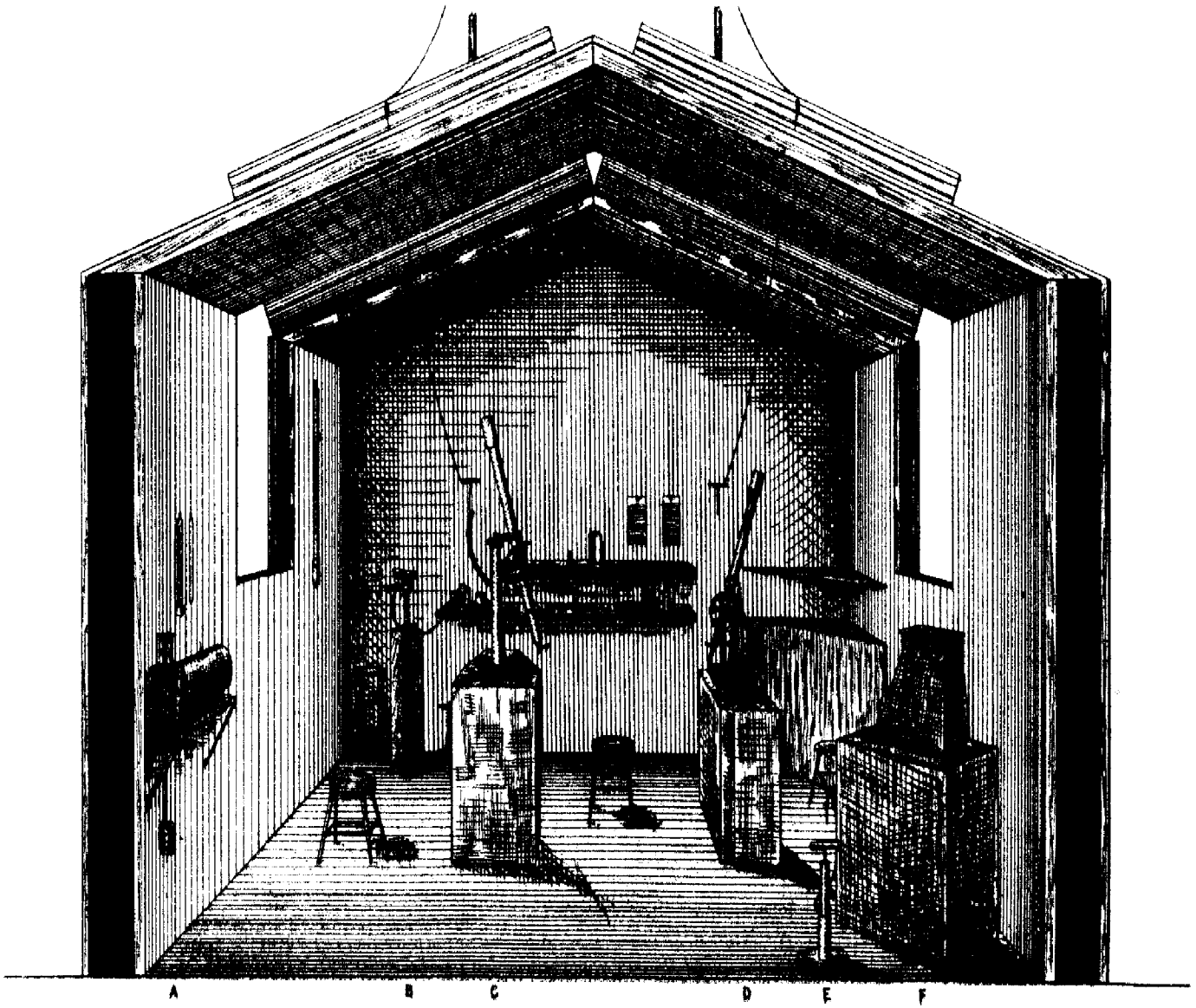
U.S. COAST AND GEODETIC SURVEY OBSERVATORY

WAIKIKI, H.I.



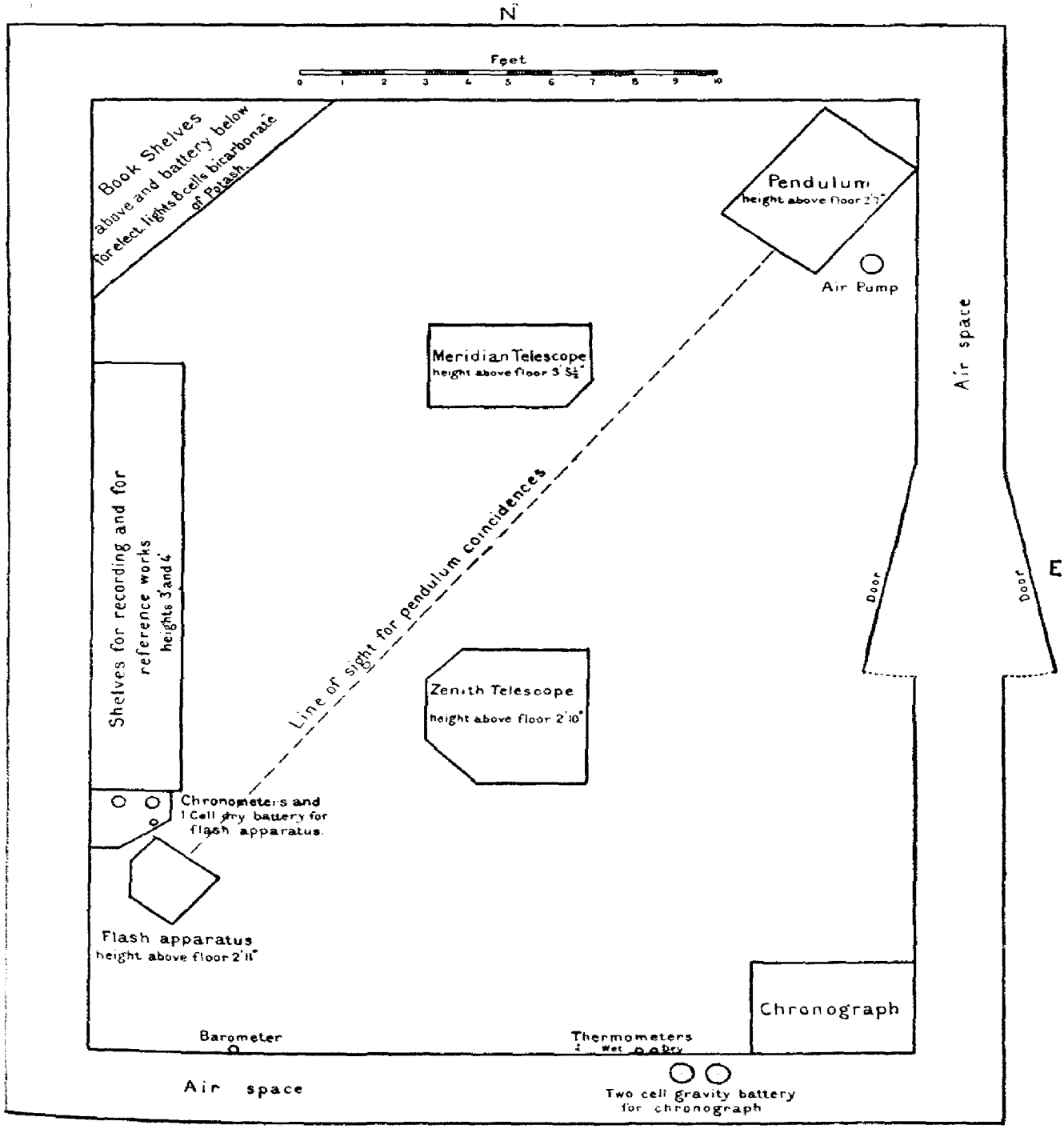
ZENITH TELESCOPE No. 2.


Puu Ohia
Tantalus
Meridian Mark
Punchbowl
Punchbowl
KING STREET
PALACE
GOV. SURVEY OBSERVATORY
INTERNATIONAL ASSOCIATION OBSERVATORIES
Diamond Head
KALIHI
NUUANU
PALOLO
MANOA
WAIKIKI
HONOLULU
AND VICINITY
Reduced from maps of Hawaiian Government Survey



A - CHRONOGRAPH (TIME)
B - FLASH APPARATUS (GRAVITY)
C - ZENITH TELESCOPE (LATITUDE)
D - MERIDIAN TELESCOPE COMBINATION INSTRUMENT (TIME AND LATITUDE)
E - AIR PUMP FOR VACUUM CHAMBER.
F - PENDULUM RECEIVER (GRAVITY)

Interior View of Observatory



Plan of Observatory

On the variation of latitude at Waikiki, Hawaiian Islands.

was done, the stops were replaced by means of the graduation of the horizontal circle. Indeed the south stop was always so set. The slight deviation of the instrument from the plane of the meridian has very little effect on the resulting value for the latitude if the north and south azimuth are the same, the greatest correction at Waikiki for a zenith distance of 30° being less than 0″.01 for an azimuth of 10ˢ. When the two azimuths are different the correction is greater; but even in this case if the azimuths do not exceed 10ˢ the correction is hardly worth taking into account. The horizontal circle of the telescope reads to 30″, and the most violent assumption could never place the instrument out of the meridian more than one minute of arc. Admitting the north azimuth to be practically zero, the correction to the latitude for south stars would still be less than one-hundredth of a second.

The following table gives the correction to the latitude for various zenith distances at Waikiki. The first column is on the supposition that both the N & S azimuths are 10ˢ; in the second the N one is considered zero and the S one 10ˢ.

Zenith distance.	Corrections to latitude.	
°	″	″
0	0·000	0·001
5	·000	·003
10	·000	·005
15	·002	·008
20	·003	·011
25	·005	·016
30	·007	·020

The corrections to the observed latitudes on account of errors of level and collimation were sensibly constant throughout the year and practically insignificant.

The effect of an inclination in the horizontal axis is to make the telescope describe a great circle whose inclination to the plane of the meridian is given by the level readings on the horizontal axis. The star will be observed when it has an hour angle equal to the inclination of the axis in seconds of time multiplied by the level factor B, depending on the star's declination and zenith distance. The effect of an error in collimation is to make the line of light describe a small circle whose distance from the plane of the meridian is measured by the angle between the line of sight and the axis of collimation. The hour angle of an observed star is the distance on the equator in seconds of time multiplied by the secant of the declination. Now the effect on a measured zenith distance will be given in either of these cases by the formula

$$\frac{2 \sin^2 \frac{t}{2}}{\sin 1'' \cdot \frac{1}{2} \sin 2\delta}$$

Illustration No. 13 shows the values of the azimuth, level, and collimation factors for Waikiki for every 10° of declination. The greatest value for the level factor for the latitude stars is 1·35, while that of the collimation factor is 1·56. In order to produce a correction to the latitude exceeding 0″.01 for the highest stars, it would be necessary to suppose a value for the inclination of the horizontal axis of about 1 minute of arc and of several seconds for the error of collimation, neither, of which suppositions is admissible. The factors for relative weights, azimuth, level, and collimation in the time reductions were taken from the plotted sheet, and considerably facilitated the reductions.

REDUCTION TO THE MERIDIAN.

In the field reduction of the latitudes the correction for meridian distance was made on the supposition that the star's path from A to B and from B to C in Fig. 1 was a straight line.

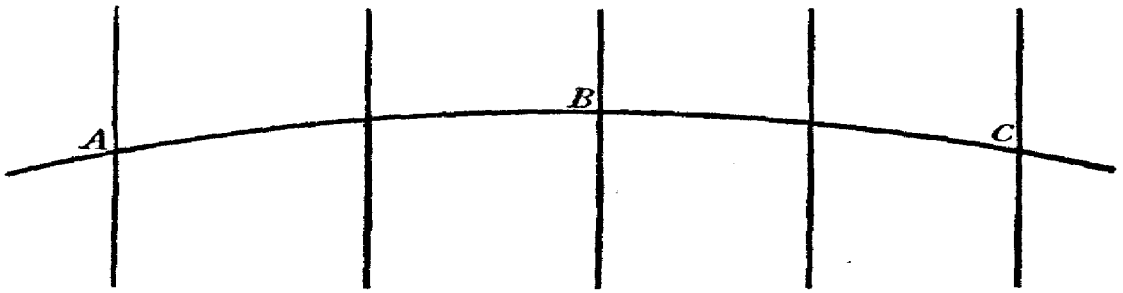
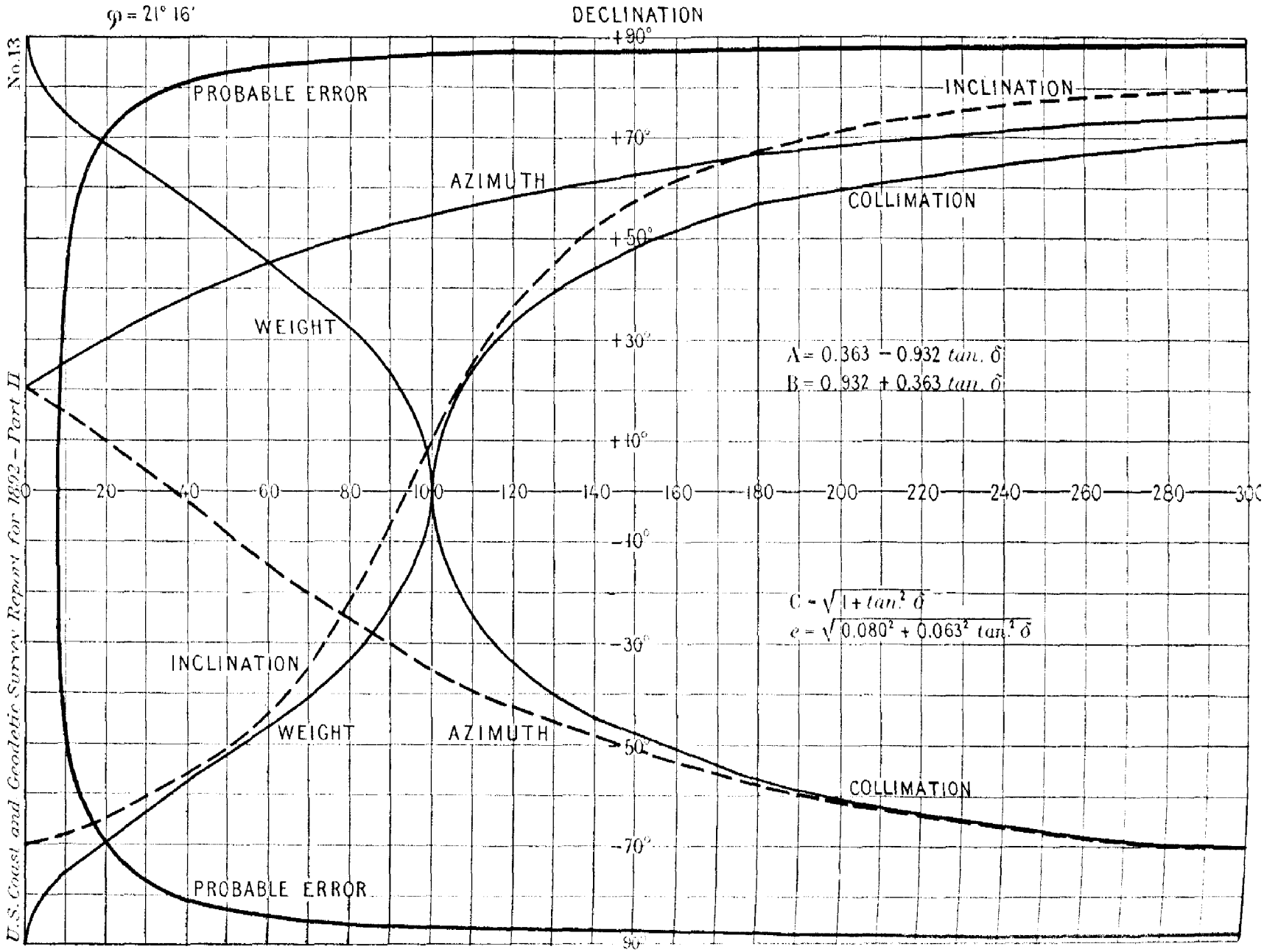


FIG. 1.

The equatorial interval of the vertical threads is 9ˢ·5. The bisections were made at A, B, and C, or rather the first bisection was never perfected until the star had passed A and the third was finished before it arrived at C, but it is obviously impracticable to designate within several seconds for each particular star just when the pointing was concluded. The error introduced by the above supposition is very much inside that of observation and vastly inferior to the probable error of the declination, and was therefore not considered important in the field reduction. In the final computation, however, a rigorous correction was applied, and the quantity $\frac{2\sin^2\frac{t}{2}}{\sin 1''}\frac{1}{2}\sin 2\delta$ being represented by m, a correction of $\frac{m}{3}$ was applied to the mean of the three micrometer readings, $\frac{2m}{3}$ being the reduction for a measured zenith distance, and one-half of this being the correction to the latitude.



FACTORS AND WEIGHTS FOR REDUCTION OF STAR OBSERVATIONS

WAIKIKI, H.I.

On the variation of latitude at Waikiki, Hawaiian Islands.

THE MICROMETER.

The value of one revolution of the micrometer screw was determined 12 times with the following results:

Date.	Star.	Intervals.	Elongation.	Value.	Probable error.
1891.				″	″
Aug. 21	Polaris.	Half turns.	East.	46·402	± ·013
30	do.	do.	do.	·400	·012
Sept. 17	do.	do.	do.	·348	·018
Nov. 8	δ Ursæ Min.	Whole turns.	West.	·322	·015
8	51 Cephei.	do.	East.	·413	·014
22	δ Ursæ Min.	do.	West	·342	·015
22	51 Cephei.	do.	East.	·433	·024
1892.					
May 21	δ Ursæ Min.	do.	East.	·452	·017
21	51 Cephei.	do.	West.	·439	·012
June 5	δ Ursæ Min.	do.	East.	·401	·026
10	do.	do.	do.	·391	.025
12	do.	do.	do.	·427	·028

Mean value 46″·397 ±″·008

With regard to these determinations it should be remarked that the situation of Waikiki is very unfavorable for such an investigation. The altitude of the stars used was only 21°. This generally would give an unsteady atmosphere, but in addition we had high mountains immediately north of the station, around which the clouds continually hung. The probable error of the determination is therefore large. But when we consider that the zenith distances of the latitude stars are almost perfectly balanced for each group, thus eliminating in the final result any error in the value of the micrometer screw, this slight uncertainty becomes of no importance.

The screw was examined for irregularities in the whole turns in the following manner: The reduced times from 33 turns to 3 turns were tabulated and from them the mean time for one turn was determined. This value being applied to the mean of all the tabulated times, we have the theoretical values corresponding to the different turns, and the difference between these theoretical values and those actually obtained by observation gives the irregularities of the screw at the different turns.

No very decided irregularity was discovered except at the seventh and eighth and thirty-second turns, and even here the correction to the resulting latitudes was too small to be of real importance. Moreover, the number of observations which during the course of the work actually fell on these particular divisions was almost insignificant. No correction was adopted for the periodic inequality, as the observations were scanty and failed to bring out any pronounced fact in regard to it. The range of temperature for the entire year was not more than 11° C., so that the changes of temperature can hardly be said to have influenced the micrometer values.

INCLINATION OF THE MICROMETER THREAD.

For a short time after the beginning of the work the micrometer thread was slightly inclined to the tangent to the star's path at culmination. An adjustment was made on June 20, and its position was not subsequently disturbed. Before this date, all zenith distances which have not corresponding observations before and after the meridian require a very appreciable correction from the above-mentioned cause. After June 20 the adjustment was such that no inclination could be detected by a cursory inspection of the observations. Nevertheless, by combining many of the bisections it was found that the mean inclination gave a value which would still be appreciable when the observation was some distance from the meridian.

Two values for the inclination were therefore deduced—one applicable before and the other after June 20. It is assumed that the inclination did not change between this date and the close of the work more than a year later. This was tested by a careful scrutiny of the record at several epochs during the interval, the greatest difference between the mean value adopted for the inclination and that deduced from individual groups only causing a difference of one or two hundredths of a second in the correction to the latitude.

VALUES OF THE LEVELS.

Zenith telescope No. 2 was provided with two levels made by Reichel. Dr. Marcuse kindly brought these to Washington on his way to Honolulu. They are distinguished as A and B. The values of one division were determined by the maker, by Dr. Marcuse at Berlin and by Mr. Fischer in Washington.

The following were the results:

Values of levels.

Observer.	A.	B.
Reichel	1″·190	1″·276
Marcuse	1″·132	1″·076
Fischer	1″·211	1″·318
Mean	1″·178	1″·223

In December, 1891, a very exhaustive determination was made by myself at Waikiki, using the micrometer attached to the zenith telescope. All parts of the screw were used from 5 turns to 31 turns, and in all about 70 individual values were obtained. When the mean was taken the result was, for A, 1″·194; for B, 1″·230. This was so near the mean of all the others that it was adopted in the latitude computation.

On the variation of latitude at Waikiki, Hawaiian Islands.

The level vial as mounted on the telescope is incased in a hollow glass tube, to protect it against sudden changes of temperature. This glass tube is itself covered, except directly above the level scale, by a blanket of thick baize. Attention has sometimes been called to an error of reading from parallax on account of this outer glass tube. An inspection of the formula for the level correction shows the effect of errors from imperfect readings is indeed very small.

If N and S are the readings of the north and south end of the level before reversal, and n and s the readings after reversal, the correction to the latitude, disregarding for the moment the sign, is

$$\frac{(N + S) - (n + s)}{2} \cdot \frac{1}{2}$$

the first factor being the mean motion of the bubble. Now if we suppose one of these readings liable to an error of one-quarter of a division—*i. e.*, supposing the reading was only made to the nearest half division and 1 division=1″·21—the error in the latitude correction is 0″·07, and when combined with the result from another level the error would only be 0″·03. Each latitude correction depends on 8 readings (each end of both levels before and after reversal) so that in the mean reading there would necessarily be some compensation of errors, and the effect on the final latitude would certainly not be more than 0″·01, even if the readings were only made to the nearest half division. Moreover, the above formula shows that if the north end was read too large both before and after reversal the latitude correction is unchanged, and similarly for the south end. This explains to a certain extent the fact that the deduced latitudes are about as concordant where only one reading was made for each star as where the level was read both before and after making the bisections with the micrometer. It is evident that if the influence on the level, consequent upon making the bisections, is in the same direction and of the same amount for both stars the resulting value of the latitude is unchanged.

The large range in the individual values is due principally to the inaccuracies of the level indications. In several cases the original notes remark that the level was moving at the time of reading. These values, however, were retained because the discordant value of the latitude arises in this case from a condition inherent to the instrument, and there is no valid reason for rejecting them.

SPECIMEN OF PROGRAMME OF WORK AND RECORD OF OBSERVATIONS.

The determinations of latitude, time, and gravity during the same evening made it necessary to utilize the intervals between the star pairs, and sometimes between individual stars of the same pair, by observing for time with the meridian telescope and noting the pendulum coincidences with the gravity apparatus.

The following schedule gives a specimen of the programme from $18^h\ 40^m\ 10^s$, the time of observation of the first star of pair 1 of Group III, to $18^h\ 55^m\ 21^s$, the time of the first star of pair 2 of the same group:

Time.			Occupation.
h.	*m.*	*s.*	
18	40	10	Observe star No. 2633 (Pulkowa).
	40	40	Record bisections, read levels and record.
	42	00	Read inside and outside thermometers and wet and dry bulb.
	44	00	Observe star No. 2647.
	44	50	Start chronograph and switch in chronometer.
	45	50	Observe β Lyræ for time.
	46	10	Set zenith telescope for next pair of latitude stars.
	48	00	Compare chronometers and mark sheet.
	48	45	Point telescope for o Draconis.
	49	15	Observe o Draconis.
	50	00	Set telescope for ϑ Serpentis (pr.).
	50	30	Observe ϑ Serpentis.
	51	00	Stop chronograph, switch out chronograph and start flash apparatus.
18	51	45	Start pendulum.
	52	00	Read manometer and thermometer on dummy and record them.
	54	00	Observe first coincidence and make record.
	55	00	Observe star No. 2681 for latitude.

Specimen of latitude record, $18^h\ 40^m\ 10^s$ to $19^h\ 3^m\ 7^s$.

Group III.—July 4.

Pair.	Micrometer.				Level A.		Level B.	
	Turns.	Divisions.			N.	S.	N.	S.
1	25	82·5	81·0	81·0	11·9	34·0	62·8	82·6
	13	98·0	2·0	0·0	31·0	9·0	79·9	60·0
2	29	0·0	0·0	99·0	9·0	31·0	60·3	80·0
	11	62·8	62·7	62·5	31·0	9·0	80·0	60·1

From June 6, 1891, to November 15, 1891, the level was only read after the bisections of the star. From November 15, 1891, to June 25, 1892, readings were made before, as well. This course was necessary because no electric lights were provided until November 15, and the great sensitiveness of the levels made it inadvisable to hold the highly heated bull's-eye lantern near enough to read them before the bisection was made.

COMPARISON OF THE LEVELS.

On the whole, level A appeared to be more sensitive than B. They cannot be compared, however, in regard to delicacy by reference to the individual latitude results except in so far as the magnitude of the discrepancies is concerned. If one level is more sensitive than the other,

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whether the resulting latitude is greater or less, depends on the direction of the vertical axis of the instrument. If A has moved farther than B after reversal and the axis of the instrument points to the north of the zenith the latitude from A will be less than from B, whereas in case the instrument points to the south the reverse will be the case. So that the only way is to take actual level readings and compare them. The means for the readings of the two ends of the bubble being taken, and the means for the motion before and after bisections of the star, we have the following table from Group VII. + indicates that A has moved farther than B.

Date.	2	4	5	6	7	8
1892.	*d*	*d*	*d*	*d*	*d*	*d*
Feb. 24	+ 0·28	+ 0·25	+ 0·02	+ 0·50	− 0·22	0·00
27	+ ·05	+ ·52	+ ·80	+ ·07	+ ·45	+ ·27
28	+ ·30	+ ·28	+ ·35	+ ·30	+ ·30	− ·18
29	+ ·32	+ ·25	+ ·20	+ ·65	+ ·07	+ ·22
March 1	− ·15	+ ·10	+ ·18	+ ·42	+ ·12	+ ·05
2	+ ·25	+ ·18	+ ·55	+ ·45	+ ·42	+ ·10
5	+ ·10	− ·15	+ ·30	+ ·17	+ ·05	+ ·25
9	+ ·22	+ ·25	+ ·22	− ·12	− ·12	+ ·10
10	+ ·05	− ·05	+ ·22	+ ·22	− ·25	·00
16	+ ·10	+ ·15	+ ·10	+ ·07	+ ·22	− ·18
21	− ·48	− ·20	− ·20	+ ·22	+ ·10	+ ·15
24	+ ·25	− ·10	− ·28	+ ·30	+ ·02	− ·15

The figures are the differences between the motions of the two bubbles, and the table shows that A is in general more sensitive than B.

In order now to see the effect of the discrepancies of the indications of the two levels on the final latitude results, the following summary is given. It is made up from Groups III and VII, involving observations in the warmest part of the year and the coolest. The quantities are the latitude from level A minus that from level B. The mean for the results in August and September is — 0″.062, and from February and March we get — 0″.022. The weighted mean is — 0″.045.

III. Date. Star first.	Pair— 1. N $\varphi_A - \varphi_B$	2. N $\varphi_A - \varphi_B$	3. N $\varphi_A - \varphi_B$	4. S $\varphi_A - \varphi_B$	5. N $\varphi_A - \varphi_B$	6. N $\varphi_A - \varphi_B$	7. S $\varphi_A - \varphi_B$	8. N $\varphi_A - \varphi_B$
1891.	″	″	″	″	″	″	″	″
Aug. 5	+·14	+·17	+·11	—·21	+·16	—·13	—·43	—·06
10	—·18	+·28	—·15	—·39	—·09	+·08	—·34	—·14
14	+·09	+·06	—·04	—·21	—·02	—·38	—·28	—·21
15	—·15	—·05	—·49	—·04	+·13	—·16	—·10	+·07
16	—·18	+·07	+·03	—·13	·00	—·05	—·42	+·30
26	—·23	+·35	—·04	—·16	—·20	+·01	—·23	+·27
29	—·05	+·02	+·09	—·18	·00	+·06	—·17	+·10
31	—·40	+·10	+·12	—·21	+·06	—·01	—·43	+·30
Sept. 2	—·33	—·09	+·04	+·05	+·21	+·14	—·13	+·18
9	—·11	+·10	—·06	—·34	—·12	+·13	—·59	—·05
12	—·19	+·23	—·10	—·11	—·16	+·10	—·02	+·24
18	+·06	—·01	+·09	—·14	—·22	—·19	—·04	—·32
	″	″	″	″	″	″	″	″
	—·128	+·102	—·033	—·172	—.021	—·033	—·265	+·057

Mean of results for 8 pairs = —0″·062.

VII. Date. Star first.	Pair— 2. N $\varphi_A - \varphi_B$	4. N $\varphi_A - \varphi_B$	5. S $\varphi_A - \varphi_B$	6. S $\varphi_A - \varphi_B$	7. N $\varphi_A - \varphi_B$	8. N $\varphi_A - \varphi_B$	
1892.	″	″	″	″	″	″	
Feb. 24	+·11	—·15	·00	—·24	+.14	—·04	
27	·00	—·32	—·39	—·03	+·20	+·11	
28	—·14	—·11	—·12	—·16	+·18	—·12	
29	+·17	—·13	—·20	—·33	—·04	—·13	
Mar. 1	+·19	—·06	—·07	—·21	—·04	+·02	
2	+·14	+·11	—·30	—·22	—·22	—·03	
5	—·04	+·11	—·10	—·05	—·01	—·13	
9	+·12	+·14	—·12	+·08	+·09	—·04	
10	·00	+·04	—·12	—·11	+·15	·00	
16	+·04	+·09	—·03	+·01	—·09	+·08	
21	+·14	+·14	+·03	—·04	—·04	—·06	
24	+·14	+·06	+·22	—·14	—·02	+·10	
	+·072	+·010	—·100	—·120	+·025	—·020	Mean = —″·022

	″
6 pairs give	—·022
8 pairs give	—·062
Weighted mean =	—·045

The smallness of this quantity is due to the fact that there are about as many positive as negative corrections. If B is always more sluggish than A and to the same degree, and the vertical axis is inclined as often to the north as to the south and by the same amount, it is plain that

the means of all the latitudes from each level will be equal. We may reasonably suppose that the average difference between A and B as affecting the final latitudes is considerably less than the weighted mean given above.

ATMOSPHERIC PRESSURE AND TEMPERATURE.

The barometer and thermometers were used several times every night in connection with the determinations of the force of gravity. The range during the whole year is so small that these factors can have no influence on the latitude observations as far as changes in the refraction are concerned. The inside temperature of the observatory was about a degree higher than the outside temperature and the range for the whole year during the hours of observation was from 15° C. in January to 26° in August. The barometer ran from 758^{mm} to 768^{mm}.

BATTERIES.

The following batteries were employed in the observatory work. See illustration No. 12.

I. 5 cells bichromate of potash battery for zenith telescope electric lights. Three cells were used for the hand lamps and one cell for the axis lamp. This battery required renewing every two months.

II. 2 cells gravity battery (sulphate of copper) for chronographic registration of star transits. The battery required attention twice during the year.

III. 1 cell Burnley dry battery for gravity flash apparatus. Three of these cells sufficed for the entire occupation of Waikiki involving observations on nearly 200 nights.

RELATIVE PROBABLE ERRORS.

Since different opinions are held among observers as to the relative merits of increasing the number of bisections on a star or increasing the number of levels to be read, or of reading the levels before and after the bisection, it is worth while to examine this point in the light of the observations themselves. The case will also be considered where the levels are read only to the nearest half division.

It would evidently be a waste of labor to work out all the different cases or combinations that could arise under the supposition of one, two, or three bisections, one or two levels, readings made before bisection or after, or readings made with two different degrees of accuracy. This would require 24 cases, being the number of combinations of 4 things, taking 1 out of each of 4 collections, the different collections containing 3, 2, 2, and 2 things, respectively, or, which is the same thing if we consider the bisection and the inclination as the necessary requisites for the result, we have two collections with 3 and 8 things, respectively, which, as before, gives 24 cases. Of these cases the following are worth investigating and are here given with the respective probable errors of observations and that of the mean result. The ob-

servations chosen are those of Group VII, extending over one month, from February 24, 1892, to March 24, 1892, giving 12 values for each pair. Pairs 1 and 3 were not included in the investigation. On account of the short time between the two stars of pair 1, and between pairs 2 and 3 the full complement of level readings could not be made. As one of the objects is to compare results from readings before and after, only pairs were taken giving the necessary data.

Probable errors of observation.

Group VII.	Pair.						Mean.	Probable error from one night's work.
	2.	4.	5.	6.	7.	8.		
	//	//	//	//	//	//	//	//
a	·15	·17	·18	·15	·15	·13	·155	·039
b	·17	·17	·19	·15	·15	·14	·162	·040
c	·23	·25	·26	·17	·14	·21	·210	·052
d	·17	·19	·21	·18	·18	·14	·178	·045
e	·17	·16	·18	·15	·14	·12	·153	·038
f	·17	·16	·22	·16	·17	·12	·167	·042
g	·27	·22	·24	·15	·15	·21	·207	·052

a = 3 bisections, 2 levels, before and after, nearest tenth division.
b = 3 bisections, 2 levels, before and after, nearest half division.
c = 1 bisection, 2 levels, before and after, nearest tenth division.
d = 3 bisections, 2 levels, before, nearest tenth division.
e = 3 bisections, 2 levels, after, nearest tenth division.
f = 3 bisections, 1 level, before and after, nearest tenth division.
g = 1 bisection, 1 level, after, nearest tenth division.
a and *b* have reference to the accuracy necessary in reading the levels.
a and *c* have reference to the number of bisections.
a and *d*, *a* and *e*, have reference to the number of readings to be made of level.
a and *f*, have reference to the number of levels to be read.
d and *e*, have reference to the time of reading level.
g is the case ordinarily employed in the field of work of the C. and G. S.
a is the case employed in the Waikiki work after November 15.

It is not supposed that the conclusions drawn from this investigation apply rigorously to all classes of instruments or to widely different conditions of work, but they certainly form a criterion for judging of the most advantageous way of observing with portable instruments as pursued in the field work of our own service.

THE STAR LIST.

The stars observed for the variation of latitude were identical for the two observers and were selected by Dr. Marcuse before leaving Germany. They were divided into 8 groups, making a total number of 63 pairs. This list was prepared with great care, and combines all the conditions necessary for precise latitude work.

The zenith distances are all within 29°, and the stars are so selected that the sums of the north and south zenith distances for each group

are practically equal. The declinations of the stars are well determined. Their proper motions are for the most part known, the stars having been observed by Bradley. No bright stars were taken, and those of the same pair were as near as possible of the same magnitude. Moreover, the differences of zenith distances for any pair did not exceed 18′, and the difference of right ascension between stars of the same pair was never more than 17 minutes.

The mean places of these stars, as far as concerns the observations made on the part of the Coast and Geodetic Survey, were derived by Mr. Farquhar, of the Computing Division. The same system was followed as in the case of the Rockville observations, and the following extract from Assistant Schott's report on that work is a statement by Mr. Farquhar of the principles on which the reductions were made:

"Combination weights used for adopted polar distances in Rockville latitude list: Twenty catalogues used in computing latitudes on this Survey were tested and their probable errors reported in June, 1890. These results were made the basis of the following series of weights: the unit of weight corresponding to a probable error of $\pm \sqrt{0''.1}$ and the ratio of observation error to systematic error being taken (for all catalogues alike) $= \sqrt{5}$, so that if w_∞ be the weight of an infinite number of observations and w_1 of one, $w = \frac{n}{n+5} w_\infty = \frac{6n}{n+5} w_1$.

The weight of no polar distance therefore can exceed six times that of a single observation, given in the table below:

Catalogue.	w_1
Lalande δ, Weisse-Bessel ε	0·015
d'Agelet α, Piazzi β	·02
Rümker ε	·025
Taylor β	·04
Groombridge β	·06
Armagh '75	·07
Armagh '40, Jacob β, Smyth β	·08
Auwers-Bradley α, Paris '45, Main β, Glasgow, Cape '40 ε	·09
Radcliffe '60	·10
Radcliffe '45	·12
Pond β, Cambridge '30 β, Greenwich 6 yr.	·14
Washington	·16
Cape '50 ε, Bonn β, Paris '60, Rome	·18
Paris '75	·20
Henderson, Greenwich 7y1, Melb. β, Cape '80 ε, Ann Arb. ε	·25
Struve Pos. M. β Greenwich 12y (1 + 2), Brussels, Becker ε	·3
Polkowa merid. circle, Greenwich 7y2, Cordoba γ	·35
Abo β, Harv. '85 ε	·4
Greenwich 9y., Harv. '75	·5
Leiden β	·6
Romberg α	·7
Pulkowa vert. circle β, Greenwich 10y.	·8

In this table weights were deduced for catalogues marked:

α..... from probable errors of observation given in the prefaces to the catalogues.

β..... from Boss's investigations, $\frac{2}{3}$ of his weights being taken—*i. e.*, his unit being supposed to correspond to a probable error of $\pm \sqrt{0''\cdot15}$.

γ..... from a determination of systematic error by myself, using the method of Boss.

δ..... from determination of observation error by myself and formula above as in (α).

ε..... from simple estimate, the places being too few for better methods.

Others, from the results obtained in 1890.

Piazzi, Taylor, Jacob, Main, and a few others used by Safford, I have to take at second hand, being without the originals. For stars of the Berliner Jahrbuch, Dr. Auwers's combination of the authorities used by him was usually accepted, $\frac{5}{8}$ of his total weights being allowed them (*i. e.*, his probable error taken $= \pm \sqrt{0''\cdot16}$ for $w = 1$) and the weights of this table used for the remaining authorities."

The following is the list with the squares of the probable errors of declination (ε^2) and the proper motion in declination (μ):

GROUP I.

Pair.	Right ascension.					Declination.				
	No. Pulkowa.	1892·0.				1892·0.			ε^2.	μ.
		h.	*m.*	*s.*		°	′	″	″	″′
1	2021	13	27	46		— 7	4	3·21	·07	+0·021
	2027		30	2		+49	34	5·82	·02	+ ·021
2	2056		41	42		26	14	38·75	·03	— ·077
	2064		44	16		16	20	1·53	·02	+ ·029
3	2093	14	3	37		44	22	5·68	·03	— ·032
	2113		13	59		— 1	45	57·56	·05	— ·079
4	2126		21	26		+19	42	45·63	·03	+ ·017
	2134		27	40		22	44	8·58	·04	+ ·030
5	2139		29	59		30	12	51·90	·03	+ ·119
	2146		36	32		12	7	34·79	·02	— ·105
6	2163		46	14		37	42	55·02	·04	+ ·090
	2177		54	0		4	59	56·95	·05	— ·023
7	2204	15	9	58		29	33	55·05	·04	+ ·019
	2214		17	16		12	57	15·47	·04	— ·032

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GROUP II.

Pair.	Right ascension.				Declination.				
	No. Pulkowa.	1892·0.			1892·0.			ϵ^2.	μ.
		h.	*m.*	*s.*	°	′	″	″	″
1	2387	10	37	14	27	7	30·74	·07	—·062
	2409		47	10	15	9	20·51	·04	—·014
2	2420		56	24	22	47	29.61	·05	—·043
	2430		59	59	19	44	54·97	·04	—·016
3	2437	17	4	6	24	37	38·76	·08	—·060
	2456		15	33	18	10	7·51	·06	—·037
4	2464		19	41	16	24	2·32	·05	—·044
	2479		26	22	26	11	32·19	·03	+·010
5	2501		38	3	24	37	7·43	·03	—·118
	2509		42	22	17	44	14·00	·04	—·017
6	2513		44	26	25	39	32·49	·03	—·049
	2532		55	15	16	45	26·80	·09	—·004
7	2571	18	13	5	13	44	11·14	1·00	—·02
	2580		16	13	28	56	8·76	·14	+·001
8	2595		22	20	26	23	5·89	·04	—·027
	2621		32	18	16	6	21·20	·04	+·040

GROUP III.

1	2633	18	40	10	23	28	54·41	·06	—·085
	2647		44	11	19	12	28·93	·06	—·02
2	2681		55	21	26	3	52·78	·06	—·017
	2703	19	3	7	16	41	33·76	·05	—·319
3	2718		7	37	31	6	12·17	·03	—·009
	2743		14	48	11	20	6·62	·18	+·029
4	2798		33	52	5	9	6·99	·03	—·009
	2822		40	23	37	5	37·24	·02	+·038
5	2824		42	27	32	37	25·31	·20	—·01
	2832		45	51	10	8	43·83	·05	—·163
6	2862		54	33	22	48	27·34	·02	+·007
	2886	20	0	22	19	40	54·32	·05	+·081
7	2893		2	39	9	5	10·33	·06	+·005
	2925		11	12	33	24	8·09	·08	—·102
8	2936		13	48	37	41	49·86	·06	—·010
	2949		17	50	4	59	53·39	·05	—·036

GROUP IV.

1	3155	21	34	5	1	45	29·83	·09	—·083
	3170		38	46	40	39	41·17	·07	·000
2	3182		41	45	2	11	11·62	·08	—·022
	3192		45	17	40	38	43·24	·07	·000
3	2899 Bradley	22	2	0	24	49	3·28	·02	+·006
	3256		8	0	17	44	46·56	·90	—·075
4	3274		11	16	37	12	39·17	·04	—·005
	3279		15	1	5	14	48·37	·04	—·018
5	3315		30	37	19	43	7·78	·09	—·101
	3010 Bradley		41	20	22	59	50·50	·02	—·016
6	3345		47	10	42	44	17·70	·03	+·009
	3367		55	6	—0	23	38·49	·03	+·008
7	3390	23	3	9	+1	32	23·89	·05	+·108
	3419		13	16	41	11	1·85	·04	—·004
8	3436		17	38	11	43	18·36	·03	—·032
	3463		28	36	30	43	44·90	·02	—·019

GROUP V.

Pair.	Right ascension.				Declination.				
	No. Pulkowa.	1892·0.			1892·0.			ϵ^2.	μ.
		h.	*m.*	*s.*	°	′	″	″	″
1	88	0	40	12	44	16	15·13	·06	—·014
	111		47	29	— 1	43	51·11	·03	—·020
2	122		51	59	28	24	29·44	·03	—·025
	138		59	23	14	21	53·99	·05	+·033
3	170	1	7	53	24	0	42·00	·05	—·030
	199		20	26	18	36	36·52	·04	+·017
4	211		24	31	5	35	13·03	·03	—·044
	216		28	2	36	40	59·73	·03	—·016
5	235		34	12	40	1	47·52	·03	—·026
	251 Bradley		47	58	2	39	14·74	·03	+·008
6	270		51	27	17	17	23·94	·04	—·032
	296	2	0	42	25	11	20·46	·06	—·031
7	331		12	7	19	24	4·56	·03	—·002
	358		23	4	22	59	11·79	·06	—·018
8	367		27	34	18	24	12·55	·12	+·005
	375		30	47	24	10	36·09	·02	—·019

GROUP VI.

1	598	4	1	7	37	26	36·80	·07	—·180
	606		5	35	5	14	29·71	·07	+·010
2	629		13	4	20	52	49·46	·06	—·050
	662		21	36	21	22	42·17	·06	—·066
3	686		28	24	5	20	29·01	·05	—·062
	725		42	38	37	17	48·94	·03	+·033
4	732		45	3	18	39	19·68	·04	—·046
	751		51	15	23	46	45·41	·10	—·033
5	796	5	7	39	2	43	55·50	·09	—·013
	810		11	33	40	0	8·44	·04	—·667
6	824		15	51	8	19	15·77	·07	+·005
	841		19	40	34	17	45·71	·07	—·015
7	866		25	53	18	30	47·88	·04	—·014
	874		28	51	23	58	1·37	·04	—·031
8	939		47	59	20	15	19·68	·03	—·108
	960		55	10	22	23	50·48	·04	—·024

GROUP VII.

1	1164	7	4	41	27	2	0·15	·04	—·058
	1173		6	39	15	21	32·51	·15	+·015
2	1198		16	53	25	15	26·93	·03	—·029
	1217		25	35	17	18	54·88	·07	—·088
3	1223		28	41	46	25	3·51	·07	—·040
	1232		31	55	—3	52	13·01	·04	+·020
4	1250		40	33	33	40	48·22	·02	—·042
	529 Greenwich 87		51	24	8	55	46·55	·25	—·025
5	1291		55	44	—1	5	35·72	·06	—·078
	1298		59	41	43	34	10·39	·07	—·056
6	1345	8	21	3	—3	37	56·96	·04	—·045
	1376		33	33	46	12	44·13	·04	+·083
7	1410		45	54	32	52	42·29	·05	+·014
	1429		51	53	9	48	12·15	·07	·000
8	1453	9	2	26	27	4	32·24	·04	—·384
	1464		6	23	15	25	50·97	·05	+·236

On the variation of latitude at Waikiki, Hawaiian Islands.

Pair.	Right ascension.				Declination.				
	No. Pulkowa.	1892·0.			1892·0.			ϵ^2.	μ.
		h.	*m.*	*s.*	°	′	″	″	″
1	1585	10	2	10	10	31	36·16	·03	—·062
	1598		8	54	32	00	14·13	·03	—·003
2	1624		19	34	9	20	1·16	·04	—·035
	1640		25	43	32	56	1·22	·03	+·011
3	1656		32	58	38	28	22·62	·05	—·030
	1667		37	3	4	8	49·99	·04	+·020
4	1713		54	48	39	47	31·41	·05	—·022
	1727	11	1	24	2	32	30·35	·03	—·072
5	1743		13	15	38	46	40·06	·06	—·080
	1754		20	6	3	53	45·71	·09	—·043
6	4875 Yarnall		22	23	—1	6	20·37	·11	—·002
	1763		24	40	43	45	57·42	·04	+·063
7	1739		40	18	7	8	4·47	·04	—·179
	1795		44	5	35	31	54·18	·11	—·010
8	1802		50	7	16	14	51·97	·04	+·005
	1827		6	22	26	28	18·52	·05	—·050

METHOD OF FINDING APPARENT POSITIONS OF THE STARS.

The reductions from mean to apparent declination were made by means of the independent star numbers.

In Appendix No. 13, Report for 1888, a differential method is described. Its application reduced the time and labor about one-half as compared with the usual way. But this method was devised to meet those cases where a large number of pairs had been observed on a few consecutive nights. The time of occupation of a station was supposed to be so short that after one date had been computed all others for that particular station could be derived with sufficient accuracy by considering the finite differences of the variables as differentials.

In the present case, however, the problem is entirely different. Here we have one station occupied for more than a year, and each group of stars was continuously observed for three months.

The difference in the two cases is that in the former many stars were observed a few nights, in the latter a few stars were observed on many nights.

It is evident at the outset that whether we use Bessel's star numbers or the independent ones some sort of check computation must be made. Moreover, when observations are made on every clear night and in a locality where more than 200 nights can be obtained in a year, it is probably immaterial whether we compute reductions for necessary dates or whether we simply compute for every other day. It is desirable to have the apparent declination of the stars with no greater error than one or two hundredths of a second for latitude work of the highest precision. A comparison of the changes in the star's position between successive dates, as deduced by the independent numbers and the method here presented of applying Bessel's numbers, shows that in the latter case the errors are absolutely without significance, whereas

the time of reduction is only about one-fourth that required by the usual independent numbers.

The present method was suggested by the following facts:

First, inasmuch as the same star must be reduced for about fifty different dates, a method should be employed in which the quantities depending on the star's position and those depending only on the date of observation are handled separately. This avoids the introduction as a distinct quantity, into each day's computation, of those terms which do not change from day to day. For example, in the use of the independent numbers the logarithms of sine and cosine of the declination are added to the others for every new date computed. Since the star constants do not vary, these are first computed and then applied to the differences of the varying day numbers. Besides, since a number of stars were observed on the same date, we only have to apply the different star constants to the same differences of the day numbers throughout the three months' period.

Of the day numbers it is to be remarked that the principal term of A depends on the sine of the moon's ascending node, and the corresponding one of B depends on the cosine of the same function. The values of A and B can therefore never exceed very much the numerical factors of their principal terms, so that three significant figures need only be considered and they may be dealt with by means of Crelle's tables. C and D vary as the cosine and sine of the sun's longitude, and only change rapidly when they are small in amount, so that their changes also are such that Crelle's tables give all the accuracy necessary. Since the obliquity is practically constant as far as these reductions are concerned, and as the sun's longitude does not change more than a few degrees between the dates chosen, the changes in C and D are either small or quite uniform. It is therefore never necessary to use logarithms to get all the accuracy desirable for the reductions to apparent place.

The star constants having been calculated for the star to be reduced, the values of dA and dB are taken from the Ephemeris, and dC and dB are calculated by using the formulæ

$$dC = 18{\cdot}7\ d \cos \odot$$
$$dB = 20{\cdot}4\ d \sin \odot$$

the values for dA, dB, dC, and D being tabulated, the star constants a, b, c, and d are applied and the sum of the products gives the reduction from one date to another.

The following table gives the reductions from June 6 to June 28 for star I_1a,* $\alpha = 13^h\ 27^m\ 46^s$, $\delta = -\ 7^\circ\ 4'$, and a comparison of differences for the two methods. To compute the places by the independent star numbers for one group for three months, required generally about eight days, and a check computation using the same method would take as long. By the method of applying differences for a check we get at the same result in about two days.

*Group I, Pair 1, Star *a*.

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

Date.	$a \cdot dA$	$b \cdot dB$	$c \cdot dC$	$d \cdot dD$	Σ	Ind. Nos.	Δ
	″	″	″	″	″	″	″
June 6	—0·195	—0·002	+0·234	—0·017	+0·020	+0·022	—0·002
8	— ·162	— ·025	+ ·237	— ·015	+ ·035	+ ·032	+ 3
10	— ·102	— ·029	+ ·238	— ·012	+ ·095	+ ·097	— 2
12	— ·061	— ·011	+ ·239	— ·010	+ ·157	+ ·157	0
14	— ·073	+ ·013	+ ·239	— ·007	+ ·172	+ ·174	— 2
16	— ·125	+ ·026	+ ·240	— ·004	+ ·137	+ ·139	— 2
18	— ·186	+ ·014	+ ·240	— ·002	+ ·066	+ ·062	+ 4
20	— ·203	— ·012	+ ·239	— ·001	+ ·023	+ ·013	+ 10
22	— ·175	— ·033	+ ·240	+ ·003	+ ·035	+ ·041	— 6
24	— ·117	— 034	+ ·239	+ ·006	+ ·094	+ ·095	— 1
26	— ·080	— ·014	+ ·239	+ ·008	+ ·153	+ .153	0
28							Σ +0·002

The sum of the residuals shows that the apparent places for this whole month could have been obtained by simply adding the differences. The accumulation of error for the entire period is much less than 0″·01, so that this short method is quite accurate enough to check the results.

In the computations I had the help of the following persons attached to the Computing Division of the Survey: Mr. H. Farquhar furnished the mean north polar distances for all the stars. Mr. A. L. Baldwin computed the apparent places. Mr. H. L. Stidham checked these, and assisted in the final reductions; and Mr. H. F. Flynn applied the micrometer and level corrections and deduced the latitudes. The final adjustment by least squares, the reductions to the mean declination systems and the derivation of the daily means was participated in by both Mr. Flynn and Mr. Stidham. At the beginning of the work Subassistant John Nelson assisted for a short time. I desire to express my obligations to all these gentlemen, as well as to Assistant C. A. Schott, in charge of the Computing Division, who did everything in his power to facilitate the work.

Illustration No. 14 shows the time of beginning and ending the observations of each group and the period during which any two or three successive groups were observed at the same evening. This diagram is inserted in order to avoid the necessity of publishing the individual results in this particular form on a large sheet for the sake of showing the group connections.

In order to condense as much as possible the publication of the latitudes now under discussion, the following scheme is adopted. First, a specimen record will be given which is copied from the book of observation. Second, a computation of one latitude showing the details and giving the exact form followed throughout the office work, and finally a general table containing all that part of the reduction necessary for a rediscussion should such a step ever be thought advisable, either from improved values in the star places or in the instrumental constants.

The correction for inclination of the micrometer thread is applied in the refraction column.

Specimen of record.

GROUP VII, PAIR 7.

Date.	Micrometer reading.				Level A.				Level B.				Remarks.
	Turns.	Divisions.			Before.		After.		Before.		After.		
1892.					*d.*	*d.*	*d.*	*d.*	*d.*	*d.*	*d.*	*d.*	
Feb. 20	23	83·0	82·2	82·0	9.9	34·5	10·2	34·9	59·3	82·8	60·0	83·2	Distinct and steady.
	13	27·4	27·5	28·3	34·8	10·1	35·2	10·8	83·3	60·0	83·7	60·2	

Specimen of computation.

GROUP VII, PAIR 7.

Date.	Pos.	Micrometer.		Level.			Mer.	Declination.
		Reading.	Diff. Z. D.	N.	S.	Diff.	Dist.	
1892.		*t.* *d.*	*t.* *d.*	*d.*	*d.*	*d.*	*s.*	° ′ ″
Feb. 20				44·4	44·9	+0·5		
	N	23 82·4		45·1	46·0	0·9	15	32 52 45·46
	S	13 27·7	—10 54·7	142·1	143·3	1·2	13	9 48 11·83
				143·2	143·9	0·7		

Sum and half sum.	Corrections.				Latitude.	Mean.
	Micrometer.	Level.	Ref.	Mer.		
° ′ ″	′ ″	″	″	″	″	
		+0·15				21° 16′
42 40 57·29		·27		+0·04	24·16	
21 20 28·64	— 4 04·67	·37	—0·07	+ ·01	·25	24″·20
		·22				

GROUP I, PAIR 1.

The latitude value may differ in some cases from that which would apparently result from the corrections given. This comes from neglected decimals. In the computations four values were derived corresponding to the level readings before and after bisection and to

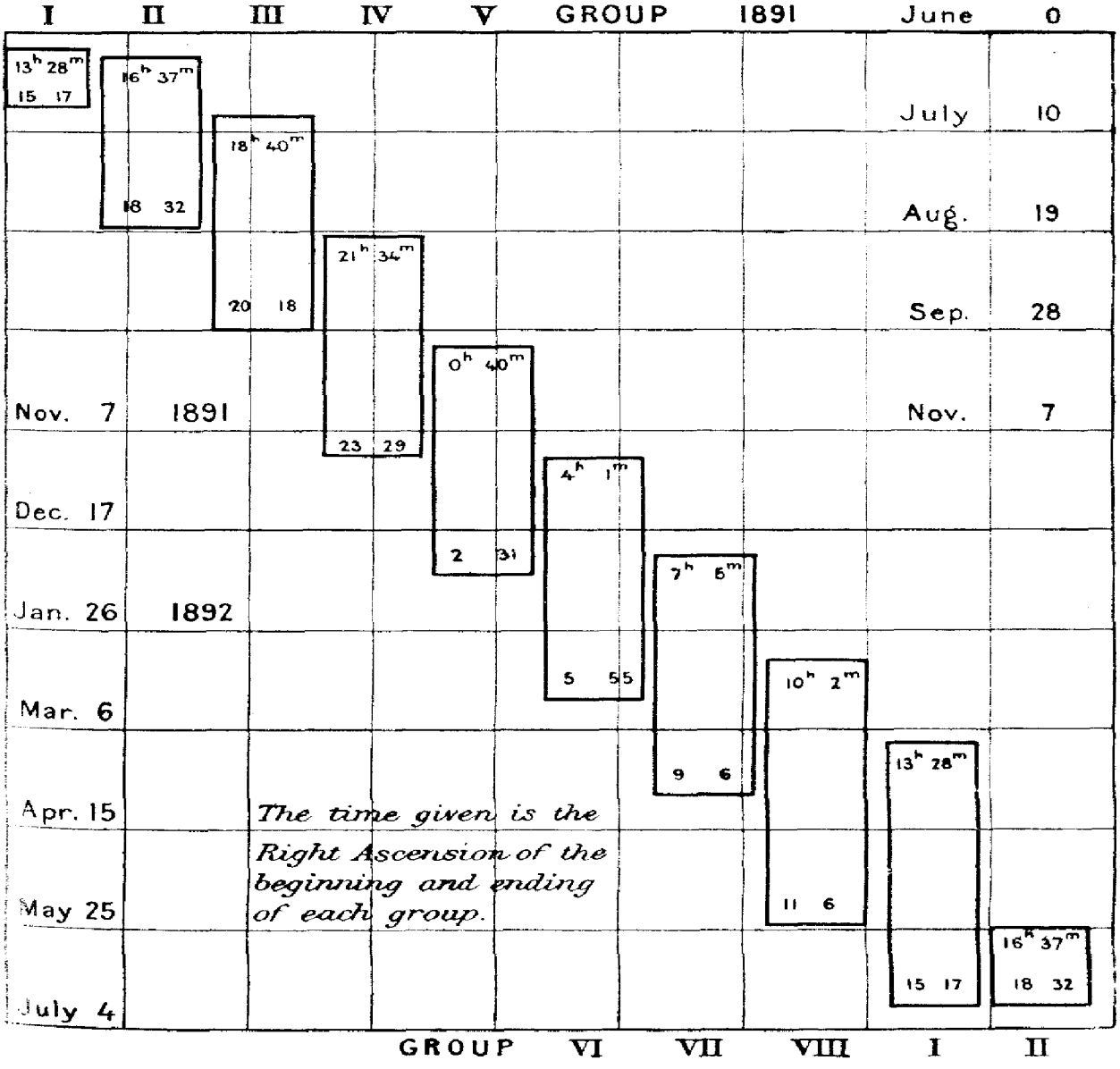


Diagram showing the method of connecting the Groups.

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

different levels and the adopted latitude is a mean between the two readings of the same level and then between the two levels. The level correction given is a mean one which would give a latitude differing very slightly in a few cases from that obtained in the usual way. This difference is entirely without significance.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
June 7	21·07·9	18·27·2	21 15 20·23	+1 5·12	—0·41	—0·33	+0·06	21 16 24·67
8	21·84·1	19·05·8	20·30	4·56	—1·08	+0·02	+0·04	23·84
9	22.96·0	20·21·1	20·38	3·77	+0·63	+0·02	+0·04	24·84
11	21·95·3	19·15·3	20·54	5·00	—1·46	+0·02	+0·04	24·14
13	21·94·5	19·34·0	20·77	0·43	+3·56	+0·02	+0·04	24·82
15	21·18·9	18·32·8	21·02	6·37	—2·81	—0·13	+0·04	24·50
16	21·15·7	18·44·0	21·15	3·05	+0·32	+0·02	+0·04	24·58
17	21·40·7	17·67·5	21·26	3·37	+0·05	+0·02	+0·05	24·75
18	21·52·7	18·70·7	21·36	5·42	—2·77	+0·02	+0·09	24·12
22	22·52·3	19·74·6	21·56	4·42	—1·20	—0·04	+0·07	24·81
26	21·11·7	18·34·5	21·78	4·30	—1·93	+0·02	+0·09	24·26

GROUP I, PAIR 2.

1891.	*t. d.*	*t. d.*	° ′ ″	′ ″	″	″	″	° ′ ″
June 6	22·12·9	18·96·2	21 17 38·66	—1 13·47	—0·06	—·32	+·04	21 16 24·85
7	22·29·3	19·04·1	38·76	15·42	+1·15	—·02	+·03	24·50
8	22·27·4	19·04·2	38·85	14·98	+0·30	—·02	+·03	24·18
10	21·83·1	18·55·0	39·06	16·11	+1·92	—·02	+·03	24·88
11	21·74·2	18·46·7	39·18	15·98	+0·37	—·02	+·03	23·58
12	21·09·2	17·85·2	39·30	15·16	+0·01	—·02	+·03	24·16
13	21·73·0	18·47·7	39·45	15·46	+0·56	—·02	+·03	24·56
15	21·92·6	18·64·0	39 76	16·23	+0·85	—·20	+·03	24·21
16	22·23·1	18·91·2	39·92	17·00	+2·20	—·02	+·03	25·13
17	21·82·5	18·48·0	40·05	17·60	+1·80	—·02	+·05	24·28
18	21·88·9	18·54·4	40·18	17·60	+1·97	—·02	+·05	24·58
19	21·32·9	17·98·3	40·28	17·62	+1.71	—·02	+·05	24·40
22	21·46·2	18·09·3	40·50	18·16	+1·91	—·02	+·05	24·28
24	21·76·5	18·40·4	40·64	17.97	+1·96	—·02	+·05	24·66
26	21·16·0	17·84·6	40·81	16·88	+0·16	—·02	+·05	24·12
27	21·33·8	18·02·4	40·92	16·88	+0·39	—·02	+·05	24·48
28	21·98·3	18·58·2	41·04	18·90	+2·09	—·02	+·05	24·26
30	20·75·6	17·33·9	41·27	19·27	+2·17	—·02	+·05	24·20

GROUP I, PAIR 3.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ″ ″
June 6	21·03·1	16·04·0	20 18 20·59	—1 55·78	— ·77	—·04	+·04	21 16 24·04
7	23·12·0	18·09·0	20·69	56·69	+ ·52	—·04	+·04	24·52
8	16·78·0	10·81·5	20·78	55·18	—1·04	—·22	+·03	24·37
9	21·90·8	16·75·8	20·87	59·47	+3·42	—·04	+·04	24·82
10	22·81·7	17·77·0	20·97	57·08	+0·14	—·04	+·04	24·03
11	22·21·8	17·19·0	21·08	56·64	+0·58	—·04	+·04	25·02
12	21·62·3	16·60·5	21·20	56·41	—0·70	—·04	+·04	24·09
13	21·31·7	16·27·7	21·34	56·92	+0·06	—·04	+·04	24·48
15	22·97·7	16·90·2	21·64	57·73	+0·43	—·04	+·04	24·34
16	20·65·7	15·56·5	21·80	58·13	+0·83	—·04	+·04	24·50
17	21·43·6	16·32·9	21·93	58·47	+0·84	—·04	+·06	24·32
18	20·83·4	15·71·8	22·07	58·68	+0·97	—·04	+·06	24·38
19	21·76·9	16·62·8	22·17	59·26	+1·13	—·04	+·06	24·06
22	21·03·9	15·86·2	22·38	—2 00·10	+2·25	—·04	+·06	24·54
24	21·50·8	16·29·7	22·50	00·89	+1·99	—·04	+·06	23·62
26	21·60·2	16·49·6	22·68	—1 58·45	+0·01	—·04	+·06	24·26
27	22·06·0	16·95·0	22·79	58·54	—0·31	—·04	+·06	23·96
28	21·62·4	16·44·5	22·90	58·98	+1·37	—·01	+·06	25·34
30	21·96·7	16·74·0	23·14	—2 01·26	+2·68	—·04	+·06	24·58

GROUP I, PAIR 4.

1893.	*t. d.*	*t. d.*	° ′ ″	′ ″	″	″	″	° ′ ″
June 6	24·02·0	17·01·7	21 13 42·48	+2 42·46	—1·45	+·04	+·03	21 16 23·56
7	23·20·5	16·22·7	42·60	41·88	—0·09	+·04	+·03	24·46
8	23·24·7	16·19·3	42·72	43·64	—2·18	+·04	+·03	24·25
9	23·05·7	15·90·8	42·84	45·85	—4·92	+·04	+·03	23·84
10	22·56·2	16·61·4	42·96	41·18	+0·14	+·04	+·03	24·35
11	22·45·8	15·44·7	43·10	42·64	—0·95	+·04	+·03	24·80
12	22·61·3	15·60·0	43·24	42·69	—0·88	+·04	+·03	25·12
13	22·60·2	15·63·1	43·41	41·72	—0·70	+·04	+·03	24·50
15	23·96·2	16·93·0	43·76	43·13	—3·04	+·04	+·03	23·92
16	22·95·7	16·01·7	43·94	41·00	—0·11	+·04	+·03	24·90
17	23·10·2	16·19·2	44·11	40·30	+0·44	+·04	+·04	24·92
18	22·53·0	15·63·4	44·27	39·98	+1·01	+·04	+·05	25·35
19	22·24·7	15·33·5	44·38	40·35	+0·01	—·44	+·06	24·36
22	22·89·6	16·03·6	44·68	39·14	+0·47	+·04	+·05	24·38
24	23·03·2	16·21·2	44·86	38·21	+1·42	+·04	+·05	24·58
25	22·52·5	15·73·0	44·98	37·63	+2·27	+·04	+·05	24·96
26	23·04·0	16·13·7	45·08	40·14	—0·69	+·04	+·05	24·62
27	23·45·2	16·56·3	45·22	39·82	—0·73	+·04	+·05	24·40
28	23·13·9	16·31·2	45·35	38·38	+1·25	+·04	+·05	25·07
30	23·23·8	16·33·4	45·64	40·16	—1·08	+·04	+·05	24·81

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP I, PAIR 5.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
June 6	12·95·5	28·36·5	21 10 28·00	+5 57·49	—0·38	—·40	+·04	21 16 24·75
7	12·28·0	27·64·0	28·12	56·33	+0·95	—·08	+·03	25·35
8	10·57·9	25·96·8	28 24	57·00	—0·09	—·08	+·03	25·10
9	10·08·5	26·52·8	28·36	58·26	—2·12	—·08	+·03	24·45
10	11·22·2	26·54·2	28·48	55·40	+1·43	—·08	+·03	25·26
11	11·65·5	27·02·9	28·62	56·66	—0·05	+·10	+·03	25·36
12	10·94·7	26·31·2	28·76	56·45	—0·78	+·10	+·03	24·56
13	10·45·2	25·77·1	28·93	55·38	+0·84	+.10	+·03	25·28
15	11·48·9	26·88·2	29·28	57·10	—1·59	+·10	+·03	24·92
16	11·37·2	26·65·5	29·46	54·54	+0·79	+·10	+·03	24·92
17	11·54·9	26·81·7	29·63	54·20	+1·08	+·10	+·05	25·06
18	11·69·8	26·94·2	29·80	53·64	+1·68	+·10	+·05	25·27
19	10·07·4	26·33·1	29·92	53·94	+0·60	+·10	+·05	24·61
22	11·61·8	26·81·5	30·24	52·55	+1·90	+·10	+·05	24·84
24	11·50·5	26·67·0	30·41	51·80	+2·82	+·10	+·05	25·18
25	10·72·0	25·84·8	30·49	50·95	+3·21	+·10	+·05	24·80
26	10·35·9	25·58·2	30·62	53·15	+1·03	+·10	+·05	24·95
27	11·22·7	26·46·8	30·76	53·57	+0·71	+·10	+·05	25·19
28	10·44·4	25·60·6	30·90	51·74	+2·05	+·10	+·05	24·84
30	10·76·1	25·98·7	31·18	53·22	+0·45	+·10	+·05	25·00

GROUP I, PAIR 6.

1891.	*t. d.*	*t. d.*	° ′ ″	′ ″	″	″	″	° ′ ″
June 6	28·82·5	15·25·0	21 21 39·17	—5 14·92	— ·27	—·45	+·04	21 16 23·57
7	27·35·2	13·74·9	39·30	15·57	+ ·54	—·10	+·03	24·20
8	27·93·8	14·38·0	39·42	14·53	— ·46	—·10	+·03	24·36
9	27·43·3	13·74·5	39·54	17·54	+2·44	—·10	+·03	24·37
10	28·26·4	14·62·7	39·66	16·36	+0·87	—·10	+·03	24·10
11	27·87·7	14·27·5	39·80	15·55	—0·04	—·10	+·03	24·14
13	26·90·4	13·29·6	40·12	15.69	—0·28	—·10	+·03	24·08
16	28·49·3	14·83·8	40·66	16·78	+0·59	—·10	+·03	24·40
17	26·36·5	12·69·2	40·82	17·19	+0·97	—·10	+·06	24·56
18	27·69·5	13·98·3	40·99	18·10	+1·77	—·10	+·06	24·62
19	27·24·1	13·53·7	41·12	17·91	+0·91	—·10	+·06	24·08
22	28·27·5	13·52·6	41·44	18·96	+1·73	—·06	+·05	24·20
24	27·04·0	13·25·2	41·63	19·86	+2·50	—·06	+·05	24·26
26	27·37·9	13·66·7	41·85	18·10	+0·71	—·10	+·06	24·42
27	27·77·3	14·02·4	41·99	18·96	+0·63	—·10	+·06	23·62
28	26·68·3	12·88·2	42·13	20·16	+2·31	—·10	+·06	24·24
30	27·20·7	13·49·5	42·43	18·10	+0·15	—·10	+·06	24·44

GROUP I, PAIR 7.

Date.	Micrometer.		Mean app. Dec.			Corrections.				Latitude.		
1891.	*N.* *t. d.*	*S.* *t. d.*	°	′	″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	°	′	″
June 6	20·34·3	19·96·7	21	15	46·30	+0 37·68	+0·33	+·01	+·03	21	16	24·35
7	20·40·7	20·96·8			46·45	36·21	+1·76	+·01	+·03			24·46
8	20·04·4	21·68·7			46·60	38·12	—·34	+·01	+·03			24·42
9	20·40·8	22·01·4			46·74	37·26	+·79	+·19	+·03			25·01
10	19·38·8	20·93·1			46·88	35·80	+·79	+·01	+·03			23·51
11	19·75·5	21·38·4			47·03	37·79	—·48	+·01	+·03			24·38
12	20·38·4	22·00·8			47·18	37·68	—·71	+·01	+·03			24·19
13	19·83·4	20·39·0			47·38	36·10	+·90	+·01	+·03			24·42
15	20·68·7	21·26·6			47·76	36·63	—·19	+·01	+·03			24·24
17	19·07·8	20·57·1			48·15	34·64	+1·37	+·01	+·06			24·23
18	18·25·7	19·80·0			48·34	35·80	+·39	+·01	+·06			24·60
19	19·12·0	20·60·2			48·49	34·38	+·82	+·01	+·06			23·76
22	19·92·7	21·32·2			48·88	32·36	+2·71	+·01	+·06			24·02
24	19·17·7	20·60·2			49·10	33·06	+2·18	+·01	+·06			24·41
26	18·70·1	20·13·8			49·36	33·34	+1·39	+·01	+·06			24·16
27	17·96·8	19·43·0			49·52	33·92	+·85	+·01	+·06			24·36
28	19·90·0	21·38·5			49·67	32·59	+2·03	+·01	+·06			24·36
30	19·23·1	20·67·5			50·00	33·50	+·56	+·01	+·04			24·11

GROUP II, PAIR 1.

1891.	*t. d.*	*t. d.*	°	′	″	′	″	″	″	″	°	′	″
June 10	10·21·8	30·74·1	21	08	28·22	+7	56·10	+·51	—·01	+·03	21	16	24·85
11	10·13·5	30·69·5			28·40		56·96	—·82	—·01	+·03			24·56
13	9·87·3	30·36·2			28·79		55·31	+·12	+·14	+·03			24·39
15	9·84·5	30·30·0			29·25		54·52	+·67	—·21	+·03			24·20
17	10·40·6	30·81·3			29·74		53·41	+1·39	+·14	+·05			24·73
18	10·32·2	30·76·7			29·96		54·29	+0·41	+·14	+·05			24·85
19	10·28·4	30·72·1			30·20		54·11	+0·14	+·14	+·05			24·64
22	10·14·8	30·66·8			30·73		56·03	—2·58	+·14	+·05			24·37
24	9·02·9	29·35·5			31·04		51·53	+1·88	+·14	+·03			24·62
26	9·40·0	29·72·5			31·38		51·51	+1·18	+·14	+·05			24·26
27	9·26·4	29·62·4			31·58		52·32	+·43	+·14	+·05			24·52
28	9·68·0	29·93·8			31·77		49·96	+2·78	+·14	+·05			24·70
30	8·46·5	28·85·1			32·20		52·92	—·23	+·14	+·05			25·08
July 4	8·99·0	29·36·0			33·01		52·55	—1·79	+·14	+·05			23·96
7	9·11·8	29·46·7			33·42		52·07	—1·05	+·16	+·04			24·04
8	9·24·7	29·57·0			33·58		51·46	—·97	+·14	+·05			24·26
17	10·58·0	30·69·1			35·14		46·54	+1·94	+·14	+·05			23·81
18	8·89·5	29·16·9			35·28		50·33	—1·35	+·06	+·16			24·48
22	9·99·8	29·21·8			35·70		49·07	—·65	+·14	+·06			24·32
23	9·06·0	29·22·0			35·82		47·68	+1·06	+·14	+·05			24·75
25	8·49·2	28·65·2			36·07		47·68	+0·08	+·06	+·07			23·90
26	9·38·3	29·52·8			36·22		47·33	+1·34	+·14	+·05			25·08
27	9·88·3	29·08·5			36·38		48·66	—·64	+·14	+·05			24·59
29	8·91·2	29·99·1			36·71		45·80	+·97	+·18	+·04			23·70
Aug. 2	8·80·7	28·91·9			37·12		46·57	+·52	+·14	+·05			24·40
3	8·59·0	28·73·6			37·18		47·36	—·06	+·14	+·05			24·67
4	9·13·9	29·25·8			37·25		46·73	—·51	+·14	+·05			23·66
5	8·95·2	29·02·4			37·32		45·64	+1·57	+·14	+·05			24·72
6	8·90·4	28·93·6			37·40		44·71	+1·99	+·16	+·06			24·32
10	9·54·2	29·65·3			37·83		46·54	0·00	+·14	+·05			24·56
12	8·88·4	28·94·0			38·06		45·27	+0·47	+·14	+·05			23·99
13	9·67·3	29·70·1			38·14		44·62	+1·34	+·14	+·05			24·29
14	8·31·9	28·47·4			38·22		47·57	—1·42	+·14	+·05			24·56
15	9·61·4	29·67·1			38·26		45·29	+·41	+·14	+·05			24·15
16	9·57·7	29·61·2			38·30		44·78	+1·29	+·14	+·05			24·56

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP II, PAIR 2.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t.* *d.*	*S.* *t.* *d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
June 10	20·48·0	20·89·0	21 16 13·12	+0 09·51	+1·11	+·15	+·03	21 16 23·92
11	21·00·7	21·50·0	13·30	11·44	—0·63	+·00	+·03	24·14
13	19·77·4	20·21·0	13·70	10·11	+0·46	+·00	+·03	24·30
17	20·47·0	20·83·5	14·66	08·47	+2·11	—·47	+·04	24·81
18	20·53·2	21·93·4	14·91	09·32	+0·32	·00	+·05	24·60
19	18·85·9	19·18·5	15·16	07·56	+1·44	·00	+·05	24·21
24	19·76·6	20·04·3	16·06	06·43	+1·65	·00	+·05	24·19
26	18·85·0	19·14·6	16·40	06·87	+1·40	·00	+·06	24·73
27	20·55·3	20·85·6	16·60	06·89	+0·48	·00	+·05	24·02
28	18·93·9	19·14·8	16·81	04·85	+2·27	·00	+·05	23·98
30	19·46·2	19·73·2	17·26	06·26	+0·32	·00	+·07	23·91
July 4	20·56·7	20·89·0	18·11	07·49	—1·75	·00	+·05	23·90
7	19·66·1	19·95·5	18·59	06·82	—0·78	·00	+·05	24·68
8	19·73·3	19·90·5	18·73	03·99	+1·08	·00	+·05	23·85
17	19·86·5	20·03·8	20·40	04·01	—0·32	·00	+·05	24·14
18	19·60·8	19·73·9	20·54	03·04	—0·22	—·08	+·06	23·34
22	19·64·7	19·81·7	21·02	03·94	—0·85	·00	+·05	24·16
23	18·62·9	18·68·4	21·14	01·28	+1·59	·00	+·05	24·06
24	18·92·2	19·00·0	21·26	01·81	+ ·57	·00	+·06	23·70
25	19·03·7	19·08·0	21·42	01·00	+ ·91	·00	+·03	23·36
26	19·25·2	19·27·3	21·57	00·49	+1·85	·00	+·05	23·96
27	19·78·7	19·86·3	21·74	01·76	+ ·32	·00	+·05	23·87
29	19·66·6	19·75·8	22·08	02·13	— ·67	·00	+·05	23·59
31	19·39·3	19·48·5	22·36	02·13	— ·88	—·02	+·05	23·64
Aug. 2	19·27·6	19·29·1	22·56	00·35	+ ·98	·00	+·05	23·94
3	19·45·0	19·50·8	22·64	01·35	— ·34	·00	+·05	23·70
4	19·35·3	19·36·0	22·72	+0 00·16	+1·23	—·03	+·07	24·15
5	19·34·3	19·37·6	22·80	+0 00·77	+ ·03	·00	+·05	23·65
6	19·21·5	19·17·0	22·88	—0 01·04	+2·45	·00	+·05	24·34
7	18·61·4	18·59·3	22·99	—0 00·49	+1·19	·00	+·05	23·74
10	19·55·0	19·58·9	23·36	+0 00·90	— ·43	·00	+·05	23·88
13	19·40·1	19·30·7	23·72	—0 02·18	+2·07	·00	+·05	23·66
14	18·99·9	18·99·0	23·79	—0 00·21	+0·05	·00	+·05	23·68
15	19·26·7	19·38·4	23·87	+0 02·71	—2·85	·00	+·05	23·78
16	19·50·0	19·47·2	23·92	—0 00·65	— ·02	·00	+·05	23·30

GROUP II, PAIR 3.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t.* *d.*	*S.* *t.* *d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
June 13	29·19·2	9·86·1	21 23 53·32	−7 28·45	+ ·95	−·13	+·03	21 16 25·72
17	29·64·2	10·22·2	54·32	30·52	+1·16	−·13	+·03	24·86
18	29·75·9	10·38·3	54·56	29·49	− ·07	−·13	+·03	24·90
22	28·71·0	9·26·5	55·39	31·10	+ ·96	−·17	+·02	25·10
24	29·14·2	9·63·7	55·74	32·49	+1·97	−·17	+·02	25·07
26	28·03·1	8·54·2	56·09	32·12	+ ·88	−·13	+·03	24·75
27	28·10·6	8·64·8	56·30	31·40	− ·08	−·13	+·03	24·72
28	28·20·6	8·71·2	56·50	32·23	+ ·08	−·17	+·02	24·20
30	28·05·0	8·55·1	56·97	32·35	+ ·09	−·17	+·02	24·58
July 4	28·35·3	8·90·4	23 57·85	31·19	−2·00	−·13	+·03	24·56
17	28·11·8	9·50·5	24 00·21	34·99	− ·46	−·14	+·04	24·66
18	27·62·8	8·01·7	00·36	34·95	− ·61	−·17	+·02	24·65
22	28·71·5	9·12·4	00·87	34·48	−1·36	−·16	+·05	24·92
23	28·88·0	9·14·3	01·00	37·87	+1·55	−·13	+·03	24·58
26	28·84·0	8·35·7	01·44	37·64	+ ·90	−·13	+·03	24·60
27	28·38·2	8·76·8	01·63	35.02	−1·23	−·13	+·05	25·30
29	28·25·6	8·58·9	01·98	36·24	−1·45	−·17	+·02	24·14
Aug. 2	28·89·4	9·10·0	02·50	39·19	+ ·88	−·17	+·02	24·04
3	28·09·6	8·40·3	02·58	36·85	− ·65	−·13	+·03	24·98
4	28·81·2	9·08·3	02·66	37·68	− ·45	−·15	+·02	24·40
5	28·37·1	8·61·0	02·76	38·42	+ ·10	−·13	+·03	24·34
6	27·82·0	8.01·8	02·84	39·38	+1·24	−·13	+·03	24·60
7	27·75·8	8·08·9	02·95	36·29	−1·67	−·13	+·03	24·89
10	28·91·5	9·16·6	03·34	38·15	− ·57	−·13	+·03	24·52
11	29·83·5	9·01·9	03·98	39·70	+ ·48	−·13	+·01	24·64
13	28·47·5	8·59·8	03·72	41·12	+1·70	−·13	+·03	24·20
14	28·85·8	9·06·2	03·82	39·24	−0·02	−·13	+·05	24·48
16	28·46·7	8·63·7	03·95	40·03	+ ·24	−·13	+·03	24.06

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP II, PAIR 4.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t.* *d.*	*S.* *t.* *d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
June 10	21·11·4	17·62·2	21 17 45·34	—1 21·01	00	—·03	+·03	21 19 24·33
11	20·03·2	16·58·2	45·54	20·04	—1·79	—·18	+·03	23·56
13	20·29·1	16·76·6	45·94	21·78	00	—·03	+·03	24·16
17	21·91·4	18·34·4	46·92	22·82	+·04	—·03	+·05	24·16
18	20·55·9	17·02·9	47·18	21·89	—1·15	—·03	+·05	24·16
19	20·53·5	16·89·6	47·44	24·42	+1·20	—·03	+·05	24·30
22	19·84·3	16·20·1	48·04	24·49	+·17	—·03	+·05	23·74
26	19·64·3	15·98·1	48·77	24·95	+·03	—·03	+·05	23·87
27	19·84·0	16·21·9	48·98	24·00	—·66	—·03	+·05	24·34
28	20·54 9	16·92·0	49·18	24·19	—·97	—·03	+·05	24·04
30	20·00·7	16·32·5	49·66	25·42	—·51	—·01	+·04	23·76
July 4	20·93·6	17·29·9	50·58	24·37	—2·41	—·03	+·05	23·82
7	19·55·6	15·84·0	51·09	26·20	—·70	+·00	+·06	24·25
8	20·34·4	16·49·7	51·24	29·24	+1·66	—·03	+·05	23·68
17	20·64·2	16·83·9	53·02	28·22	—1·10	—·03	+·05	23·72
18	19·34·7	15·51·8	53·18	28·83	—1·20	—·03	+·06	23·18
22	20·25·8	16·47·9	53·72	27·67	—2·15	—·03	+·05	23·92
23	19·93·6	16·03·7	53·86	30·45	+·67	—·03	+·05	24·10
24	19·86·0	16·01·3	54·00	29·24	—1·20	—·03	+·07	23·60
26	20·38·3	16·44·2	54·33	31·42	+·83	—·03	+·05	23·76
27	19·73·7	15·87·9	54·51	29·50	—1·37	—·03	+·05	23·66
29	20·65·3	16·76·1	54·88	30·29	—·87	—·03	+·05	23·74
31	20·32·5	16·47·9	55·20	29·22	—2·46	—·03	+·05	23·54
Aug. 2	20·35·8	16·41·6	55·44	31·45	—·01	—·03	+·05	24·00
4	20·74·3	16·82·3	55·63	30·94	—·99	—·03	+·05	23·72
5	20·62·8	16·69·5	55·72	31·24	—·73	—·03	+·05	23·77
6	20·47·5	16·47·2	55·82	32·86	+·24	—·03	+·05	23·22
7	20·33·9	16·45·5	55·93	30·10	—2·65	+·01	+·04	23·23
10	20·20·1	16·28·1	56·35	30·94	—1·47	—·03	+·05	23·96
11	20·16·6	16·17·9	56·50	32·49	—·12	—·03	+·05	23·91
13	20·55·3	16·50·2	56·76	33·98	+·58	—·03	+·05	23·38
14	20·44·0	16·30·0	56·88	36·04	+2·44	—·03	+·05	23·30
15	20·48·5	16·43·4	56·95	33·98	+0·79	—·03	+·05	23·78
16	20·35·7	16·28·3	57·02	34·51	+1·29	—·03	+·05	23·82

GROUP II, PAIR 5.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
June 11	12·26·5	27·24·1	21 10 37·17	+5 47·42	— ·61	+·10	+·03	21 16 24·11
13	12·74·3	27·65·8	37·59	46·01	+ ·52	+·10	+·03	24·25
17	13·93·8	28·77·4	38·58	44·17	+1·64	—·37	+·04	24·06
18	13·69·2	28·56 8	38·84	45·10	+ ·33	+·10	+·05	24·42
22	12·79·0	27·59·3	39·74	43·41	+1·23	+·10	+·05	24·53
24	11·59·2	26·31·2	40·12	41·48	+2·19	+·12	+·05	23·96
26	12·93·8	27·71·8	40·50	42·87	+1·28	+·10	+·05	24·80
27	13·47·2	28·25·9	40·72	43·04	+ ·32	+·14	+·04	24·26
28	12·81·5	27·61·3	40·93	43·29	+ ·12	+·10	+·05	24·49
30	11·51·3	26·27·2	41·42	42·39	+ ·36	+·10	+·05	24·32
July 4	12·51·1	27·32·9	42·36	43·76	—1·56	+·10	+·05	24·71
8	12·80·4	27·48·7	43·06	40·62	+ ·11	+·10	+.05	23·94
17	12·01·7	26·63·2	44·90	39·05	— ·38	+·07	+·05	23·69
18	12·19·1	26·81·8	45·10	39·32	— ·47	+·09	+·05	24·09
26	12·51·1	26·55·1	46·30	36·38	+1·30	+·09	+·05	24·12
27	11·84·9	26·40·0	46·50	37·56	— ·30	+·09	+·05	23·90
29	11·69·4	26·23·1	46·84	37·24	— ·33	+·07	+·04	23·86
Aug. 2	12·31·6	26·78·5	47·50	35·66	+ ·92	+·09	+·05	24·22
3	12·03·3	26·55·2	47·60	36·82	—1·16	+·09	+·05	23·40
4	12·54·0	27·05·9	47·71	36·82	— ·27	+·11	+·05	24·42
5	11·77·9	26·26·1	47·82	35·96	+ ·18	+·09	+·05	24·10
6	12·63·3	27·05·3	47·92	34·52	+1·10	+·09	+·05	23·68
7	11·74·5	26·30·9	48·04	37·86	—1·96	+·09	+·05	24·08
10	12·79·0	27·28·8	48·48	36·33	—1·05	+·09	+·05	23·90
13	12·46·9	26·81·1	48·93	32·71	+2·00	+·09	+·05	23·78
14	11·96·7	26·39·9	49·06	34·80	— ·34	+·09	+·05	23·66
15	12·41·0	26·76·3	49·15	32·97	+1·48	+·09	+·05	23·74
16	11·72·4	26·13·1	49·23	34·22	+ ·29	+·09	+·05	23·88

GROUP II, PAIR 6.

1891.	*t. d.*	*t. d.*	° ′ ″	′ ″	″	″	″	° ″ ″
June 13	14·82·8	25·12·4	21 12 25·48	+3 58·85	+ ·68	—·11	+·02	21 16 24.92
17	15·03·0	25·26·2	26·50	57·37	+1·23	—·41	+·07	24·76
18	14·18·8	24·46·6	26·76	58·44	— ·20	+·07	+·05	25·12
19	13·51·8	23·73·3	27·02	56·97	+1·55	—·18	+·04	25·40
22	14·03·1	24·20·7	27·66	56·07	+1·32	+·03	+·04	25·12
26	14·87·7	25·02·2	28·43	55·35	+1·24	+·07	+·05	25·14
27	14·00·6	24·17·7	28·66	55·95	+ ·23	+·07	+·05	24·96
28	14·87·1	25·06·5	28·88	56·49	— ·24	+·07	+·05	25·25
30	13·82·9	23·97·0	29·36	55·26	+ ·55	+·05	+·04	25·26
July 4	14·22·2	24·41·5	30·33	56·47	—2·08	+·07	+·05	24·84
8	14·43.6	24·49·7	31·06	53·40	+ ·47	+·07	+·05	25·05
18	14·53·2	24·48·3	33·12	50·85	+ ·32	+·07	+·05	24·41
26	13·38·6	23·25·2	34·38	48·88	+1·26	+·07	+·05	24·64
27	13·17·0	23·12·7	34·58	50·99	— ·59	+·07	+·05	25·10
29	13·37·6	23·31·2	34·98	50·50	— ·68	+·10	+·08	24·98
31	13·42·1	23·36·0	35·34	50·57	—1·50	+·03	+·04	24·48
Aug. 3	14·26·4	24·17·6	35·74	49·94	— ·61	+·07	+·05	25·19
4	13·53·2	23·42·4	35·84	49·48	— ·76	+·10	+·00	24·72
5	13·92·5	23·82·0	35·94	49·55	— ·44	+·03	+·04	25·12
6	13·55·2	23·29·2	36·06	45·95	+2·12	+·03	+·04	24·20
7	14·67·5	24·60·6	36·18	50·38	—2·29	+·07	+·05	24·39
10	12·76·3	22·63·0	36·64	48·90	— ·99	+·07	+·05	24·67
13	13·44·6	23·16·4	37·10	45·44	+2·24	+·07	+·05	24·90
14	13·46·2	23·27·2	37·24	47·58	— ·31	—·05	+·12	24·58
15	13·28·9	23·05·0	37·34	46·44	+ ·70	+·08	+·08	24·64
16	13·73·3	23·51·8	37·43	47·00	+ ·32	+·07	+·05	24·87

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP II, PAIR 7.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
	N.	*S.*		*Mic.*	*Level.*	*Ref.*	*Mer.*	
1891.	*t. d.*	*t. d.*	° ′ ″	′ ″	″	″	″	° ′ ″
June 10	24·37·6	15·99·5	21 20 02·46	−3 37·62	— ·73	—·06	+·03	21 16 24·08
13	24·72·7	15·33·1	03·08	37·97	— ·46	—·06	+·03	24·62
18	24·86·2	15·43·5	04·40	38·69	— ·69	—.31	+·05	24·76
19	24·04·1	15·49·7	04·66	41.41	+1·07	—·06	+·06	24·32
22	25·17·7	15·65·7	05·33	40·85	— ·09	—·06	+·06	24·39
26	24·53·7	14·97·0	06·13	41·94	+ ·41	—·06	+·06	24·60
27	25·05·3	15·53·1	06·36	40·90	— ·70	—·06	+·06	24·76
28	24·00·0	14·39·5	06·58	42·82	+ ·06	—·06	+·06	23·82
30	25·11·0	15·55·8	07·08	41·59	— ·95	—·06	+·06	24·54
July 4	23·37·3	13·87·5	08·10	41·04	−2·87	—·03	+.06	24·22
8	24·45·5	14·82·0	08·82	43·52	— ·67	—·10	+·05	24·58
18	24·19·1	14·47·8	11·05	45·33	−1·31	—·06	+·07	24·42
22	24·64·3	14·91·6	11·71	45·65	−1·96	—·06	+·04	24·08
23	24·64·4	14·79·9	11·87	48·39	+ ·34	—·12	+·04	23·74
24	24·10·8	14·28·3	12·03	47·92	— ·09	—·06	+·07	24·03
26	24·41·8	14·56·1	12·41	48·67	+ ·64	—·06	+·06	24·38
27	24·26·2	14·47·9	12·62	46·95	−1·39	—·06	+·06	24·28
29	24·64·3	14·85·5	13·05	47·07	−1·38	—·06	+·06	24·60
30	23·60·5	13·76·4	13·26	48·30	— ·72	—·06	+·06	24·24
Aug. 2	23·44·6	13·56·7	13·76	49·18	— ·41	—·10	+·05	24·12
3	24·35·1	14·53·1	13·89	47·81	−1·70	—·06	+·06	24·38
4	23·59·3	13·74·4	14·02	48·48	−1·59	—·06	+·06	23·95
5	24·14·3	14·31·0	14·14	48·11	−1·99	—·06	+·06	24·04
6	24·02·7	14·08·3	14·26	50.69	+1·08	—·06	+·06	24·65
7	24·06·7	14·73·5	14·39	50·41	+ ·09	—·06	+·06	24·07
10	23·33·3	13·46·6	14·88	48·90	−1·94	—·06	+·06	24·04
13	23·66·3	13·63·0	15·40	52·75	+1·25	—·06	+·06	23·90
14	24·60·4	14·67·0	15·56	50·46	−1·03	—·06	+·06	24·07
15	24·20·4	14·26·9	15·68	50·48	−1·26	—·06	+·06	23·94
16	23·52·0	13·54·8	15·78	51·34	— ·40	—·06	+·06	24·04

GROUP II, PAIR 8.

1891.	*t. d.*	*t. d.*	° ′ ″	′ ″	″	″	″	° ′ ″
June 10	18·12·3	22·78·8	21 14 34·76	+1 48·22	+0·23	+·03	+·03	21 16 23·27
17	17·38·2	21·93·7	36·42	45·67	+1·55	+·51	+·07	24·22
18	17·43·7	22·06·9	36·70	47·46	— ·22	+·03	+·05	24·02
19	17·83·5	22·39·2	36·98	45·72	+1·83	—·45	+·16	24·24
22	18·55·5	22·13·2	37·64	46·18	+ ·30	+·03	+·05	24·20
26	17·70·6	22·22·0	38·50	44·72	+1·05	+·03	+·05	24·35
27	18·00·4	22·53·2	38·72	45·04	— ·01	+·03	+·05	23·83
28	16·58·6	21·03·0	38·96	43·09	+1·17	+·03	+·08	23·33
30	17·23·2	21·72·5	39·46	44·23	+ ·36	+·03	+·05	24·13
July 8	17·22·7	21·64·7	41·30	42·54	— ·02	+·03	+·05	23·90
18	16·55·3	20·89·7	43·54	40·77	— ·59	+·03	+·05	23·80
22	16·84·5	21·16·8	44·23	40·29	— ·58	+·06	+·13	24·13
24	16·63·1	20·89·9	44·56	39·01	+ ·39	+·03	+·05	24·04
26	17·11·5	21·28·1	44·95	36·64	+1·55	+·03	+·05	23·22
29	15·74·9	20·01·6	45·61	38·99	— ·98	+·03	+·05	23·70
30	18·35·8	21·58·7	45·82	38·11	— ·26	+·07	+·04	23·78
Aug. 2	17·02·6	21·21·6	46·36	37·20	+ ·76	—·00	+·08	24·40
3	16·83·1	21·05·6	46·50	38·01	−1·06	+·03	+·05	23·53
5	16·74·4	20·95·4	46·70	37·67	— ·93	+·03	+·05	23·52
6	16·60·6	20·68·6	46·89	34·65	+2·24	+·03	+.05	23·86
7	16·70·7	20·78·4	47·02	34·58	+1·74	+·03	+·05	23·42
10	16·63·0	20·82·8	47·54	37·39	−1·32	+·03	+·05	23·69
13	16·65·0	20·75·6	48·08	35·25	+ ·67	+·03	+·05	24·08
14	17·24·6	21·37·6	48·24	35·81	— .19	+·03	+·05	23·94
15	17·04·6	21·16·8	48·36	35·62	— ·38	+·03	+·05	23·68
16	17·17·6	21·24·7	48·49	34·44	+ ·38	+·03	+·05	23·39

GROUP III, PAIR 1.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
July 4	25·81·5	14·00·0	21 20 37·18	—4 10·89	—1·73	—·07	+·05	21 16 24·54
8	24·72·2	13·80·4	38·02	13·28	— ·12	—·07	+·05	24·60
17	24·78·9	13·80·0	40·10	14·93	— ·65	—·07	+·05	24·50
18	24·36·2	13·35·5	40·32	15·34	— ·32	—·07	+·05	24·64
23	25·22·1	13·08·6	41·22	18·32	+1·78	—·05	+·05	24·68
24	24·90·5	13·82·2	41·40	17·11	+ ·50	—·07	+·05	24·77
26	24·22·9	13·05·0	41·80	19·34	+1·94	—·07	+·05	24·38
27	24·87·7	13·79·4	42·02	17·11	— ·65	—·07	+·05	24·24
29	24·48·8	13·39·0	42·49	17·46	— ·91	—·07	+·05	24·10
30	24·32·0	13·18·4	42·71	18·36	·00	—·07	+·05	24·33
Aug. 3	24·48·3	13·31·0	43·44	19·20	— ·15	—·07	+·05	24·07
5	24·15·1	13·00·3	43·70	18·62	— ·52	—·07	+·05	24·54
6	24·83·5	13·57·6	43·85	21·19	+1·88	—·07	+·05	24·52
7	24·76·6	13·48·4	43·99	21·72	+1·87	—·07	+·05	24·12
10	24·67·5	13·49·9	44·53	19·27	— ·70	—·07	+·05	24·54
13	24·53·4	13·22·9	45·10	22·26	+1·02	—·07	+·05	23·84
14	25·12·8	13·88·4	45·26	20·84	— ·04	—·07	+·05	24·36
15	24·66·2	13·41·0	45·42	21·03	— ·30	+·01	+·08	24·18
16	24·74·2	13·48·5	45·54	21·14	·00	—·07	+·05	24·38
22	24·54·8	13·15·1	46·21	24·39	+2·44	—·07	+·05	24·24
23	24·90·4	13·58·6	46·36	22·56	+ ·64	—·07	+·05	24·42
25	25·12·6	13·81·0	46·68	22·51	+ ·62	—·07	+·05	24·77
26	24·96·5	13·51·3	46·83	25·67	+3.62	—·07	+·05	24·70
29	24·87·9	13·50·0	47·19	23·98	+ ·87	—·07	+·05	24·06
Sept. 2	25·98·7	14·49·3	47·44	26·64	+4·12	—·07	+·05	24·90
9	25·61·1	14·25·3	48·11	23·49	— ·30	—·07	+·05	24·30
12	24·94·1	14·57·8	48·32	23·60	— ·30	—·07	+·05	24·40
18	25·11·4	13·69·1	48·44	25·00	+1·17	—·07	+·05	24·59
19	25·50·6	14·11·1	48·50	24·35	+ .57	—·07	+·05	24·70
26	25·16·0	13·76·8	48·80	24·28	+ .38	—·07	+·05	24·88
28	24·85·4	13·44·0	48·74	24·79	+ .58	—·07	+·05	24·51

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP III, PAIR 2.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
July 4	28·99·7	11·62·7	21 23 07·24	−6 42·96	— ·01	—·12	+·05	21 16 24·20
8	29·49·6	12·07·5	08·10	44·14	+ ·20	—·12	+·05	24·09
17	29·64·3	12·14·3	10·22	45·97	— ·18	—·12	+·05	24·00
18	28·81·6	12·34·9	10·45	45·21	— ·88	—·12	+·05	24·29
22	28·27·5	10·79·0	11·22	45·63	—1·34	—·12	+·05	24·18
23	28·51·6	10·90·5	11·40	48·55	+1·97	—·10	+·05	24·77
26	28·47·9	10·90·1	11·99	47·78	— ·34	—·12	+·05	23·80
27	30·11·6	12·54·0	12·22	47·74	— ·41	—·20	+·13	24·00
29	29·13·0	11·52·6	12·70	48·39	— ·46	—·12	+·05	23·78
30	29·16·8	11·55·2	12·92	48·66	— ·15	—·12	+·05	24·04
31	29·18 1	11·60·0	13·16	48·85	— ·90	—·12	+·05	24·34
Aug. 2	28·60·6	10·97·6	13·54	48·99	— ·28	—·12	+·05	24·20
3	29·68·2	12·05·4	13·70	48·94	— ·82	—·15	+·06	23·85
5	28·85·3	11·19·4	13·99	49·66	— ·50	—·12	+·05	23·76
6	29·50·9	11·82·4	14·14	50·27	+ ·42	—·12	+·05	24·22
7	29·38·7	11·62·7	14·28	52·00	+1·97	—·12	+·05	24·18
10	29·25·6	11·63·3	14·84	48·83	—2·08	—·12	+·05	23·86
13	29·61·1	11·85·1	15·44	52·00	+ ·51	—·12	+·05	23·88
14	29·75·7	11·99·6	15·61	52·03	— ·12	—·12	+·05	23·39
15	28·85·8	11·12·8	15·78	51·31	— ·58	—·12	+·05	23·82
16	29·82·9	12·05·5	15·92	52·33	+ ·72	—·12	+·05	24·24
20	28·88·7	11·06·4	16·38	53·47	+1·29	—·12	+·05	24·13
22	29·34·4	11·53·1	16·64	53·24	+ ·55	—·12	+·05	23·88
23	28·89·8	11·07·8	16·80	53·40	+ ·57	—·12	+·05	23·90
25	29·43·9	11·62·2	17·14	53·33	+ ·84	—·12	+·05	24·58
26	29·23·1	11·32·2	17·29	55·46	+2·46	—·12	+·05	24·22
29	29·73·9	11·93·4	17·70	53·05	— ·31	—·12	+·05	24·27
31	29·29·9	11·48·9	17·86	53·16	— ·38	—·12	+·05	24·25
Sept. 2	29·33·8	11·55·1	17·98	52·63	—1·12	—·12	+·05	24.16
6	29·96·3	12·08·3	18·36	54·79	+ ·92	—·12	+·05	24·42
9	28·71·8	10·95·8	18·72	52·00	—2·14	—·12	+·05	24·51
15	29·17·7	11·21·1	19·07	56·78	+1·94	—·12	+·05	24·16
18	29·23·7	11·29·2	19·17	56·30	+1·50	—·12	+·05	24·30
19	29·87·6	11·99·6	19·22	54·79	— ·22	—·12	+·05	24·14
26	29·75·7	11·82·5	19·62	56·00	+ ·35	—·12	+·05	23·90
27	29·28·1	11·28·2	19·60	57·55	+1·72	—·12	+·03	23·68
28	28·17·3	10·25·9	19·59	55·58	+ ·24	—·12	+·05	24·18

GROUP III, PAIR 3.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* t. d.	*S.* t. d.	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
July 4	14·66·8	23·40·1	21 13 02·18	+3 22·59	— ·26	+·06	+·04	21 16 24·62
8	14·53·2	23·21·8	03·04	21·50	+ ·12	+·06	+·05	24·76
17	15·05·1	23·67·2	05·16	19·99	—1·62	+·06	+·05	23·64
18	14·13·4	22·73·0	05·40	19·41	— ·32	+·06	+·05	24·60
22	14·63·4	23·24·3	06·19	19·72	—1·68	+·06	+·05	24·34
23	14·93·0	23·43·8	06·37	17·37	+1·20	+·09	+·06	25·09
26	14·79·0	23·30·7	06·98	17·58	+ ·24	+·10	+·04	24·94
27	15·06·7	23·60·7	07·20	18·12	—1·35	+·06	+·05	24·08
29	14·89·2	23·38·0	07·68	16·91	— ·45	+·10	+·04	24·28
30	14·20·7	22·69·3	07·91	16·86	— ·42	+·06	+·05	24·46
31	15·34·1	23·84·9	08·14	17·37	—1·40	+·08	+·05	24·24
Aug. 2	14·23·8	22·70·9	08·54	16·51	— ·64	+·06	+·05	24·52
3	15·49·5	23·99·0	08·70	17·07	—1·06	+·09	+·06	24·86
5	15·04·3	23·50·2	09·00	16·24	— ·63	+·06	+·05	24·72
6	15·18·7	23·61·4	09·16	15·49	— ·54	+·06	+·05	24·22
7	15·55·8	23·92·1	09·30	14·01	+ ·98	+·06	+·05	24·40
10	15·00·9	23·46·4	09·86	16·14	—1·87	+·06	+·05	24·24
13	14·87·9	23·20·6	10·48	13·17	+ ·36	+·06	+·05	24·12
14	15·25·0	23·62·3	10·65	14·24	— ·97	+·06	+·05	24·03
15	15·29·7	23·72·0	10·83	15·40	—1·72	+·06	+·05	24·62
16	15·54·3	23·88·5	10·98	13·52	— ·13	+·06	+·05	24·48
20	15·55·1	23·85·6	11·46	12·66	+ ·27	+·06	+·05	24·50
22	14·81·6	23·13·6	11·73	13·01	— ·31	+·06	+·05	24·54
23	14·51·2	23·82·1	11·89	12·76	— ·34	+·06	+·05	24·42
26	15·36·9	23·57·3	12·40	10·32	+1·40	+·06	+·05	24·23
29	15·53·5	23·82·3	12·82	12·27	— ·98	+·06	+·05	24·22
31	15·56·5	23·86·2	13·00	12·48	— ·93	+·06	+·05	24·66
Sept. 2	14·76·1	23·03·5	13·14	11·94	—1·40	+·03	+·05	23·70
6	15·29·8	23·55·1	13·53	11·46	—1·02	+·05	+·05	24·07
9	14·86·8	23·15·1	13·91	12·15	—1·91	+·05	+·05	24·25
12	15·41·9	23·66·7	14·18	11·34	—1·65	+·05	+·05	23·07
15	15·17·3	23·33·7	14·31	09·39	+ ·14	+·05	+·05	23·04
18	14·98·6	23·15·1	14·43	09·42	— ·05	+·05	+·05	23·90
19	14·62·7	22·82·6	14·50	10·20	— ·45	+·01	+·04	24·30
26	14·80·7	22·92·3	14·94	08·28	+ ·53	+·05	+·05	23·85
27	14·84·2	22·98·9	14·94	09·00	+ ·52	+·05	+·05	24·56
28	14·97·4	23·09·1	14·93	08·30	+ ·79	+·05	+·05	24·12

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP III, PAIR 4.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
July 4	8·47·0	31·29·2	21 07 12·46	+9 12·63	— ·76	+·19	+·05	21 16 24·57
8	7·00·3	30·83·3	13·34	12·82	—1·12	+·17	+·06	25·26
17	7·67·8	31·42·3	15·47	10·85	—2·19	+·17	+·06	24·36
18	8·41·6	31·12·4	15·72	09·99	— ·96	+·17	+·06	24·98
23	6·65·8	30·26·9	16·71	07·74	+ ·17	+·12	+·09	24·83
26	6·93·9	30·53·1	17·32	07·30	— ·49	+·17	+·06	24·36
27	6·73·5	30·36·6	17·56	08·27	—1·58	+·17	+·06	24·48
29	6·40·0	29·99·0	18·06	07·25	—1·46	+·23	+·05	24·13
30	7·73·1	31·32·6	18·30	07·37	—1·39	+·16	+·06	24·50
Aug. 2	7·81·2	31·37·7	18·95	06·67	—1·23	+·12	+·04	24·55
3	6·62·6	30·22·2	19·12	07·39	—1·81	+·10	+·04	24·84
5	6·97·7	30·53·9	19·44	06·60	—1·96	+·16	+·06	24·30
7	6·37·3	29·84·5	19·76	04·51	+ ·20	+·16	+·06	24·69
10	6·53·7	30·07·2	20·33	05·98	—1·99	+·16	+·06	24·54
13	7·07·3	30·50·1	20·97	03·50	— ·21	+·16	+·06	24·48
14	5·74·2	29·17·9	21·16	03·70	—1·16	+.16	+·06	23·92
15	6·36·2	29·89·0	21·35	05·82	—2·64	+·16	+·06	24·75
16	7·02·6	30·44·6	21·52	03·31	— ·71	+·16	+·06	24·34
20	6·03·9	29·41·5	22·04	02·29	— ·24	+·16	+·06	24·31
22	6·66·5	30·10·0	22·33	03·66	—1·51	+·16	+·06	24·70
23	7·11·2	30·50·4	22·50	02·66	— ·68	+·16	+·06	24·70
25	6·84·5	30·17·3	22·86	01·18	— ·12	+·16	+·06	24·14
26	6.14·4	30·44·1	23·05	00·46	+ ·41	+·16	+·06	24·14
29	7·10·0	30·44·7	23·52	01·62	—1·83	+·16	+·06	23·53
31	7·07·5	30·43·5	23·74	01·92	—2·08	+·16	+·06	23·80
Sept. 2	7·07·1	30·38·7	23·90	00·90	— ·84	+·16	+·06	24·18
6	7·05·9	30·40·6	24·34	01·62	—2·04	+·16	+·06	24·14
9	7·09.4	30·46·2	24·76	02·10	—2·76	+·16	+·06	24·32
12	7·07·9	30·40·8	25·09	01·20	—2·33	+·16	+·06	24·18
15	7·42·6	30·65·8	25·26	+8 58·95	— ·17	+·16	+·06	24·26
18	7·30·3	30·52·1	25·44	58·62	— ·44	+·16	+·06	23·84
19	6·75·5	30·00·4	25·52	59·34	—1·36	+·16	+·06	23·72
26	6·98·8	30·18·6	26·10	58·16	— ·65	+·16	+·06	23·83
27	7·50·4	30·68·9	26·12	57·86	— ·48	+·16	+·06	23·72
28	5·96·5	29·19·5	26·13	58·90	— ·83	+·16	+·06	24·42

GROUP III, PAIR 5.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
July 4	29·70·7	11·87·7	21 22 54·44	—6 30·43	— ·79	—·11	+·05	21 16 23·16
8	27·71·5	10·83·8	55·36	31·52	— ·32	—·11	+·05	23·46
17	27·25·5	10·30·3	57·56	33·26	— ·76	—·11	+·05	23·48
18	28·09·6	11·12·5	57·80	33·70	— ·66	—·11	+·05	23·38
22	27·53·0	10·55·0	58·64	33·91	—1·27	—·11	+·05	23·40
23	27·45·5	10·16·5	58·84	36·46	+1·30	—·22	+·08	23·54
26	27·98·2	10·88·6	59·47	36.60	+ ·53	—·11	+·05	23·34
27	27·98·9	10·95·5	59·71	35·16	— ·75	—·11	+·05	23·74
Aug. 2	27·89·4	10·79·8	23 01·14	36·60	— ·57	—·15	+·04	23·86
5	27·94·5	10·79·3	01·66	37·90	— ·38	—·11	+·05	23·32
7	27·64·8	10·41·9	01.98	39·69	+1·36	—·15	+·04	23·54
10	27·52·8	10·39·8	02·58	37·39	—1·27	—·11	+·04	23·86
13	27·62·2	10·34·7	03·25	40·75	+ ·83	—·11	+·05	23·28
14	27·26·2	10·04·3	03·52	39·46	— ·50	—·11	+·05	23·50
15	27·82·7	10·64·3	03·67	38·64	—1·35	—·11	+·05	23·62
16	27·84·4	10·24·9	03·82	40·15	+ ·18	—·11	+·05	23·79
20	28·59·7	11·29·7	04·36	41·33	+ ·55	—·11	+·05	23·52
22	27·90·3	10·63·4	04·68	40·62	— ·48	—·11	+·05	23·52
23	28·03·5	10·73·4	04·84	41·36	+ ·23	—·11	+·05	23·65
25	28·56·8	11·23·6	05·22	42·08	+ ·67	—·11	+·05	23·75
26	28·91·7	11·58·1	05·42	42·17	+ ·52	—·11	+·05	23·71
29	28·23·7	11·01·3	05·90	39·57	—2·00	—·11	+·05	24·27
31	28·42·0	11·12·6	06·14	41·20	—1·05	—·11	+·05	23·83
Sept. 2	29·14·7	11·71·1	06·32	44·49	+2·19	—·11	+·05	23·96
6	28·02·8	10·75·5	06·76	40·71	—1·74	—·15	+·04	24·20
9	27·54·9	10·27·4	07·20	40·75	—2·00	—·11	+·05	24·39
12	28·27·6	10·89·9	07·56	43·12	— ·76	—·11	+·05	23·62
15	28·39·0	11·95·8	07·75	44·40	+ ·19	—·13	+·03	23·44
18	28·81·2	11·38·6	07·94	44·26	+ ·26	—·11	+·05	23·88
19	27·71·5	10·29·3	08·02	44·16	— ·12	—·11	+·05	23·68
26	28·77·5	10·33·1	08·64	44·68	— ·32	—·15	+·04	23·53
27	28·95·3	10·51·2	08·66	44·60	+ ·18	—·11	+·05	24·18
28	27·87·6	10·44·2	08·68	44·44	— ·10	—·11	+·05	24·08

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP III, PAIR 6.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* t. d.	*S.* t. d.	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
July 4	16·47·5	21·41·0	21 14 29·50	+1 54·48	+ ·20	+ ·03	+ ·05	21 16 24·26
8	15·86·6	20·77·7	30·46	53·93	— ·03	+ ·03	+ ·05	24·44
17	15·64·2	20·49·6	32·72	52·61	— ·83	+ ·03	+ ·05	24·58
18	16·70·6	21·49·7	32·97	51·14	— ·03	+ ·03	+ ·05	24·16
22	15·47·8	20·30·5	33·86	51·98	— ·91	— ·01	+ ·04	24·96
23	16·07·3	20·73·1	34·06	48·06	+2·03	+ ·03	+ ·05	24·23
26	16·52·8	21·22·7	34·72	49·01	+1·07	+ ·03	+ ·05	24·88
27	15·89·5	20·62·1	34·96	49·64	— ·58	+ ·03	+ ·05	24·10
30	16·51·8	21·19·7	35·75	48·55	+ ·12	+ ·03	+ ·05	24·50
Aug. 2	15·97·2	20·61·5	36·46	47·71	— ·06	+ ·03	+ ·05	24·19
5	16·57·8	21·24·2	37·00	48·20	— ·83	— ·05	+ ·06	24·38
6	15·78·1	20·39·3	37·17	46·99	+ ·18	+ ·03	+ ·05	24·42
10	16·38·5	21·01·2	37·96	47·34	— ·88	+ ·03	+ ·05	24·50
13	16·09·6	20·60·3	38·66	44·56	+1·30	+ ·03	+ ·05	24·60
14	15·75·6	20·31·3	38·86	45·72	— ·56	+ ·03	+ ·05	24·10
15	16·66·0	21·22·0	39·07	45·78	— ·88	+ ·03	+ ·05	24·05
16	16·58·2	21·08·0	39·26	44·35	+ ·77	+ ·03	+ ·05	24·46
20	17·25·1	21·72·9	39·86	43·88	+ ·21	— ·01	+ ·04	23·98
23	16·32·3	20·76·5	40·36	43·05	+ ·79	+ ·03	+ ·05	24·28
25	16·27·9	20·65·3	40·75	41.47	+1.94	+ ·03	+ ·05	24·24
26	16·13·3	20·56·1	40·96	42·72	+ ·60	+ ·03	+ ·05	24·36
29	16·62·3	20·90·0	41.38	39·22	+3·26	+ ·03	+ ·05	23·94
31	16·50·8	20·95·4	41·74	43·14	— ·96	+ ·03	+ ·05	24·00
Sept. 2	16·40·3	20·65·2	41·94	38·57	+3·40	+ ·03	+ ·05	23·99
6	16·22·2	20·64·1	42·42	42·51	— ·69	+ ·03	+ ·05	24·32
9	15·93·3	20·22·3	42·88	39·52	+1·46	+ ·03	+ ·05	23·94
12	15·86·4	20·13·1	43·27	38·99	+1·74	+ ·03	+ ·05	24·08
18	16·87·5	21·18·0	43·71	39.87	+ ·40	+ ·03	+ ·05	24·06
19	16·53·9	20·84·4	43·80	39·87	+ ·06	+ ·03	+ ·05	23·81
23	16·64·4	20·85·9	44·26	37·78	+1·85	+ ·03	+ ·05	23·97
26	16·27·9	20·56·5	44·50	39·43	+ ·35	+ ·03	+ ·05	24·36
27	16·46·3	20·70·1	44·52	38·32	+ ·84	+ ·03	+ ·05	23·76
28	15·88·3	20·14·8	44·56	38·94	+ ·46	+ ·03	+ ·05	24·04

GROUP III, PAIR 7.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
	N.	*S.*		*Mic.*	*Level.*	*Ref.*	*Mer.*	
1891.	*t. d.*	*t. d.*	° ′ ″	′ ″	″	″	″	° ′ ″
July 8	17·57·7	22·60·4	21 14 28·04	+1 56·62	— ·16	+·05	+·05	21 16 24·60
17	16·59·7	21·56·3	30·26	55·20	—1·52	+·03	+·05	24·02
18	16·70·7	21·60·9	30·52	53·72	— ·85	+·03	+·06	23·48
23	16·27·2	21·05·8	31·60	51·03	+1·06	+·05	+·08	23·82
26	17·01·4	21·81·5	32·25	51·38	+ ·06	+·03	+·05	23·77
27	16·56·8	21·40·9	32·48	52·30	—1·32	+·03	+·04	23·53
30	16·78·2	21·61·5	33·26	52·12	—1·75	+·03	+·06	23·72
Aug. 2	15·73·6	21·51·4	33·98	50·84	—1·08	+·03	+·05	23·82
3	16·14·1	21·91·4	34·17	50·73	—1·23	+·03	+·05	23·75
5	16·66·9	21·42·9	34·52	50·42	—1·10	+·03	+·05	23·92
7	16·49·0	21·20·3	34·86	49·34	+ ·35	+·03	+·05	24·03
13	16·62·6	21·26·4	36·16	47·60	+ ·10	+·03	+·05	23·94
14	17·34·9	21·99·2	36·36	47·71	— ·97	+·03	+·05	23·18
15	16·65·2	21·38·9	36·59	49·89	—2·82	+·03	+·05	23·74
16	16·97·1	21·57·7	36·78	46·85	— ·18	+·03	+·05	23·53
20	17·65·3	22·27·2	37·38	47·15	— ·85	+·03	+·05	23·76
22	17·10·0	21·73·6	37·70	47·55	—1·11	+·03	+·05	24·22
25	18·21·0	22·62·7	38·28	46·85	—1·19	+·03	+·05	24·02
26	17·57·2	22·12·3	38·48	45·58	— ·42	+·03	+·05	23·72
29	17·54·5	22·01·0	39·04	43·58	+ ·80	+·03	+·05	23·50
31	17·07·8	21·66·3	39·28	46·36	—1·64	+·03	+·05	24·08
Sept. 2	17·91·3	22·30·9	39·48	41·98	+1·56	+·03	+·05	23·10
6	17·32·1	21·83·5	39·98	44·72	—1·17	+·03	+·05	23·61
9	17·89·6	22·26·9	40·45	41·45	+1·74	+·03	+·05	23·72
12	18·00·7	22·38·8	40·86	41·63	+ ·94	+·05	+·05	23·53
15	17·43·9	21·37·8	41·10	42·98	— ·18	+·03	+·05	23·98
18	16·92·5	21·34·7	41·32	42·58	— ·20	+·05	+·05	23·80
19	17·49·2	21·93·1	41·42	42·98	— ·71	+·03	+·05	23·77
23	17·85·7	22·20·8	41·90	40·94	+ ·42	+·01	+·05	23·32
26	18·05·1	22·47·0	42·16	42·51	— ·70	+·03	+·05	24·05
27	17·35·3	21·73·9	42·19	41·75	— ·66	+·06	+·06	23·40
28	17·54·1	21·92·5	42·23	41·70	— ·19	+·03	+·05	23·82

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP III, PAIR 8.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
July. 4	25·78·4	14·82·3	21 20 38·92	−4 14·28	— ·08	—·08	+·06	21 16 24·54
8	24·95·5	13·94·1	39·86	15·51	— ·25	—·08	+·06	24·08
17	24·10·4	13·04·9	42·06	16·46	—1·42	—·08	+·06	24·16
18	24·82·0	13·71·0	42·32	17·74	— ·47	—·08	+·06	24·09
22	26·02·8	14·93·0	43·21	17·46	—1·45	—·08	+·06	24·28
23	25·03·1	13·78·5	43·40	20·89	+1·29	—·08	+·06	23·78
26	24·34·0	13·10·9	44·05	20·54	+ ·41	—·08	+·06	23·90
27	24·66·1	13·48·0	44·28	19·38	— ·68	—·08	+·06	24·20
30	24·92·0	13·70·9	45·56	20·08	— ·66	—·08	+·06	24·80
Aug. 2	24·72·8	13·47·2	45·77	21·12	— ·70	—·03	+·08	24·00
3	25·29·3	14·04·0	45·97	21·05	— ·36	—·08	+·06	24·54
5	24·75·8	13·50·3	46·34	21·10	—1·17	—·08	+·06	24·05
6	24·80·0	13·49·5	46·49	22·26	+ ·27	—·13	+·09	24·46
7	24·06·7	12·69·7	46·66	23·77	+ ·97	—·08	+·06	23·84
10	25·59·3	14·27·8	47·27	22·49	—1·08	—·08	+·06	23·68
13	24·61·9	13·22·4	47·96	24·35	+ ·82	·08	+·06	24·41
14	24·51·3	13·12·7	48·17	24·14	— ·25	—·08	+·06	23·76
15	24·27·2	13·97·4	48·38	22·10	—1·38	—·08	+·06	24·88
16	24·90·5	13·50·7	48·57	24·42	+ ·15	—·08	+·06	24·28
20	24·82·1	13·37·9	49·18	25·44	+ ·44	—·06	+·05	24·17
22	25·21·4	13·82·7	49·50	24·16	—1·02	—·08	+·06	24·30
23	25·26·6	13·80·5	49·68	25·88	+ ·26	—·08	+·06	24·04
25	25·00·2	13·55·0	50·08	25·67	+ ·36	—·08	+ 06	24·75
26	25·43·2	13·96·6	50·28	25·99	+ ·29	—·08	+·06	24·56
29	25·36·2	13·88·8	50·82	26·18	— ·20	—·08	+·06	24·42
31	24·18·9	13·73·8	51·09	25·65	— ·74	—·08	+·06	24·68
Sept. 2	24·68·9	13·09·4	51·30	28·99	+2·15	—·08	+·06	24·44
9	25·32·6	13·75·1	52·27	28·52	+ ·75	—·08	+·06	24·48
12	24·77·3	13·14·8	52·69	29·68	+1·26	—·08	+·06	24·25
15	24·05·8	13·46·2	52·94	29·01	+ ·39	—·08	+·06	24·30
18	25·61·0	14·04·3	53·18	28·34	— ·56	—·08	+·06	24·26
19	24·90·8	13·32·4	53·27	28·73	+ ·14	—·08	+·06	24·66
23	24·95·4	13·28·7	53·75	30·66	+1·07	—·08	+·06	24·14
26	25·53·8	13·90·9	54·02	29·78	— ·02	—·08	+·06	24·20
27	24·55·9	12·92·4	54·06	29·92	+ ·08	—·08	+·06	24·20
28	25·65·7	14·03·0	54·11	29·73	— ·06	—·08	+·06	24·30

GROUP IV, PAIR 1.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Aug. 20	13·77·2	23·96·7	21 12 28·52	+3 56·51	—1·00	+·08	+·06	21 16 24·17
22	13·08·0	23·37·7	28·88	58·87	—3·41	+·08	+·06	24·48
23	14·16·7	24·36·0	29·08	56·46	— ·88	+·08	+·06	24·80
25	13·66·3	23·81·4	29·52	55·49	— ·50	+·08	+·06	24·65
29	13·73·1	23·84·0	30·40	54·51	—1·39	+·08	+·06	23·60
31	13·57·0	23·79·1	30·75	57·11	—2·84	+·08	+·06	25·16
Sept. 2	14·07·9	24·16·8	31·04	54·05	— ·57	+·08	+·06	24·66
6	13·65·7	23·65·5	31·62	51·94	+ ·50	+·08	+·06	24·20
9	13·41·4	23·49·7	32·19	53·91	—1·58	+·08	+·06	24·66
12	13·75·6	23·77·8	32·73	52·49	—1·02	+·08	+·06	24·34
15	14·25·1	24·24·4	33·14	51·82	— ·91	+·08	+·06	24·19
19	12·97·3	23·00·1	33·60	52·63	—2·31	+·08	+·06	24·06
23	13·66·8	23·62·2	34·22	50·92	—1·47	+·08	+·06	23·81
26	13·77·4	23·73·9	34·64	51·17	—1·68	+·08	+·06	24·27
27	13·95·4	23·87·5	34·75	50·15	—1·16	+·08	+·06	23·88
28	13·69·7	23·61·6	34·83	50·11	—1·12	+·08	+·06	23·96
Oct. 4	14·77·1	24·66·3	35·28	49·48	— ·58	+·08	+·06	24·32
5	14·70·3	24·57·6	35·39	49·04	— ·39	+·08	+·06	24·18
8	14·25·7	24·05·4	35·74	47·28	+ ·93	+·08	+·06	24·09
10	14·63·7	24·48·5	35·93	48·46	— ·39	+·08	+·06	24·14
16	15·28·3	25·10·0	36·19	47·74	— ·01	+·08	+·06	24·06
17	14·59·7	24·42·3	36·25	47·95	— ·72	+·08	+·06	23·62
28	13·70·4	23·62·5	36·69	50·15	—2·42	+·06	+·06	24·54
30	15·20·6	25·12·1	36·67	50·01	—2·14	+·08	+·06	24·68
31	15·22·8	25·02·1	36·68	47·18	+ ·12	+·08	+·06	24·12
Nov. 1	14·45·4	24·33·7	36·70	49·27	—1·88	+·08	+·06	24·23
4	15·43·6	25·17·4	36·81	45·91	+1·32	+·08	+·06	24·18
6	14·59·8	24·38·2	36·85	46·97	+ ·55	+·08	+·06	24·51
7	14·24·8	24·05·8	36·83	47·58	— ·59	+·08	+·06	23·96
12	14·37·6	24·23·7	36·61	48·76	— ·75	+·08	+·08	24·78
14	14·30·8	24·05·2	36·56	46·05	+1·15	+·08	+·06	23·90
16	15·33·2	25·23·0	36·55	49·62	—2·73	+·08	+·06	23·58
17	14·26·2	24.14·8	36·55	49·34	—1·63	+·08	+·06	24·40

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP IV, PAIR 2.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
	N.	*S.*		*Mic.*	*Level.*	*Ref.*	*Mer.*	
1891.	*t. d.*	*t. d.*	° ′ ″	′ ″	″	″	″	° ′ ″
Aug. 20	28·84·5	7·05·6	21 24 50·15	—8 25·47	—1·39	—·16	+·06	21 16 23·19
22	28·58·5	6·66·8	50·52	28·44	+1·10	—·14	+·06	23·10
23	29·20·2	7·33·3	50·72	27·33	— ·06	—·16	+·06	23·23
25	29·13·9	7·31·3	51·16	26·33	— ·93	—·16	+·06	23·80
29	29·15·2	7·21·6	52·06	28·88	+ ·29	—·16	+·06	23·37
31	29·41·3	7·55·1	52·42	27·16	—1·26	—·16	+·06	23·90
Sept. 2	29·25·4	7·31·0	52·70	29·07	+ ·09	—·16	+·06	23·62
6	29·32·9	7·35·7	53·30	29·72	+ ·53	—·16	+·06	24·01
9	29·38·4	7·38·7	53·88	30·30	+ ·08	—·16	+·06	23·56
12	28·67·6	6·66·8	54·44	30·55	— ·67	—·16	+·06	23·12
15	29·38·3	7·41·3	54·85	29·67	—1·58	—·20	+·04	23·44
19	29·01·2	7·01·5	55·33	30·30	—1·93	—·16	+·06	23·00
23	28·13·8	6·07·6	55·97	31·80	—1·23	—·16	+·06	22·84
26	29·18·1	7·14·9	56·40	31·11	—1·48	—·16	+·06	23·71
27	29·01·7	6·94·1	56·51	32·13	—1·40	—·16	+·06	22·88
28	29·50·3	7·42·6	56·60	32·15	— ·97	—·16	+·06	23·38
Oct. 4	29·46·2	7·35·5	57·07	32·85	— ·62	—·13	+·06	23·53
5	29·36·6	7·24·9	57·19	33·08	— ·81	—·16	+·06	23·20
8	29·24·3	7·07·0	57·50	34·38	+ ·68	—·16	+·06	23·70
10	29·89·3	7·73·3	57·75	34·08	+ ·07	—·16	+·06	23·64
17	29·72·8	7·56·3	58·09	34·20	— ·25	—·24	+·08	23·48
28	29·30·2	7·19·2	58·58	32·92	—2·52	—·20	+·04	22·98
30	29·54·0	7·44·7	58·57	32·52	—1·98	—·16	+·06	23·96
31	29·31·5	7·11·1	58·58	35·10	+ ·58	—·16	+·06	23·96
Nov. 1	29·19·4	7·08·8	58·61	32·83	—1·94	—·16	+·06	23·74
4	29·72·1	7·44·3	58·73	36·82	+1·79	—·20	+·04	23·54
6	29·54·7	7·34·4	58·78	35·08	— ·29	—·16	+·06	23·31
7	29·41·2	7·24·5	58·77	34·24	—1·03	—·24	+·08	23·34
10	29·15·8	6·98·3	58·66	34·43	— ·57	—·16	+·06	23·56
12	29·04·2	6·84·5	58·57	34·94	— ·55	—·16	+·08	23·00
14	30·29·0	8·07·0	58·53	35·47	+1·24	—·16	+·08	24·22
15	30·00·9	7·84·0	58·53	34·29	— ·61	—·26	+·08	23·45
17	30·84·7	8·58·9	58·53	36·35	+1·72	—·16	+·06	23·80

GROUP IV, PAIR 3.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* t. d.	*S.* t. d.	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Aug. 20	19·62·1	18·54·8	21 16 47·14	—0 24·89	+ ·47	—·01	+·05	21 16 22·76
23	20·46·0	19·37·9	47·75	25·08	+ ·89	—·01	+·05	23·60
25	19·98·1	18·82·7	48·21	26·77	+1·14	—·01	+·05	22·62
29	19·75·1	18·52·4	49·16	28·46	+1·79	—·01	+·05	22·53
31	19·64·8	18·44·6	49·55	27·88	+1·19	—·01	+·05	22·90
Sept. 2	19·82·9	18·64·3	49·88	27·51	+ ·79	—·01	+·05	23·20
6	19·53·0	18·25·8	50·52	29·51	+2·81	—·01	+·05	23·86
9	19·69·6	18·42·7	51·13	29·44	+ ·62	—·05	+·04	22·30
12	19·72·5	18·41·6	51·73	30·37	+1·42	—·01	+·05	22·82
15	19·93·5	18·64·2	52·20	30·00	+ ·78	—·01	+·05	23·02
19	19·80·0	18·52·9	52·73	29·48	— ·44	—·01	+·05	22·85
23	19·95·9	18·57·4	53·40	32·13	+1·09	—·01	+·05	22·40
26	19·32·5	17·95·5	53·89	31·78	+1·02	—·01	+·05	23·17
27	19·45·8	17·09·5	54·02	31·62	+ ·59	—·01	+·05	23·03
28	19·54·2	18·12·8	54·12	32·80	+1·53	—·01	+·05	22·80
Oct. 4	20·09·7	19·67·2	54·66	33·06	+1·84	—·01	+·05	23·48
5	19·78·2	18·35·6	54·78	33·08	+ ·96	—·01	+·05	22·70
8	19·54·1	18·04·5	55·18	34·70	+2·48	—·01	+·05	23·00
10	19·65·0	18·22·9	55·40	32·96	+ ·58	—·01	+·05	23·06
14	19·65·7	18·15·2	55·65	34·91	+2·20	—·01	+·05	22·98
16	19·80·2	18·31·0	55·76	34·61	+1·22	—·05	+·04	22·36
22	19·93·1	18·49·0	56·28	33·43	+ ·44	—·03	+·05	23·31
28	20·27·8	18·91·5	56·43	31·62	— ·93	—·01	+·06	23·93
30	20·01·6	18·61·3	56·44	32·55	— ·45	—·01	+·05	23·48
31	19·71·1	18·16·2	56·45	35·94	+2·38	—·01	+·05	22·93
Nov. 1	19·82·2	18·39·6	56·48	33·08	— ·58	—·01	+·05	22·86
4	19·89·5	18·34·2	56·63	36·03	+2·64	—·01	+·05	23·28
6	19·94·3	18·47·7	56·70	34·01	+ ·47	—·01	+·05	23·20
7	19·94·0	18·46·7	56·70	34·17	+ ·12	—·01	+·03	22·67
10	19·88·3	18·40·4	56·62	34·31	+ ·50	— ·01	+·05	22·85
14	20·65·7	19·15·5	56·51	34·84	+1·68	—·01	+·05	23·39
17	20·49·7	19·00·5	56·53	34·61	+1·23	—·01	+·05	23·19

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP IV, PAIR 4.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t.* *d.*	*S.* *t.* *d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Aug. 20	14·26·9	21·55·5	21 13 35·54	+2 49·02	— ·55	+·05	+·06	21 16 24·12
23	14·66·6	21·92·3	36·13	48·35	— ·23	+·05	+·06	24·36
25	15·16·6	22·37·3	36.58	47·19	+ ·10	+·05	+·06	23·98
29	14·05·6	21·21·2	37·51	46·01	+ ·44	+·05	+·06	24·07
31	13·64·9	20·84·8	37·89	47·01	— ·65	+·05	+·06	24·36
Sept. 2	14·27·1	21·42·9	38·21	46·06	— ·23	+·05	+·06	24·15
9	14.58·4	21·72·9	39·43	45·75	—1·32	+·05	+·06	23·97
12	14·76·0	21·80·3	40·02	43·39	+ ·58	+·05	+·06	24·10
15	13·86·4	20·91·4	40·49	43·55	— ·18	+·01	+·05	23·92
19	15.03·0	22·10·2	41·00	44·06	—1·07	+·05	+·06	24·10
23	14·24·5	21·24·6	41·67	42·41	— ·38	+·05	+·06	23·81
26	14·69·9	21·68·5	42·16	42·06	— ·33	+·05	+·06	24·00
27	13·82·0	20·81·0	42·29	42·16	— ·20	+·05	+·06	24·36
28	14·77·5	21·72·9	42·39	41·32	— ·26	+·05	+·06	23·56
Oct. 4	15·17·6	22·08·2	42·92	40·21	+ ·86	+·05	+·06	24·10
5	14·52·9	21·45·5	43·04	40·67	+ ·08	+·05	+·06	23·90
8	15·01·4	21·87·1	43·44	39·07	+1·18	+·05	+·06	23·80
14	15·21·1	22·04·6	43·94	38·56	+1·23	+·05	+·06	23·84
16	15·96·6	21·80·5	44·05	38·66	+1·02	+·05	+·06	23·84
22	15·02·7	21·89·9	44·58	39·42	·00	+·05	+·06	24·11
28	15·02·2	21·92·4	44·75	40·12	— ·84	+·05	+·06	24·14
30	14·27·0	21·16·5	44·77	39·95	— ·42	+·09	+·05	24·44
31	14·57·9	21·34·5	44·79	36·96	+2·21	+·05	+·06	24·07
Nov. 1	15·02·2	21·89·5	44·83	39·44	— ·55	+·05	+·06	23·83
4	14·94·2	21·81·7	44·98	39·49	— ·68	+·05	+·06	23·90
6	15·55·6	22·38·1	45·06	38·33	+1·18	+.05	+·06	24·68
7	14·79·4	21·65·7	45·07	39·21	— ·76	+·01	+·05	23·58
10	14·59·2	21·43·1	45·02	38·66	— ·41	+·05	+·06	23·38
14	15·59·8	22·39·2	44·93	37·61	+ ·86	+·01	+·05	23.46
15	16·24·6	23·01·7	44·94	37·08	+1·39	+·05	+·06	23·52
17	16·14·0	22·98·2	44·97	38·72	+ ·18	+·05	+·06	23·98

GROUP IV, PAIR 5.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Aug. 20	25·46·0	12·78·2	21 21 20·33	−4 54·11	−1·56	−·08	+·05	21 16 24·63
23	25·63·2	12·85·3	20·94	56·45	+ ·30	−·08	+·05	24·76
25	25·80·2	13·00·9	21·41	56·78	− ·08	−·08	+·05	24·52
29	25·76·5	12·90·2	22·39	58·40	+ ·46	−·08	+·05	24·42
31	25·18·7	13·38·9	22·81	57·06	− ·90	−·08	+·05	24·82
Sept. 2	26·23·0	13·42·9	23·14	56·96	−1·11	−·08	+·05	25·04
6	25·91·6	12·98·6	23·81	59·96	+ ·50	−·08	+·05	24·32
9	25·92·4	13·09·5	24·43	57·61	−2·12	−·08	+·05	24·67
12	25·74·4	12·78·5	25·07	60·63	+ ·49	−·08	+·05	24·90
15	25·45·3	12·53·9	25·58	59·59	−1·74	−·08	+·05	24·22
23	26·40·2	13·40·6	26·86	−5 01·49	−1·22	−·08	+·05	24·12
26	25·78·8	12·78·2	27·38	01·72	−1·45	−·08	+·05	24·18
28	25·20·5	12·16·4	27·64	02·53	− ·50	−·08	+·05	24·58
Oct. 4	26·40·4	13·32·3	28·24	03·46	+ ·25	−·08	+·05	25·00
5	26·24·3	13·17·2	28·37	03·23	− ·43	−·08	+·05	24·68
8	25·89·4	12·73·2	28·81	05·34	+1·11	−·08	+·05	24·55
10	26·05·7	12·94·9	29·07	04·09	− ·32	−·08	+·05	24·63
16	25·40·6	12·25·1	29·52	05·18	+ ·45	−·08	+·05	24·76
22	25·63·2	12·49·5	30·12	04·76	−1·22	−·05	+·07	24·16
28	25·59·4	12·51·7	30·38	03·37	−1·92	−·08	+·05	25·06
30	25·51·1	12·41·0	30·42	03·92	−1·77	−·08	+·05	24·70
31	26·28·1	13·01·2	30·45	07·82	+1·73	−·08	+·05	24·33
Nov. 1	26·72·8	13·62·5	30·49	03·97	−1·79	−·08	+·05	24·70
4	25·70·3	12·48·7	30·68	06·59	+ ·28	−·08	+·05	24·34
6	25·79·8	12·60·3	30·79	06·10	− ·24	−·08	+·05	24·42
7	25·53·7	12·39·3	30·81	04·92	−1·42	−·08	+·05	24·44
13	26·21·2	13·06·0	30·74	05·11	−1·00	−·08	+·05	24·60
14	26·46·7	13·25·7	30·75	06·45	+ ·19	−·08	+·05	24·46
17	27·23·6	14·07·6	30·82	05·29	- ·67	·08	+·05	24·83

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP IV, PAIR 6.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* t. d.	*S.* t. d.	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Aug. 20	10·06·2	26·23·0	21 10 10·18	+6 15·07	—1·41	+·11	+·07	21 16 24·02
23	10·30·9	26·35·0	10·75	12·13	+ ·88	+·11	+·07	23·94
25	10·16·8	26·21·2	11·20	12·20	+ ·50	+·11	+·07	24·08
29	10·77·4	26·76·7	12·13	11·01	+ ·26	+·11	+·07	23·58
31	10·76·0	26·75·1	12·53	10·97	+ ·55	+·11	+·07	24·23
Sept. 2	10·53·0	26·56·5	12·86	11·99	—1·25	+·11	+·07	23·78
6	10·66·9	26·58·4	13·48	09·20	+1·16	+·11	+·07	24·02
9	11·04·1	27·06·0	14·06	11·62	—2·08	+·11	+·07	23·78
12	10·78·8	26·69·6	14·68	09·04	+ ·02	+·11	+·07	23·92
15	10·26·5	26·13·8	15·18	08·23	+ ·13	+·11	+·07	23·72
19	10·51·3	26·42·6	15·71	09·16	—1·55	+·11	+·07	23·50
23	10·57·7	26·30·3	16·39	04·82	+2·49	+·11	+·07	23·88
26	10·11·4	25·98·7	16·90	08·23	—1·80	+·11	+·07	23·51
27	10·51·4	26·31·5	17·04	06·56	— ·34	+·11	+·07	23·44
28	9·92·2	25·69·3	17·16	05·86	— ·18	+·07	+·05	22·96
Oct. 4	10·69·8	26·45·1	17·73	05·45	+ ·33	+·11	+·07	23·69
5	10·75·1	26·52·9	17·86	06·03	+ ·12	+·11	+·07	24·19
8	10·57·8	26·26·3	18·29	03·87	+1·17	+·11	+·07	23·51
10	9·77·5	25·54·3	18·54	05·79	— ·59	+·11	+·07	23·92
16	10·90·4	26·54·4	19·00	02·82	+1·18	+·11	+·07	23·18
22	10·75·0	26·45·0	19·61	04·22	— ·65	+·13	+·07	23·38
28	10·53·0	26·24·2	19·90	04·49	—1·74	+·11	+·07	22·83
30	10·24·2	25·99·1	19·94	05·35	—1·45	+·11	+·07	24·02
31	10·80·0	26·37·7	19·97	01·36	+2·12	+·15	+·05	23·65
Nov. 1	10·36·5	26·12·4	20·02	05·58	—2·26	+·11	+·07	23·52
4	10·88·5	26·51·2	20·22	02·52	+ ·38	+·11	+·07	23·30
6	10·73·6	26·38·3	20·35	02·99	+ ·22	+·11	+·07	23·74
10	10·62·6	26·28·0	20·40	03·15	— ·97	+·11	+·07	22·76
12	10·38·5	26·06·0	20·37	03·64	—1·34	+·11	+·07	22·85
13	11·34·7	26·99·4	20·38	02·99	+ ·13	+·11	+·07	23·68
14	12·44·5	28·06·3	20·38	02·31	+1·07	+·11	+·07	23·94
17	12·25·3	27·88·6	20·48	02·66	+ ·12	+·11	+·07	23·44

GROUP IV, PAIR 7.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t.* *d.*	*S.* *t.* *d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Aug. 20	25·46·6	12·22·7	21 21 32·86	−5 07·12	−1·51	−·10	+·06	21 16 24·19
23	25·10·6	11·77·1	33·43	09·35	+ ·35	−·10	+·06	24·39
25	25·59·4	12·27·5	33·87	08·98	− ·06	−·10	+·06	24·79
29	25·96·5	12·56·8	34·82	10·79	+ ·31	−·14	+·04	24·24
31	25·67·4	12·24·9	35·24	11·44	+ ·47	−·10	+·06	24·23
Sept. 2	25·84·4	12·51·7	35·57	09·17	−1·54	−·10	+·06	24·82
6	25·80·3	12·34·5	36·21	12·21	+ ·42	−·10	+·06	24·38
9	26·06·9	12·66·2	36·81	11·02	−1·05	−·10	+·06	24·70
12	25·69·5	12·21·7	37·43	12·67	− ·60	−·10	+·06	24·12
15	25·78·8	12·30·1	37·96	12·88	− ·60	−·10	+·06	24·44
19	25·66·5	12·23·4	38·52	11·58	−2·06	−·10	+·06	24·84
23	25·42·8	11·81·4	39·22	15·82	+ ·96	−·10	+·06	24·32
26	25·29·7	11·76·4	39·75	13·95	−1·42	−·10	+·06	24·34
27	25·64·9	12·09·4	39·91	14·46	− ·95	−·14	+·04	24·40
28	25·26·3	11·67·1	40·03	15·31	− ·28	−·10	+·06	24·40
Oct. 4	26·07·1	12·44·2	40·65	16·17	+ ·04	−·10	+·06	24·48
5	26·11·0	12·45·0	40·78	16·89	+ ·45	−·10	+·06	24·30
8	26·23·5	12·55·2	41·23	17·43	+ ·84	−·10	+·06	24·60
10	25·69·5	12·08·4	41·51	15·76	− ·99	−·10	+·06	24·72
16	25·36·9	11·65·4	42·04	18·17	+ ·59	−·10	+·06	24·42
22	25·84·4	12·16·8	42·70	17·26	−1·42	−·10	+·06	23·98
28	25·80·2	12·15·1	43·07	16·68	−2·51	−·06	+·04	23·86
30	26·47·5	12·81·1	43·12	16·98	−1·40	−·10	+·06	24·70
31	26·14·7	12·31·8	43·16	20·81	+1·80	−·10	+·06	24·11
Nov. 1	26·05·8	12·43·9	43·22	15·94	−2·53	−·10	+·06	24·71
4	26·45·3	12·71·3	43·45	18·75	+ ·14	−·10	+·06	24·80
6	26·07·2	12·34·5	43·61	18·44	− ·45	−·10	+·06	24·68
7	25·93·5	12·24·1	43·66	17·68	−1·88	−·12	+·06	24·04
12	25·88·2	12·16·2	43·71	18·28	−1·56	−·16	+·08	23·79
13	26·24·1	12·47·8	43·72	19·28	− ·26	−·10	+·06	24·14
14	26·89·8	13·13·8	43·74	19·21	·00	−·10	+·06	24·49
17	27·45·4	13·71·7	43·87	18·68	− ·56	−·10	+·06	24·59

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP IV, PAIR 8.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t.* *d.*	*S.* *t.* *d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Aug. 20	15·52·6	23·45·1	21 13 21·66	+3 03·85	— 1·21	+·05	+·05	21 16 24·40
23	15·18·8	23·00·6	22·24	01·37	·00	+·05	+·05	23·71
25	15·60·7	23·40·1	22·70	00·81	+ ·12	+·05	+·05	23·73
29	15·93·3	23·64·3	23·67	+2 58·86	+ ·81	+·05	+·05	23·44
31	14·83·0	22·50·7	24·12	58·10	+ ·19	+·01	+·04	22·46
Sept. 2	15·36·9	23·14·8	24·47	60·46	—1·21	+·05	+·05	23·82
6	15·54·3	23·22.5	25·13	58·21	+ ·50	+·05	+·05	23·94
9	15·67·9	23·32·8	25·74	57·45	+ ·20	+·05	+·05	23·49
12	15·17·8	22·82·6	26·40	57·42	— ·20	+·05	+·05	23·72
15	15·23·6	22·88·7	26·95	57·49	—1·17	+·05	+·05	23·37
19	16·02·5	23·66·1	27·54	57·14	—1·70	+·05	+·05	23·08
23	15·68·7	23·21·5	28·25	54·64	+ ·85	+·05	+·05	23·84
26	15·28·0	22·84·7	28·81	55·54	—1·13	+·05	+·05	23·32
27	14·31·9	21·87·2	28·98	55·22	— ·65	+·05	+·05	23·65
28	15·41·9	22·94·4	29·11	54·57	— ·34	+·05	+·05	23·44
Oct. 4	15·76·3	23·26·1	29·76	53·94	+ ·09	+·05	+·05	23·89
5	15·95·7	23·42·3	29·90	53·20	+ ·40	+·05	+·05	23·60
8	15·47·9	22·92·1	30·37	52·64	+ ·53	+·05	+·05	23·64
16	14·85·4	22·26·7	31·23	51·97	+ ·52	+·05	+·05	23·82
22	15·55·0	23·04·2	31·90	53·80	—1·71	+·05	+·05	24·09
28	15·66·1	23·15·9	32·31	53·94	—2·81	+·05	+·05	23·54
30	15·63·9	23·08·6	32·37	52·76	—1·67	+·05	+·05	23·56
31	15·66·4	22·97·1	32·42	49·51	+2·19	+·05	+·05	24·22
Nov. 1	17·63·0	25·10·8	32·47	53·27	—2·17	+·05	+·05	23·67
4	15·59·5	22·95·9	32·71	50·83	— ·46	+·05	+·05	23·18
6	15·69·0	23·03·4	32·88	50·37	+ ·43	+·05	+·05	23·78
7	15·29·4	22·71·3	32·93	52·11	—2·20	+·05	+·07	22·96
1_2	15·20·2	22·61·5	33·01	51·97	—1·41	+·05	+·05	23·67
1^2	15·66·1	23·03·9	33·02	51·16	+ ·44	+·05	+·05	24·72
1^3	16·10·9	23·42·5	33·04	49·72	+ ·60	+.05	+·05	23·46
1^4	16·89·9	23·30·0	33·18	51·69	—1·03	+·05	+·05	23·94
7								

GROUP V, PAIR 1.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
	N.	*S.*		*Mic.*	*Level.*	*Ref.*	*Mer.*	
1891.	*t.* *d.*	*t.* *d.*	° ′ ″	′ ″	″	″	″	° ′ ″
Oct. 4	19·45·8	20·01·3	21 16 09·44	+0 12·88	+1·54	·00	+·06	21 16 23·92
5	18·96·9	19·53·7	09·57	13·18	+1·01	·00	+·06	23·82
8	19·51·5	20·05·7	10·02	12·57	+1·49	·00	+·06	24·14
16	19·43·9	19·93·3	10·98	11·46	+1·45	·00	+·06	23·95
30	19·72·8	20·25·3	12·29	12·18	—1·26	·00	+·06	23·27
31	19·90·3	20·27·5	12·34	08·63	+3·05	·00	+·06	24·08
Nov. 1	17·90·2	18·42·3	12·39	12·09	— ·86	·00	+·06	23·68
6	17·92·6	18·40·3	12·88	11·07	·00	·00	+·06	24·01
12	18·13·4	18·60·8	13·20	11·00	— ·22	—·08	+·10	24·00
13	18·33·2	19·75·8	13·24	9·88	+ ·91	·00	+·06	24·09
14	18·09·2	18·47·7	13·27	8·93	+1·66	·00	+·06	23·92
17	18·97·6	19·41·7	13·44	10·23	+0·53	·00	+·06	24·26
19	18·70·6	19·09·4	13·60	9·00	+1·31	·00	+·06	23·97
21	18·74·0	19·17·5	13·71	10·09	+ ·62	—·08	+·10	24·44
23	19·12·6	19·42·6	13·76	6·96	+3·06	·00	+·06	23·84
30	18·80·3	19·27·5	13·76	10·95	— ·99	·00	+·06	23·78
Dec. 5	18·94·5	19·35·1	13·96	09·42	+ ·24	·00	+·06	23·68
10	19·37·2	19·82·9	13·86	10·60	— ·29	·00	+·06	24·23
12	18·89·3	19·34·4	13·81	10·46	— ·39	·00	+·06	23·94
13	19·33·2	19·75·6	13·82	09·84	+ ·32	·00	+·06	24·04
15	18·43·4	18·78·7	13·84	08·19	+1·91	·00	+·06	24·00
17	18·48·0	18·83·4	13·88	08·21	+1·71	·00	+·06	23·86
21	17·94·8	18·32·2	13·76	08·68	+1·84	·00	+·06	24·34
23	18·64·0	19·03·6	13·63	09·19	+0·88	—·04	+·04	23·70
24	18·59·5	18·98·8	13·57	09·12	+0·59	·00	+·06	23·34
25	18·69·5	19·07·4	13·50	08·79	+1·51	·00	+·06	23·86
26	18·88·4	19·34·1	13·46	10·60	— ·03	·00	+·06	24·09
27	18·63·5	19·16·5	13·42	09·98	+ ·36	—·02	+·06	23·80
29	18·35·5	18·68·4	13·40	07·63	+2·80	·00	+·06	23·89
30	18·44·0	18·85·8	13·38	09·70	+ ·62	·00	+·06	23·76
1892.								
Jan. 1	18·74·2	19·16·8	13·34	09·88	+ ·31	—·10	+·10	23·53

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP V, PAIR 2.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
	N.	*S.*		*Mic.*	*Level.*	*Ref.*	*Mer.*	
1891.	*t. d.*	*t. d.*	° ′ ″	′ ″	″	″	″	° ′ ″
Oct. 4	28·48·4	10·96·6	21 23 09·66	−6 46·39	+ ·83	−·12	+·06	21 16 24·04
5	28·54·7	10·96·7	09·78	47·83	+2·01	−·12	+·06	23·90
8	28·69·0	11·11·5	10·26	47·71	+1·77	−·10	+·05	24·27
16	29·36·1	11·71·0	11·28	50·48	+2·72	−·12	+·06	23·46
28	29·12·3	11·53·1	12·57	48·11	− ·03	−·12	+·06	24·37
30	29·09·1	11·47·8	12·66	48·60	+ ·04	−·12	+·06	24·04
31	28·66·2	10·92·3	12·72	51·52	+3·10	−·12	+·06	24·24
Nov. 1	28·76·3	11·12·2	12·78	49·24	+ ·60	−·12	+·06	24·08
4	28·73·4	11·08·1	13·06	49·52	+ ·36	−·12	+·06	23·84
6	29·86·3	12·21·0	13·28	49·52	+ ·06	−·12	+·06	23·76
13	28·92·2	11·23·8	13·46	50·24	+ ·28	−·12	+·06	23·44
14	29·80·1	12·06·1	13·70	51·54	+1·78	−·12	+·06	23·88
17	29·63·7	11·94·2	13·88	50·50	+ ·53	−·16	+·04	23·79
19	29·42·5	11·70·2	14·04	51·15	+1·03	−·12	+·06	23·86
21	30·18·8	12·58·4	14·16	48·39	−1·97	−·12	+·06	23·74
23	29·11·8	11·34·5	14·22	52·31	+1·94	−·12	+·06	23·79
30	29·03·3	11·35·4	14·22	50·13	− ·67	−·12	+·06	23·36
Dec. 5	28·66·6	10·99·8	14·43	49·87	− ·52	−·12	+·06	23·98
6	28·21·6	10·56·1	14·44	49·57	−1·23	−·12	+·06	23·58
9	29·07·2	11·37·8	14·38	50·48	+ ·25	−·12	+·06	24·09
10	29·33·5	11·72·1	14·34	48·62	−1·46	−·10	+·06	24·22
12	28·89·0	11·24·2	14·30	49·41	−1·06	−·12	+·06	23·77
13	28·49·7	10·81·5	14·30	50·20	− ·01	−·12	+·06	24·03
15	29·81·8	12·07·0	14·32	51·73	+1·17	−·12	+·06	23·70
17	28·34·5	10·62·9	14·36	50·98	+ ·68	−·12	+·06	24·00
19	28·42·8	10·63·1	14·34	52·86	+2·19	−·11	+·03	23·59
21	29·44·5	11·73·4	14·25	50·87	+ ·67	−·12	+·06	23·99
23	29·00·7	11·31·7	14·11	50·38	+ ·57	−·12	+·06	24·24
24	28·91·6	11·23·2	14·05	50·24	+ ·29	−·12	+·06	24·04
25	29·26·2	11·57·5	13·98	50·31	+ ·47	−·12	+·06	24·08
26	29·66·0	11·99·2	13·94	49·87	+ ·08	−·12	+·06	24·09
27	28·60·6	10·91·1	13·89	50·50	− ·24	−·12	+·06	23·09
28	28·62·1	10·94·3	13·88	50·10	+ ·12	−·12	+·06	23·84
29	28·23·4	10·50·0	13·86	51·40	+1·06	−·12	+·06	23·46
30	29·28·9	11·62·0	13·84	49·89	+ ·05	−·12	+·06	23·94
1892.								
Jan. 1	28·97·0	11·29·0	13·81	50·15	+ ·36	−·12	+·02	23·92
5	28·58·6	10·95·9	13·54	48·92	+ ·03	−·12	+·06	24·59

GROUP V, PAIR 3.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* t. d.	*S.* t. d.	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Oct. 4	22·81·2	17·06·8	21 18 37·36	—2 13·25	+ ·38	—·04	+·05	21 16 24·50
5	23·07·6	17·29·5	37·48	14·11	+1·30	—·04	+·05	24·68
8	22·56·4	16·75·1	37·94	14·85	+1·38	—·04	+·05	24·48
16	21·89·3	16·09·9	38·96	14·41	+0·41	—·04	+·05	24·97
22	22·32·4	16·45·7	39·66	16·10	+1·22	—·00	+·04	24·82
30	23·06·1	17·22·8	40·35	15·32	+ ·15	—·04	+·05	25·19
31	22·64·0	16·78·2	40·40	15·90	+ ·06	—·04	+·05	24·57
Nov. 1	22·74·9	16·91·0	40·46	15·46	— ·27	—·04	+·05	24·74
4	22·32·0	16·49·3	40·74	15·18	— ·83	—·04	+·05	24·74
6	22·74·7	16·86·5	40·97	16·45	+ ·13	—·04	+·05	24·66
12	22·81·5	16·95·0	41·34	16·06	— ·21	—·06	+·05	25·06
13	22·33·1	16·41·1	41·37	17·34	+1·43	—·04	+·05	25·47
14	23·27·3	17·32·6	41·40	17·96	+1·40	—·04	+·05	24·85
17	22·77·5	16·86·6	41·60	17·08	+ ·24	—·04	+·05	24·77
19	22·79·0	16·83·6	41·76	18·12	+1·10	—·08	+ ·04	24·70
21	23·03·7	17·22·4	41·89	14·85	—2·34	—·04	+·05	24·71
23	23·16·9	17·23·2	41·96	17·73	+1·06	—·04	+·05	25·30
30	21·83·5	15·93·9	41·99	16·78	— ·39	—·04	+·05	24·83
Dec. 5	22·67·5	16·71·8	42·22	18·19	+ ·41	—·04	+·05	24·45
9	22·07·2	16·16·2	42·20	17·10	— ·01	·04	+·05	25·10
10	22·35·7	16·47·1	42·17	16·55	— ·77	—·04	+·05	24·86
12	22·43·3	16·56·9	42·14	16·04	— ·71	— ·04	+·05	25·40
13	22·20·8	16·29·6	42·14	17·15	— ·19	—·04	+·05	24·81
15	22·92·5	16·96·8	42·16	18·19	+1·08	—·04	+·05	25·06
17	22·26·7	16·31·5	42·22	18·08	+ ·81	—·04	+·05	24·96
21	22·21·1	16·27·2	42·16	17·78	+ ·30	—·04	+·05	24·69
23	22·43·8	16·49·5	42·04	17·87	+1·08	—·04	+·05	25·26
24	22·57·5	16·63·3	41·97	17·85	+ ·61	—·04	+·05	24·74
25	22·27·1	16·32·8	41·90	17·87	+ ·98	—·04	+·05	25·02
26	22·01·5	16·14·1	41·86	16·27	— ·47	—·04	+·05	25·13
29	22·81·4	16·85·8	41·82	18·17	+ ·99	—·04	+·05	24·65
30	22·56·9	16·65·4	41·82	17·22	+ ·07	—·04	+·05	24·68
1892.								
Jan. 2	23·18·0	17·30·3	41·76	16·34	+ ·09	—·12	+·06	25·45
5	22·51·9	16·70·5	41·58	14·88	—1·16	—·04	+·05	25·55

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP V, PAIR 4.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Oct. 4	7·78·7	29·33·1	21 08 04·32	+8 19·79	— ·53	+·15	+·06	21 16 23·79
5	7·92·7	29·42·8	04·44	18·79	+ ·32	+·15	+·06	23·76
8	8.77·2	30·26·3	04·88	18·56	+ ·41	+·15	+·06	24·06
30	7·89·7	29·34·5	07·20	17·56	—1·30	+·15	+·06	23·67
31	7·87·7	29·32·3	07·26	17·51	—1·34	+·15	+·06	23·64
Nov. 1	7·92·2	29·41·0	07·30	18·49	—1·42	+·15	+·06	24·58
4	8·09·3	29·52·7	07·58	17·24	—1·21	+·15	+·06	23·82
6	7·80·2	29·25·1	07·80	17·58	—1·48	+·15	+·06	24·11
13	8·01·8	29·34·8	08·21	14·82	+ ·30	+·15	+·06	23·54
14	8·36·1	29·69·3	08·25	14·87	+ ·76	+·15	+·06	24·09
17	8·31·1	29·69·2	08·44	16·01	— ·59	+·15	+·06	24·07
19	8·78·8	30·13·9	08·60	15·31	— ·16	+·17	+·05	23·97
21	8·84·6	30·33·6	08·74	18·54	—3·29	+·15	+·06	24·20
23	8·85·9	30·18·2	08·82	14·66	+ ·11	+·15	+·06	23·80
30	7·88·4	29·31·4	08·88	17·14	—2·27	+·15	+·06	23·96
Dec. 5	8·09·0	29·46·0	09·14	15·75	—1·22	+·15	+·06	23·88
6	8·27·4	29·65·3	09·16	15·96	—1·30	+·12	+·06	24·00
9	8·40·0	29·80·8	09·14	16·63	—1·79	+·15	+·06	24·19
10	7·67·5	29·12·7	09·12	17·65	—2·39	+·17	+·05	24·60
12	7·37·3	28·82·1	09·09	17·56	—2·42	+·15	+·06	24·44
15	7·75·7	29·08·6	09·14	14·80	— ·27	+·15	+·06	23·88
19	8·33·3	29·61·1	09·22	13·62	+ ·81	+·11	+·06	23·82
21	8·29·7	29·62·3	09·19	14·73	— ·49	+·15	+·06	23·64
23	7·96·8	29·31·9	09·08	15·31	— ·72	+·15	+·06	23·88
25	9·57·3	29·96·3	08·97	16·22	—1·54	+·15	+·06	23·86
26	8·14·3	29·53·3	08·93	16·22	—1·13	+·15	+·06	24·23
29	8·23·2	29·58·6	08·91	15·38	— ·26	+·17	+·05	24·25
30	8·53·6	29·95·7	08·92	16·94	—1·56	+·15	+·06	24·51
1892.								
Jan. 5	8·32·1	29·77·7	08·74	17·75	—1·88	+·11	+·04	24·76

GROUP V, PAIR 5.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
	N.	*S.*		*Mic.*	*Level.*	*Ref.*	*Mer.*	
1891.	*t.* *d.*	*t.* *d.*	° ′ ″	′ ″	″	″	″	° ′ ″
Oct. 4	25·44·7	14·87·7	21 20 29·33	−4 05·21	— ·68	—·08	+·06	21 16 23·42
5	24·77·2	14·15·0	29·44	06·41	+ ·69	—·08	+·06	23·70
8	25·30·6	14·67·9	29·88	06·53	+ ·53	—·08	+·06	23·86
16	24·63·2	14·03·0	30·84	05·95	— ·49	—·08	+·06	24·38
28	25·63·3	14·99·7	32·09	06·74	—1·80	—·08	+·06	23·53
30	24·97·3	14·32·3	32·18	07·06	— ·96	—·08	+·06	24·14
31	25·17·7	14·53·7	32·23	06·83	—1·46	—·08	+·06	23·92
Nov. 1	24·99·2	14·27·8	32·28	08·55	+ ·09	—·08	+·06	23·80
4	25·14·5	14·45·2	32·54	08·06	— ·72	—·08	+·06	23·74
6	25·76·5	15·05·8	32·77	08·39	— ·59	—·08	+·06	23·77
13	25·06·2	14·28·7	33·18	09·96	+ ·42	—·08	+·06	23·62
14	25·61·3	14·84·2	33·22	09·87	+ ·56	—·08	+·06	23·89
17	25·88·0	15·09·5	33·40	10·20	+ ·94	—·08	+·06	24·12
19	25·94·5	15·12·7	33·57	10·96	+1·45	—·08	+·06	24·04
21	25·45·1	14·80·1	33·72	07·06	—2·39	—·08	+·06	24·25
23	25·49·8	14·71·9	33·80	10·06	+ ·48	—·08	+·06	24·20
30	26·03·5	15·30·0	33·86	09·04	—1·40	—·08	+·06	23·40
Dec. 1	25·29·5	14·42·9	33·91	12·07	+2·04	—·08	+·06	23·86
5	25·37·0	14·63·0	34·14	09·15	—1·17	—·16	+·08	23·74
6	25·33·0	14·60·2	34·16	18·87	—1·28	—·08	+·06	23·99
9	25·11·1	14·38·1	34·16	08·92	—1·54	—·08	+·06	23·68
10	25·48·1	14·71·3	34·14	09·80	— ·37	—·08	+·06	23·95
12	24·68·8	13·99·0	34·12	08·18	—1·66	—·08	+·06	24·26
15	25·93·6	15·11·6	34·17	11·01	+ ·69	—·08	+·06	23·83
19	24·98·7	14·15·1	34·27	11·38	+1·16	—·08	+·06	24·03
21	25·26·4	14·48·5	34·24	10·06	— ·32	—·08	+·06	23·84
23	25·37·9	14·61·5	34·14	09·71	— ·85	—·08	+·06	23·56
24	25·70·2	14·92·6	34·10	09·99	— ·24	—·08	+·06	23·85
25	26·04·8	15·28·8	34·04	09·62	— ·06	—·08	+·06	24·34
26	25·61·2	14·86·4	34·01	09·34	— ·45	—·08	+·06	24·20
29	25·05·7	14·25·3	33·99	10·64	+ ·92	—·08	+·06	24·25
30	25·98·9	15·20·8	34·00	10·10	— ·55	—·08	+·06	23·33
1892.								
Jan. 5	25·36·0	14·66·3	33·88	08·15	—1·12	—·08	+·06	24·59

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP V, PAIR 6.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Oct. 4	16·96·8	22·28·7	21 14 20·74	+2 03·39	— ·55	+·03	+·05	21 16 23·66
5	16·63·0	21·92·5	20·85	02·84	— ·06	+·03	+·05	23·71
8	16·85·2	22·09·4	21·27	01·61	+ ·84	+·03	+·05	23·80
16	16·99·2	22·23·7	22·25	01·68	—1·06	+·03	+·05	22·95
28	17·32·5	22·57·1	23·50	01·70	—2·02	+·03	+·05	23·26
30	16·54·9	21·77·6	23·59	01·26	—1·53	+·03	+·05	23·40
31	16·61·3	21·83·8	23·64	01·21	—1·63	+·03	+·05	23·30
Nov. 1	16·55·9	21·73·3	23·70	00·03	— ·33	+·03	+·05	23·48
4	17·19·8	22·39·1	23·96	60·47	— ·97	+·03	+·05	23·54
6	16·83·2	22·00·8	24·18	00·08	—1·00	+·03	+·05	23·34
13	17·21·0	22·35·5	24·62	+1 59·36	+ ·18	+·03	+·05	24·24
14	17·66·6	22·76·3	24·65	58·24	+ ·71	+·03	+·05	23·68
17	17·87·7	23·00·7	24·84	59·01	— ·12	+·03	+·05	23·81
19	17·53·1	22·61·7	25·00	57·99	+ ·75	+·03	+·05	23·82
21	17·68·6	22·84·9	25·16	59·77	—1·33	+·03	+·05	23·68
23	17·66·3	22·77·2	25·26	58·52	·00	+·03	+·05	23·86
30	17·05·8	22·21·0	25·32	59·52	—1·53	+·03	+·05	23·39
Dec. 1	17·53·0	22·55·8	25·38	56·64	+1·53	+·03	+·05	23·63
6	17·02·7	22·19·9	25·64	59·98	—2·10	+·03	+·05	23·60
9	16·84·2	22·01·7	25·65	60·05	—1·74	+·03	+·05	24·04
10	16·55·4	21·69·6	25·63	59·29	— ·98	+·03	+·05	24·02
12	16·13·9	21·21·5	25·62	57·76	+ ·22	+·03	+·05	23·68
15	16·52·3	21·62·6	25·66	58·38	— ·46	+·03	+·05	23·66
19	16·94·9	22·02·0	25·79	57·64	+ ·17	+·05	+·05	23·70
21	17·25·8	22·39·8	25·78	59·24	— ·95	+·03	+·05	24·15
23	17·10·9	22·27·1	25·70	59·75	—1·21	+·03	+·05	24·32
24	17·04·2	22·14·0	25·64	58·27	— ·33	+·03	+·05	23·66
25	16·85·8	22·03·4	25·59	60·08	—2·00	+·03	+·05	23·75
26	16·82·6	21·95·7	25·50	59·03	—1·13	+·03	+·05	23·48
29	16·87·1	21·96·3	25·54	58·13	— ·23	+·03	+·05	23·52
30	16·23·2	21·38·8	25·56	59·61	—1·16	+·03	+·05	24·09
1892.								
Jan. 5	16·95·8	22·16·7	25·46	60·84	—1·86	+·03	+·05	24·52

GROUP V, PAIR 7.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Oct. 4	13·41·0	25·77·0	21 11 37·10	+4 46·73	+ ·09	+·08	+·05	21 16 24·05
5	13·38·7	25·74·4	37·20	46·66	+ ·55	+·08	+·05	24·54
8	13·54·0	25·87·5	37·60	46·15	+ ·38	+·08	+·05	24·26
16	13·42·8	25·74·6	38·55	45·76	— ·72	+·08	+·05	23·72
30	13·74·8	26·02·2	39·84	44·74	— ·05	+·08	+·05	24·66
31	13·65·7	25·96·8	39·88	45·60	—1·19	+·08	+·05	24·42
Nov. 1	13·59·6	25·80·7	39·92	43·28	+ ·51	+·10	+·05	23·86
4	13·45·0	25·72·2	40·17	44·69	— ·49	+·08	+·05	24·50
6	13·81·8	26·07·9	40·40	44·44	— ·37	+·08	+·05	24·60
14	14·09·8	26·28·0	40·86	42·60	+ ·96	+·08	+·06	24·56
17	14·19·3	26·40·0	41·04	43·18	— ·20	+·08	+·05	24·15
19	14·11·9	26·30·8	41·20	42·77	+ ·22	+·10	+·05	24·34
21	14·43·1	26·57·3	41·36	41·68	+ ·64	+·08	+·05	23·81
23	14·54·2	26·69·2	41·46	41·86	+ ·71	+·08	+·05	24·16
30	13·73·1	25·97·9	41·54	44·14	—1·17	+·08	+·06	24·65
Dec. 1	14·17·4	26·30·2	41·58	41·35	+1·35	+·08	+·05	24·41
5	13·40·6	25·62·5	41·84	43·46	—1·47	+·06	+·05	23·94
9	13·31·8	25·59·7	41·90	44·86	—1·92	+·08	+·05	24·97
10	13·41·0	25·63·9	41·88	43·70	—1·46	+·08	+·05	24·25
12	13·59·2	25·76·6	41·87	42·42	+ ·04	+·08	+·05	24·46
15	12·98·8	25·12·1	41·93	41·47	+ ·65	+·08	+·05	24·18
17	13·54·5	25·73·7	42·02	42·84	— ·40	+·08	+·05	24·59
19	13·28·2	25·47·7	42·08	42·91	— ·50	+·08	+·05	24·62
21	13·52·9	25·72·5	42·08	42·93	— ·70	+·08	+·05	24·44
23	13·24·6	25·41·3	42·01	42·26	+ ·04	+·08	+·05	24·44
24	13·30·0	25·47·3	41·96	42·40	— ·34	+·12	+·04	24·18
25	13·26·9	25·49·7	41·92	43·67	—1·15	+·08	+·05	24·57
26	13·66·0	25·85·3	41·89	42·86	— ·88	+·08	+·05	24·00
29	13·51·3	25·69·3	41·90	42·56	— ·75	+·16	+·05	23·90
30	13·56·0	25·77·4	41·92	43·35	— ·80	+·08	+·05	24·60
1892.								
Jan. 5	13·53·6	25·75·5	41·88	43·46	—1.04	+·08	+·05	24·43

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP V, PAIR 8.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Oct. 4	19·96·9	17·41·2	21 17 23·47	—0 59·32	— ·20	—·02	+·05	21 16 23·98
5	20·20·4	17·76·6	23·56	56·56	—3·02	—·02	+·05	24·01
8	21·73·6	19·09·7	23·95	—1 01·22	+ ·73	—·02	+·05	23·49
16	21·89·7	19·32·8	24·88	0 59·60	— ·83	—·02	+·05	24·48
30	21·58·7	18·91·9	26·14	—1 01·89	— ·56	—·02	+·05	23·72
31	21·54·5	18·90·2	26·18	01·31	—1·37	—·02	+·05	23·53
Nov. 1	21·75·7	19·02·5	26·22	03·38	+ ·81	—·02	+·05	23·68
4	22·18·9	19·51·9	26·46	01·94	— ·67	—·02	+·05	23·88
6	22·29·0	19·62·5	26·68	01·82	— ·93	—·02	+·05	23·96
13	21·36·3	18·63·8	27·12	03·22	— ·06	00	+·04	23·88
14	22·72·9	19·96·6	27·14	04·10	+ ·71	—·02	+·05	23·78
17	22·47·7	19·78·5	27·31	02·45	— ·56	—·02	+·05	24·33
19	22·58·3	19·81·3	27·48	04·26	+ ·61	—·02	+·05	23·86
21	22·90·1	20·11·7	27·64	04·58	+ ·91	—·02	+·05	24·00
23	22·91·8	20·13·6	27·75	04·54	+1·05	—·02	+·05	24·29
Dec. 6	21·70·2	19·01·9	28·18	—1 02·24	—1·86	—·02	+·05	24·11
9	21·65·6	18·97·2	28·21	02·26	—1·77	—·02	+·05	24·21
10	22·61·2	19·91·5	28·20	02·57	—1·52	—·02	+·05	24·14
12	21·80·3	19·08·9	28·18	02·96	— ·79	—·02	+·05	24·46
15	21·72·1	18·94·3	28·25	04·44	— ·12	—·02	+·05	23·72
17	22·44·6	19·69·1	28·34	03·91	— ·26	—·02	+·05	24·20
19	22·07·1	19·31·8	28·40	03.87	— ·10	00	+·05	24·48
21	22·00·5	19·23·6	28·43	04·24	— ·37	—·02	+·05	23·85
23	22·43·2	19·69·2	28·36	03·56	— ·57	—·02	+·05	24·26
24	21·71·5	18·97·2	28·32	03·63	— ·92	—·02	+·02	23·77
25	22·27·4	19·54·1	28·28	03·40	—1·02	—·02	+·05	23·89
26	21·88·5	19·16·1	28·25	03·19	— ·73	—·02	+·05	24·36
29	21·50·7	18·87·6	28·26	01·04	—2·95	—·02	+·05	24·30
30	22·17·5	19·45·8	28·29	03.03	—1·28	—·02	+·05	24·01
1892.								
Jan. 5	21·70·1	18·96·3	28·28	03·52	— ·56	—·02	+·05	24·23

GROUP VI, PAIR 1.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* t. d.	*S.* t. d.	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Nov. 19	25·12·2	14·15·2	21 20 37·28	—4 14·49	+ ·62	—·08	+·06	21 16 24·39
21	24·76·6	13·78·1	37·42	14·84	+2·76	—·08	+·06	25·32
30	24·74·8	13·88·5	37·54	12·01	— ·51	—·08	+·06	25·00
Dec. 1	24·01·6	13·02·6	37·56	14·95	+2·37	—·08	+·06	24·96
5	23·99·1	13·11·3	37·79	12·35	— ·54	—·08	+·06	24·88
9	24·30·4	13·44·1	37·90	12·01	— ·89	—·08	+·06	24·98
10	24·51·5	13·60·6	37·89	13·07	— .14	—·08	+·06	24·66
12	24·69·1	13·74·9	37·86	13·84	+ ·40	—·08	+·06	24·40
15	25·02·9	14.09·0	37·90	13·77	+ ·74	—·08	+·06	24·85
19	24·77·2	13·79·7	38·08	14·60	+ ·52	—·08	+·06	24·98
23	24·49·1	13·58·1	38·14	13·10	— ·35	—·08	+·06	24·67
24	25·06·9	14·09·8	38·12	14·51	+1·03	—·08	+·06	24·62
25	24·18·0	13·25·5	38·09	13·44	+ ·34	—·18	+·17	24·98
26	24·51·0	13·57·9	38·06	13·58	— ·06	—·08	+·06	24·40
29	24·78·4	13·93·3	38·09	11·73	—1·16	—·08	+·06	25·18
30	25·28·5	14·40·8	38·12	12·33	—1·14	—·12	+·05	24·58
1892.								
Jan. 5	24·47·2	13·47·9	38·30	15·02	+1·48	—·08	+·06	24·74
6	24·96·9	13·86·0	38·28	17·71	+3·55	—·08	+·06	24·10
10	25·81·7	14·80·0	38·19	15·58	+2·16	—·08	+·06	24·75
11	24·24·6	13·30·3	38·20	13·82	+ ·84	—·08	+·06	25·20
12	24·20·6	13·29·6	38·22	13·10	+ ·30	—·08	+·06	25·40
15	24·46·0	13·49·6	38·32	14·35	+1·35	—·08	+·06	25·30
16	24·18·6	13·22·7	38·36	14·23	+ ·32	—·08	+·06	24·43
18	23·81·5	12·82·4	38·36	14·97	+1·59	—·08	+·06	24·96
20	24·16·5	13·16·2	38·30	15·25	+1·51	—·08	+·06	24·54
21	23·93·2	12·85·9	38·26	16·88	+3·19	—·08	+·06	24·55
23	23·84·1	12·69·1	38·18	18·66	+5·31	—·08	+·06	24·81
24	24·16·9	13·22·7	38·17	13·79	+ ·60	—·08	+·06	24·96
25	23·96·0	13·01·4	38·16	13·93	+1·15	—·08	+·09	25·39
27	23·87·9	12·99·2	38·21	12·56	— ·20	—·08	+·06	25·43
Feb. 3	24·48·7	13·51·0	38·19	14·65	+1·52	—·08	+·06	25·04
4	24·21·6	13·22·3	38·14	15·02	+1·92	—·08	+·06	25·02
6	24·45·5	13·38·2	38·06	16·88	+3·54	—·08	+·06	24·70
8	23·95·6	12·96·9	38·04	14·88	+2·22	—·08	+·06	25·36
11	24·84·2	13·80·5	38·09	16·04	+2·88	—·08	+·06	24·91
12	24·00·6	13·02·3	38·10	14·79	+2·23	—·08	+·06	25·52
13	24·20·7	13·22·5	38·09	14·77	+1·34	—·08	+·06	24·64
14	24·31·9	13·30·0	38·08	15·62	+2·92	—·08	+·06	25·36

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP VI, PAIR 2.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
	N.	*S.*		*Mic.*	*Level.*	*Ref.*	*Mer.*	
1891.	*t. d.*	*t. d.*	° ′ ″	′ ″	″	″	″	° ′ ″
Nov. 21	8·09·3	30·18·5	21 07 50·22	+8 32·50	+1·47	+·14	+·05	21 16 24·38
30	7·89·4	30·07·1	50·30	34·47	—1·27	+·14	+·05	23·69
Dec. 5	8·48·0	30·64·8	50·54	34·26	— ·87	+·14	+·05	24·12
9	8·43·6	30·61·9	50·63	34·61	—1·84	+·14	+·05	23·59
10	8·06·4	30·19·7	50·62	33·45	— ·45	+·14	+·05	23·81
12	7·76·6	29·86·9	50·58	32·76	+ ·07	+·14	+·05	23·60
15	8·02·6	30·11·0	50·60	32·32	+ ·33	+·14	+·05	23·44
23	8·08·1	30·20·4	50·84	33·22	— ·96	+·14	+·05	23·29
26	7·61·7	29·74·3	50·74	33·29	— ·42	+·14	+·05	23·80
28	7·96·2	30·09·7	50·74	33·50	— ·59	+·14	+·05	23·84
29	7·80·9	29·96·8	50·76	34·06	—1·17	+·14	+·05	23·84
1892.								
Jan. 5	7·89·2	29·96·6	50·96	32·08	+ ·51	+·14	+·05	23·74
6	8·24·5	30·26·2	50·94	30·76	+1·47	+·14	+·05	23·36
10	8·41·3	30·47·1	50·85	31·71	+ ·73	+·14	+·05	23·48
11	7·33·4	29·40·5	50·86	32·01	+ ·56	+·14	+·05	23·62
12	7·28·5	29·43·9	50·88	33·94	—1·13	+·14	+·05	23·88
15	7·13·3	29·19·1	50·99	31·71	+ ·55	+·16	+·05	23·46
16	6·93·5	29·04·7	51·02	32·96	— ·33	+·14	+·05	23·84
18	7·73·9	29·82·4	51·03	32·34	+ ·36	+·14	+·05	23·92
19	7·60·0	29·68·6	51·00	32·36	— ·01	+·14	+·07	23·56
20	7·48·5	29·59·7	50·97	32·96	— ·84	+·14	+·05	23·28
21	7·89·7	29·92·5	50·93	31·02	+1·95	+·14	+·05	24·09
23	7·15·9	29·21·9	50·86	31·76	+1·34	+·14	+·05	24·15
24	7·28·1	29·42·7	50·84	33·75	— ·93	+·14	+·05	23·85
27	7·73·4	29·96·3	50·89	35·68	—2·11	+·14	+·05	24·65
Feb. 2	7·76·5	29·80·9	50·94	31·39	+1·51	+·18	+·04	24·06
3	7·41·7	29·46·6	50·94	31·50	+1·31	+·14	+·05	23·94
4	7·39·2	29·48·5	50·84	32·52	+ ·57	+·14	+·05	24·14
6	8·76·3	29·84·3	50·77	32·23	+ ·99	+·14	+·05	24·18
8	8·05·8	30·15·8	50·75	32·69	+ ·63	+·14	+·05	24·26
11	7·62·7	29·68·2	50·82	31·64	+1·51	+·14	+·05	24·16
12	7·54·0	29·63·2	50·83	32·50	+1·16	+·12	+·07	24·68
13	8·98·8	30·07·1	50·83	32·29	+ ·21	+·14	+·05	23·52
14	7·20·9	29·33·5	50·83	33·29	— ·35	+·14	+·05	23·96
20	7·29·8	29·41·8	50·53	33·15	+ ·55	+·14	+·05	24·42
21	6·95·9	29·08·4	50·52	33·27	— ·03	+·14	+·05	23·95
22	6·65·7	28·75·8	50·50	32·71	+ ·62	+·14	+·05	24·02
23	7·41·1	29·59·2	50·51	34·57	— ·91	+·14	+·05	24·36

GROUP VI, PAIR 3.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
	N.	*S.*		*Mic.*	*Level.*	*Ref.*	*Mer.*	
1891.	*t. d.*	*t. d.*	° ′ ″	′ ″	″	″	″	° ′ ″
Nov. 19	23·03·6	15·67·5	21 19 13·16	−2 50·76	+ ·89	−·05	+·06	21 16 23·30
21	22·60·4	15·26·1	13·27	50·35	+ ·65	−·05	+·06	23·58
30	22·53·7	15·26·3	13·32	48·75	−1·55	−·05	+·06	23·03
Dec. 1	23·60·0	16·24·8	13·32	50·55	+1·28	−·05	+·06	24·06
5	22·10·1	14·82·3	13·52	48·84	− ·64	−·05	+·06	24·05
9	21·86·5	14·64·6	13·62	47·47	−1·99	−·05	+·06	24·17
10	21·76·0	14·53·3	13·60	47·65	−1·76	−·05	+·06	24·20
12	22·26·4	14·96·2	13·56	49·39	+ ·23	−·05	+·06	24·41
15	21·68·7	14·32·6	13·57	50·76	+ ·62	−·05	+·06	23·44
19	21·84·0	14·51·2	13·74	50·00	+ ·25	−·05	+·05	23·99
23	21·46·8	14·14·3	13·82	49·93	+ ·19	−·05	+·06	24·09
26	21·44·6	14·16·3	13·74	48·95	− ·98	−·05	+·06	23·82
28	21·59·8	14·29·9	13·74	49·33	− ·69	−·05	+·06	23·73
29	21·97·6	14·72·3	13·76	48·26	−1·49	−·05	+·06	24·02
30	21·67·1	14·39·3	13·79	48·84	−1·25	−·05	+·06	23·71
1892.								
Jan. 5	22·49·6	15·10·3	14·00	51·51	+ ·82	−·05	+·06	23·32
6	22·37·3	15·01·7	13·99	50·65	+ ·45	−·05	+·06	23·80
10	22·15·5	14·74·9	13·91	51·81	+1·79	−·05	+·06	23·90
11	21·10·1	13·74·6	13·93	50·62	+ ·56	−·05	+·06	23·88
12	21·21·3	13·96·0	13·95	48·26	−1·10	−·05	+·06	24·60
15	21·20·8	13·85·4	14·08	50·60	+ ·87	−·05	+·06	24·36
18	21·57·0	14·23·3	14·15	50·21	+ ·14	−·05	+·06	24·09
19	21·78·5	14·44·6	14·13	50·25	− ·29	−·05	+·06	23·60
20	21·11·2	13·85·7	14·12	48·31	−2·12	−·05	+·06	23·70
21	21·59·3	14·19·8	14·08	51·55	+1·02	−·05	+·06	23·56
23	21·21·6	13·80·2	14·02	51·99	+2·10	−·05	+·06	24·14
24	21·05·7	13·81·0	14·02	48·12	−1·99	−·05	+·06	23·92
25	20·72·4	14·47·1	14·02	48·26	−2·10	−·05	+·06	23·67
27	20·92·6	12·69·7	14·08	47·70	−2·02	−·05	+·06	24·37
Feb. 3	21·37·4	14·00·2	14·18	51·02	+ ·76	−·05	+·06	23·93
4	21·58·0	14·21·9	14·14	50·76	+ ·69	−·05	+·06	24·08
6	21·36·0	14·02·3	14·08	50·21	+ ·24	−·05	+·06	24·12
8	21·23·5	13·89·1	14·08	50·37	+ ·26	−·05	+·06	23·98
12	20·64·0	13·34·5	14·20	49·23	− ·97	−·05	+·06	24·01
13	21·18·9	13·85·6	14·21	50·11	− ·50	−·05	+·06	23·61
14	20·54·7	13·19·6	14·22	50·53	− ·02	−·05	+·06	23·68
21	21·25·3	13·94·9	14·00	49·44	− ·43	−·05	+·06	24·14
22	21·22·8	13·91·2	14·00	49·72	− ·17	−·05	+·06	24·12
23	20·87·2	13·56·6	14·02	49·49	− ·97	−·05	+·06	23·57

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP VI, PAIR 4.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Nov. 19	14·70·1	23·17·2	21 13 07·12	+3 16·51	+ ·37	+·08	+·06	21 16 24·14
21	15·93·5	24·36·8	07·22	15·03	+ ·99	+·05	−·05	23·94
Dec. 1	14·02·0	22·44·3	07·24	15·40	+ 1·12	+·05	−·05	23·86
5	14·10·3	22·60·8	07·40	17·30	−1·23	+·05	+·05	23·57
9	13·98·5	22·48·9	07·49	17·28	−1·33	+·05	−·05	23·54
12	14·11·3	22·59·4	07·42	16·75	− ·29	+·07	−·05	24·00
15	14·85·1	23·30·4	07·41	16·10	+ ·44	−·05	−·05	24·05
19	14·73·2	23·15·5	07·56	15·40	+ ·16	+·09	+·05	23·26
23	14·02·4	22·49·4	07·69	16·49	− ·28	+·05	−·05	24·00
26	13·90·8	22·40·9	07·54	17·21	−1·09	+·05	+·05	23·76
28	14·61·9	23·13·1	07·52	17·46	− ·90	+·05	+·05	24·18
29	14·55·8	22·08·3	07·55	17·77	−1·60	+·05	+·05	23·82
30	15·30·8	22·81·7	07·58	17·40	−1·17	+·05	−·05	23·91
1892.								
Jan. 5	15·25·3	23·67·8	07·78	15·45	+ ·50	+·05	+·05	23·83
6	14·94·8	23·36·2	07·76	15·19	+ ·35	+·05	+·05	23·40
10	15·05·0	23·48·3	07·66	15·63	+ ·03	+·08	−·05	23·45
11	13·93·1	22·39·1	07·68	16·26	+ ·12	+·05	+·06	24·17
12	14·05·3	22·57·3	07·70	17·65	−1·08	+·05	+·05	24·37
18	14·07·8	22·53·5	07·89	16·19	·00	+·05	+·05	24·18
19	14·20·0	22·66·6	07·87	16·40	− ·51	+·05	+·05	23·86
20	14·11·5	22·63·1	07·84	17·56	−1·35	+·05	+·05	24·15
21	13·93·1	22·38·3	07·82	16·07	+ ·06	+·05	+·05	24·05
23	13·84·6	22·24·0	07·76	14·73	+1·28	+·05	+·05	23·87
24	13·79·6	22·30·5	07·75	17·40	−1·67	+·05	+·05	23·58
25	13·46·3	22·96·6	07·75	17·26	−1·39	+·05	+·05	23·72
27	13·51·8	22·08·2	07·80	18·67	−2·35	+·05	+·05	24·22
Feb. 3	13·16·8	21·56·2	07·90	14·73	+ ·81	+·05	+·05	23·54
4	13·45·0	21·86·3	07·86	15·17	+ ·88	+·05	+·05	24·01
6	13·63·6	22·06·9	07·79	15·63	+ ·67	+·05	+·05	24·19
8	14·02·9	22·44·5	07·78	15·24	+ ·30	+·05	+·05	23·42
11	13·88·2	22·25·2	07·88	14·17	+1·65	+·05	+·05	23·80
12	13·76·5	22·26·2	07·90	17·12	−1·11	+·05	+·05	24·01
13	13·56·5	22·00·3	07·92	15·75	+ ·06	+·05	−·05	23·83
14	12·94·8	21·42·1	07·92	16·56	− ·22	+·05	−·05	24·36
20	14·01·3	22·47·6	07·72	16·33	− ·20	+·05	+·05	23·95
21	13·97·6	22·47·8	07·70	17·23	− ·93	+·05	+·05	24·10
22	13·26·6	21·74·4	07·70	16·68	− ·35	+·05	+·05	24·13
23	13·78·8	22·23·8	07·72	16·03	− ·61	+·05	+·05	23·24

GROUP VI, PAIR 5.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Nov. 19	27·23·9	12·45·8	21 22 06·64	−5 42·90	+ ·06	−·10	+·06	21 16 23·76
21	27·58·2	12·78·7	06·72	43·22	+ ·64	−·10	+·06	24·10
30	25·77·6	11·10·0	06·66	40·46	−2·78	−·10	+·06	23·38
Dec. 1	26·51·9	12·72·6	06·66	43·18	+ ·12	−·10	+·06	23·56
5	27·27·2	12·53·7	06·79	41·83	−1·72	−·10	+·06	23·20
9	26·39·6	11·73·0	06·86	40·23	−2·90	−·10	+·06	23·69
12	26·08·2	14·77·9	06·77	42·85	− ·32	−·10	+·06	23·56
15	25·46·7	10·72·5	06·74	41·99	− ·71	−·10	+·06	24·00
19	25·85·4	11·06·6	06·88	43·06	+ ·63	−·10	+·06	24·41
23	25·96·3	11·19·3	06·95	42·64	− ·84	−·10	+·06	23·43
25	26·08·5	11·34·4	06·89	41·97	− ·58	−·10	+·06	24·30
26	26·26·6	11·50·8	06·86	42·36	− ·50	−·10	+·06	23·96
28	26·40·6	11·68·6	06·83	41·48	− ·97	−·10	+·06	24·34
29	26·34·6	11·67·5	06·85	40·34	−2·01	−·10	+·06	24·46
30	26·65·8	11·92·8	06·87	41·71	−1·04	−·10	+·06	24·08
1892.								
Jan. 5	26·64·7	11·83·8	07·08	43·54	+ ·23	−·10	+·06	23·73
6	26·58·3	11·81·3	07·08	42·64	− ·25	−·10	+·06	24·15
10	26·87·9	12·08·9	06·98	43·11	− ·21	−·10	+·06	23·62
11	25·80·5	11·01·7	07·00	43·03	+ ·01	−·10	+·06	23·94
12	25·91·9	11·18·4	07·01	41·83	− ·80	−·10	+·06	24·34
18	25·18·9	10·41·1	07·23	42·82	− ·17	−·10	+·06	24·20
19	26·01·3	11·26·9	07·22	42·04	−1·46	−·10	+·06	23·68
20	25·82·3	11·06·5	07·22	42·36	−1·01	−·10	+·06	23·81
21	25·74·1	10·97·2	07·18	42·62	− ·90	−·10	+·06	23·62
23	25·40·9	10·56·3	07·13	44·40	+1·21	−·10	+·06	23·90
24	25·06·2	10·31·2	07·13	42·18	−1·31	−·10	+·06	23·60
25	25·72·0	10·99·6	07·12	41·57	−1·49	−·10	+·06	24·02
27	25·43·4	10·79·3	07·19	39·65	−2·65	−·10	+·06	24·85
Feb. 3	25·97·2	11·18·4	07·34	43·06	− ·28	−·10	+·06	23·96
4	25·92·0	11·12·8	07·31	43·15	·00	−·10	+·06	24·12
6	25·61·1	10·82·5	07·26	43·01	− ·11	−·10	+·06	24·10
8	25·21·4	11·42·3	07·26	43·13	+ ·06	−·10	+·06	24·15
12	25·80·4	11·04·4	07·42	42·41	−1·39	−·10	+·06	23·58
13	25·49·8	10·72·8	07·44	42·64	− ·86	−·10	+·06	23·90
20	25·33·5	10·55·5	07·31	42·87	− ·40	−·10	+·06	24·00
21	26·83·1	12·09·8	07·30	41·78	−1·76	−·10	+·06	23·72
22	25·23·8	10·48·1	07·32	42·34	−1·02	−·06	+·04	23·94
23	25·61·9	10·85·8	07·34	42·43	−1·17	−·10	+·06	23·70

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP VI, PAIR 6.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Nov. 19	23·11·0	17·43·6	21 18 35·54	—2 11·63	+ ·18	—·04	+·05	21 16 24·10
21	23·41·2	17·70·4	35·62	12·42	+ ·91	—·04	+·05	24·12
30	22·44·7	16·72·8	35·52	12·07	+1·48	—·04	+·05	24·34
Dec. 1	22·87·1	17·20·6	35·52	11·42	+ ·44	—·04	+·05	24·55
5	23·05·7	17·44·8	35·04	10·12	—1·37	—·04	+·05	24·16
9	21·94·3	16·35·8	35·70	09·56	—1·99	—·04	+·05	24·16
12	22·59·5	16·85·9	35·60	10·75	— ·11	—·06	+·05	24·73
15	22·13·4	16·48·2	35·56	11·12	— ·17	—·04	+·05	24·28
19	21·77·1	16·13·4	35·70	10·77	— ·22	—·04	+·05	24·72
23	22·28·7	16·65·1	35·76	10·75	— ·52	—·04	+·05	24·50
25	21·93·8	16·34·5	35·70	09·75	—1·38	—·04	+·05	24·58
26	21·89·1	16·29·8	35·67	09·75	— ·95	—·04	+·05	24·98
28	22·20·6	16·56·6	35·64	10·84	— ·61	—·04	+·05	24·20
29	22·06·5	16·48·0	35·66	09·56	—1·73	—·12	+·09	24·34
30	22·96·8	16·38·1	35·67	09·61	—1·68	+·08	+·09	24·51
1892.								
Jan. 5	22·73·1	17·03·7	35·88	12·09	+ ·22	—·04	+·05	24·02
6	21·91·3	16·26·4	35·86	11·04	— ·23	—·04	+·05	24·60
12	22·52·1	15·92·1	35·80	09·91	—1·07	—·04	+·05	24·83
18	21.19·9	15·56·8	36·02	10·63	— ·61	—·04	+·05	24·80
20	21·21·2	15·60·4	36·00	10·10	—1·21	—·04	+·05	24·70
21	21·40·9	15·74·3	35·98	11·44	—0·19	—·04	+·05	24·36
23	21·16·7	15·40·3	35·92	13·72	+2·13	—·04	+·05	24·34
24	21·64·3	16·03·7	35·92	10·05	—1·65	—·04	+·05	24·23
25	21·28·8	15·69·4	35·92	09·77	—1·91	—·04	+·05	24·25
27	21·23·9	15·70·0	35·98	08·50	—2·51	—·04	+·05	24·98
Feb. 3	21·44·5	15·75·6	36·14	11·98	+ ·14	—·04	+·05	24·31
4	21·62·1	15·94·6	36·34	11·65	+ ·31	—·04	+·05	25·01
6	20·61·9	14·94·6	36·30	11·61	+ ·06	—·04	+·05	24·76
8	22·04·8	16·34·1	36·07	12·39	+ ·11	—·04	+·05	23·80
12	21·53·3	15·92·1	36·23	10·19	—1·12	—·04	+·05	24·93
13	21·24·8	15·60·4	36·26	10·93	— ·67	—·04	+·05	24·67
20	21·52·0	15·87·6	36·15	10·93	— ·71	—·04	+·05	24·52
21	21·59·6	15·99·2	36·14	10·00	—1·83	—·04	+·05	24·32
22	21·12·2	15·49·8	36·16	10·47	— ·75	—·06	+·05	24·93
23	21·11·2	15·48·2	36·18	10·61	—1·01	—·04	+·05	24·57

GROUP VI, PAIR 7.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Nov. 19	15·73·0	20·65·5	21 14 29·60	+1 54·25	+ ·11	+·03	+·05	21 16 24·04
21	16·06·8	20·95·2	29·66	53·30	+ ·55	+·03	+·05	23·59
30	17·05·7	21·94·8	29·54	53·46	+1·22	+·03	+·05	24·30
Dec. 1	17·60·0	22·50·7	29·52	53·83	+ ·35	+·03	+·05	23·78
5	16·77·5	21·73·7	29·62	55·11	−1·62	+·03	+·05	23·19
9	16·02·3	21·02·1	29·67	55·95	−2·14	+·03	+·05	23·56
12	16·56·4	21·50·4	29·56	54·60	− ·22	−·01	+·05	23·98
15	16·90·9	21·84·7	29·50	54·55	− ·28	+·03	+·05	23·85
23	17·49·2	22·49·1	29·68	55·97	− ·91	−·09	+·11	24·76
25	17·06·5	22·01·8	29·61	54·90	−1·05	−·01	+·04	23·49
26	16·67·1	21·66·1	29·58	55·76	−1·41	+·03	+·05	24·01
28	17·03·5	22·00·6	29·54	55·32	−1·10	+·03	+·05	23·84
29	17·16·0	22·14·5	29·56	55·64	−1·59	+·03	+·05	23·69
30	16·88·0	21·82·2	29·57	54·65	− ·39	−·01	+·04	23·86
1892.								
Jan. 5	16·91·8	21·79·0	29·76	53·02	+ ·54	+·03	+·05	23·40
6	17·12·5	22·06·9	29·74	54·69	− ·43	+·03	+·05	24·08
10	17·60·8	22·50·8	29·64	53·67	+ ·38	+·03	+·05	23·77
12	16·55·2	21·49·2	29·66	54·60	− ·45	+·03	+·05	23·89
18	16·35·6	21·29·0	29·88	54·46	− ·45	−·01	+·05	23·93
19	16·54·4	21·49·6	29·88	54·88	−1·39	+·03	+·05	23·45
20	16·20·2	21·18·3	29·87	55·55	−1·20	+·03	+·05	24·30
21	16·36·3	21·27·9	29·84	54·04	+ ·17	+·03	+·05	24·13
23	16·83·9	21·68·8	29·78	52·49	+1·61	+·03	+·05	23·96
24	16·20·6	21·17·7	29·78	55·32	−1·37	+·03	+·05	23·81
25	16·53·5	21·51·6	29·78	55·55	−1·19	+·03	+·05	24·22
27	16·06·3	21·11·0	29·84	57·08	−2·54	+·03	+·05	24·46
Feb. 3	16·75·9	21·65·8	30·02	53·65	+ ·27	+·03	+·05	24·02
4	16·60·1	21·50·0	29·98	53·65	+ ·50	+·03	+·05	24·21
6	16·01·7	20·90·6	29·94	53·42	+ ·30	+·03	+·05	23·74
8	16·96·5	21·86·8	29·92	53·74	+ ·37	+·03	+·05	24·11
12	16·50·7	21·48·7	30·10	55·53	−1·17	+·03	+·05	24·54
13	16·57·8	21·50·0	30·13	54·18	− ·84	+·03	+·05	23·55
14	16·61·6	21·54·8	30·16	54·41	− ·97	+·03	+·05	23·68
20	16·63·2	21·60·4	30·04	55·34	− ·70	+·11	+·06	24·85
22	16·18·8	21·11·5	30·05	54·30	− ·45	−·01	+·05	23·94
23	16·10·8	21·03·7	30·08	54·35	− ·65	+·03	+·05	23·86

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP VI, PAIR 8.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Nov. 19	23·11·5	14·62·2	21 19 40·28	—3 17·02	+ ·88	—·05	+·05	21 16 24·14
21	23·71·4	15·20·3	40·32	17·44	+1·04	—·05	+·05	23·92
30	24·51·9	16·01·2	40·08	17·35	+1·82	—·05	+·05	24·55
Dec. 1	23·77·2	15·27·9	40·06	17·02	+1·02	—·05	+·05	24·06
12	23·28·0	14·84·3	40·00	15·73	+ ·27	—·05	+·05	24·54
15	23·48·8	15·06·4	39·90	15·42	— ·06	—·05	+·05	24·42
23	23·63·0	15·22·5	40·04	14·98	— ·20	—·03	+·05	24·88
25	23·01·7	14·62·8	39·96	14·61	—1·09	—·05	+·05	24·26
26	23·57·0	15·20·7	39·92	14·01	— ·88	—·05	+·02	25·00
28	23·33·0	14·97·9	39·86	13·73	— ·98	—·13	+·06	25·08
29	23·94·2	15·55·7	39·88	14·52	—1·18	—·05	+·05	24·18
30	24·26·6	15·85·7	39·88	15·08	— ·86	—·05	+·05	23·94
1892.								
Jan. 5	23·05·5	14·57·2	40·06	16·79	+1·31	—·05	+·05	24·58
6	23·67·7	15·26·1	40·04	15·24	— ·42	—·05	+·05	24·38
10	23·88·7	15·44·0	39·92	15·96	+ ·53	—·13	+·06	24·42
11	23·56·8	15·14·2	39·93	15·47	— ·22	—·03	+·05	24·26
12	22·49·7	14·07·0	39·94	15·49	— ·34	—·05	+·05	24·11
18	23·06·2	14·64·1	40·16	15·35	— ·53	—·05	+·05	24·28
20	23·30·6	14·90·7	40·16	14·84	— ·85	—·05	+·05	24·47
21	22·40·1	13·93·8	40·14	16·33	— ·14	—·05	+·05	23·67
24	22·74·8	14·36·6	40·08	14·45	—1·80	—·05	+·05	23·83
25	22·81·6	14·42·9	40·08	14·56	—1·34	—·05	+·05	24·18
27	22·06·9	14·77·3	40·14	12·45	—2·55	—·05	+·05	25·14
Feb. 3	22·78·2	14·29·9	40·34	16·79	+ ·67	—·05	+·05	24·22
6	22·90·3	14·46·1	40·28	15·84	— ·34	—·05	+·05	24·10
8	22·84·0	14·39·0	40·28	16·03	+ ·21	—·05	+·05	24·46
12	22·09·5	13·67·3	40·46	15·38	— ·28	—·05	+·05	24·80
13	22·72·4	14·29·9	40·50	15·45	— ·73	—·05	+·05	24·32
14	22·70·7	14·28·8	40·55	15·31	—1·05	—·05	+·05	24·19
20	21·76·5	14·30·4	40·48	16·28	— ·04	—·05	+·05	24·16
21	23·35·5	14·97·7	40·48	14·36	—1·56	—·05	+·05	24·56
22	22·61·0	14·17·5	40·50	15·68	— ·24	—·05	+·05	24·58
23	22·86·4	14·47·4	40·52	14·63	—1·15	—·05	+·05	24·74

GROUP VII, PAIR 1.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* t. d.	*S.* t. d.	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Dec. 29	12·06·9	23·94·4	21 11 50·28	+4 35·48	— ·70	+·10	+·04	21 16 25·20
1892.								
Jan. 5	12·36·8	24·11·0	50·33	32·40	+1·71	+·11	+·06	24·61
10	13·88·2	25·70·4	50·12	34·25	+ ·73	+·15	+·05	25·30
11	12·71·4	24·49·9	50·11	33·39	+1·43	+·08	+·05	25·06
12	12·58·2	24·40·4	50·08	34·25	+ ·47	+·08	+·05	24·93
20	13·38·7	25·17·7	50·26	33·51	+ ·44	+·08	+·05	24·34
23	12·77·3	24·50·2	50·18	32·10	+1·94	+·08	+·08	24·38
24	13·08·4	24·97·5	50·18	35·85	—1·08	+·08	+·06	25·09
25	12·33·2	24·18·3	50·16	34·92	— ·01	+·11	+·06	25·24
27	12·64·6	24·49·3	50·20	34·83	+ ·09	+·08	+·05	25·25
Feb. 2	12·75·8	24·51·9	50·44	32·84	+1·77	+·11	+·06	25·22
3	12·81·1	24·56·7	50·44	32·72	+1·53	+·08	+·05	24·82
4	12·82·2	24·61·7	50·43	33·63	+1·06	+·08	+·05	25·25
6	12·93·7	24·67·0	50·40	32·19	+1·58	+·08	+·05	24·30
8	13·03·5	24·79·0	50·40	32·70	+1·27	+·08	+·05	24·50
12	12·90·6	24·67·2	50·60	32·95	+ ·50	+·08	+·05	24·18
13	12·46·0	24·26·9	50·66	33·95	+ ·10	+·08	+·05	24·84
14	12·63·3	24·46·1	50·61	34·39	+ ·06	+·08	+·06	25·20
19	12·85·2	24·63·5	50·78	33·35	+ ·48	+·11	+·06	24·78
20	12·26·9	24·04·6	50·79	33·21	+ ·67	+·08	+·06	24·81
21	12·97·3	24·75·5	50·79	33·32	+ ·59	+·08	+·05	24·83
23	12·92·0	24·73·0	50·86	33·97	+ ·55	+·19	+·08	25·65
24	14·08·8	25·80·9	50·42	31·91	+2·46	+·08	+·05	24·92
26	14·08·2	25·82·9	51·05	32·51	+1·85	+·14	+·03	25·58
27	12·99·2	24·92·8	51·10	36·90	—2·98	+·08	+·04	25·14
28	12·57·2	24·17·9	51·16	29·26	+4·01	+·08	+·05	24·56
29	13·92·3	25·74·1	51·20	34·16	— ·37	+·08	+·05	25·12
Mar. 1	12·69·5	24·47·5	51·22	33·28	+ ·31	+·08	+·05	24·94
2	13·62·7	25·35·5	51·23	32·07	+1·55	+·08	+·05	24·98
4	13·14·1	24·90·9	51·22	33·00	+ ·95	+·08	+·05	25·30
5	13·96·6	25·78·4	51·22	34·16	— ·27	+·08	+·05	25·24
9	12·56·3	24·25·6	51·36	31·26	+1·14	+·08	+·05	23·89
10	12·32·4	24·12·6	51·44	33·79	+ ·23	+·08	+·05	25·59
13	13·47·3	25·22·0	51·62	32·51	+ ·66	+·08	+·05	24·92
16	12·93·4	24·68·4	51·68	32·58	+ ·63	+·08	+·05	25·02
19	13·50·7	25·31·8	51·66	34·00	— ·31	+·11	+·06	25·52
20	12·80·2	24·61·6	51·69	34·07	—1·14	+·08	+·05	24·75
24	12·48·1	24·22·9	51·89	32·54	+1·02	+·08	+·05	25·58
30	13·18·0	24·87·5	52·06	31·31	+1·91	+·10	+·04	25·42

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP VII, PAIR 2.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Dec. 29	21·39·5	19·28·8	21 17 14·62	—0 48·88	—1·28	—·01	+·05	21 16 24·50
1892.								
Jan. 5	20·40·9	18·14·3	14·62	52·57	+1·91	—·01	+·05	24·00
6	20·97·6	18·76·4	14·59	51·32	+ ·82	—·01	+·05	24·13
11	19·42·8	17·19·7	14·36	51·75	+1·28	+·03	+·04	23·96
12	21·41·0	19·24·5	14·34	50·22	— ·08	—·01	+·05	24·08
18	21·05·1	18·88·8	14·50	50·18	+ ·52	—·01	+·05	24·88
20	21·73·0	19·49·2	14·50	51·02	+1·50	—·01	+·05	24·12
21	22·65·9	20·46·6	14·48	50·88	— ·32	—·01	+·05	23·96
23	22·29·2	20·00·7	14·42	53·01	+3·15	—·01	+·05	24·60
24	22·57·3	20·45·6	14·38	49·11	— ·84	—·01	+·05	24·47
25	22·32·2	20·17·9	14·38	49·72	— ·21	—·01	+·05	24·49
27	22·54·6	20·35·8	14·42	50·76	+ ·58	—·01	+·05	24·28
Feb. 2	22·64·7	20·38·8	14·66	52·41	+2·40	—·01	+·05	24·69
3	22·98·8	20·77·7	14·66	51·29	+1·54	—·01	+·05	24·95
4	22·40·0	20·16·9	14·66	51·76	+1·47	—·03	+·05	24·39
6	22·73·6	20·52·6	14·62	51·27	+1·23	—·01	+·05	24·62
8	22·74·3	20·51·3	14·62	51·73	+1·53	—·01	+·05	24·46
12	22·66·3	20·45·8	14·82	51·15	+ ·85	—·01	+·05	24·56
13	22·46·1	20·21·4	14·88	52·13	+2·12	—·01	+·05	24·91
19	22·69·0	20·47·7	15·04	51·34	+ ·70	—·01	+·05	24·44
20	22.25·5	20·71·9	15·04	50·48	+ ·43	—·11	+·08	24·06
21	22·62·2	20·43·8	15·04	50·67	+ ·75	—·01	+·05	25·16
23	22·12·7	19·96·2	15·10	50·22	+ ·08	—·01	+·05	25·00
24	22·45·7	20·20·5	15·16	52·24	+1·82	—·01	+·05	24·78
26	22·73·7	20·51·4	15·30	51·57	— ·39	—·01	+·05	24·16
27	22·66·6	20·54·5	15·37	49·20	—1·44	—·01	+·05	24·77
28	22·31·1	20·14·3	15·44	50·30	— ·67	—·01	+·05	24·51
29	22·27·7	20·06·6	15·47	51·29	+ ·18	—·01	+·05	24·40
Mar. 1	22·50·3	20·32·4	15·50	50·55	— ·01	—·01	+·05	24·98
2	22·23·3	20·00·2	15·52	51·76	+ ·67	—·01	+·05	24·47
4	22·74·6	20·57·0	15·51	50·48	— ·21	—·01	+·05	24·86
5	22·68·5	20·51·9	15·50	50·25	— ·72	—·01	+·05	24·57
9	22·92·0	20·70·6	15·67	51·36	+ ·56	—·01	+·05	24·91
10	22·54·0	20·28·8	15·74	52·29	+ ·76	—·01	+·05	24·25
13	22·48·9	20·27·6	15·96	51·34	+ ·18	—·01	+·05	24·84
14	22·86·2	20·66·5	16·00	50·97	— ·07	—·01	+·05	25·00
16	22·60·7	20·34·9	16·02	52·38	+1·02	—·01	+·05	24·70
19	22·19·5	19·98·4	16·02	51·29	— ·41	—·01	+·05	24·36
20	22·38·5	20·22·9	16·06	50·02	—1·08	—·01	+·05	25·00
21	22·51·0	20·33·2	16·08	50·53	— ·70	—·01	+·05	24·89
24	22·58·8	20·34·8	16·28	51·96	+ ·17	—·01	+·05	24·53
29	24·05·2	21·85·2	16·48	51·04	— ·53	—·01	+·05	24·95
30	22·35·2	20·17·4	16·48	50·53	— ·89	—·01	+·05	25·10
31	22·65·7	20·35·9	16·48	53·31	+1·63	—·01	+·05	24.84

GROUP VII, PAIR 3.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Dec. 29	18·44·7	18·28·0	21 16 29·12	—0 03·87	— ·76	+ ·02	+ ·07	21 16 24·58
1892.								
Jan. 5	18·24·4	17·92·2	29·10	07·47	+2·08	·00	+ ·07	23·78
6	18·01·8	17·74·2	29·07	06·40	+1·36	·00	+ ·07	24·10
11	17·77·9	17·54·2	28·83	05·50	+ ·88	·00	+ ·07	24·28
12	17·80·2	17·57·5	28·80	05·27	+ ·88	+ ·04	+ ·07	24·52
18	17·61·1	17·40·4	28·94	04·80	+ ·17	·00	+ ·07	24·38
20	17·66·2	17·38·2	28·95	06·50	+1·91	·00	+ ·07	24·43
21	18·16·8	18·12·2	28·92	01·07	—3·40	·00	+ ·07	24·52
23	17·28·9	16·99·7	28·86	06·77	+2·82	·00	+ ·07	24·98
24	17·73·5	17·57·3	28·84	03·76	— ·58	·00	+ ·07	24·57
25	17·32·6	17·16·0	28·82	03·85	— ·50	·00	+ ·07	24·54
27	17·38·6	17·16·8	28·85	05·06	+ ·23	·00	+ ·07	24·09
Feb. 2	17·72·9	17·44·6	29·08	06·57	+1·71	·00	+ ·07	24·29
3	17·45·4	17·22·2	29·07	05·38	+ ·84	·00	+ ·07	24·60
4	17·88·0	17·63·1	29·06	05·78	+1·13	·00	+ ·07	24·48
6	17·69·2	17·44·8	29·03	05·66	+ ·88	·00	+ ·07	24·32
8	17·99·5	17·77·8	29·02	05·03	+ ·58	·00	+ ·07	24·64
12	17·30·5	17·10·4	29·21	04·66	+ ·14	·00	+ ·07	24·76
13	18·08·0	17·83·0	29·28	05·80	+ ·65	+ ·08	+ ·10	24·31
19	18·29·6	18·04·0	29·42	05·94	+1·02	·00	+ ·07	24·57
20	17·83·5	17·59·3	29·41	05·61	+0·55	·00	+ ·07	24·42
24	16·92·1	16·67·8	29·53	05·64	+ ·67	·00	+ ·07	24·63
26	17·82·9	17·60·5	29·67	05·20	+ ·33	·00	+ ·07	24·87
27	17·60·8	17·47·1	29·73	03·18	—1·91	·00	+ ·07	24·71
28	17·95·8	17·56·6	29·80	09·09	+3·84	·00	+ ·07	24·62
29	18·40·2	18·24·1	29·83	03·74	—1·46	·00	+ ·07	24·70
Mar. 1	17·62·5	17·38·6	29·86	05·54	+ ·24	·00	+ ·07	24·63
2	18·05·5	17·84·6	29·87	04·85	— ·21	·00	+ ·07	24·88
4	17·61·7	17·45·7	29·86	03·71	—1·30	·00	+ ·07	24·92
5	17·80·5	17·64·8	29·86	03·64	—1·20	·00	+ ·07	25·09
9	17·71·7	17·46·6	30·00	05·82	+ ·41	·00	+ ·07	24·66
10	17·99·7	17·76·7	30·08	05·34	— ·09	·00	+ ·07	24·72
13	17·51·3	17·23·8	30·29	06·38	+ ·27	·00	+ ·07	24·24
14	17·95·1	17·75·6	30·32	04·52	—1·39	·00	+ ·07	24·48
16	17·99·6	17·78·4	30·36	04·92	— ·55	— ·04	+ ·05	24·90
19	17·79·9	17·60·3	30·36	04·55	—1·19	·00	·00	24·62
20	18·13·0	17·96·9	30·39	03·74	—2·22	·00	+ ·07	24·50
21	18·02·9	17·84·5	30·41	04·27	—1·54	+ ·04	+ ·05	24·69
24	16·94·1	16·73·0	30·58	04·90	—1·02	·00	+ ·07	24·73
29	17·76·0	17·55·8	30·79	04·69	—1·17	+ ·02	+ ·07	25·02
30	18·18·2	18·01·5	30·74	03·87	—1·85	·00	+ ·07	25·09
31	18·30·9	18·03·9	30·78	06·26	+ ·47	·00	+ ·07	25·06

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP VII, PAIR 4.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Dec. 29	22·37·0	17·39·8	21 18 20·80	—1 55·34	— ·90	—·03	+·01	21 16 24·58
1892.								
Jan. 5	22·67·9	17·55·4	20·72	1 58·89	+1·82	—·03	+·05	23·67
6	22·71·5	17·60·9	20·68	58·45	—1·05	—·03	+·05	23·30
11	21·51·2	16·42·5	20·42	58·01	—1·19	—·03	+·05	23·62
12	21·34·2	16·28·9	20·39	57·22	+ ·58	—·01	+·04	23·80
20	19·86·9	14·75·5	20·50	58·64	+1·42	—·03	+·05	23·30
21	21·32·4	16·22·1	20·48	58·38	+1·66	—·03	+·05	23·78
23	21·60·6	16·57·8	20·41	56·64	— ·10	—·03	+·05	23·69
24	22·02·6	17·03·0	20·38	55·90	— ·40	—·03	+·05	24·10
25	21·32·8	16·34·7	20·36	55·55	— ·47	—·07	+·05	24·32
27	22·10·7	17·07·7	20·38	56·69	+ ·62	—·03	+·05	24·33
Feb. 2	21·74·3	16·67·3	20·62	57·62	+ ·50	—·03	+·05	24·52
3	21·95·7	16·88·8	20·61	57·59	+1·06	—·03	+·05	24·10
4	21·69·3	16·63·7	20·60	57·29	+ ·64	—·03	+·05	23·97
6	21·43·7	16·38·5	20·56	57·20	+ ·61	—·03	+·05	23·99
8	21·88·2	16·83·8	20·56	57·01	+1·00	—·03	+·05	24·57
12	21·50·9	16·50·3	20·74	56·13	+ ·01	—·03	+·05	24·64
13	21·57·2	16·52·2	20·82	57·15	+ ·42	—·03	+·05	24·11
19	21·49·0	16·44·1	20·98	57·13	+ ·31	—·03	+·05	24·18
20	21·44·2	16·40·1	20·48	56·94	+ ·16	—·03	+·05	23·72
21	22·03·0	17·00·5	20·99	56·57	+ ·26	—·03	+·05	24·70
23	22·07·4	17·05·0	21·05	56·55	+ ·26	—·03	+·05	24·78
24	20·90·1	15·89·2	21·11	56·20	— ·11	—·03	+·05	24·82
26	21·94·1	16·92·1	21·26	56·46	— ·40	—·03	+·05	24·42
27	21·72·4	16·79·5	21·25	54·35	—2·08	—·03	+·05	24·84
28	21·37·7	16·41·2	21·39	55·18	—1·65	—·03	+·05	24·58
29	22·28·0	17·26·4	21·43	56·36	— ·69	—·03	+·05	24·40
Mar. 1	21·44·0	16·40·7	21·48	56·76	— ·11	—·03	+·05	24·63
2	21·19·8	16·14·1	21·49	57·31	+ ·08	—·03	+·05	24·28
4	21·47·7	16·46·0	21·50	56·39	— ·46	—·03	+·05	24·67
5	21·86·4	16·85·5	21·50	56·20	— ·56	—·03	+·05	24·76
9	21·05·1	15·96·1	21·66	58·08	+ ·43	—·03	+·05	24·03
10	22·06·4	17·01·8	21·74	57·06	— ·14	—·03	+·05	24·56
13	21·88·2	16·78·5	21·98	58·24	+ ·59	—·11	+·08	24·30
14	21·79·5	16·72·0	22·03	57·73	+ ·23	—·03	+·05	24·55
16	21·52·5	16·45·5	22·08	57·62	+ ·27	—·03	+·05	24·75
21	21·89·8	16·87·7	22·16	56·48	— ·87	—·03	+·05	24·83
24	21·64·8	16·58·1	22·36	57·55	— ·19	—·03	+·05	24·64
29	21·86·4	16·76·2	22·62	58·36	—1·02	—·03	+·05	23·26
31	21·79·5	16·66·9	22·62	58·92	+ ·74	—·03	+·05	24·46

GROUP VII, PAIR 5.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Dec. 29	16·71·1	22·10·6	21 14 20·60	+2 05·16	—1·71	+·04	+·06	21 16 24·15
1892.								
Jan. 5	16·30·4	21·52·8	20·46	01·19	+1·05	+·04	+·06	22·80
6	16·54·5	21·82·6	20·43	02·51	+ ·25	+·04	+·06	23·29
11	16·40·1	21·72·0	20·14	03·39	— ·19	+·04	+·06	23·44
12	15·93·6	21·24·4	20·10	03·14	— ·10	+·04	+·06	23·24
20	16·24·7	21·51·7	20·18	02·26	+ ·86	+·04	+·06	23·40
21	16·20·7	21·48·4	20·16	02·42	+1·19	+·04	+·06	23·87
23	15·42·0	20·78·0	20·08	04·34	— ·69	+·04	+·06	23·83
24	16·48·7	21·90·3	20·06	05·64	—2·24	+·04	+·06	23·56
25	15·20·7	20·65·9	20·03	05·09	—1·80	+·04	+·06	23·42
27	16·45·2	21·80·0	20·04	04·07	— ·28	+·04	+·06	23·93
Feb. 2	16·30·1	21·60·2	20·25	02·98	+ ·45	+·04	+·06	23·78
3	15·68·0	20·98·2	20·24	03·00	+ ·34	+·04	+·06	23·68
4	15·94·7	21·28·0	20·24	03·72	— ·21	+·04	+·06	23·85
8	15·33·0	21·63·8	20·18	03·14	+ ·10	+·04	+·06	23·52
12	16·46·5	21·79·8	20·36	03·72	— ·85	+·04	+·06	23·33
13	16·45·8	21·78·5	20·44	03·58	— ·32	+·04	+·06	23·80
19	15·91·5	21·23·2	20·61	03·35	— ·67	+·14	+·14	23·57
21	16·65·0	21·99·8	20·62	04·07	—1·02	+·04	+·06	23·77
23	15·22·2	20·58·3	20·68	04·37	—1·25	+·04	+·06	23·90
24	16·23·2	21·58·0	20·73	04·07	—0·80	+·04	+·06	24·10
26	15·69·8	21·04·0	20·88	03·93	— ·99	+·04	+·06	23·92
27	16·27·0	21·70·1	20·94	05·99	—3·27	+·04	+·06	23·76
28	15·50·3	20·93·2	21·02	05·94	—3·22	+·04	+·06	23·84
29	16·00·3	21·20·2	21·06	00·61	+1·76	+·04	+·06	23·53
Mar. 1	16·56·3	21·91·7	21·10	04·20	—1·40	+·04	+·06	24·00
2	16·22·2	21·56·0	21·12	03·83	—1·21	+·04	+·06	23·84
4	15·55·2	20·94·2	21·13	05·04	—2·20	+·04	+·06	24·07
5	15·74·4	21·14·7	21·13	05·34	—2·35	+·04	+·06	24·22
9	16·76·6	22·06·4	21·30	02·91	— ·70	+·04	+·06	23·61
10	16·12·5	21·42·9	21·38	03·04	— ·69	+·04	+·06	23·83
13	16·16·3	21·50·7	21·62	03·97	—1·12	+·09	+·06	24·62
14	16·36·6	21·67·7	21·68	03·21	—1·13	+·04	+·06	23·86
16	15·85·7	21·17·1	21·74	03·28	—1·22	+·04	+·06	23·90
19	16·41·5	21·77·5	21·76	04·34	—2·33	+·04	+·06	23·87
20	15·81·4	21·19·0	21·80	04·72	—2·44	+·04	+·06	24·18
21	16·18·0	21·54·2	21·82	04·39	—1·87	+·04	+·06	24·44
24	15·43·0	20·76·6	22·02	03·79	—1·63	+·04	+·06	24·28
30	16·08·5	21·49·3	22·31	05·46	—3·35	—·04	+·10	24·48
31	15·71·3	20·94·3	22·32	01·33	— ·23	+·04	+·06	23·52

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP VII, PAIR 6.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Dec. 29	21·07·3	18·48·7	21 17 26·23	—0 59·99	—1·37	—·02	+·07	21 16 24·92
1892.								
Jan. 5	21·09·4	18·41·7	26·00	—1 02·10	+ ·71	—·02	+·07	24·66
6	21·02·4	18·31·6	25·95	02·82	+1·00	—·02	+·07	24·18
11	20·03·9	17·43·7	25·60	00·36	— ·71	—·02	+·07	24·58
18	19·76.5	17·14·6	25·56	00·76	— ·13	—·02	+·07	24·72
20	19·92·8	17·25·6	25·54	01·99	+1·08	—·02	+·07	24·68
21	20·23·0	17·54·2	25·52	02·36	+1·49	—·02	+·07	24·70
23	20·13·7	17·55·6	25·44	—0 59·88	—1·09	—·02	+·08	24·53
24	29·59·6	17·06·7	25·40	58·67	—2·06	—·02	+·07	24·72
27	21·64·0	19·05·8	25·34	59·90	— ·69	—·02	+·07	24·80
Feb. 2	22·22·6	19·58·2	25·52	—1 01·34	+ ·59	—·02	+·07	24·82
3	23·86·2	21·25·6	25·52	00·46	— ·31	—·02	+·07	24·80
4	23·94·7	21·31·3	25·01	01·10	+ ·60	—·02	+·07	24·56
6	23·96·8	21·36·6	25·46	00·36	— ·23	—·02	+·07	24·92
8	24·26·0	21·68·5	25·44	—0 59·74	— ·58	—·02	+·07	25·17
12	23·47·6	20·92·1	25·60	59·27	—1·59	—·02	+·07	24·79
13	23·61·3	21·04·7	25·67	59·53	—1·19	—·02	+·07	25·00
20	23·51·7	20·92·5	25·87	—1 00·13	— ·73	—·02	+·07	25·06
21	23·78·3	21·20·8	25·88	—1 59·74	—1·27	—·02	+·07	24·92
23	23·55·6	20·96·1	25·93	—1 00·20	— ·68	—·02	+·07	25·10
24	23·45·6	20·91·4	25·98	—0 58·97	—1·94	—·02	+·07	25·12
27	24·02·6	21·39·1	26·21	—1 01·13	— ·43	—·02	+·07	24·70
28	23·74·5	21·14·4	26·32	—1 00·34	— ·96	—·02	+·07	25·07
29	24·29·3	21·74·1	26·35	—0 59·20	—2·22	—·02	+·07	24·98
Mar. 1	23·86·7	21·32·0	26·38	—0 59·09	—1·98	—·02	+·07	25·36
2	23·85·0	21·29·5	26·40	59·27	—1·81	—·02	+·07	25·37
4	23·79·4	21·23·2	26·43	59·43	—1·45	—·02	+·07	25·60
5	23·90·2	21·33·5	26·44	59·55	—1·54	—·02	+·07	25·40
9	23·94·7	21·30·7	26·62	—1 01·24	— ·34	—·02	+·07	25·09
10	23·64·0	21·01·9	26·70	00·80	— ·99	—·02	+·07	24·96
13	24·26·1	21·64·4	26·97	00·71	— ·88	—·02	·00	25·36
14	23·95·6	21·31·8	27·04	01·20	— ·61	—·02	+·07	25·28
16	24·06·2	21·46·9	27·12	00·15	—1·62	—·02	+·07	25·40
19	23·77·2	21·18·8	27·18	—0 59·94	—1·83	+·02	+·05	25·48
20	23·86·3	21·29·6	27·22	59·55	—2·54	—·02	+·05	25·16
21	24·31·4	21·73·1	27·26	59·92	—2·04	—·02	+·07	25·35
24	23·75·1	21·14·4	27·48	—1 00·48	—1·69	—·00	+·07	25·38
29	23·44·3	20·83·9	27·81	00·41	—2·13	—·02	+·07	25·32
31	23·09·2	21·39·3	27·86	02·61	— ·02	—·02	+·07	25·28

GROUP VII, PAIR 7.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Dec. 29	24·95·9	14·42·5	21 20 29·27	—4 04·37	— ·60	—·07	+·05	21 16 24·28
1892								
Jan. 5	24·65·6	14·00·6	28·94	07·06	+1·82	—·07	+·05	23·68
6	24·74·6	14·08·5	28·90	07·32	+1·89	—·07	+·05	23·45
11	23·67·0	13·08·7	28·48	05·51	+1·51	—·07	+·05	24·46
20	24·09·5	13·48·2	28·38	06·21	+2·07	—·07	+·05	24·22
24	23·73·7	13·24·9	28·18	03·31	— ·28	—·07	+·05	24·57
25	23·80·8	13·27·0	28·16	04·47	+ ·27	—·07	+·05	23·94
Feb. 2	23·44·0	12·85·8	28·30	05·49	+2·15	—·07	+·05	24·94
3	24·33·8	13·77·1	28·29	05·14	+1·61	—·07	+·05	24·74
4	23·96·3	13·37·6	28·28	05·60	+1·74	—·07	+·05	24·40
8	24·35·4	13·78·2	28·20	05·25	+1·19	—·07	+·05	24·12
12	24·03·6	13·48·8	28·35	04·70	— ·73	—·07	+·05	24·36
13	24·15·2	13·61·0	28·42	04·56	+ ·47	—·07	+·05	24·31
19	23·88·9	13·30·3	28·64	05·58	+1·24	—·10	+·06	24·26
20	23·82·4	13·27·7	28·64	04·67	+ ·25	—·07	+·05	24·20
23	23·52·9	13·02·7	28·70	03·63	— ·15	—·07	+·05	24·90
24	23·46·7	12·96·4	28·76	03·65	— ·19	—·07	+·05	24·90
27	23·86·6	13·23·4	28·98	06·65	+2·41	—·07	+·05	24·72
28	24·02·2	13·47·4	29·06	04·70	+ ·39	—·07	+·05	24·73
29	23·81·2	13·28·0	29·12	04·33	— ·24	—·07	+·05	24·53
Mar. 1	23·66·3	13·12·3	29·18	04·51	— ·10	—·07	+·05	24·55
2	24·02·0	13·49·1	29·22	04·26	— ·49	—·07	+·05	24·45
4	24·02·2	13·53·3	29·25	03·33	—1·28	—·07	+·05	24·62
5	24·04·4	13·54·7	29·26	03·51	— ·97	—·07	+·05	24·76
9	23·94·1	13·39·4	29·45	04·67	— ·32	—·07	+·05	24·44
10	24·08·1	13·54·5	29·54	04·42	— ·16	—·07	+·05	24·94
13	24·07·4	13·47·9	29·83	05·79	+ ·17	—·03	+·04	24·22
14	24·02·7	13·47·3	29·91	04·84	— ·32	—·07	+·05	24·73
16	24·20·7	13·63·1	30·02	05·35	— ·21	—·07	+·05	24·44
20	23·87·3	13·35·0	30·14	04·12	—1·39	+·04	+·08	24·75
21	23·83·2	13·30·3	30·18	04·26	—1·12	—·11	+·04	24·73
24	24·09·1	13·52·9	30·42	05·02	— ·25	—·07	+·05	25·13
29	24·44·1	13·91·1	30·80	04·28	—1·47	—·07	+·05	25·03
31	23·82·5	13·20·9	30·87	06·28	+ ·67	—·07	+·05	25·24

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP VII, PAIR 8.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1891.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Dec. 29	17·21·9	20·46·2	21 15 13·18	+1 15·23	−3·91	+·04	+·04	21 16 24·58
1892.								
Jan. 5	19·17·2	22·12·4	12·80	08·48	+2·14	+·02	+·05	23·49
6	19·44·6	22·40·4	12·74	08·62	+2·18	+·02	+·05	23·61
11	18·82·1	21·89·1	12·30	11·22	+·89	+·02	+·05	24·48
18	18·35·4	21·46·6	12·14	12·19	+·48	+·02	+·05	24·88
20	18·45·1	21·46·6	12·11	09·94	+1·75	+·02	+·05	23·87
23	18·07·7	21·17·3	11·98	11·82	+·21	+·02	+·05	24·08
24	18·34·7	21·47·3	11·92	12·52	−·38	+·02	+·05	24·13
25	18·16·1	21·29·8	11·87	12·77	−·04	+·02	+·05	24·75
Feb. 2	18·19·6	21·20·3	11·95	09·76	+2·49	+·02	+·05	24·27
3	17·90·8	20·96·2	11·94	10·85	+1·70	+·02	+·05	24·56
4	18·45·6	21·52·6	11·94	11·22	+1·23	+·02	+·05	24·46
8	18·34·6	21·41·4	11·84	11·17	+1·31	+·02	+·05	24·39
12	18·11·7	21·20·3	11·98	11·59	+·44	+·02	+·05	24·08
13	18·99·4	22·07·4	12·04	11·45	+·51	+·02	+·05	24·07
20	18·80·6	21·88·0	12·28	11·31	−·60	+·02	+·05	24·26
23	18·62·1	21·71·2	12·33	11·71	−·63	+·02	+·05	24·74
24	18·25·2	21·32·7	12·38	11·34	+·78	+·02	+·05	24·57
27	18·55·9	21·56·0	12·61	09·62	+2·30	+·02	+·05	24·60
28	17·97·4	21·04·1	12·69	11·15	+·52	+·02	+·05	24·43
29	18·15·3	21·27·3	12·75	12·38	−·36	+·02	+·05	24·84
Mar. 1	18·42·6	21·50·0	12·82	11·31	+·33	+·02	+·05	24·53
2	18·49·1	21·58·9	12·85	11·87	−·33	+·02	+·05	24·46
5	17·62·6	20·76·3	12·90	12·77	−·90	+·02	+·05	24·84
9	18·33·9	21·41·5	13·10	11·36	−·26	−·02	+·04	24·22
10	17·99·0	21·06·1	13·20	11·24	+·26	+·02	+·05	24·77
13	18·33·3	21·36·8	13·60	10·41	+·32	+·02	+·05	24·40
14	17·92·2	20·99·8	13·75	11·36	−·12	+·02	+·05	25·06
16	18·11·2	21·16·3	13·70	10·78	+·02	+·02	+·05	24·57
21	18·55·6	21·65·6	13·90	11·92	−1·11	+·02	+·05	24·78
24	18·22·9	21·27·4	14·14	10·64	−·37	+·02	+·05	24·48
29	18·27·0	21·33·7	14·56	11·15	−1·08	+·02	+·05	24·70
31	18·53·8	21·50·6	14·64	08·85	+·85	−·06	+·07	24·35

GROUP VIII, PAIR 1.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1892.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Feb. 3	23·34·0	24·64·3	21 15 52·92	+0 30·23	+ ·65	+·01	+·05	21 16 23·86
8	23·80·5	25·18·2	52·74	31·94	— ·80	+·01	+·05	23·94
12	23·87·9	25·25·4	52·77	31·90	— ·74	+·01	+·05	23·99
21	24·77·6	26·17·5	53·04	32·45	— ·47	+·01	+·05	25·08
23	23·70·7	25·09·7	53·07	32·25	—1·16	+·01	+·05	24·22
27	23·96·7	25·25·8	53·30	29·95	+ ·81	+·01	+·05	24.12
28	23·76·8	25·16·7	53·38	32·45	—1·27	+·01	+·05	24·62
29	23·49·1	24·90·4	53·46	32·78	—2·03	+·01	—·05	24·27
Mar. 2	23·97·6	25·36·7	53·57	32·27	— ·98	+·01	+·05	24·92
5	22·99·7	24·45·3	53·64	33·78	—2·58	+·01	+·05	24·90
8	22·87·2	24·20·0	53·75	30·81	— ·11	+·01	+·07	24·53
9	23·97·9	25·34·4	53·84	31·67	— ·85	+·01	+·05	24·72
10	23·45·8	24·80·1	53·92	31·16	— ·82	+·01	+·05	24·32
13	23·53·4	24·91·6	54·24	32·06	—1·63	+·04	+·06	24·77
14	23·68·7	25·05·0	54·35	31·62	—1·45	+·01	+·05	24·58
16	23·59·0	24·92·8	54·50	31·04	—1·57	+·04	+·06	24·07
21	23·58·2	24·94·6	54·76	31·64	—1·70	+·01	+·05	24·76
23	23·56·1	24·91·5	54·91	31·41	—1·68	+·01	+·07	24·72
24	23·41·2	24·74·9	55·01	31·02	—1·38	+·01	+·05	24·71
29	24·16·2	25·52·3	55·54	31.57	—2·11	—·07	+·07	25·00
30	23·77·8	25·15·4	55·60	31·92	—2·79	+·01	+·05	24·79
31	23·66·5	24·94·5	55·66	29·69	— ·65	+·01	·05	24·76
Apr. 6	24·39·2	25·65·3	56·04	29·25	— ·90	+·03	+·06	24·48
9	24·04·5	25·29·8	56·42	29·07	— ·17	+·01	+·05	25·38
10	24·27·7	25·55·3	56·54	29·60	—1·06	+·01	+·05	25·14
15	23·79·0	25·04·8	56·92	29·18	—2·16	—·07	+·07	23·94
16	24·60·4	25·80·1	56·97	27·77	— ·42	+·01	+·05	24·38
17	24·37·1	25·44·7	57·02	24·96	+2·27	—·03	+·04	24·26
19	24·68·3	25·88·2	57·18	27·81	— ·62	+·01	+·05	24·43
20	24·69·7	25·91·1	57·28	28·16	—1·12	+·01	+·05	24·38
21	25·08·9	26·31·7	57·38	28·49	— ·97	+·01	+·05	24·96
23	23·81·6	25·00·8	57·60	27·65	— ·72	+·01	+·05	24·59
24	23·75·4	24·90·9	57·70	26·79	+ ·17	+·01	+·05	24·72
25	24·16·7	25·38·0	57·80	28·14	—1·57	+·01	+·05	24·43
29	24·34·0	25·47·3	57·98	26·28	+ ·04	+·01	+·05	24·36
May 2	24·60·9	25·81·0	58·13	27·86	—1·20	+·01	+·05	24·85
4	24·63·4	25·86·6	58·31	28·58	—2·49	+·01	+·05	24·46
5	24·79·5	25·98·0	58·42	27·49	—1·63	+·01	+·05	24·34
6	23·48·0	24·58·9	58·53	25·73	+ ·24	+·01	+·05	24·56
9	24·60·8	25·71·9	58·80	25·77	·00	+·01	+·05	24·63
10	24·77·7	25·89·7	58·86	25·98	— ·40	+·01	+·05	24·50
11	25·20·7	26·35·1	58·90	26·54	— ·31	+·01	+·07	25·21
15	24·48·0	25·65·2	59·03	27·19	— ·40	+·01	+·01	25·84

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP VIII, PAIR 2.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1892.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Feb. 3	7·10·3	28·86·5	21 07 58·23	+8 24·84	+ ·91	+ ·15	+ ·05	21 16 24·18
8	7·10·2	28·93·7	58·02	26·54	+ ·13	+ ·15	+ ·05	24·89
12	6·92·7	28·80·0	58·03	27·42	—1·47	+ ·15	+ ·05	24·18
21	7·09·2	28·92·9	58·26	26·58	— ·49	+ ·19	+ ·04	24·58
23	6·81·8	28·65·7	58·28	26·63	— ·76	+ ·15	+ ·05	24·35
27	7·17·2	28·94·6	58·50	25·12	+ ·50	+ ·15	+ ·05	24·32
28	7·11·3	28·96·4	58·57	26·91	— ·92	+ ·15	+ ·05	24·76
29	7·24·1	29·15·1	58·65	28·28	—1·99	+ ·15	+ ·05	25·14
Mar. 2	7·14·2	29·01·8	58·76	27·49	—1·66	+ ·15	+ ·05	24·79
5	7·18·5	29·10·1	58·83	28·42	—2·65	+ ·15	+ ·05	24·80
8	6·94·9	28·79·1	58·94	26·70	—1·04	+ ·15	+ ·05	24·80
9	6·72·2	28·53·5	59·02	26·03	— ·67	+ ·15	+ ·05	24·58
10	7·09·7	28·94·7	59·09	26·89	—1·50	+ ·15	+ ·05	24·68
13	7·69·0	28·47·2	59·42	25·31	— ·74	+ ·04	+ ·08	24·11
14	7·65·5	29·47·2	59·52	26·12	—1·16	+ ·15	+ ·05	24·68
23	6·81.2	28·58·6	00·10	25·12	—1·36	+ ·15	+ ·05	24·06
24	6·94·5	28·76·4	00·20	26·17	—1·71	+ ·15	+ ·05	24·86
29	7·00·1	28·71·6	00·74	23·75	— ·11	+ ·15	+ ·05	24·58
30	7·00·6	28·71·3	00·80	23·57	— ·11	+ ·15	+ ·05	24·46
31	7·39·5	29·11·7	00·87	23·92	— ·33	+ ·15	+ ·05	24·66
Apr. 6	8·50·9	29·23·7	01·28	24·06	— ·78	+ ·15	+ ·07	24·78
9	7·23·1	28·97·2	01·66	24·36	—1·43	+ ·15	+ ·05	24·79
14	7·11·8	28·84·9	02·14	24·12	—1·68	+ ·15	+ ·05	24·78
17	7·03·1	28·77·4	02·32	24·40	—2·48	+ ·15	+ ·05	24·44
19	7·61·8	29·31·7	02·48	23·38	—1·15	+ ·15	+ ·05	24·91
23	7·40·0	29·05·8	02·92	22·43	—1·00	+ ·15	+ ·05	24·55
24	7·27·7	28·91·8	03·02	22·04	— ·78	+ ·15	+ ·05	24·48
25	7·29·7	28·97·6	03·12	22·92	—1·38	+ ·15	+ ·05	24·86
29	7·32·9	28·96·3	03·34	21·88	— ·88	+ ·15	+ ·05	24·54
May 2	7·50·6	29·12·4	03·50	21·50	— ·18	+ ·15	+ ·05	25·02
4	7·48·1	29·20·0	03·70	23·85	—3·37	+ ·15	+ ·05	24·38
5	7·28·2	28·93·0	03·80	22·20	—1·64	+ ·15	+ ·05	24·56
6	7·67·0	29·27·1	03·92	21·11	— ·45	+ ·15	+ ·05	24·78
9	7·12·9	28·71·0	04·22	20·65	— ·20	+ ·15	+ ·05	24·87
10	7·53·6	29·11·3	04·28	20·55	— ·46	+ ·15	+ ·05	24·57
11	7·25·8	28·80·9	04·34	19·95	+ ·04	+ ·15	+ ·05	24·53
14	7·53·2	29·13·2	04·46	21·09	— ·73	+ ·15	+ ·05	25·02
15	7·40·8	28·96·2	04·50	20·02	— ·22	+ ·15	+ ·05	24·50
18	7·55·2	29·09·5	04·70	19·77	+ ·17	+ ·15	+ ·05	24·84

GROUP VIII, PAIR 3.

Date.	Micrometer.		Mean app. Dec.			Corrections.				Latitude.		
1892.	*N.* *t. d.*	*S.* *t. d.*	°	′	″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	°	′	″
Feb. 3	20·54·9	14·97·5	21	18	33·06	−2 09·31	+1·19	−·04	+·05	21	16	24·95
12	21·64·4	16·14·2			32·82	07·64	−·19	−·04	+·05			25·00
21	21·70·4	16·19·7			33·04	07·75	00	−·04	+·05			25·30
23	21·67·7	16·15·3			33·04	08·15	−·13	−·04	+·05			24·77
27	21·64·1	16·04·2			33·24	09·89	+1·50	−·04	+·05			24·86
28	22·25·4	16·75·5			33·32	07·57	−·47	−·04	+·05			25·29
29	22·26·7	16·83·0			33·39	06·13	−1·73	·00	+·06			25·59
Mar. 2	21·42·2	15·90·7			33·50	07·94	−·60	−·04	+·05			24·97
5	21·55·4	16·09·8			33·57	06·57	−1·44	−·04	+·05			25·57
8	21·62·5	16·10·1			33·66	08·15	−·16	−·04	+·05			25·36
9	21·76·9	16·23·2			33·74	08·45	+·13	−·00	+·06			25·48
10	21·46·0	15·94·4			33·82	07·96	−·49	−·04	+·05			25·38
13	22·21·7	16·64·5			34·14	09·26	+·16	−·04	+·06			25·06
23	21·88·4	16·34·5			34·81	08·50	−1·06	−·04	+·05			25·26
24	21·91·9	16·36·9			34·91	08·75	−1·08	−·04	+·05			25·09
29	22·06·1	16·45·0			35·44	10·17	+·19	−·04	+·05			25·47
30	21·53·2	15·91·5			35·51	10·31	−·11	−·01	+·06			25·14
31	22·24·5	16·65·0			35·58	09·80	−·24	−·04	+·05			25·55
Apr. 9	21·64·6	16·01·4			36·36	10·65	−·35	−·06	+·04			25·34
14	21·50·6	15·85·6			36·85	11·07	−·42	−·04	+·05			25·37
17	22·28·1	16·55·0			37·02	12·95	+1·15	−·04	+·05			25·23
18	21·50·1	15·77·0			37·10	12·95	+·75	−·01	+·06			24·85
19	22·13·3	16·45·2			37·18	11·79	−·06	−·04	+·05			25·34
20	22·85·5	17·19·2			37·28	11·37	−·18	−·12	+·07			25·68
21	21·71·0	16·01·5			37·39	12·12	−·26	−·09	+·09			25·01
23	21·57·4	15·87·7			37·62	12·16	−·35	−·04	+·05			25·12
24	21·74·8	16·01·9			27·73	12·90	+·08	−·04	+·05			24·92
25	22·05·6	16·37·6			37·83	11·77	−·89	−·04	+·05			25·18
29	21·82·7	16·07·0			38·06	13·55	+·21	−·04	+·00			24·68
May 1	22·03·1	16·30·6			38·15	12·81	−·15	−·04	+·05			25·20
2	22·24·8	16·45·7			38·22	14·34	+·59	−·01	+·06			24·52
4	21·63·2	16·99·4			38·40	10·79	−2·26	−·04	+·05			25·36
5	22·22·4	16·52·5			38·52	12·21	−·97	−·04	+·05			25·35
6	22·20·3	16·45·3			38·63	13·39	+·17	−·04	+·05			25·42
10	22·26·0	16·48·2			39·01	14·04	+·25	−·04	+·05			25·23
11	21·50·2	15·71·0			39·06	14·36	+·17	−·02	+·05			24·90
14	21·85·6	16·05·9			39·18	14·48	+·26	−·04	+·05			24·97
15	22·08·0	16·29·5			39·22	14·20	+·51	−·04	+·05			25·54
18	22·82·7	16·98·9			39·42	15·43	+1·44	−·04	+·05			25·44

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP VIII, PAIR 4.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1892.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Feb. 3	8·23·3	24·88·0	21 09 56·68	+6 26·18	+1·50	+·12	+·05	21 16 24·53
8	9·20·9	25·90·3	56·43	27·28	+ ·84	+·12	+·05	24·72
12	8·66·8	25·37·3	56·36	27·53	+ ·31	+·12	+·05	24·37
21	8·47·6	25·21·9	56·53	28·41	— ·30	+·12	+·05	24·81
27	8·38·8	25·04·8	56·70	26·49	+1·67	+·12	+·05	25·03
28	8·91·6	25·62·4	56·76	27·60	+ ·17	+·12	+·05	24·70
29	8·51·6	25·28·7	56·84	29·06	—1·41	+·12	+·05	24·66
Mar. 2	7·95·5	24·66·9	56·94	27·74	— ·46	+·12	+·05	24·39
5	8·59·5	25·36·9	57·00	29·13	—1·26	+·12	+·05	25·04
8	9·53·7	26·29·5	57·08	28·76	— ·95	+·12	+·05	25·06
9	8·31·8	25·05·0	57·13	28·16	— ·32	+·12	+·05	25·14
10	8·98·8	25·69·1	57·22	27·48	— ·28	+·12	+·05	24·59
13	9·29·7	25·96·0	57·54	26·56	+ ·10	+·12	+·05	24·37
21	8·41·1	25·18·7	57·57	29·18	—2·11	+·08	+·05	24·77
23	8·16·0	24·86·9	58·21	27·62	—1·20	+·12	+·05	24·80
24	8·01·5	24·70·5	58·31	27·18	— ·90	+·12	+·05	24·76
29	8·18·4	24·80·1	58·86	25·49	+ ·40	+·12	+·05	24·92
30	8·09·8	24·73·7	58·93	26·00	— ·19	+·12	+·05	24·91
31	9·91·1	26·50·5	59·00	24·96	+ ·67	+·12	+·05	24·80
Apr. 6	8·49·5	25·10·7	59·42	25·37	— ·25	+·06	+·08	24·68
9	9·17·0	25·79·0	59·80	25·56	— ·48	+·12	+·05	25·05
13	8·55.8	25·11·5	10 00·26	24·10	+ ·12	+·12	+·05	24·65
14	8·39·7	24·98·8	00·33	24·89	— ·31	+·12	+·05	25·08
17	8·74·9	25·29·6	00·52	23·86	+ ·10	+·12	+·05	24·65
18	8·83·1	25·38·1	00·60	23·93	+ ·33	+·12	+·05	25·03
19	8·92·3	25·46·6	00·68	23·77	+ ·19	+·12	+·05	24·81
20	9·45·4	26·03·9	00·78	24·75	— ·40	+·12	+·05	25·30
23	8·97·6	25·50·7	01·15	23·49	— ·19	+·12	+·05	24·62
24	8·29·8	24·81·0	01·27	23·05	— ·05	+·12	+·05	24·44
25	8·50·8	25·05·5	01·37	23·98	— ·74	+·12	+·05	24·78
May 1	9·18·6	25·70·2	01·74	23·15	— ·12	+·12	+·05	24·94
2	8·94·4	25·42·2	01·81	22·26	+ ·70	+·12	+·05	24·94
4	9·00·3	25·46·5	02·00	21·89	+ ·60	+·12	+·05	24·66
5	9·48·9	26·03·9	02·11	23·93	—1·11	+·14	+·05	25·12
6	9·20·6	25·65·6	02·24	21·61	+ ·84	+·12	+·05	24·86
7	8·50·5	25·01·0	02·37	22·89	— ·37	+·12	+·05	25·06
9	8·27·7	24·73·7	02·58	21·85	— ·02	+·10	+·05	24·56
11	9·01·0	25·45·5	02·73	21·50	+ ·39	+·12	+·05	24·79
14	8·38·7	24·84·0	02·86	21·68	— ·18	+·12	+·05	24·53
15	9·05·7	25·49·7	02·90	21·38	+ ·07	+·12	+·05	24·52
18	8·50·9	24·89·6	03·13	20·15	+1·35	+·12	+·05	24·80
22	8·57·1	24·97·7	03·50	20·59	+ ·76	+·12	+·05	25·02
23	9·25·1	25·74·0	03·58	22·52	— ·69	+·12	+·05	25·58

GROUP VIII, PAIR 5.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1892.	*N.* *t.* *d.*	*S.* *t.* *d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Feb. 3	23·34·2	13·70·3	21 20 07·75	—3 43·61	+ ·96	—·07	+·05	21 16 25·08
8	23·86·3	14·18·0	07·46	44·63	+1·71	—·07	+·05	24·52
12	23·58·1	13·93·9	07·37	43·68	+1·02	—·07	+·05	24·69
21	23·84·0	14·23·7	07·51	42·78	+ ·14	—·09	+·05	24·83
23	23·57·8	13·97·6	07·48	42·75	— ·29	—·05	+·05	24·44
27	23·78·2	14·11·1	07·64	44·35	+1·49	—·07	+·05	24·76
28	23·66·0	14·10.5	07·70	41·66	—0·96	—·07	+·05	25·06
29	23·67·3	14·11·7	07·78	41·68	—1·13	—·15	+·08	24·90
Mar. 2	23·38·3	13·81·3	07·89	42·01	— ·75	—·07	+·05	25·11
5	24·05·4	14·48·1	07·96	42·08	—1·04	—·07	+·05	24·82
9	22·68·4	13·05·9	08·08	43·29	+ ·19	—·07	+·05	24·96
10	24·57·5	14·94·6	08·17	43·38	— ·41	—·07	+·05	24·36
13	23·52·8	13·91·1	08·49	43·10	— ·24	—·09	+·05	25·11
21	23·36·0	13·81·6	09·05	41·41	—2·38	—·11	+·04	25·19
23	23·68·9	14·08·2	09·20	42·87	—1·39	—·07	+·05	24·92
24	23·82·3	14·22·1	09·29	42·75	1·75	—·07	+·05	24·77
29	24·33·7	13·59·3	09·86	46·05	+ ·62	—·07	+·05	24·41
30	23·80·8	14·06·6	09·95	46·00	+ ·43	—·07	+·05	24·36
31	23·31·0	13·60·0	10·03	45·26	+ ·45	—·07	+·05	25·20
Apr. 6	23·44·6	14·72·4	10·47	45·53	— ·11	—·11	+·04	24·76
9	23·60·4	13·89·0	10·87	45·35	— ·31	—·09	+·05	25·17
14	23·87·1	14·10·1	11·45	46·65	— ·32	—·07	+·05	24·46
15	23·36·0	13·59·7	11·52	46·49	+ ·04	—·12	+·09	25·04
16	23·31·7	13·57·2	11·58	46·07	— ·70	—·07	+·05	24·79
18	23·06·2	13·27·0	11·74	47·16	+ ·65	—·15	+·08	25·16
19	23·75·5	13·95·4	11·83	47·37	— ·06	—·07	+·05	24·38
23	23·70·2	13·93·9	12·32	46·49	— ·27	—·07	+·05	25·54
24	23·35·2	13·54·0	12·46	47·62	— ·07	—·07	+·05	24·75
25	23·46·7	13·68·5	12·56	46·93	— ·60	—·07	+·05	25·01
May 2	23·56·9	13·71·8	13·06	48·53	+ ·15	—·07	+·05	24·66
4	23·48·2	13·54·4	13·26	50·55	+1·99	—·07	+·05	24·68
5	23·83·9	14·04·2	13·39	47·27	—1·24	—·07	+·05	24·86
9	23·71·6	13·80·3	14·07	51·08	+1·24	—·07	+·05	24·21
14	23·58·4	13·66·5	14·23	50·11	+ ·32	—·04	+·06	24·46
18	23·36·6	13·39·8	14·53	51·24	+1·59	—·07	+·05	24·86
22	23·59·0	13·70·2	14·94	49·39	—0·97	—·07	+·05	24·56
23	23·33·4	13·46·7	15·02	48·90	—1·07	—·07	+·05	25·03

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP VIII, PAIR 6.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1892.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Feb. 3	22·78·3	14·19·6	21 19 43·28	−3 19·20	— ·19	—·07	+·06	21 16 23·88
8	22·44·8	13·82·2	42·98	20·11	+1·02	—·07	+·06	23·88
12	22·08·5	13·57·2	42·86	17·49	—1·14	—·07	+·06	24·22
23	22·35·2	13·85·1	42·95	17·21	—1·57	—·07	+·06	24·16
27	22·59·6	13·98·9	43·08	19·67	+ ·55	—·07	+·06	23·95
28	22·35·3	13·86·5	43·14	16·91	—1·66	—·07	+·06	24·56
Mar. 2	22·23·5	13·72·3	43·31	17·46	—1·28	—·07	+·06	24·56
5	22·40·5	13·93·9	43·37	16·40	—1·94	—·07	+·06	25·02
8	22·32·1	13·81·2	43·44	17·40	—1·29	+·01	+·06	24·82
9	21·93·9	13·42·2	43·47	17·58	—1·68	—·07	+·06	24·20
10	22·08·4	13·55·0	43·57	17·98	— ·84	—·07	+·06	24·74
13	22·50·7	13·94·6	43·87	18·60	—1·26	—·07	+·06	24·00
21	22·46·0	13·92·0	44·41	18·11	—1·88	—·05	+·06	24·43
23	22·76·1	14·24·2	44·55	17·63	—2·38	—·07	+·06	24·53
24	22·23·8	13·69·5	44·64	18·18	—1·73	—·07	+·06	24·72
29	22·42·3	13·78·0	45·20	20·50	+ ·09	—·05	+·06	24·80
30	22·27·5	13·63·4	45·29	20·46	— ·46	—·07	+·06	24·36
31	22·31·0	13·69·0	45·36	19·97	— ·90	—·07	+·06	24·46
Apr. 9	22·57·7	13·95·2	46·18	20·09	—1·37	—·07	+·06	24·71
15	22·22·2	13·54·8	46·83	21·22	—1·42	—·15	+·10	24·14
16	22·68·2	14·01·6	46·89	21·04	—1·90	—·07	+·06	23·94
19	23·06·4	14·35·8	47·13	21·97	— ·81	—·07	+·06	24·34
21	21·84·6	13·17·1	47·35	21·25	—1·56	—·07	+·06	24·53
23	22·37·8	13·67·3	47·61	21·94	— ·96	—·07	+·06	24·70
24	23·04·3	14·31·0	47·75	22·59	— ·77	—·07	+·06	24·38
25	22·57·8	13·87·9	47·85	21·80	—1·40	—·07	+·06	24·64
May 2	22·93·6	14·17·3	48·35	23·29	— ·63	—·07	+·06	24·42
5	21·97·8	13·25·0	48·67	22·48	—2·06	—·07	+·06	24·12
9	22·65·6	13·86·2	49·18	24·01	— ·26	—·04	+·06	24·93
14	22·05·3	13·24·3	49·52	24·38	—1·02	—·07	+·06	24·11
15	22·55·4	13·76·2	49·57	23·96	— ·74	—·15	+·10	24·82
18	22·33·7	13·44·5	49·82	26·28	+ ·44	—·07	+·06	23·97
22	22·91·0	14·09·2	50·23	24·56	—1·23	—·07	+·06	24·43
23	22·01·9	13·24·1	50·31	23·64	—2·30	—·07	+·06	24·36

GROUP VIII, PAIR 7.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1892.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Feb. 3	23·05·0	14·03·8	21 19 52·77	−3 29·06	+ ·90	−·06	+·06	21 16 24·61
8	22·22·2	13·22·7	52·43	28·67	+ ·39	−·06	+·06	24·15
12	22·62·8	13·70·8	52·28	26·93	−1·05	−·06	+·06	24·30
21	21·86·2	12·93·5	52·37	27·09	−1·21	−·06	+·06	24·07
23	22·33·1	13·42·5	52·33	26·61	−1·36	−·06	+·06	24·36
27	22·37·5	13·37·5	52·45	28·79	+ ·57	−·06	+·06	24·23
28	21·97·0	13·05·4	52·50	26·84	−1·14	−·06	+·06	24·52
29	22·16·2	13·29·4	52·57	25·72	−2·11	−·06	+·06	24·74
Mar. 2	22·29·9	13·42·9	52·68	25·77	−1·77	−·06	+·06	25·14
5	21·85·2	13·01·9	52·76	24·91	−2·52	−·06	+·06	25·33
8	21·82·1	12·82·1	52·83	28·79	+ ·06	−·04	+·06	24·12
9	22·57·6	13·67·3	52·87	26·54	−1·69	−·06	+·06	24·64
10	22·64·9	13·75·0	52·94	26·44	−1·38	−·06	+·06	25·12
13	22·91·7	13·97·5	53·25	27·44	−1·50	−·06	+·06	24·31
21	22·25·7	13·33·3	53·85	27·02	−2·08	−·06	+·06	24·75
23	22·06·1	13·13·4	54·00	27·09	−2·23	−·06	+·06	24·68
24	22·24·2	13·30·0	54·10	27·44	−2·20	−·06	+·06	24·46
29	23·09·5	14·05·5	54·70	29·71	− ·19	−·06	+·06	24·80
30	22·98·1	13·93·7	54·80	29·81	− ·37	−·06	+·06	24·62
31	22·79·1	13·74·4	54·88	29·88	− ·62	−·06	+·06	24·38
Apr. 6	22·72·4	13·68·2	55·36	29·76	−1·04	−·10	+·04	24·50
9	23·23·7	14·21·9	55·78	29·20	−1·82	−·06	+·06	24·76
16	22·93·0	13·86·1	56·59	30·39	−2·08	−·10	+·04	24·06
17	22·99·2	13·82·6	56·68	32·64	− ·14	−·06	+·06	23·90
18	22·89·7	13·78·7	56·76	31·34	− ·78	−·06	+·06	24·64
19	22·76·6	13·66·6	56·86	31·11	−1·17	−·06	+·06	24·58
21	22·66·6	13·55·2	57·10	31·43	−1·18	−·03	+·06	24·52
24	22·53·1	13·39·3	57·53	31·99	− ·54	−·06	+·06	25·00
May 1	22·60·7	13·44·2	58·18	32·61	−1·03	−·06	+·06	24·54
2	22·85·7	13·68·3	58·27	32·82	− ·58	−·06	+·06	24·87
4	22·76·7	13·47·5	58·49	35·56	+ ·65	−·06	+·06	23·58
5	22·73·1	13·60·1	58·62	31·80	−2·02	−·06	+·06	24·80
10	22·93·0	13·70·1	59·31	34·10	− ·63	−·06	+·06	24·58
15	22·72·5	13·47·5	59·68	34·59	− ·91	−·06	+·06	24·18
18	22·39·5	13·08·3	59·96	36·02	+ ·66	−·06	+·06	24·60
22	22·13·4	12·89·5	20 00 00·43	34·33	−1·37	−·06	+·06	24·73
23	22·17·0	12·95·9	00·53	33·68	−2·34	−·06	+·06	24·51

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP VIII, PAIR 8.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1892.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Feb. 3	24·63·1	11·53·3	21 21 27·66	−5 03·85	+1·07	−·08	+·05	21 16 24·85
8	24·72·5	11·61·5	27·29	04·13	+1·10	−·08	+·05	24·23
12	24·69·8	11·62·6	27·12	03·25	+ ·60	−·08	+·05	24·44
21	23·44·8	10·41·1	27·17	02·44	− ·29	−·08	+·05	24·41
23	24·32·4	11·28·3	27·13	02·53	− ·31	−·08	+·05	24·26
27	25·38·1	11·30·0	27·23	03·46	+1·44	−·06	+·04	25·19
28	24·06·2	11·02·8	27·28	02·37	− ·24	−·08	+·05	24·64
29	23·85·7	10·85·0	27·35	01·74	− ·83	−·08	+·05	24·76
Mar. 2	23·93·9	10·92·1	27·45	02·00	−1·08	−·08	+·05	24·34
5	23·94·3	11·00·4	27·53	00·17	−2·23	−·08	+·05	25·10
8	24·05·5	11·01·8	27·60	02·44	− ·45	−·08	+·05	24·68
9	24·30·5	11·29·5	27·65	01·81	−1·22	−·08	+·05	24·59
10	23·85·5	10·80·7	27·72	02·69	− ·94	−·08	+·05	24·06
21	24·03·5	11·01·1	28·67	02·14	−1·96	−·08	+·05	24·54
23	24·11·5	11·08·6	28·82	02·25	−1·52	−·08	+·05	25·02
24	24·38·1	11·31·5	28·91	03·11	−1·34	−·08	+·05	24·43
29	24·03·6	10·86·9	29·55	05·45	+ ·03	−·08	+·05	24·10
30	24·32·4	11·22·5	29·65	03·88	− ·71	−·08	+·05	25·03
31	24·63.1	11·47·2	29·74	05·27	+ ·24	−·08	+·05	24·68
Apr. 6	23·80·0	10·63·8	30·26	05·34	− ·05	−·08	+·05	24·84
9	24·18·9	11·01·5	30·70	05·62	− ·45	−·08	+·05	24·60
13	23·52·0	10·29·7	31·29	06·75	− ·32	−·08	+·05	24·19
14	24·60·5	11·41·7	31·40	05·94	− ·59	−·08	+·05	24·84
16	24·36·1	11·15·2	31·58	06·43	− ·80	−·08	+·05	24·32
18	23·93·5	10·67·3	31·77	07·66	+ ·85	−·08	+·03	24·91
19	24·21·8	10·98·6	31·89	06·96	− ·22	−·08	+·05	24·68
20	23·95·1	10·72·1	32·00	06·92	− ·79	−·05	+·06	24·30
21	24·03·3	10·82·4	32·13	06·43	− ·71	−·08	+·05	24·96
24	23·73·3	10·46·8	32·59	07·73	− ·24	−·08	+·05	24·59
25	23·95·5	10·71·0	32·73	07·26	− ·42	−·08	+·03	25·00
May 1	24·48·1	11·20·2	33·33	08·05	− ·69	−·10	+·04	24·53
2	24·22·1	10·92·5	33·42	08·45	− ·24	−·08	+·05	24·70
4	24·33·0	10·93·8	33·67	10·67	+1·57	−·08	+·05	24·54
5	23·97·7	10·69·2	33·81	8·19	−1·09	−·08	+·05	24·50
9	23·74·4	10·40·8	34·43	09·37	− ·62	−·08	+·05	24·40
10	23·95·0	10·60·0	34·56	9·70	·00	−·08	+·05	24·83
14	24·00·0	10·64·2	34·92	9·89	− ·13	−·12	+·04	24·82
22	24·31·8	10·93·5	35·82	10·47	− ·92	−·08	+·05	24·40
23	23·74·2	10·40·8	35·94	9·33	−2·26	−·08	+·05	24·32

GROUP I, PAIR 1.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1892.	*N.* t. d.	*S.* t. d.	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Apr. 9	10·52·2	14·54·8	21 14 52·41	+1 33·40	— ·61	+·05	+·07	21 16 25·32
19	13·76·8	17·75·0	53·59	32·38	—1·19	+·03	+·07	24·88
24	13·62·2	17·56·9	54·28	31·56	— ·82	+·03	+·07	25·12
25	13·88·2	17·86·3	54·43	32·35	—1·49	+·01	+·04	25·34
May 1	13·70·5	17·65·9	55·11	31·73	—1·86	+·03	+·07	25·08
4	14·28·5	18·04·4	55·44	27·20	+1·48	+·03	+·07	24·22
5	14·17·2	18·08·7	55·58	30·82	—1·43	+·03	+·07	25·07
9	13·60·7	17·42·1	56·24	28·48	— ·26	+·03	+·07	24·56
10	13·69·8	17·54·0	56·40	29·13	— ·79	+·03	+·07	24·84
13	13·25·0	17·03·8	56·76	27·88	— ·10	+·03	+·07	24·64
14	13·92·5	17·74·6	56·86	28·64	—1·08	+·03	+·07	24·52
15	15·09·0	18·89·2	56·95	28·20	— ·38	+·03	+·07	24·87
18	13·60·9	17·42·3	57·26	28·48	— ·54	+·03	+·07	25·30
21	13·42·1	17·23·8	57·70	28·55	—1·49	+·03	+·07	24·86
22	14·01·3	17·85·7	57·84	29·18	—2·09	+·03	+·07	25·03
24	14·45·9	18·15·4	58·11	25·72	+ ·85	+·03	+·07	24·78
25	13·69·1	17·48·1	58·23	27·92	—1·55	+·03	+·07	24·70
26	13·65·3	17·39·5	58·32	26·81	— ·84	—·01	+·05	24·34
28	14·43·6	18·17·5	58·48	26·74	— ·68	+·03	+·07	24·64
29	15·18·2	19·03·6	58·56	29·41	—2·97	+·03	+·07	25·10
June 3	13·82·5	17·41·7	59·14	23·33	+1·95	+·03	+·07	24·52
4	14·60·5	18·40·3	59·28	28·11	—2·59	+·03	+·07	24·90
5	15·07·4	18·80·5	59·42	26·55	—1·25	+·03	+·07	24·82
7	14·36·3	17·95·9	59·68	23·42	+1·22	+·03	+·07	24·42
11	14·44·8	18·06·9	59·98	24·00	+ ·82	+·03	+·07	24·90
13	13·89·2	17·51·0	15 00·10	23·93	— ·06	+·03	+·07	24·07
17	14·57·0	18·16·7	00·49	23·44	+ ·63	+·01	+·05	24·62
18	14·40·0	18·09·0	00·60	25·60	—1·12	+·03	+·07	25·18
19	13·81·9	17·57·8	00·71	27·20	—2·87	+·03	+·07	25·14
22	14·25·4	17·81·4	00·94	22·59	+1·02	+·03	+·07	24·65
24	13·51·3	17·11·6	01·02	23·58	— ·18	+·03	+·07	24·52
25	15·07·1	18·71·3	01·04	24·49	— ·89	+·03	+·07	24·74

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP I, PAIR 2.

Date.		Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1892.		*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Apr.	9	13·74·3	15·75·3	21 17 09·70	−0 46·65	+1·23	−·02	+·05	21 16 24·31
	16	14·58·5	16·65·6	10·76	48·04	+1·92	−·02	+·05	24·67
	19	14·38·3	16·43·4	11·10	47·58	+ ·63	−·02	+·05	24·18
	23	14·76·8	16·79·3	11·72	46·98	− ·13	−·02	+·05	24·64
	24	14·16·3	16·22·6	11·90	47·86	+ ·66	−·02	+·05	24·73
	25	13·80·1	16·84·1	12·08	47·32	− ·21	−·02	+·05	24·58
May	4	14·31·3	16·56·6	13·34	52·27	+3·42	−·02	+·05	24·52
	5	14·64·5	16·76·1	13·51	49·09	− ·13	−·02	+·05	24·32
	9	14·27·3	16·49·8	14·28	51·62	+1·19	−·02	+·05	23·88
	10	14·34·6	16·54·8	14·47	51·08	+ ·68	−·02	+·05	24·10
	13	14·13·7	16·38·8	14·92	52·22	+2·06	−·02	+·05	24·79
	14	14·46·5	16·68·1	15·04	51·41	+ ·90	−·02	+·05	24·56
	15	13·99·1	16·23·7	15·16	52·10	+1·23	−·02	+·05	24·32
	18	13·91·5	16·17·7	15·57	52·48	+1·52	−·02	+·05	24·64
	21	14·39·9	16·62·1	16·08	51·55	+ ·16	−·02	+·05	24·72
	22	15·68·3	17·87·1	16·26	50·76	− ·73	−·02	+·05	24·80
	23	14·16·2	16·34·7	16·44	50·69	−1·21	−·02	+·05	24·57
	24	13·94·4	16·30·3	16·59	54·72	+2·13	−·02	+·05	24·03
	25	14·42·9	16·75·0	16·74	53·84	+1·54	−·02	+·05	24·47
	26	13·06·0	15·36·2	16·86	53·40	+1·28	−·10	+·07	24·71
	28	14·40·3	16·73·7	17·09	54·14	+1·14	−·02	+·05	24·12
	29	13·87·8	16·23·3	17·20	54·63	+2·16	−·02	+·05	24·76
	30	13·76·6	16·12·0	17·31	54·61	+1·47	+·02	+·04	24·23
June	3	14·03·6	16·43·9	17·90	55·75	+2·81	−·02	+·05	24·99
	4	14·09·1	16·38·4	18·08	52·36	−1·12	−·00	+·04	24·64
	5	14·32·2	16·66·0	18·25	54·24	+ ·10	−·02	+·05	24·14
	6	14·54·4	16·89·4	18·41	54·52	+ ·50	−·02	+·05	24·42
	7	14·14·3	16·66·6	18·56	58·53	+3·62	−·02	+·05	23·68
	11	14·59·9	17·05·8	18·98	57·04	+2·14	−·02	+·04	24·10
	13	14·37·9	16·80·1	19·16	56·19	+1·50	−·02	+·05	24·50
	14	14·50·2	16·90·7	19·26	55·79	+ ·76	−·02	+·05	24·26
	17	13·83·4	16·33·8	19·65	58·09	+2·86	−·02	+·05	24·45
	18	13·89·6	16·27·7	19·80	55·24	+ ·12	−·02	+·05	24·71
	22	14·83·0	17·34·4	20·24	58·32	+2·33	−·02	+·05	24·28
	24	13·79·5	16·36·9	20·38	59·71	+2·92	−·02	+·05	23·62
	25	14·69·8	17·13·5	20·43	56·53	+ ·36	−·02	+·05	24·29

GROUP I, PAIR 3.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1892.	*N.* *t.* *d.*	*S.* *t.* *d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Apr. 9	19·26·6	15·45·3	21 17 52·96	—1 28·46	+ ·29	—·03	+·06	21 16 24·82
16	20·70·1	17·81·1	53·97	30·24	+ ·99	—·03	+·06	24·75
19	21·01·7	17·11·9	54·27	30·43	+ ·78	—·03	+·06	24·65
23	20·51·8	16·61·7	54·83	30·50	+ ·34	—·03	+·06	24·70
24	20·52·8	16·59·1	55·00	31·33	+ ·86	—·03	+·06	24·56
25	21·10·7	17·17·7	55·18	31·17	+ ·42	—·03	+·06	24·46
May 1	21·66·6	17·69·1	56·00	32·21	— ·23	—·23	+·29	23·62
4	21·07·1	16·97·4	56·37	35·04	+3·52	—·03	+·06	24·88
5	20·84·3	16·88·6	56·52	31·80	+ ·11	—·03	+·06	24·86
9	21·39·7	17·38·6	57·26	33·05	+ ·14	—·03	+·06	24·38
13	20·89·1	16·85·9	57·88	33·54	— ·35	—·03	+·06	24·02
14	21·15·7	17·06·0	58·00	35·04	+1·52	—·03	+·06	24·51
15	20·73·0	16·67·3	58·11	34·12	+ ·19	—·03	+·06	24·21
18	20·78·5	16·69·1	58·48	34·97	+1·26	—·03	+·06	24·80
21	20·79·1	16·76·3	58·96	33·44	— ·88	—·03	+·06	24·67
22	21·13·7	17·12·0	59·14	33·19	— ·90	—·03	+·06	25·08
23	20·75·2	16·72·1	59·31	33·51	—1·41	—·03	+·06	24·42
24	20·89·3	16·82·7	59·46	34·32	— ·42	—·03	+·06	24·75
25	20·51·6	16·37·6	59·62	36·04	+ ·77	—·03	+·06	24·38
26	20·04·9	15·91·2	59·74	35·97	+ ·59	+·01	+·05	24·42
27	21·13·6	17·03·1	59·86	35·23	— ·36	—·03	+·06	24·30
28	20·55·5	16·39·1	59·96	36·60	+1·18	—·03	+·06	24·57
29	20·76·1	16·55·5	18 00·06	37·57	+2·28	—·03	+·06	24·80
June 3	21·68·7	17·44·5	00·72	38·41	+2·06	—·03	+·06	24·40
4	21·01·7	16·92·3	00·89	34·98	—1·49	—·03	+·06	24·45
5	21·64·0	17·54·4	01·06	35·02	—1·15	—·03	+·06	24·92
7	21·20·5	16·90·6	01·38	39·73	+2·61	—·03	+·06	24·29
11	21·80·2	17·53·6	01·79	38·96	+1·46	—·03	+·06	24·32
12	21·18·5	16·96·6	01·88	37·88	+ ·44	—·03	+·06	24·47
13	21·38·4	17·27·8	01·96	37·57	+ ·64	—·03	+·06	25·06
14	21·02·3	16·80·2	02·06	37·92	+ ·38	—·03	+·06	24·55
17	22·02·7	17·71·9	02·42	39·94	+1·97	—·03	+·06	24·48
18	20·99·2	16·81·3	02·56	36·95	— ·84	—·03	+·06	24·80
22	20·55·7	16·22·2	03·02	40·56	+1·30	+·05	+·10	23·91
24	20·73·4	16·46·1	03·16	39·13	+ ·06	—·03	+·06	24·12
25	21·11·3	16·84·0	03·20	39·13	+ ·16	—·03	+·06	24·26

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP 1, PAIR 4.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1892.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Apr. 9	13·99·4	22·18·5	21 13 14·39	+3 10·02	+ ·21	+·05	+·05	21 16 24·72
16	14·19·3	22·27·0	15·50	07·37	+1·71	+·05	+·05	24·68
19	13·83·4	21·98·0	15·85	08·98	— ·13	+·05	+·05	24·80
23	13·58·2	21·69·8	16·48	08·28	— ·18	+·05	+·05	24·68
24	13·81·8	21·91·9	16·66	07·93	+ ·07	+·05	+·05	24·76
May 1	14·03·5	22·11·6	17·80	07·46	— ·32	+·05	+·05	25·04
4	13·44·7	21·53·5	18·22	07·63	—1·26	+·05	+·05	24·69
5	13·73·5	21·78·6	18·45	06·77	— ·50	+·05	+·05	24·82
9	13·95·3	21·93·4	19·22	05·15	— ·18	+·05	+·05	24·29
13	13·85·2	21·77·5	19·94	03·80	+ ·47	+·05	+·05	24·31
14	14·29·0	22·22·7	20·08	04·13	+ ·20	+·05	+·05	24·51
15	13·51·7	21·44·0	20·22	03·80	+ ·18	+·05	+·05	24·30
18	13·88·7	21·77·5	20·66	02·99	+1·01	+·05	+·05	24·76
21	13·42·5	21·39·8	21·22	04·96	—1·28	+·05	+·05	25·00
22	13·99·4	21·91·7	21·41	03·80	—1·16	+·05	+·05	24·15
23	13·75·7	21·71·9	21·61	04·71	—1·56	+·05	+·05	24·86
24	13·50·3	21·39·9	21·79	03·18	— ·49	+·05	+·05	24·58
25	13·47·9	21·35·0	21·97	02·60	+ ·15	+·05	+·05	24·82
26	13·60·8	21·45·7	22·12	02·08	+ ·16	+·05	+·05	24·46
27	13·94·9	21·78·7	22·26	01·83	+ ·52	+·05	+·06	24·72
28	14·32·0	22·17·7	22·39	02·27	— ·04	+·05	+·05	24·72
29	13·69·9	22·48·7	22·52	00·67	+1·13	+·05	+·05	24·42
June 3	14·56·1	22·28·5	23·30	+2 59·18	+1·89	+·05	+·05	24·47
4	14·34·6	22·22·2	23·50	+3 02·71	—1·37	+·05	+·05	24·94
5	14·19·3	22·04·8	23·69	02·22	—1·05	+·05	+·05	24·96
7	14·36·9	22·18·4	24·06	01·30	— ·55	+·05	+·05	24·91
11	14·51·6	22·17·9	24·59	+2 57·77	+1·18	+·05	+·05	23·64
12	14·06·2	21·79·7	24·70	59·44	+ ·28	+·05	+·05	24·52
13	14·69·1	22·42·2	24·81	59·35	+ ·67	+·05	+·05	24·93
14	14·08·8	21·81·0	24·94	59·14	+ ·36	+·05	+·05	24·54
17	14·12·3	21·91·8	25·38	+3 00·83	—1·26	+·05	+·05	25·05
18	13·89·2	21·66·7	25·54	00·37	—1·42	+·05	+·05	24·59
22	14·58·3	22·22·0	26·11	+2 57·17	+1·12	+·05	+·05	24·50
24	13·96·4	21·67·8	26·30	58·95	— ·55	+·05	+·05	24·80
25	14·26·8	21·96·8	26·38	58·63	— ·28	+·05	+·05	24·83

GROUP 1, PAIR 5.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1892.	*N.* t. d.	*S.* t. d.	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
Apr. 9	8·79·8	25·36·1	21 10 00·33	+6 24·24	+ ·72	+·10	+·05	21 16 25·44
16	9·05·7	25·50·3	01·43	21·52	+2·22	+·10	+·05	25·32
19	9·66·6	26·10·8	01·78	21·43	+2·04	+·10	+·05	25·40
23	8·77·0	25·22·9	12·39	21·82	+ ·68	+·10	+·05	25·04
24	9·32·7	25·79·1	02·58	21·94	+ ·61	+·10	+·05	25·28
25	9·23·5	25·69·1	12·77	21·75	+ ·15	+·10	+·05	24·82
May 1	9·24·4	25·66·3	03·71	20·90	+ ·27	+·06	+·04	24·98
4	9·33·4	25·75·9	04·14	21·04	— ·41	+·10	+·05	24·92
5	9·08·9	25·49·8	04·30	20·66	+ ·20	+·06	+·06	25·28
9	8·57·8	24·93·8	05·12	19·53	+ ·26	+·10	+·05	25·06
13	9·11·4	25·41·6	05·85	18·18	+ ·67	+·10	+·05	24·85
14	8·90·4	25·21·6	05·98	18·41	+ ·10	+·10	+·05	24·64
15	8·27·1	24·56·0	06·12	17·88	+ ·48	+·06	+·04	24·58
18	9·38·1	25·62·4	06·56	16·81	+1·44	+·10	+·05	24·96
21	9·09·9	25·21·1	07·09	13·77	+3·89	+·10	+·05	24·90
22	9·42·8	25·76·8	07·30	19·06	—1·47	+·10	+·05	25·04
23	8·54·1	24·90·7	07·52	19·67	—1·98	+·10	+·05	25·36
24	9·63·1	25·89·2	07·70	17·23	+ ·14	+·10	+·05	25·22
25	8·98·9	25·20·8	07·88	16·26	+ ·76	+·10	+·05	25·05
26	9·16·8	25·39·6	08·03	16·47	+ ·31	+·10	+·05	24·96
27	8·78·3	25·00·0	08·18	16·21	— ·03	+·10	+·05	24·51
28	9·09·2	25·32·0	08·30	16·47	+ ·40	+·10	+·05	25·32
29	9·15·8	25·35·0	08·43	15·63	+ ·93	+·10	+·05	25·14
June 3	9·45·0	25·59·7	09·22	14·59	+1·02	+·10	+·05	24·98
4	9·80·2	26·02·9	09·41	16·44	—1·10	+·10	+·05	24·90
5	9·53·3	25·76·0	09·61	16·44	— ·96	+·10	+·05	25·24
7	9·61·5	25·78·3	09·98	15·07	+ ·06	+·10	+·05	25·26
11	9·31·2	25·40·6	10·52	13·36	+1·01	+·10	+·05	25·04
12	9·33·9	25·49·2	10·63	14·73	— ·05	+·10	+·05	25·46
13	9·78·7	25·87·5	10·74	13·22	+ ·74	+·10	+·05	24·85
14	9·12·5	25·24·7	10·86	14·01	·00	+·10	+·05	25·02
17	9·62·1	25·78·3	11·30	14·93	—1·28	+·10	+·05	25·10
18	8·95·6	25·10·4	11·48	14·61	—1·04	+·10	+·05	25·20
22	9·61·7	25·65·9	12·05	12·15	+ ·77	+·10	+·05	25·12
24	9·40·9	25·46·0	12·24	12·36	+ ·01	+·10	+·05	24·76
25	9·23·2	25·30·1	12·33	12·78	— ·06	+·10	+·05	25·20

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP I, PAIR 6.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1892.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
April 9	25·50·6	13·06·4	21 21 12·44	—4 48·64	+ ·77	—·09	+·06	21 16 24·54
19	25·20·9	12·70·7	13·86	50·03	+ ·56	—·09	+·06	24·36
23	26·10·2	13·59·9	14·46	50·05	+ ·29	—·09	+·06	24·67
24	25·33·6	12·77·9	14·64	51·30	+1·10	—·09	+·06	24·41
25	25·33·5	12·83·9	14·84	49·89	— ·07	—·07	+·05	24·86
May 1	25·81·5	13·27·0	15·78	51·02	— ·30	—·13	+·05	24·38
4	26·16·5	13·65·0	16·19	50·33	— ·89	—·09	+·06	24·94
5	25·50·1	12·94·3	16·36	51·33	— ·22	—·09	+·06	24·78
9	25·31·9	12·70·0	17·16	52·74	+ ·15	—·09	+·06	24·54
13	25·53·7	12·85·3	17·90	54·25	+ ·82	—·09	+·06	24·44
14	25·34·1	12·66·4	18·04	54·09	+ ·38	—·09	+·06	24·30
15	25·44·0	12·77·1	18·18	53·90	+ ·38	—·09	+·06	24·63
18	25·66·6	12·94·9	18·61	55·02	+1·12	—·09	+·06	24·68
21	25·60·1	12·94·8	19·16	53·53	— ·77	—·09	+·06	24·83
22	25·56·1	12·91·6	19·36	53·34	—1·33	—·09	+·06	24·66
23	25·15·6	12·51·3	19·56	53·30	—1·49	—·09	+·06	24·74
24	25·13·8	12·43·5	19·75	54·69	— ·49	—·09	+·06	24·54
25	25·76·9	13·02·2	19·93	55·71	+ ·55	—·09	+·06	24·74
26	25·54·7	12·78·6	20·08	56·04	+ ·58	—·09	+·06	24·59
27	24·95·0	12·18·6	20·24	56·11	+ ·13	—·09	+·01	24·18
28	25·42·7	12·67·8	20·36	55·76	·00	—·09	+·06	24·57
29	25·27·6	12·47·4	20·49	56·99	+1·15	—·09	+·06	24·62
June 3	26·03·8	13·25·9	21·28	56·45	— ·14	—·09	+·06	24·66
4	25·78·4	13·03·2	21·48	55·83	— ·93	—·09	+·06	24·69
5	26·06·7	13·29·5	21·68	56·29	— ·65	—·09	+·06	24·71
11	26·74·3	13·84·8	22·61	59·14	+1·04	—·09	+·06	24·48
13	25·81·2	12·93·7	22·84	58·68	+ ·45	—·09	+·06	24·58
14	26·17·0	13·29·7	22·96	58·64	+ ·12	—·09	+·06	24·41
17	25·88·2	13·02·4	23·41	58·29	— ·92	—·05	+·05	24·20
18	25·69·3	12·84·9	23·58	—4 57·96	— ·89	—·09	+·06	24·70
22	25·88·0	12·90·7	24·18	—5 00·96	+1·56	—·09	+·06	24·75
24	25·99·5	13·11·3	24·38	—4 58·84	— ·78	—·09	+·06	24·73
25	25·47·4	12·56·2	24·48	59·54	— ·13	—·09	+·06	24·78

GROUP I, PAIR 7.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1892.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
April 9	15·49·8	18·22·5	21 15 20·28	+1 03·26	+ ·54	+·02	+·06	21 16 24·16
19	14·83·8	17·50·4	21·74	01·85	+ ·85	+·02	+·06	24·52
23	15·27·4	17·91·9	22·36	01·36	+ ·65	+·02	+·06	24·45
24	14·95·5	17·55·7	22·56	00·36	+1·19	+·02	+·06	24·19
25	14·97·8	17·59·1	22·75	00·62	+ ·45	+·02	+·06	23·90
May 1	15·82·0	18·44·8	23·76	00·97	— ·09	+·02	+·06	24·72
4	15·51·7	18·12·4	24·20	00·48	— ·13	+·02	+·06	24·63
5	14·87·1	17·45·7	24·37	59·99	— ·15	+·02	+·06	24·29
9	15·03·0	17·57·3	25·22	58·99	— ·12	+·02	+·06	24·17
13	14·65·6	17·12·4	26·02	57·25	+ ·63	+·02	+·06	23·98
14	14·65·0	17·10·4	26·17	56·93	+ ·92	+·02	+·06	24·10
15	15·42·9	17·90·3	26·32	57·39	+ ·11	+·02	+·06	23·90
18	14·90·2	17·31·7	26·78	56·02	+1·62	+·02	+·06	24·50
21	14·62·1	17·09·7	27·38	57·44	— ·27	+·02	+·06	24·63
22	15·31·7	17·33·1	27·60	58·32	—1·23	+·02	+·06	24·77
23	14·78·9	17·27·2	27·81	57·60	—1·13	+·02	+·06	24·36
24	15·80·4	18·26·3	28·02	57·04	— ·78	+·02	+·06	24·36
25	15·31·2	17·66·0	28·22	54·47	+1·26	+·02	+·06	24·03
26	15·07·0	17·44·8	28·39	55·17	+ ·77	+·02	+·06	24·41
27	15·31·1	17·70·6	28·56	55·56	+ ·14	+·09	+·05	24·40
28	15·28·4	17·63·3	28·71	54·49	+1·10	+·02	+·06	24·38
29	15·19·9	17·53·2	28·86	54·12	+1·28	+·02	+·06	24·34
June 3	15·56·2	17·91·1	29·70	54·49	+ ·31	+·02	+·06	24·58
4	14·99·3	17·33·1	29·92	54·24	— ·10	+·02	+·06	24·14
5	15·83·2	18·16·3	30·14	54·08	— ·07	+·02	+·06	24·23
7	15·30·7	17·60·5	30·56	53·31	+ ·22	+·02	+·06	24·17
11	15·32·3	17·53·3	31·20	51·27	+1·80	+·02	+·06	24·35
13	15·41·5	17·66·2	31·46	52·13	+ ·87	+·02	+·06	24·54
17	15·70·8	17·98·1	32·10	52·73	— ·53	+·02	+·06	24·38
18	15·47·0	17·69·0	32·28	51·50	+ ·18	+·02	+·04	24·02
22	15·27·1	17·38·8	32·98	49·11	+1·85	+·02	+·06	24·02
24	15·30·1	17·44·7	33·22	49·78	+ ·24	+·02	+·06	23·32
25	15·80·5	17·95·4	33·34	49·85	+ ·37	+·02	+·06	23·64

GROUP II, PAIR 1.

1892.	*t. d.*	*t. d.*	° ′ ″	′ ″	″		″	° ′ ″
May 22	7·38·6	28·52·8	21 08 14·84	+8 10·46	— ·63	+·14	+·05	21 16 24·86
25	7·65·5	28·64·9	15·57	07·03	+ ·85	+·14	+·05	23·64
28	8·13·5	29·08·8	16·19	06·08	+1·81	+·14	+·05	24·27
June 7	9·42·4	30·35·9	18·33	05·66	+ ·06	+·14	+·05	24·24
11	8·69·9	29·52·6	19·16	03·16	+1·00	+·14	+·05	23·51
13	7·74·4	28·61·0	19·51	04·06	+ ·64	+·14	+·05	24·40
14	7·32·1	28·18·4	19·68	03·99	+ ·44	+·14	+·05	24·30
18	7·64·0	28·53·7	20·52	04·78	—1·02	+·14	+·05	24·47
23	7·76·4	28·51·7	21·62	01·44	+ ·54	+·14	+·05	23·79
25	7·67·9	28·45·4	21·96	01·95	— ·01	+·14	+·05	24·09

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

GROUP II, PAIR 2.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1892.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
May 22	16·93·3	17·95·0	21 16 00·98	+0 23·59	— ·24	+·01	+·05	21 16 24·39
25	16·51·2	17·41·4	01·74	20·92	+1·41	+·01	+·05	24·13
28	16·77·4	17·62·6	02·38	19·76	+1·94	+·01	+·05	24·14
June 7	17·21·6	18·02·7	04·58	18·81	+ ·68	+·01	+·05	24·13
11	17·12·0	17·86·3	05·44	17·24	+1·32	+·01	+·05	24·06
13	17·19·7	17·94·7	05·80	17·40	+1·19	+·01	+·05	24·45
14	17·77·1	18·50·6	05·99	17·05	+ ·96	+·03	+·05	24·08
17	16·83·9	17·64·0	06·61	18·58	— ·65	+·01	+·03	24·58
18	17·02·0	17·77·6	06·82	17·54	— ·13	+·01	+·05	24·29
25	17·00·5	17·67·0	08·36	15·43	+ ·11	+·01	+·05	23·96

GROUP II, PAIR 3.

1892.								
May 22	27·87·9	9·11·9	21 23 41·53	—7 15·20	— ·88	—·13	+·05	21 16 25·37
25	27·12·3	8·20·1	42·29	18·96	+1·99	—·13	+·05	25·24
28	27·34·6	8·44·5	42·96	18·48	+ ·78	—·13	+·05	25·18
June 7	27·24·7	8·30·4	45·17	19·45	— ·32	—·13	+·05	25·32
11	27·51·1	8·44·8	46·08	22·23	+ ·85	—·13	+·05	24·62
13	27·49·5	8·48·0	46·44	21·12	+ ·58	—·13	+·05	25·82
18	27·79·5	8·76·5	47·50	21·47	—0·56	—·13	+·05	25·39
25	27·70·2	8·58·3	49·06	23·53	— ·33	—·13	+·05	25·12

GROUP II, PAIR 4.

1892.								
May 22	18·88·7	15·88·7	26 17 35·30	—1 09·60	—1·19	—·02	+·05	21 16 24·54
25	18·22·8	15·10·3	36·06	12·50	+ ·50	—·02	+·05	24·09
28	18·16·1	14·97·3	36·72	13·96	+1·17	—·02	+·05	23·96
June 7	18·14·5	14·92·7	38·98	14·65	— ·04	—·02	+·05	24·32
11	18·15·8	14·81·7	39·89	17·51	+1·25	—·02	+·05	23·66
13	18·68·6	15·39·9	40·26	16·25	+ ·21	—·02	+·05	24·25
18	18·59·7	15·31·8	41·34	16·07	— ·91	—·02	+·05	24·39
25	18·45·0	15·16·8	42·95	18·46	— ·44	—·02	+·05	24·08

GROUP II, PAIR 5.

1892.								
May 22	11·08·8	26·47·8	21 10 28·40	+5 57·02	— ·77	+·10	+·05	21 16 24·80
25	10·97·5	26·19·5	29·17	53·08	+1·69	+·10	+·05	24·09
28	10·77·0	25·95·9	29·85	52·36	+1·25	+·10	+·05	23·61
June 7	10·91·1	26·06·8	32·12	51·62	+ ·38	+·10	+·05	24·27
11	11·20·7	26·26·8	33·06	49·39	+1·54	+·10	+·05	24·14
13	11·60·7	26·66·5	33·45	49·32	+1·20	+·10	+·05	24·12
18	11·55·1	26·65·2	34·04	50·32	—0·61	+·10	+·05	23·90
25	11·39·4	26·33·2	36·24	46·54	+1·38	+·08	+·04	24·28

GROUP II, PAIR 6.

Date.	Micrometer.		Mean app. Dec.	Corrections.				Latitude.
1892.	*N.* *t. d.*	*S.* *t. d.*	° ′ ″	*Mic.* ′ ″	*Level.* ″	*Ref.* ″	*Mer.* ″	° ′ ″
May 22	12·88·4	23·60·0	21 12 17·18	+4 08·60	— ·58	+·07	+·05	21 16 25·32
25	12·74·6	23·29·2	17·96	04·65	+1·85	+·07	+·05	24·58
28	12·22·8	22·80·8	18·65	05·44	+ ·70	+·07	+·05	24·91
June 7	13·49·7	23·95·2	20·92	02·54	+1·08	+·07	+·05	24·66
11	13·33·3	23·75·1	21·88	01·68	+1·18	+·07	+·05	24·86
18	12·85·9	23·32·0	23·38	02·68	—1·06	+·07	+·05	25·12
25	12·75·9	23·09·2	25·08	+3 59·71	+ ·20	+·03	+·04	25·06

GROUP II, PAIR 7.

1892.								
May 22	22·67·1	13·61·3	21 19 57·05	—3 30·13	—1·62	—·06	+·06	21 16 25·30
25	22·90·6	13·70·5	57·84	33·45	+ ·27	—·06	+·06	24·66
28	23·00·9	13·73·9	58·54	35·05	+ ·86	—·06	+·06	24·35
June 7	23·31·7	14·02·1	60·82	35·65	— ·31	—·06	+·06	24·86
11	23·35·7	13·97·7	20 01·82	37·60	+ ·53	—·06	+·06	24·75
18	22·39·1	13·07·2	03·35	36·19	—1·89	—·06	+·06	25·27
25	23·08·6	13·64·1	05·11	39·11	—1·04	—·06	+·06	24·96

GROUP II, PAIR 8.

1892.								
May 22	15·53·4	20·50·1	21 14 30·50	+1 55·23	—1·44	+·03	+·05	21 16 24·37
25	13·79·9	18·63·5	31·29	52·19	+ ·74	+·03	+·05	24·30
28	12·32·1	17·06·4	32·01	50·03	+1·83	+·03	+·05	23·95
June 7	12·69·4	17·39·5	34·32	49·06	+ ·22	+·03	+·05	23·68
11	12·42·9	17·05·2	35·32	47·25	+ ·64	+·03	+·05	23·29
18	12·58·2	17·30·7	36·88	49·61	—1·79	+·03	+·05	24·78
25	12·93·8	17·45·2	38·68	44·72	+ ·40	+·03	+·05	23·88

From the preceding individual values, all those nights were chosen on which complete series were obtained. These complete nights were collated for each group, and the daily means taken as furnished by each group. This made it possible to reduce each pair to a mean declination system by subtracting each separate value of the latitude on any one night from the mean of all the pairs in the group for that night. Every night being treated in this way, and the mean being taken, we have a system of corrections which, being applied to their appropriate pairs, reduce the individual values of the group to one homogeneous system.

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

These reductions are given in the following tables:

Reduction to mean declination system.

GROUP I.

Date.	1.	2.	3.	4.	5.	6.	7.	8.	Sum.
1891.	″	″	″	″	″	″	″		″
June 7	—0·08	+0·09	+0·07	+0·13	—0·76	+0·39	+0·13		—0·03
8	+ ·49	+ ·15	— ·04	+ ·08	— ·77	— ·03	+ ·11		— ·01
11	+ ·36	+ ·92	— ·52	— ·36	— ·86	+ ·36	+ ·12		+ ·04
13	— ·23	+ ·03	+ ·11	+ ·09	— ·69	+ ·51	+ ·17		— ·01
17	— ·16	+ ·31	+ ·27	— ·33	— ·47	+ ·03	+ ·36		+ ·01
18	+ ·58	+ ·12	+ ·32	— ·65	— ·57	+ ·08	+ ·10		— ·02
22	— ·37	+ ·16	— ·10	+ ·06	— ·40	+ ·24	+ ·42		+ ·01
26	+ ·14	+ ·28	+ ·14	— ·22	— ·55	— ·02	+ ·24		+ ·01
1892.									
Apr. 9	— ·56	+ ·45	— ·06	+ ·04	— ·68	+ ·22	+ ·60		+ ·01
19	— ·20	+ ·50	+ ·03	— ·12	— ·72	+ ·32	+ ·16		— ·03
24	— ·40	— ·01	+ ·16	— ·04	— ·56	+ ·31	+ ·53		— ·01
May 4	+ ·47	+ ·17	— ·19	·00	— ·23	— ·25	+ ·06		+ ·03
5	— ·30	+ ·45	— ·09	— ·05	— ·51	— ·01	+ ·48		— ·03
9	— ·15	+ ·53	+ ·03	+ ·12	— ·65	— ·13	+ ·24		— ·01
13	— ·21	— ·36	+ ·41	+ ·12	— ·42	— ·01	+ ·45		— ·02
14	— ·07	— ·11	— ·06	— ·06	— ·19	+ ·15	+ ·35		+ ·01
15	— ·47	+ ·08	+ ·19	+ ·10	— ·18	— ·23	+ ·50		— ·01
18	— ·50	+ ·16	·00	+ ·04	— ·16	+ ·12	+ ·30		— ·04
21	— ·06	+ ·08	+ ·13	— ·20	— ·10	— ·03	+ ·17		— ·01
22	— ·24	— ·01	— ·29	+ ·64	— ·25	+ ·13	+ ·02		·00
24	— ·17	+ ·58	— ·14	+ ·03	— ·61	+ ·07	+ ·25		+ ·01
25	— ·10	+ ·13	+ ·22	— ·22	— ·45	— ·14	+ ·57		+ ·01
26	+ ·22	— ·15	+ ·14	+ ·10	— ·40	— ·03	+ ·15		+ ·03
28	— ·02	+ ·50	+ ·05	— ·10	— ·70	+ ·05	+ ·24		+ ·02
29	— ·36	— ·02	— ·06	+ ·32	— ·40	+ ·12	+ ·40		·00
June 3	+ ·14	— ·33	+ ·26	+ ·19	— ·32	·00	+ ·08		+ ·02
4	— ·24	+ ·02	+ ·21	— ·28	— ·24	— ·03	+ ·52		— ·04
5	— ·10	+ ·58	— ·20	— ·24	— ·52	+ ·01	+ ·49		+ ·02
11	— ·50	+ ·30	+ ·08	+ ·76	— ·64	— ·08	+ ·05		— ·03
13	+ ·58	+ ·15	— ·41	— ·28	— ·20	+ ·07	+ ·11		+ ·02
17	— ·01	+ ·16	+ ·13	— ·44	— ·49	+ ·41	+ ·23		— ·01
18	— ·44	+ ·03	— ·06	+ ·15	— ·46	+ ·04	+ ·72		— ·02
22	— ·19	+ ·18	+ ·55	— ·04	— ·66	— ·29	+ ·44		— ·01
24	— ·25	+ ·65	+ ·15	— ·53	— ·49	— ·46	+ ·95		+ ·02
25	— ·21	+ ·24	+ ·27	— ·30	— ·67	— ·25	+ ·89		— ·03
Means	— .103	+ .200	+ .049	— .043	— .485	+ .047	+ .331		— .004

Reduction to mean declination system.

GROUP II.

Date.	1.	2.	3.	4.	5.	6.	7.	8.	Sum.
1891.	″	″	″	″	″	″	″	″	″
June 18	—0·25	—0·00	—0·30	+0.44	+0·18	—0·52	—0·16	+0·58	—0·03
26	+ ·30	— ·17	— ·19	+ ·69	— ·24	— ·58	— ·04	+ ·21	— ·02
27	— ·09	+ ·41	— ·29	+ ·09	+ ·17	— ·53	— ·33	+ ·60	+ ·03
28	— ·47	+ ·25	+ ·03	+ ·19	— ·26	—1·02	+ ·41	+ ·90	+ ·03
30	— ·63	+ ·54	— ·13	+ ·69	+ ·13	— ·81	— ·09	+ ·32	+ ·02
July 18	— ·43	+ ·71	— ·60	+ ·87	— ·04	— ·36	— ·37	+ ·25	+ ·03
26	— ·86	+ ·26	— ·38	+ ·46	+ ·10	— ·42	— ·16	+1·00	·00
29	+ ·34	+ ·45	— ·10	+ ·30	+ ·18	— ·94	— ·56	+ ·34	+ ·01
Aug. 5	— ·56	+ ·51	— ·18	+ ·39	+ ·06	— ·96	+ ·12	+ ·64	+ ·02
6	— ·21	— ·23	— ·49	+ ·89	+ ·43	— ·09	— ·54	+ ·25	+ ·01
10	— ·41	+ ·27	— ·37	+ ·19	+ ·25	— ·52	+ ·11	+ ·46	— ·02
13	— ·27	+ ·36	— ·18	+ ·64	+ ·24	— ·88	+ ·12	— ·06	— ·03
14	— ·53	+ ·35	— ·45	+ ·73	+ ·37	— ·55	— ·04	+ ·09	— ·03
16	— ·57	+ ·69	— ·07	+ ·17	+ ·11	— ·88	— ·05	+ ·60	·00
1892.									
May 22	+ ·01	+ ·48	— ·50	+ ·33	+ ·07	— ·45	— ·43	+ ·50	+ ·01
25	+ ·70	+ ·21	— ·90	+ ·25	+ ·25	— ·24	— ·32	+ ·04	— ·01
28	+ ·03	+ ·16	— ·88	+ ·34	+ ·69	— ·61	— ·05	+ ·35	+ ·03
June 7	+ ·19	+ ·30	— ·89	+ ·11	+ ·16	— ·23	— ·43	+ ·75	— ·04
11	+ ·60	+ ·05	— ·51	+ ·45	— ·03	— ·75	— ·64	+ ·82	— ·01
18	+ ·23	+ ·41	— ·69	+ ·31	+ ·80	— ·42	— ·57	— ·08	— ·01
25	+ ·34	+ ·47	— ·69	+ ·35	+ ·15	— ·63	— ·53	+ ·55	+ ·01
Means	— ·121	+ ·309	— ·417	+ ·423	+ ·180	— ·590	— ·217	+ ·434	

GROUP III.

1891.	″	″	″	″	″	″	″	″	″
July 8	—0·19	+0·32	—0·35	—0·85	+0·95	—0·03	—0·19	+0·33	—0·01
17	— ·41	+ ·09	+ ·45	— ·27	+ ·61	— ·49	+ ·07	— ·07	— ·02
18	— ·44	— ·09	— ·40	— ·78	+ ·82	+ ·04	+ ·72	+ ·11	— ·02
23	— ·34	— ·43	— ·75	— ·49	+ ·80	+ ·11	+ ·52	+ ·56	— ·02
26	— ·21	+ ·37	— ·77	— ·19	+ ·83	— ·71	+ ·40	+ ·27	— ·01
27	— ·19	+ ·05	— ·03	— ·43	+ ·31	— ·05	+ ·52	— ·15	+ ·03
Aug. 5	— ·42	+ ·36	— ·60	— ·18	+ ·80	— ·26	+ ·20	+ ·07	— ·03
13	+ ·23	+ ·19	— ·05	— ·41	+ ·79	— ·53	+ ·13	— ·34	+ ·01
14	— ·58	+ ·39	— ·25	— ·14	+ ·28	— ·32	+ ·60	+ ·02	·00
15	+ ·03	+ ·39	— ·41	— ·54	+ ·59	+ ·16	+ ·47	— ·67	+ ·02
16	— ·19	— ·05	— ·29	— ·15	+ ·40	— ·27	+ ·66	— ·09	+ ·02
26	— ·55	— ·01	— ·02	+ ·07	+ ·50	— ·15	+ ·49	— ·35	— ·02
29	— ·03	— ·24	— ·19	+ ·50	— ·24	+ ·09	+ ·53	— ·39	+ ·03
Sept. 2	— ·84	— ·10	+ ·30	— ·12	+ ·10	+ ·07	+ ·96	— ·38	— ·01
9	— ·06	— ·27	— ·01	— ·08	— ·15	+ ·30	+ ·52	— ·24	+ ·01
18	— ·51	— ·22	+ ·18	+ ·24	+ ·20	+ ·02	+ ·28	— ·18	+ ·01
19	— ·60	— ·04	— ·20	+ ·38	+ ·42	+ ·29	+ ·33	— ·56	+ ·02
26	— ·80	+ ·18	+ ·23	+ ·25	+ ·55	— ·28	+ ·03	— ·12	+ ·04
28	— ·33	·00	+ ·06	— ·24	+ ·10	+ ·14	+ ·36	— ·12	— ·03
Means	— ·338	+ ·047	— ·163	— ·181	+ ·456	—·098	+·400	—·120	

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

Reductions to mean declination system.

GROUP IV.

Date.	1.	2.	3.	4.	5.	6.	7.	8.	Sum.
1891.	″	″	″	″	″	″	″	″	″
Aug. 20	—0·24	+0·74	+1·17	—0·19	—0·70	—0·09	—0·26	—0·47	— ·04
23	— ·70	+ ·87	+ ·50	— ·26	— ·66	+ ·16	— ·29	+ ·39	+ ·01
25	— ·63	+ ·22	+1·40	+ ·04	— ·50	— ·06	— ·77	+ ·29	— .01
29	·00	+ ·29	+1·13	— ·41	— ·76	+ ·08	— ·58	+ ·22	— ·03
Sept. 2	— ·52	+ ·52	+ ·94	— ·01	— ·90	— ·36	— ·68	+ ·32	+ ·03
9	— ·77	+ ·33	+1·59	— ·08	— ·78	+ ·11	— ·81	+ ·40	— ·01
12	— ·46	+ ·76	+1·06	— ·22	—1·02	— ·04	— ·24	+ ·16	·00
15	— ·40	+ ·35	+ ·77	— ·13	— ·43	+ ·07	— ·65	+ ·42	·00
23	— ·18	+ ·79	+1·23	— ·18	— ·49	— ·25	— ·69	— ·21	+ ·02
26	— ·46	+ ·10	+ ·64	— ·19	— ·37	+ ·30	— ·53	+ ·49	— ·02
28	— ·31	+ ·27	+ ·76	+ ·09	— ·93	+ ·69	— ·75	+ ·21	+ ·03
Oct. 4	— ·26	+ ·53	+ ·58	— ·04	— ·94	+ ·37	— ·42	+ ·17	— ·01
5	— ·34	+ ·64	+1·14	— ·06	— ·84	— ·35	— ·46	+ ·24	— ·03
8	— ·23	+ ·16	+ ·86	+ ·06	— ·69	+ ·35	— ·74	+ ·22	— ·01
28	— ·68	+ ·88	— ·07	— ·28	—1·20	+1·03	— ·00	+ ·32	·00
30	— ·49	+ ·23	+ ·71	— ·25	— ·51	+ ·17	— ·51	+ ·63	— ·02
31	— ·20	— ·04	+ ·99	— ·15	— ·41	+ ·27	— ·19	— ·30	— ·03
Nov. 1	— ·32	+ ·17	+1·05	+ ·08	— ·79	+ ·39	— ·80	+ ·24	+ ·02
4	— ·37	+ ·27	+ ·53	— ·09	— ·53	+ ·51	— ·99	+ ·63	— ·04
6	— ·47	+ ·73	+ ·84	— ·64	— ·38	+ ·30	— ·64	+ ·26	·00
14	+ ·01	— ·31	+ ·52	+ ·45	— ·55	— ·03	— ·58	+ ·45	— ·04
17	— ·38	+ ·22	+ ·83	+ ·04	— ·81	+ ·58	— ·57	+ ·08	— ·01
Means	— ·382	+ ·396	+ ·871	— ·110	— ·690	+ ·224	— ·552	+ ·235	

GROUP V.

1891.	″	″	″	″	″	″	″	″	″
Oct. 4	0·00	—0·12	—0·58	+0·13	+0·50	+0·26	—0·13	—0·06	0.00
5	+ ·19	+ ·11	— ·67	+ ·25	+ ·31	+ ·30	— ·53	·00	— .04
8	— ·10	— ·23	— ·44	— ·02	+ ·18	+ ·24	— ·22	+ ·55	— ·04
30	+ ·74	— ·03	—1·18	+ ·34	— ·13	+ ·61	— ·65	+ ·29	— ·01
31	— ·12	— ·28	— ·61	+ ·32	+ ·04	+ ·66	— ·46	+ ·43	— ·02
Nov. 1	+ ·31	— ·09	— ·75	— ·59	+ ·19	+ ·51	+ ·13	+ ·31	+ ·02
6	+ ·02	+ ·27	— ·63	— ·08	+ ·26	+ ·69	— ·57	+ ·07	+ ·03
14	+ ·16	+ ·20	— ·77	— ·01	+ ·19	+ ·40	— ·48	+ ·30	— ·01
17	— ·10	+ ·37	— ·61	+ ·09	+ ·04	+ ·35	+ ·01	— ·17	— ·02
19	+ ·10	+ ·21	— ·63	+ ·10	+ ·03	+ ·25	— ·27	+ ·21	·00
21	— ·34	+ ·36	— ·61	— ·10	— ·15	+ ·42	+ ·29	+ ·10	— ·03
23	+ ·31	+ ·36	—1·15	+ ·35	— ·05	+ ·29	— ·01	— ·14	— ·04
Dec. 10	+ ·05	+ ·06	— ·58	— ·32	+ ·33	+ ·26	+ ·03	+ ·14	— ·03
12	+ ·36	+ ·53	—1·10	— ·14	+ ·04	+ ·62	— ·16	— ·16	— ·01
15	·00	+ ·30	—1·06	+ ·12	+ ·17	+ ·34	— ·18	+ ·28	— ·03
21	— ·22	+ ·13	— ·57	+ ·48	+ ·28	— ·03	— ·32	+ ·27	+ ·02
23	+ ·51	— ·03	—1·05	+ ·33	+ ·65	— ·11	— ·23	— ·05	+ ·02
25	+ ·31	+ ·09	— ·85	+ ·31	— ·17	+ ·42	— ·40	+ ·28	— ·01
26	+ ·21	+ ·11	— ·93	— ·03	·00	+ ·72	+ ·20	— ·16	+ ·02
29	+ ·14	+ ·57	— ·62	— ·22	— ·22	+ ·51	+ ·13	— ·27	+ ·02
30	+ ·35	+ ·17	— ·57	— ·40	+ ·78	+ ·02	— ·49	+ ·10	— ·04
Means	+ ·137	+ ·146	— ·760	+ ·043	+ ·156	+ ·368	— ·205	+ ·110	

Reductions to mean declination system.

GROUP VI.

Date.	1.	2.	3.	4.	5.	6.	7.	8.	Sum.
1891.	″	″	″	″	″	″	″	″	″
Nov. 21	—1·20	—0·26	+0·54	+0·18	+0·02	—0·00	+0·53	+0·20	+0·01
Dec. 12	— ·25	+ ·55	— ·26	+ ·15	+ ·59	— ·58	+ ·17	— ·39	— ·02
15	— ·81	+ ·60	+ ·60	— ·01	+ ·04	— ·24	+ ·19	— ·38	— ·01
23	— ·47	+ ·91	+ ·11	+ ·20	+ ·77	— ·30	— ·56	— ·68	— ·02
26	— ·18	+ ·42	+ ·40	+ ·46	+ ·26	— ·76	+ ·21	— ·78	+ ·03
29	— ·99	+ ·35	+ ·17	+ ·37	— ·27	— ·15	+ ·50	+ ·01	— ·01
1892.									
Jan. 5	— ·82	+ ·18	+ ·60	+ ·09	+ ·19	— ·10	+ ·52	— ·66	·00
6	— ·12	+ ·62	+ ·18	+ ·58	— ·17	— ·62	— ·10	— ·40	— ·03
12	— ·97	+ ·55	— ·17	+ ·06	+ ·09	— ·40	+ ·54	+ ·32	+ ·02
18	— ·67	+ ·37	+ ·20	+ ·11	+ ·09	— ·51	+ ·36	+ ·01	— ·04
20	— ·42	+ ·84	+ ·42	— ·03	+ ·31	— ·58	— ·18	— ·35	+ ·01
21	— ·55	— .09	+ ·44	— ·05	+ ·38	— ·36	— ·13	+ ·33	— ·03
24	— ·99	+ ·12	+ ·05	+ ·39	+ ·37	— ·26	+ ·16	+ ·14	— ·02
27	— ·67	+ ·11	+ ·39	+ ·54	— ·09	— ·22	+ ·30	— ·38	— ·02
Feb. 3	— ·92	+ ·18	+ ·19	+ ·58	+ ·16	— ·19	+ ·10	— ·10	·00
6	— ·46	+ ·06	+ ·12	+ ·05	+ ·14	— ·52	+ ·50	+ ·14	+ ·03
8	—1·17	— ·07	+ ·21	+ ·77	+ ·04	+ ·39	+ ·08	— ·27	— ·02
12	—1·01	— ·17	+ ·50	+ ·50	+ ·93	— ·42	— ·03	— ·29	+ ·01
13	— ·64	+ ·48	+ ·39	+ ·17	+ ·10	— ·67	+ ·45	— ·32	— ·04
Means	— ·701	+ ·303	+ ·267	+ ·269	+ ·208	— ·342	+ ·190	— ·203	

GROUP VII.

1891.	″	″	″	″	″	″	″	″	″
Dec. 29	—0·60	+0·10	+0·02	+0·02	+0·45	—0·32	+0·32	+0·02	+0·01
1892.									
Jan. 5	— ·77	— ·16	+ ·06	+ ·17	+1·04	— ·82	+ ·16	+ ·35	+ ·03
11	— ·83	+ ·27	— ·05	+ ·61	+ ·79	— ·35	— ·23	— ·25	— ·04
20	— ·30	— ·08	— ·39	+ ·74	+ ·64	— ·64	— ·18	+ ·17	— ·04
24	— ·69	— ·07	— ·17	+ ·30	+ ·84	— ·32	— ·17	+ ·27	— ·01
Feb. 2	— ·65	— ·12	+ ·28	+ ·05	+ ·79	— ·25	— ·37	+ ·30	+ ·03
3	— ·29	— ·42	— ·07	+ ·43	+ ·85	— ·27	— ·21	— ·03	— ·01
4	— ·83	+ ·03	— ·06	+ ·45	+ ·57	— ·14	+ ·02	— ·04	·00
8	— ·08	— ·04	— ·22	— ·15	+ ·90	— ·75	+ ·30	+ ·03	— ·01
12	+ ·16	— ·22	— ·42	— ·30	+1·01	— ·45	— ·02	+ ·26	+ ·02
13	— ·42	— ·49	+ ·11	+ ·31	+ ·62	— .58	+ ·11	+ ·35	+ ·01
24	— ·19	— ·05	+ ·10	— ·09	+ ·63	— ·39	— ·17	+ ·16	·00
27	— ·49	— ·12	— ·06	— ·19	+ ·89	— ·05	— ·07	+ .05	— ·04
28	— ·02	+ ·03	— ·08	— ·04	+ ·70	— ·53	— ·19	+ ·11	— ·02
29	— ·56	+ ·16	— ·14	+ ·16	+1·03	— ·42	+ ·63	— ·28	— ·02
Mar. 1	— ·24	— ·28	+ ·07	+ ·07	+ ·70	— ·66	+ .15	+ ·17	— ·02
2	— ·39	+ ·12	— ·29	+ .31	+ ·75	— ·78	+ ·14	+ ·13	— ·01
5	— ·38	+ ·29	— ·23	+ ·10	+ ·64	— ·54	+ ·10	+ ·02	·00
9	+ ·47	— ·55	— ·30	+ ·33	+ ·75	— ·73	— ·08	+ ·14	+ ·03
10	— ·89	+ ·45	— ·02	+ ·14	+ ·87	— ·26	— ·24	— ·07	— ·02
13	— ·31	— ·23	+ ·37	+ ·31	— ·01	— ·75	+ ·39	+ ·21	— ·02
16	— ·31	+ ·01	— ·19	— ·04	+ ·81	— ·69	+ ·27	+ ·14	·00
24	— ·74	+ ·31	+ ·11	+ ·20	+ ·56	— ·54	— ·29	+ ·36	— ·03
Means	— ·406	— ·046	— ·068	+ ·169	+ ·731	— ·488	— ·010	+ ·112	

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

Reductions to mean declination system.

GROUP VIII.

Date.	1.	2.	3.	4.	5.	6.	7.	8.	Sum.
1892.	″	″	″	″	″	″	″	″	″
Feb. 3	+0·63	+0·31	—0·46	—0·04	—0·59	+0·61	—0·12	—0·36	—0·02
12	+ ·41	+ ·22	— ·60	+ ·03	— ·29	+ ·18	+ ·10	— ·04	+ ·01
27	+ ·44	+ ·24	— ·30	— ·47	— ·20	+ ·61	+ ·33	— ·63	+ ·02
28	+ ·15	+ ·01	— ·52	+ ·07	— ·29	+ ·21	+ ·25	+ ·13	+ ·01
Mar. 2	— ·14	— ·01	— ·19	+ ·39	— ·33	+ ·22	— ·36	+ ·44	+ ·02
5	+ ·17	+ ·27	— ·50	+ ·03	+ ·25	+ ·05	— ·26	— ·03	— ·02
9	+ ·07	+ ·21	— ·69	— ·35	— ·17	+ ·59	+ ·15	+ ·20	+ ·01
10	+ ·34	— ·02	— ·72	+ ·07	+ ·30	— ·08	— ·46	+ ·60	+ ·03
23	+ ·03	+ ·69	— ·51	— ·05	— ·17	+ ·22	+ ·07	— ·27	+ ·01
24	+ ·01	— ·14	— ·37	— .04	— ·05	·00	+ ·26	+ ·29	— ·04
29	— ·24	+ ·18	— ·71	— ·16	+ ·35	— ·04	— ·04	+ ·66	·00
30	— ·08	+ ·25	— ·43	— ·20	+ ·35	+ ·35	+ ·09	— ·32	+ ·01
31	+ ·05	+ ·15	— ·74	+ ·01	— ·39	+ ·35	+ ·43	+ ·13	— ·01
Apr. 9	— ·41	+ ·18	— ·37	— ·08	— ·20	+ ·26	+ ·21	+ ·37	— ·04
19	+ ·25	— ·23	— ·66	— ·13	+ ·30	+ ·34	+ ·10	·00	— ·03
24	— ·06	+ ·18	— ·26	+ ·22	— ·09	+ ·28	— ·34	+ ·07	·00
May 2	— ·10	— ·27	+ ·23	— ·19	+ ·09	+ ·33	— ·12	+ ·05	+ ·02
5	+ ·37	+ ·15	— ·64	— ·41	— ·15	+ ·59	— ·09	+ ·21	+ ·03
Means	+ ·111	+ ·132	— ·469	— ·072	— ·071	+ ·282	+ ·011	+ ·083	

The preceding tables give the following summary:

Reductions to mean declination system.

Group.	Pair—							
	1.	2.	3.	4	5.	6.	7.	8.
	″	″	″	″	″	″	″	″
I	— ·10	+ ·20	+ ·05	— ·04	— ·48	+ ·05	+ ·33	
II	— ·12	+ ·31	— ·42	+ ·42	+ ·18	— ·59	— ·22	+ ·43
III	— ·34	+ ·05	— ·16	— ·18	+ ·46	— ·10	+ ·40	— ·12
IV	— ·38	+ ·40	+ ·87	— ·11	— ·69	+ ·22	— ·55	+ ·24
V	+ ·14	+ ·15	— ·76	+ ·04	+ ·16	+ ·37	— ·20	+ ·11
VI	— ·70	+ ·30	+ ·27	+ ·27	+ ·21	— ·34	+ ·19	— ·20
VII	— ·41	— ·05	— ·07	+ ·17	+ ·73	— ·49	— ·01	+ ·11
VIII	+ ·11	+ ·13	— ·47	— ·07	— ·07	+ ·28	+ ·01	+ ·08

The foregoing mean corrections being applied to the individual values for each day, and the daily means being taken, we have the following reduced values for each group and date of observation:

Corrected daily means.

Date.	Group I.	No. of pairs.	Group II.	No. of pairs.	Group III.	No. of pairs.
1891.	//		//		//	
June 6	4·20	6				
7	4·59	7				
8	4·33	7				
9	4·52	6				
10	4·37	6	4·26	5		
11	4·50	7	4·29	4		
12	4·43	5				
13	4·59	7	4·56	7		
15	4·35	6	4·14	1		
16	4·68	6				
17	4·59	7	4·54	7		
18	4·70	7	4·60	8		
19	4·23	6	4·56	6		
22	4·44	7	4·45	7		
24	4·47	6	4·45	4		
25	4·62	2				
26	4·40	7	4·56	8		
27	4·35	6	4·43	8		
28	4·70	6	4·23	8		
30	4·54	6	4·45	8		
July 4			4·22	7	4·21	7
7			4·73	3		
8			4·24	7	4·41	8
17			4·08	5	4·09	8
18			4·05	8	4·20	8
22			4·32	6	4·26	5
23			4·24	5	4·34	8
24			4·08	4	4·43	1
25			3·75	2		
26			4·22	8	4·17	8
27			4·32	7	4·05	8
29			4·04	8	3·91	4
30			4·12	2	4·27	7
31			3·93	3	4·23	2

Date.	Group II.	No. of pairs.	Group III.	No. of pairs.	Group IV.	No. of pairs.
1891.	//		//		//	
Aug. 2	4·24	7	4·21	7		
3	4·20	7	4·26	6		
4	4·08	7				
5	4·16	8	4·12	8		
6	4·11	8	4·23	5		
7	3·99	7	4·21	7		
10	4·15	8	4·12	7		
11	4·28	2				
12	3·87	1				
13	4·02	8	4·07	8		
14	4·03	8	3·78	8		
15	4·02	7	4·21	8		
16	3·99	8	4·19	8		
20			4·10	7	3·94	8
22			4·21	7	3·80	2
23			4·14	7	4·10	8
Aug. 25			4·34	7	4·02	8
26			4·21	8		
29			4·03	8	3·66	8
31			4·24	7	4·19	7
Sept. 2			4·06	8	4·14	8
6			4·20	6	4·12	7
9			4·24	8	3·89	8
12			4·00	7	3·88	8
15			4·09	6	3·79	8
18			4·08	8		
19			4·10	8	3·73	7
23			3·87	3	3·63	8
26			4·08	8	3·81	8
27			3·98	7	3·76	7
28			4·18	8	3·65	8

Date.	Group IV.	No. of pairs.	Group V.	No. of pairs.	Group VI.	No. of pairs.
1891.	//		//		//	
Oct. 4	4·06	8	3·92	8		
5	3·84	8	4·01	8		
8	3·86	8	4·04	8		
10	4·00	6				
14	3·79	2				
16	3·72	7	3·98	7		
17	3·56	2				
22	3·83	6	4·06	1		
28	3·86	8	3·94	3		
30	4·19	8	4·01	8		
31	3·92	8	3·96	8		
Nov. 1	3·91	8	3·99	8		
4	3·82	8	3·99	7		
6	4·04	8	4·03	8		
7	3·54	7				
10	3·48	4				
12	3·60	5	4·22	2		
Nov. 13	4·09	4	4·07	7		
14	3·92	8	4·08	8		
15	3·63	2				
16	3·20	1				
17	4·02	8	4·16	8		
19			4·07	8	3·94	7
21			4·10	8	4·12	8
23			4·15	8		
30			3·89	7	4·00	7
Dec. 1			4·07	3	4·07	7
5			3·87	5	3·91	7
6			3·99	6		
9			4·31	7	3·98	7
10			4·28	8	4·18	3
12			4·30	8	4·15	8
13			4·13	3		

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

Corrected daily means.

Date.	Group V.	No. of pairs.	Group VI.	No. of pairs.	Group VII.	No. of pairs.	Group VIII.	No. of pairs.
1891.	"		"		"		"	
Dec. 15	4·00	8	4·04	8				
17	4·21	5						
19	4·14	6	4·21	5				
21	4·12	8						
23	4·21	8	4·20	8				
24	3·93	7	3·92	1				
25	4·17	8	4·15	5				
26	4·20	8	4·22	8				
27	3·59	2						
28	3·99	1	4·27	7				
29	4·03	8	4·19	8	4·60	8		
30	4·12	8	4·04	7				
1892.								
Jan. 1	3·87	2						
2	4·69	1						
5	4·65	7	3·92	8	3·84	8		
6			3·98	8	3·78	7		
10			3·96	7	4·89	1		
11			4·20	6	4·24	8		
12			4·43	8	4·19	5		
15			4·33	3				
16			3·94	2				
18			4·30	8	4·59	4		
19			3·88	5				
20			4·12	8	4·04	8		
21			4·00	8	4·23	5		
23			4·20	7	4·30	7		
24			3·97	8	4·40	8		
25			4·16	7	4·46	7		
27			4·76	8	4·43	6		
Feb. 2			4·36	1	4·57	8		
3			4·12	8	4·53	8	4·49	8
4			4·40	7	4·42	8		
6			4·24	8	4·26	5		
8			4·19	8	4·42	8	4·40	7

Date.	Group VI.	No. of pairs.	Group VII.	No. of pairs.	Group VIII.	No. of pairs.	Date.	Group VI.	No. of pairs.	Group VII.	No. of pairs.	Group VIII.	No. of pairs.
1892.	"		"		"		1892.	"		"		"	
Feb. 11	4·25	3					Mar. 5			4·86	8	5·07	8
12	4·51	8	4·34	8	4·40	8	8					4·78	7
13	4·00	8	4·42	8			9			4·36	8	4·79	8
14	4·23	6	4·79	1			10			4·70	8	4·66	8
19			4·36	6			13			4·61	8	4·52	7
20	4·39	6	4·38	7			14			4·77	7	4·75	2
21	4·22	6	4·67	5	4·69	7	16			4·71	8	4·18	1
22	4·34	7					19			4·72	5		
23	4·10	7	4·88	7	4·38	7	20			4·68	6		
24			4·73	8			21			4·87	7	4·80	6
26			4·67	5			23					4·75	8
27			4·66	8	4·56	8	24			4·84	8	4·72	8
28			4·54	8	4·77	8	29			4·66	6	4·76	8
29			4·56	8	4·83	7	30			5·08	4	4·71	8
Mar. 1			4·70	8			31			4·74	7	4·81	8
2			4·59	8	4·78	8	Apr. 6					4·70	6
4			4·85	7									

Corrected daily means.

Date.	Group VIII.	No. of pairs.	Group I.	No. of pairs.	Group II.	No. of pairs.
1892.	//		//		//	
Apr. 9	4·98	8	4·76	7		
10	5·25	1				
13	4·42	2				
14	4·83	5				
15	4·48	3				
16	4·38	5	4·78	4		
17	4·44	5				
18	4·81	5				
19	4·68	8	4·68	7		
20	4·83	4				
21	4·80	5				
23	4·84	6	4·71	6		
24	4·66	8	4·72	7		
25	4·84	7	4·67	6		
29	4·45	3				
May 1	4·69	4	4·60	6		

Date.	Group VIII.	No. of pairs.	Group I.	No. of pairs.	Group II.	No. of pairs.
1892.	//		//		//	
May 2	4·75	8				
4	4·48	7	4·69	7		
5	4·71	8	4·77	7		
6	4·83	4				
7	4·99	1				
9	4·78	5	4·41	7		
10	4·72	5	4·52	2		
11	4·65	5				
13			4·43	7		
14	4·63	6	4·45	7		
15	4·69	5	4·40	7		
18	4·72	6	4·80	7		
21			4·80	7		
22	4·68	5	4·79	7	4·87	8
23	4·81	5	4·73	6		

Date.	Group I.	No. of pairs.	Group II.	No. of pairs.
1892.	//		//	
May 24	4·61	7		
25	4·60	7	4·34	8
26	4·56	7		
27	4·40	5		
28	4·62	7	4·30	8
29	4·74	7		
30	4·43	1		
June 3	4·66	7		
4	4·66	7		
5	4·72	7		
6	4·62	1		
7	4·45	6	4·44	8

Date.	Group I.	No. of pairs.	Group II.	No. of pairs.
1892.	//		//	
June 11	4·40	7	4·11	8
12	4·66	3		
13	4·65	7	4·68	5
14	4·51	5	4·28	2
17	4·61	7	4·89	1
18	4·74	7	4·70	8
19	5·04	1		
22	4·46	7		
23			3·67	1
24	4·27	7		
25	4·53	7	4·43	8

In order now to connect the groups and reduce each one to the first taken as a basis of comparison, the weighted means are taken for those parts of the preceding table which overlap, and we thus get the following group connections with their respective weights. The numbers of pairs indicate the weight, and are placed to the right of the quantities to which they apply.

GROUPS—									
I.	II.	III.	IV.	V.	VI.	VII.	VIII.	I.	II.
4·50 (97)	4·45 (81) 4·13 (161)	4·17 (146) 4·13 (128)	3·88 (118) 3·87 (134)	4·02 (99) 4·13 (158) 4·24 (26)	4·09 (119) 4·21 (176) 4·25 (82)	4·22 (16) 4·38 (145) 4·63 (221)	4·47 (37) 4·68 (161) 4·71 (149) 4·74 (10)	4·65 (114) 4·59 (140)	4·87 (8) 4·46 (65)

From the above group connections the following conditional equations arise:

$$v_{2\cdot1} + v_{3\cdot2} + v_{4\cdot3} + v_{5\cdot4} + v_{6\cdot5} + v_{7\cdot6} + v_{8\cdot7} + v_{1\cdot8} = +\,''\cdot01$$

$$v_{2\cdot1} - v_{2\cdot1} = +\,''\cdot08$$

$$+v_{6\cdot5} + v_{7\cdot6} + v_{7\cdot5} = +\,''\cdot15$$

$$+v_{7\cdot6} + v_{8\cdot7} - v_{8\cdot6} = ''\cdot00$$

$$v_{2\cdot1} + v_{1\cdot8} - v_{2\cdot8} = -\,''\cdot32$$

Where $v_{2\cdot1}$ is correction to be applied to the connection between Group II and Group I,

and so on

The correlative equations are—

v	k_1	k_2	k_3	k_4	k_5	$\frac{1}{p}$
2·1	+1	+1				0·226
3·2	+1					·131
4·3	+1					·163
5·4	+1					·175
6·5	+1		+1			·147
7·6	+1		+1	+1		·126
8·7	+1			+1		·107
1·8	+1				+1	·155
2·1		—1				·225
7·5			—1			1·010
8·6				—1		·392
2·8					—1	2·250

From which the following normal equations are derived:

k_1	k_2	k_3	k_4	k_5	
					″
+1·23	+0·23	+0·27	+0·23	+0·38	=+·01
	+0·45	·00	·00	+0·23	=+·08
		+1·28	+ ·13	·00	=+·15
			+ ·62	·00	= ·00
				+2·63	=—·32

The solution gives—

$k_1 = -0.0193$
$k_2 = +\ .257$
$k_3 = +\ .119$
$k_4 = -\ .0192$
$k_5 = -\ .140$

The adjustment of the outstanding differences brings out the following equations:

	″		″
II — I	= —·05 (1891)	II — I	= —·13 (1892)
III — II	= +·04		
IV — III	= —·25	VII — V	= —·02
V — IV	= +·15		
VI — V	= —·04	VIII — VI	= +·22
VII — VI	= +·17		
VIII — VII	= +·05	II — VIII	= +·13
I — VIII	= —·06		
Σ	= +·01		

The error of closing may result from errors of observation or from a slight error in the constant of aberration. If the accepted constant (20″·445) is too small, as seems to be indicated by other observations, then there must have been some compensation in the observing, because the outstanding error, +·01, is practically insignificant.

As several determinations of the constant of aberration give a value not far from 20″·50, it is worth while to see what effect this value would

have if introduced in the computation of the apparent places of the stars.

To this end all those values of the terms involving the constant of aberration for the dates of observation common to two successive groups were tabulated. Taking a mean for the two stars of each pair and the difference between the corresponding pairs of successive groups, the final mean values gave the following effect of those terms on the group connections:

I	II	III	IV	V	VI	VII	VIII	I	II
4″·5	3″·6	4″·3	4″·0	4″·9	4″·5	4″·3	3″·5	5″·2	

Had we used a value of 20″·50 in the reduction of the star places the effect on the group connections would have been—

$\frac{d\,A}{A}$ multiplied by the above numbers, or $\frac{\cdot 055}{20\cdot 45} \times 4''\cdot 5$, etc., as follows:

I	II	III	IV	V	VI	VII	VIII	I	II
″·012	″·010	″·012	″·011	″·013	″·012	″·012	″·009	″·014	

The sum of these is +″·10, so that if we accept the supposed value of the aberration constant there would remain an outstanding error of +″·11 from observation arising from the uncertainty of the group connections. If we admit a certain compensation in the errors of observation then the above investigation would rather confirm the value 20″·445, or at least indicate that it should not be changed more than one or two hundredths.

The values of k_1, k_2, k_3, k_4, and k_5, being substituted in the correlatives, give the following values for V:

V	″	V	″
2·1	= +·022	8·7	= —·004
3·2	= —·003	1·8	= —·025
4·3	= —·003	(2·1)	= —·058
5·4	= —·003	7·5	= —·120
6·5	= +·015	8·6	= +·008
7·6	= +·010	2·8	= +·314

and the corrected reductions between the groups are—

R	+	—
	″	″
2·1	0·072	
3·2		0·043
4·3	·247	
5·4		·153
6·5	·055	
7·6		·160
8·7		·054
1·8	·035	
Sum	0·409	0·410

The final corrections therefore to reduce the different groups to one homogeneous system are—

	"		"
I =	0·00	V = +	0·12
II = +	·07	VI = +	·18
III = +	·03	VII = +	·02
IV = +	·28	VIII = −	·04

Applying these to the daily means of the groups, each one of which has been reduced to its own mean declination system, we get the following table. Each individual value is introduced with its appropriate weight.

Date.	Lat.	Pairs.	Date.	Lat.	Pairs.	Date.	Lat.	Pairs.
1891.	"		1891.	"		1891.	"	
June 6	4·20	6	Aug. 20	4·18	15	Dec. 10	4·39	11
7	4·59	7	22	4·20	9	12	4·38	14
8	4·33	7	23	4·28	15	13	4·25	3
9	4·52	6	25	4·33	15	15	4·17	16
10	4·35	11	26	4·24	8	17	4·33	5
11	4·45	11	29	4·00	16	19	4·32	11
12	4·43	5	31	4·37	14	21	4·24	8
13	4·61	14	Sept. 2	4·26	16	23	4·36	16
15	4·33	7	6	4·32	13	24	4·06	8
16	4·68	6	9	4·22	16	25	4·31	13
17	4·60	14	12	4·10	15	26	4·36	16
18	4·68	15	15	4·09	14	27	3·71	2
19	4·43	12	18	4·11	8	28	4·41	8
22	4·48	14	19	4·07	15	29	4·38	24
24	4·49	10	23	3·91	11	30	4·23	15
25	4·62	2	26	4·10	16	1892.		
26	4·52	15	27	4·03	14	Jan. 1	3·99	2
27	4·44	14	28	4·07	16	2	4·81	1
28	4·47	14	Oct. 4	4·19	16	5	4·22	23
30	4·53	14	5	4·13	16	6	3·99	15
July 4	4·27	14	8	4·15	16	10	4·24	8
7	4·80	3	10	4·28	6	11	4·31	14
8	4·38	15	14	4·07	2	12	4·46	13
17	4·13	13	16	4·05	14	15	4·51	3
18	4·18	16	17	3·84	2	16	4·12	2
22	4·34	11	22	4·12	7	18	4·52	12
23	4·35	13	28	4·12	11	19	4·06	5
24	4·21	5	30	4·30	16	20	4·18	16
25	3·82	2	31	4·14	16	21	4·21	13
26	4·25	16	Nov. 1	4·15	16	23	4·35	14
27	4·23	15	4	4·10	15	24	4·29	16
29	4·05	12	6	4·24	16	25.	4·41	14
30	4·28	9	7	3·82	7	27	4·73	14
31	4·10	5	10	3·76	4	Feb. 2	4·58	9
Aug. 2	4·28	14	12	4·01	7	3	4·43	24
3	4·28	13	13	4·26	11	4	4·51	15
4	4·15	7	14	4·20	16	6	4·37	13
			15	3·91	2	8	4·39	23
5	4·19	16	16	3·48	1	11	4·43	3
6	4·21	13	17	4·29	16	12	4·47	24
7	4·15	14	19	4·16	15	13	4·31	16
10	4·19	15	21	4·26	16	14	4·47	7
11	4·35	2	23	4·27	8	19	4·38	6
12	3·94	1	30	4·10	14	20	4·48	13
13	4·10	16	Dec. 1	4·23	10	21	4·58	18
14	3·96	16	5	4·05	12	22	4·52	7
15	4·17	15	6	4·11	6	23	4·51	21
16	4·14	16	9	4·30	14	24	4·75	8

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

Date.	Lat.	Pairs.	Date.	Lat.	Pairs.	Date.	Lat.	Pairs.
1892.	″		1892.	″		1892.	″	
Feb. 26	4·69	5	Apr. 14	4·79	5	May 22	4·81	20
27	4·60	16	15	4·44	3	23	4·75	11
28	4·65	16	16	4·54	9	24	4·61	7
29	4·68	15	17	4·40	5	25	4·50	15
Mar. 1	4·72	8	18	4·77	5	26	4·56	7
2	4·68	16	19	4·66	15	27	4·40	5
4	4·87	7	20	4·79	4	28	4·49	15
5	4·96	16	21	4·76	5	29	4·74	7
8	4·74	7	23	4·76	12	30	4·43	1
9	4·57	16	24	4·67	15	June 3	4·66	7
10	4·67	16	25	4·74	13	4	4·66	7
13	4·56	15	29	4·41	3	5	4·72	7
14	4·77	9	May 1	4·62	10	6	4·62	1
16	4·66	9	2	4·71	8	7	4·48	14
19	4·74	5	4	4·57	14	11	4·28	15
20	4·70	6	5	4·72	15	12	4·66	3
21	4·83	13	6	4·79	4	13	4·69	12
23	4·71	8	7	4·95	1	14	4·46	7
24	4·77	16	9	4·55	12	17	4·65	8
29	4·70	14	10	4·63	7	18	4·76	15
30	4·81	12	11	4·61	5	19	5·04	1
31	4·77	15	13	4·43	7	22	4·46	7
Apr. 6	4·66	6	14	4·51	13	23	3·74	1
9	4·86	15	15	4·50	12	24	4·27	7
10	5·21	1	18	4·74	13	25	4·51	15
13	4·38	2	21	4·80	7			

These being collected for short intervals in such a way as to give about equal weights for each period as regards nights and pairs, we have the following table for the variation of the latitude of Waikiki from June 8, 1891, to June 21, 1892.

From the preceding method of reduction it will be seen that these values, as far as concerns the changes of latitude, are entirely free from errors of declination.

Date.	Lat.	Prs.	Nts.	Date.	Lat.	Prs.	Nts.	Date.	Lat.	Prs.	Nts.
1891.	″			1891.	″			1892.	″		
June 8	4·41	48	6	Oct. 24	4·17	52	5	Feb. 25	4·59	57	5
14	4·55	46	5	Nov. 3	4·16	47	3	29	4·68	55	4
21	4·53	51	4	10	4·09	45	5	Mar. 7	4·78	46	4
27	4·48	45	4	18	4·21	50	5	12	4·65	40	3
July 8	4·36	59	5	30	4·14	50	5	20	4·74	41	5
21	4·28	40	3	Dec. 13	4·30	58	5	27	4·76	42	3
26	4·21	38	4	21	4·28	61	6	Apr. 7	4·76	47	7
Aug. 1	4·21	53	5	27	4·35	50	4	18	4·64	43	6
6	4·18	50	4	1892.				26	4·70	43	4
12	4·09	50	5	Jan. 2	4·23	41	4	May 3	4·65	47	4
18	4·16	46	3	11	4·25	55	6	10	4·57	49	7
24	4·27	47	4	20	4·26	46	4	18	4·72	52	4
Sept. 2	4·23	59	4	24	4·35	44	3	25	4·58	45	5
13	4·14	53	4	31	4·55	47	3	June 2	4·62	45	7
23	4·04	56	4	Feb. 6	4·42	51	3	11	4·48	51	5
Oct. 1	4·13	48	3	12	4·41	43	3	21	4·56	54	7
12	4·13	38	4	18	4·51	44	4				

The preceding values are shown graphically in illustration No. 15.

It is evident that a sine curve follows in a general way the law of the variation of the latitude. Coefficients for this equation were first determined assuming a period of 415 days for the complete revolution of the axis of rotation about that of the earth's figure. This brought out the fact that the period chosen was entirely too long, and, moreover, that a second periodic term, depending on the cosine, would materially improve the agreement between the empirical formula and the actual observations. This change being made, revealed the necessity of another slight change in the period, and one of 378 days was used.

The equation of the curve is:

$$\varphi = \varphi_0 - 0''.311 \sin. nt + 0''.037 \cos. nt.$$

This was deduced by a method of approximation devised to suit this particular case, and very similar in its application to Canchy's method. It was hardly thought worth while to make a rigorous adjustment of the 49 conditional equations involved, because this curve fits the observations passably well. There are 22 above, 25 below, and 2 on the line.

In order to apply the method of least squares, and to test whether there are periodic terms depending on the sines and cosines of multiple angles of sufficient magnitude to influence the curve within the limit of errors of observation, a still further contraction of the above values was made. Taking the means of successive groups of four and allowing them equal weight we have the following mean dates and values of the latitude:

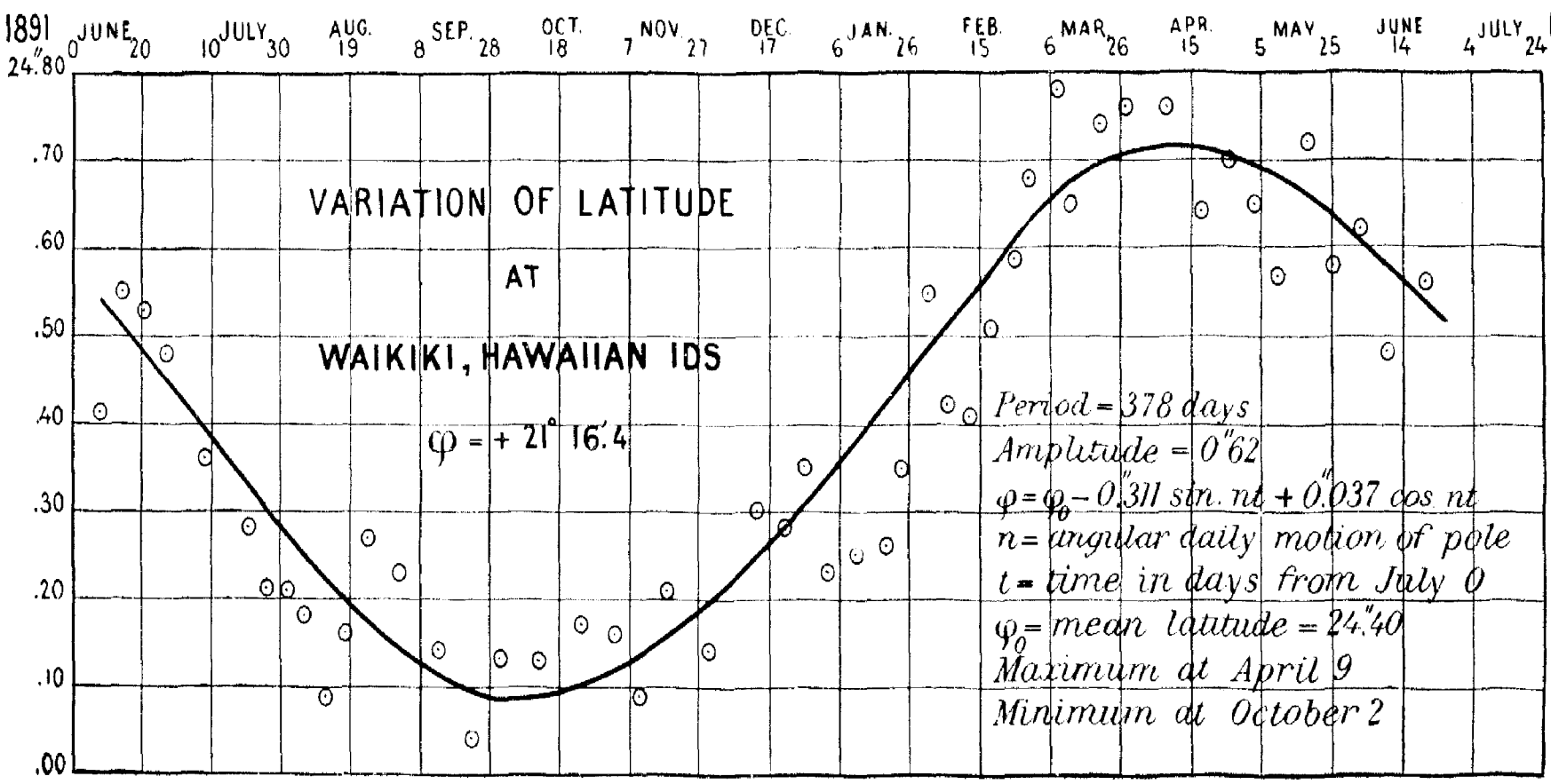
Date.	Latitude.	Date.	Latitude.
1891	"	1892	"
June 18	24·492	Jan. 7	24·272
July 22	·265	Feb. 3	·432
Aug. 15	·175	Feb. 27	·640
Sept. 17	·135	Mar. 24	·728
Oct. 28	·138	May 3	·656
Dec. 5	·232	June 7	·560

According to Fourier's theorem

$$f(\varphi) = X + h_1 \sin (a_1 + \varphi) + h_2 \sin (a_2 + 2\varphi) + h_3 \sin (a_3 + 3\varphi) + \text{etc.}$$

Substituting the above values of the latitude for $f(\varphi)$, the mean value plus a small correction (x) for X, and putting the angular value of φ in degrees for φ at the different dates counted from July 0, 1891, on the supposition of a period of 381 days, we have 12 conditional equations, of which the first is:

$$\begin{aligned} +.098'' = x &+ h_1 \sin a_1 \cos 349^\circ + h_1 \cos a_1 \sin 349^\circ \\ &+ h_2 \sin a_2 \cos 338^\circ + h_2 \cos a_2 \sin 338^\circ \\ &+ h_3 \sin a_3 \cos 327^\circ + h_3 \cos a_3 \sin 327^\circ \end{aligned}$$


1891
JUNE 20
JULY 10
30
AUG. 19
SEP. 8
28
OCT. 18
NOV. 7
27
DEC. 17
JAN. 6
26
FEB. 15
MAR. 6
26
APR. 15
MAY 5
25
JUNE 14
JULY 4
24
189
24".80
.70
.60
.50
.40
.30
.20
.10
.00
VARIATION OF LATITUDE
AT
WAIKIKI, HAWAIIAN IDS
φ = + 21° 16'.4
Period = 378 days
Amplitude = 0".62
φ = φ0 − 0".311 sin. nt + 0".037 cos nt
n = angular daily motion of pole
t = time in days from July 0
φ0 = mean latitude = 24".40
Maximum at April 9
Minimum at October 2

Results from observations for the variation of latitude at Waikiki, Hawaiian Islands.

When the numerical values for the trigonometrical functions are employed and we use y for $h \sin a$ and z for $h \cos a$ the equations take the following form:

$''$												
$+ \cdot098 =$	$x +$	$\cdot982 y_1$	$- \cdot191 z_1 +$	$\cdot927 y_2$	$- \cdot375 z_2 +$	$\cdot839 y_3$	$- \cdot545 z_3$
$- \cdot129 =$	$x +$	$\cdot934$	$+ \cdot358 +$	$\cdot743$	$+ \cdot669 +$	$\cdot454$	$+ \cdot891$
$- \cdot219 =$	$x +$	$\cdot731$	$+ \cdot682 +$	$\cdot070$	$+ \cdot998 -$	$\cdot629$	$+ \cdot777$
$- \cdot259 =$	$x +$	$\cdot259$	$+ \cdot966 -$	$\cdot866$	$+ \cdot500 -$	$\cdot707$	$- \cdot707$
$- \cdot256 =$	$x -$	$\cdot391$	$+ \cdot920 -$	$\cdot695$	$- \cdot719 +$	$\cdot934$	$- \cdot358$
$- \cdot162 =$	$x -$	$\cdot857$	$+ \cdot515 +$	$\cdot469$	$- \cdot883 +$	$\cdot052$	$+ \cdot999$
$- \cdot122 =$	$x -$	$1 \cdot000$	$\cdot000 +$	$1 \cdot000$	$\cdot000 -$	$1 \cdot000$	$\cdot000$
$+ \cdot038 =$	$x -$	$\cdot899$	$- \cdot438 +$	$\cdot616$	$+ \cdot788 -$	$\cdot208$	$- \cdot978$
$+ \cdot246 =$	$x -$	$\cdot656$	$- \cdot755 -$	$\cdot139$	$+ \cdot990 +$	$\cdot839$	$- \cdot545$
$+ \cdot334 =$	$x -$	$\cdot292$	$- \cdot956 -$	$\cdot829$	$+ \cdot559 +$	$\cdot777$	$+ \cdot629$
$+ \cdot262 =$	$x +$	$\cdot358$	$- \cdot934 -$	$\cdot743$	$- \cdot669 -$	$\cdot891$	$+ \cdot454$
$+ \cdot166 =$	$x +$	$\cdot809$	$- \cdot588 +$	$\cdot309$	$- \cdot951 -$	$\cdot309$	$- \cdot951$

From these the following normal equations are derived:

x	y_1	z_1	y_2	z_2	y_3	z_3	$= l$
$+12$	$- \cdot022$	$- \cdot421$	$+ \cdot826$	$+ \cdot907$	$+ \cdot151$	$- \cdot334$	$-0 \cdot003$
	$+6 \cdot43$	$+ \cdot451$	$+ \cdot064$	$- \cdot374$	$+ \cdot036$	$+ \cdot413$	$+0 \cdot045$
		$+5 \cdot57$	$+ \cdot043$	$- \cdot082$	$- \cdot494$	$+ \cdot828$	$-1 \cdot648$
			$+5 \cdot60$	$- \cdot040$	$- \cdot264$	$- \cdot137$	$-0 \cdot248$
				$+6 \cdot40$	$+ \cdot280$	$+ \cdot237$	$-0 \cdot017$
					$+5 \cdot92$	$- \cdot199$	$+0 \cdot393$
						$+6 \cdot08$	$-0 \cdot225$

The diagonal coefficients are so much superior to any of the side ones that the indirect method of elimination* can be readily applied, and we have, accurately enough for our purpose, the following values:

$$x = - \cdot008 \qquad z_2 = - \cdot006$$
$$y_1 = + \cdot028 \qquad y_3 = + \cdot040$$
$$z_1 = - \cdot295 \qquad z_3 = + \cdot001$$
$$y_2 = - \cdot039$$

substituting these we get

	$^\circ$	$'$		
$a_1 =$	174	35	$h_1 =$	$+ \cdot297$
$a_2 =$	81	15	$h_2 =$	$- \cdot039$
$a_3 =$	88	34	$h_3 =$	$+ \cdot040$

The substitution of the above quantities in the 12 conditional equations gives the following residuals:

0·024	0·007
·027	·020
·003	·017
·022	·007
·032	·013
·032	·003

If m = the number of equations and
μ = the number of constants to be determined,

* Die Ausgleichungsrechnungen, etc., von Gerling, p. 386.

the probable error of one determination is

$$0{\cdot}675\sqrt{\frac{[vv]}{m-\mu}}$$

from the above residuals we get $r = \pm 0.''021$
Substituting the values of the constants in Fourier's theorem

$$\begin{aligned} f(\varphi) = 24{\cdot}386 &+ 0{\cdot}297 \sin (174\ 35 + \varphi) \\ &- {\cdot}039 \sin (\ 81\ 15 + 2\varphi) \\ &+ {\cdot}040 \sin (\ 88\ 34 + 3\varphi) \end{aligned}$$

and the formula becomes

$$\begin{aligned} \text{Latitude} = 24{\cdot}386 &+ 0{\cdot}028 \cos \varphi - 0{\cdot}297 \sin \varphi. \\ &- {\cdot}039 \cos 2\varphi - {\cdot}006 \sin 2\varphi. \\ &+ {\cdot}040 \cos 3\varphi + .001 \sin 3\varphi. \end{aligned}$$

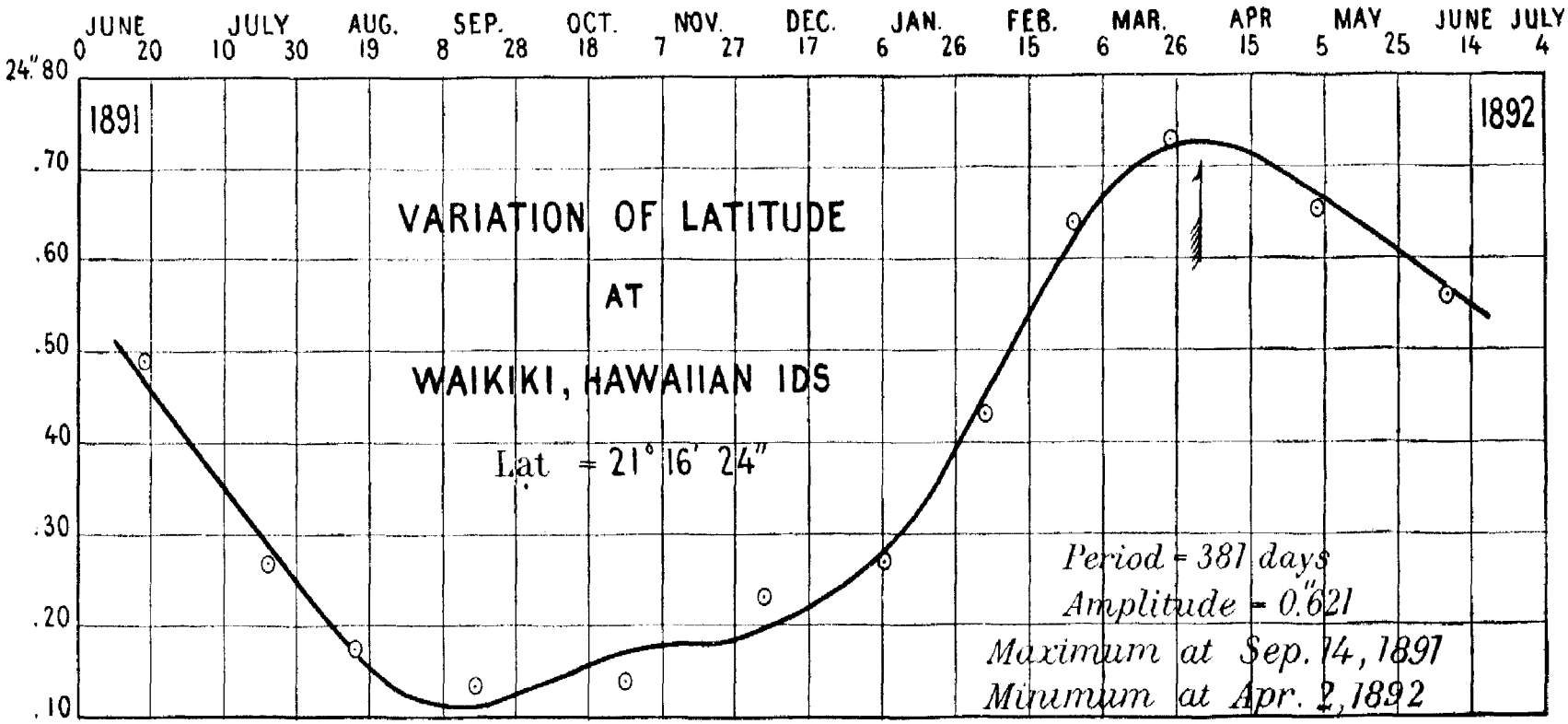
This equation is shown graphically in illustration No. 16 as well as the observational values from which it was derived.

The first differential coefficient becomes zero at 71.4° and at 261.3°. Hence there is a minimum at September 14, 1891, and a maximum at April 2, 1892. The amplitude is 0.″621.

Of course a still closer agreement could be made by using the quadruple angle and so on until we should arrive at the number of conditional equations. Just how far this approximation must be carried in order to bring out the real facts of the case can only be determined by appealing to the observations themselves. Some light is thrown on the subject by noting the concordance between theory and observation for successive applications of multiple angles. To this end are shown in illustrations No. 17 and No. 18 the curves for equations depending on the single and double angles, and also a superposition of one over the other for the sake of direct comparison.

In the adjustment by least squares, when we have arrived at such a point that the probable error of one of the conditional equations is practically equal to that calculated independently from the errors of observation and declination, we may safely conclude that a sufficient number of constants have been employed, and that all facts have been brought out which the accuracy of the observation warrants.

In the present case we have the following criterion. From the 63 pairs of stars used, the average probable error of a declination is 0.″257 and that of the mean of two declinations would be 0.″181. The probable error of observation as already given is 0.″155, so that the result of one determination of latitude would be subject to an error of $\sqrt{0.181^2 + 0.^2155}$ or 0.″238. Each one of the latitudes used in forming the conditional equations is the result of combining about 200 individual determinations, so that we should expect, since the precision increases

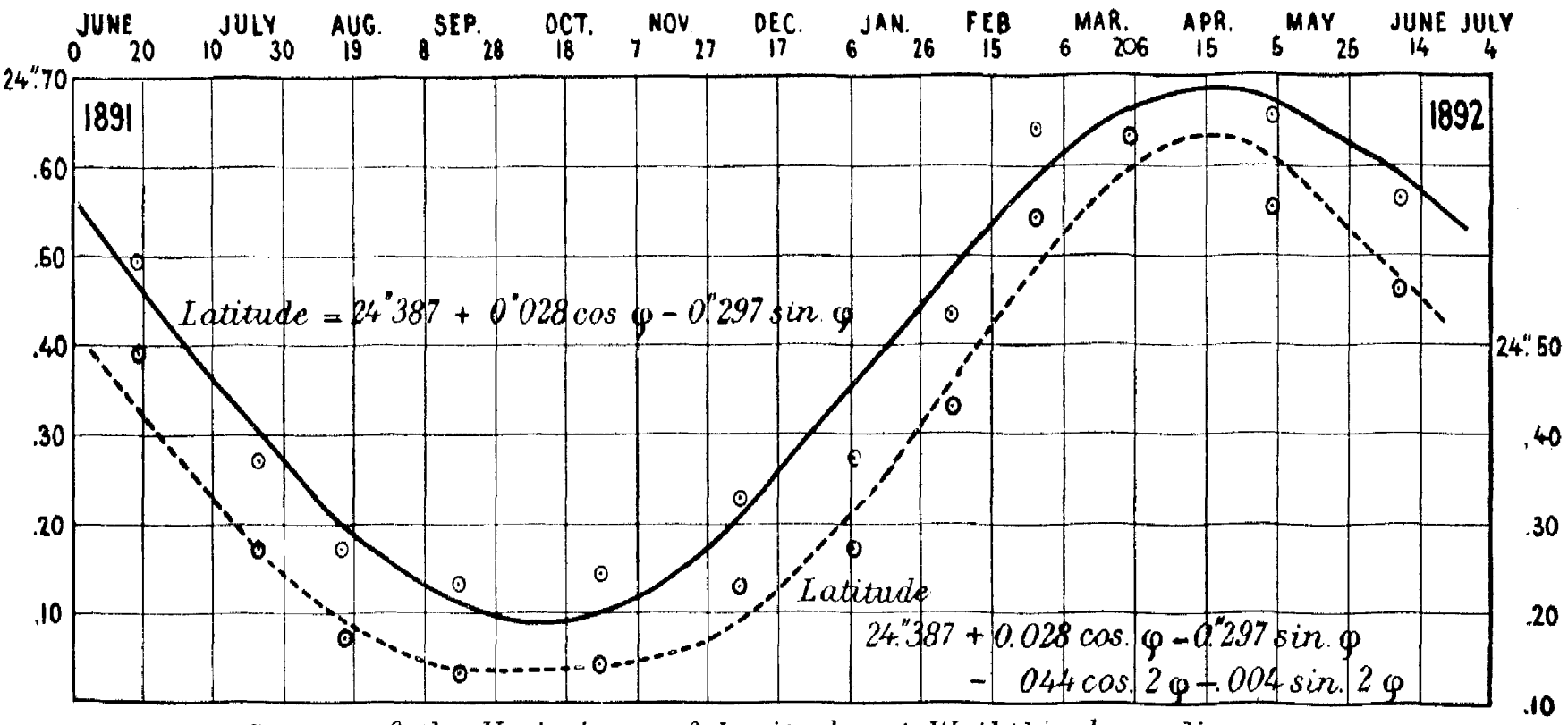


$$\text{Equation of Curve} = \left\{ \begin{aligned} \text{Latitude} = {} & 24.''386 + 0.''028 \cos.\varphi - 0.''297 \sin.\varphi \\ & - .039 \cos.2\varphi - .006 \sin.2\varphi \\ & + .040 \cos.3\varphi + .001 \sin.3\varphi \end{aligned} \right.$$

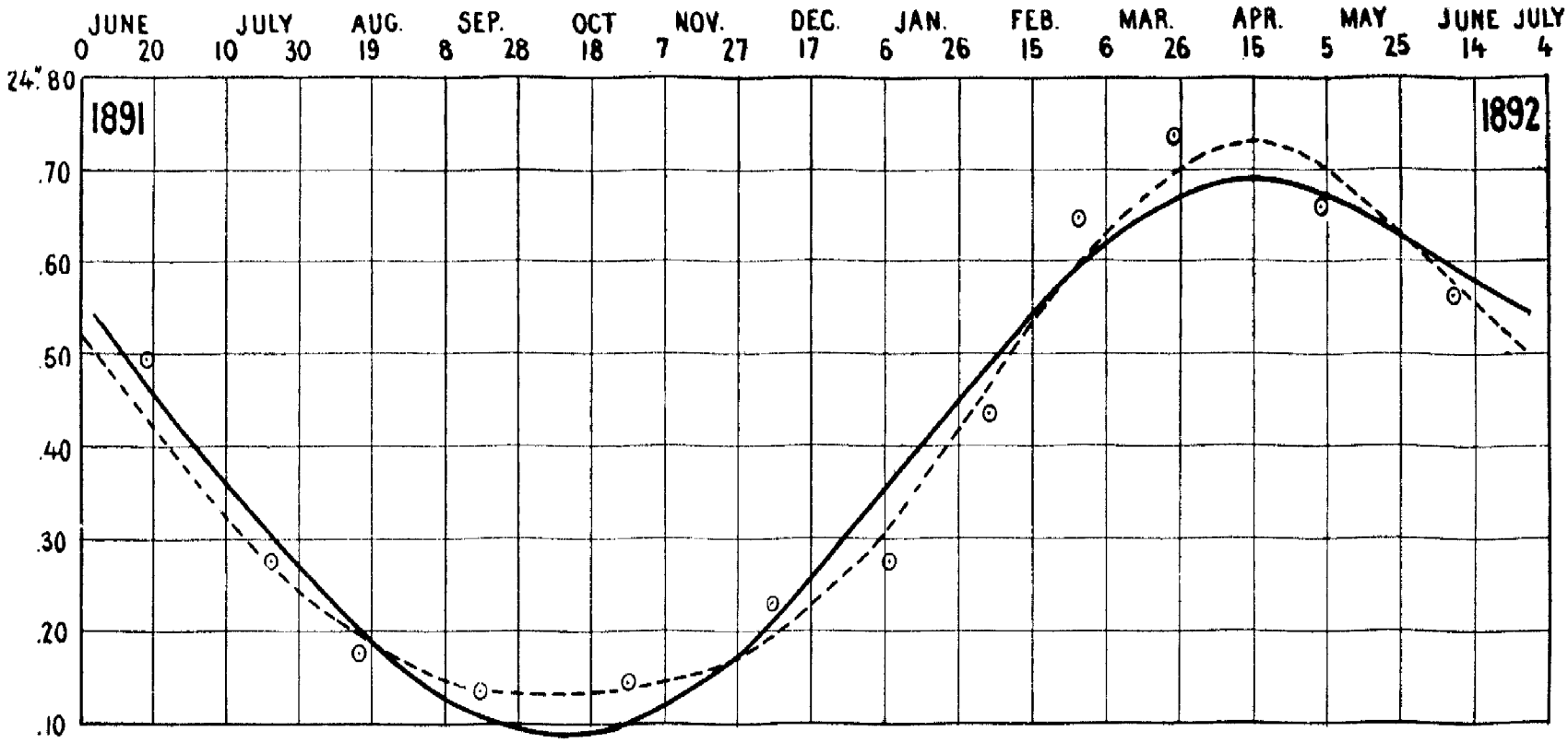
φ = angular distance of pole from July 0, 1891.

as the square root of the number of observations, a probable error of 0.″017 for each one of these collective results. As a matter of fact the least square work gives 0.″021 for this quantity, so that we may consider that the constants employed are sufficient and necessary to express the observations. The use of the triple angle is therefore justified, and the introduction of constants terminates here.

Before closing, it should be added that the difference of about 0.″5 between the determinations of Dr. Marcuse and my own is due to two causes: the position of the observatories and the adopted star places. The Coast and Geodetic Survey observatory was 0.″3 south of that of the International Association (see illustration No. 11), and the declinations deduced by Mr. Farquhar for the Bradley–Auwers stars account for the remainder.



Curves of the Variations of Latitude at Waikiki depending on single and double angles.



Superposition of Curves depending on single and double angles.

APPENDIX No. 3—1892.

ON THE RESULTS OF SPIRIT LEVELING OF PRECISION BETWEEN OKOLONA, MISS., AND ODIN, ILL., FROM OBSERVATIONS MADE BY J. B. WEIR, ASSISTANT; ISAAC WINSTON AND P. A. WELKER, SUBASSISTANTS, AND F. A. YOUNG, AID.

A report by CHARLES A. SCHOTT, Assistant, and Chief of the Computing Division.
Submitted for publication December 20, 1892.

I have the honor to submit herewith a report on the results of spirit leveling of precision between Okolona, Miss., and Odin, Ill. This is in continuation of the exact leveling between the Gulf of Mexico (at Biloxi and Mobile) and St. Louis, Mo. The latter place is reached from Odin by the levels along the the thirty-ninth parallel. Respecting the starting point, Okolona, see my report of September 28, 1888, and for Odin and St. Louis, see Appendix No. 8, C. and G. S. Rep. for 1882, p. 555.

Route line.—From Okolona the line follows the Mobile and Ohio Railroad to Corinth, Miss., and Jackson, Tenn.; thence by the Illinois Central Railroad to Cairo, Ill., and Villa Ridge, Ill. (close to it); from here it continues by the same road to Odin, Ill.; thence to St. Louis, Mo., along the Ohio and Mississippi Railroad. See accompanying diagram (illustration No. 19).

The distances are as follows:

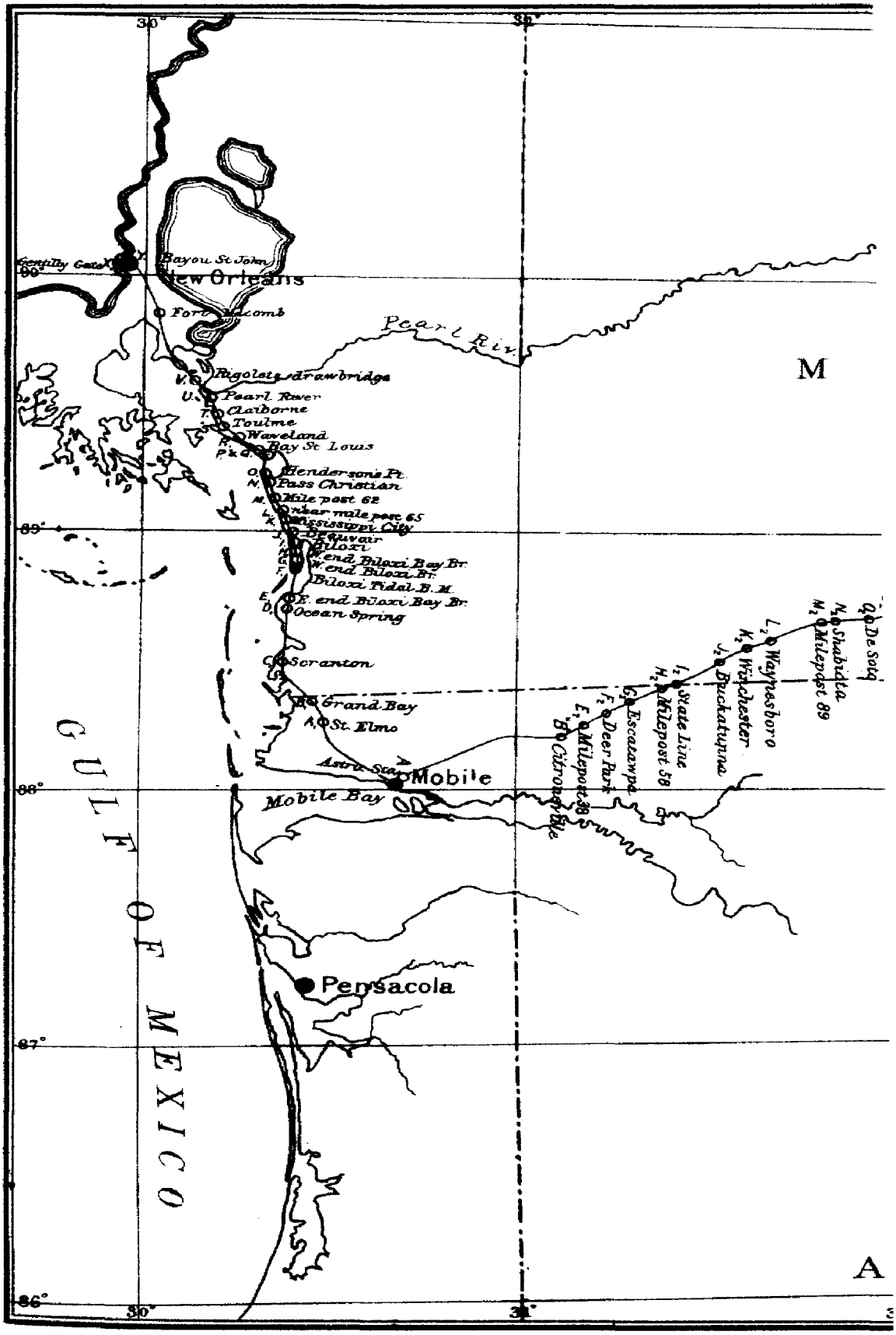
Biloxi (Tidal mark) to Mobile (A)	94·3 km. or	58·6 st. miles.
Mobile (A) to Okolona (K)	423·5	263·1
Okolona (K) to Greenfield (XV)	265·4	164·9
Greenfield (XV) to Villa Ridge (Z_3)	128·1	79·6
Villa Ridge (Z_3) to Odin (M_3 or V)	176·0	109·4
Total from Biloxi to Odin,	1 087·3	675·6

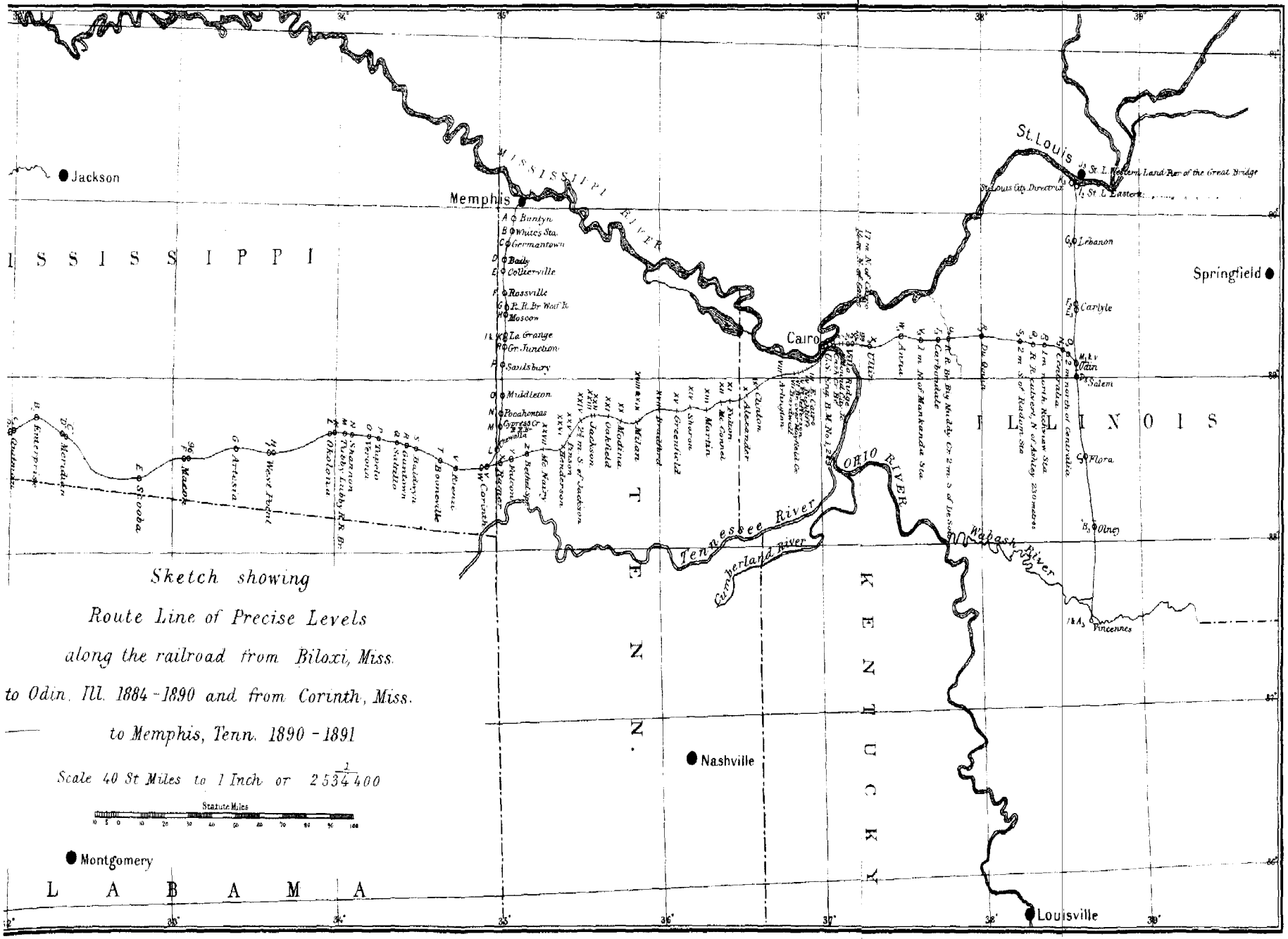
Observers and dates.—Section I, from Okolona, Miss., to Jackson, Tenn.: forward measure by Subassistant J. Winston, backward measure by Mr. F. A. Young, and from Jackson, Tenn., to Greenfield, Tenn., backward by Winston and forward by Young; between October, 1889, and March, 1890. Section II, from Greenfield to Villa Ridge, Ill.: forward and backward measures by Subassistants Winston and P. A.

Welker, between October, 1888, and January, 1889. Section III, from Villa Ridge to Odin, Ill., between April and June, 1885, Assistant J. B. Weir, observer.

Instruments and method of observing.—Section 1 was run independently forward and backward, the observers changing direction at Jackson. Winston used geodetic micrometer level No. 5 and Young used No. 6; these levels were made at the Survey Office in 1888. Section II was run twice, independently, once forward and once backward, the observers changing directions frequently; the observers used geodetic micrometer levels No. 2 and No. 3. Section III was the last line run by the old method of two parallel and simultaneous lines, alternate stretches (of 5 to 10 miles) being run in opposite directions; micrometer level No. 1 was employed. The rods were generally 200 metres apart, with the instrument mounted midway between them.

The instrumental constants are as follows:


30°
35°
90°
89°
88°
87°
86°
30°
31°
Gentilly Gate
Bayou St. John
New Orleans
Fort Pike
Pearl Riv.
Rigolets drawbridge
Pearl River
Claiborne
Toulme
Waveland
Bay St. Louis
Pk d.
Henderson's Pt.
Pass Christian
Mile post 62
Mississippi City
Biloxi
E. end Biloxi Bay Br.
E. end Biloxi Br.
Biloxi Tidal B.M.
E. end Biloxi Bay Br.
Ocean Spring
Scranton
Grand Bay
St. Elmo
Astro. Sta.
Mobile
Mobile Bay
Pensacola
GULF OF MEXICO
De Soto
Shubuta
Milepost 89
Waynesboro
Wahchester
Pachatuna
State Line
Milepost 58
Escatawpa
Deer Park
Milepost 38
Citronelle
M
A


Jackson
Memphis
MISSISSIPPI RIVER
MISSISSIPPI
Cairo
St. Louis
Springfield
ILLINOIS
Ohio River
Tennessee River
Cumberland river
Wabash River
TENN.
KENTUCKY
Nashville
Louisville
Montgomery
ALABAMA
Sketch showing
Route Line of Precise Levels
along the railroad from Biloxi, Miss.
to Odin, Ill. 1884-1890 and from Corinth, Miss.
to Memphis, Tenn. 1890-1891
Scale 40 St Miles to 1 Inch or 2534400
Statute Miles

Spirit leveling instruments	No. 1.	No. 2.	No. 3.	No. 5.	No. 6.
Aperture of telescope	3·5cm	4·3cm	4·3cm	2·9cm	2·9cm
Focal length of telescope	40·7cm	41·0cm	41·0cm	34·8cm	34·8cm
Magnifying power of telescope	26	24	25	28	28
Low-power eyepiece inserted		Sept., 1887	Sept., 1887		
Value of 1 division of striding level	5·60 5·70 5·79	3·37	2·73	2·11 2·10 2·08	2·44 2·53 2·62
when determined	July 23, 1883 Nov. 21, 1885	Apr. 2, 1887	June 2, 1887	Sept. 26, 1888 Apr. 8, 1890	May 23, 1888 Apr. 8, 1890
Inequality of collars (a ± sign indicates object end larger / smaller)	—2·47 —1·17	+0·12 (+0·2) +0·28	—0·11 (+0·1) +0·22	—0·25 (—0·42) —0·48	+0·18 (+0·10) +0·07
when determined	Apr. 4, 1885 June 12, 25, 1885	Oct. 4, 1888 Dec. 8, 1888	Oct. 11, 1888 Dec. 12, 1888	July 24, 1889 Mar. 5, 1890	Aug. 10, 1889 Mar. 15, 1890
Value of 100 div. of micrometer screw	443·1	257·5*	257·5*	313·1 312·0 311·6	310·6 312·0 312·0
when determined	Aug. and Sept., 1877	Apr., 1887	June, 1880	Sept. 19, 22, 1888 July 22, 23, 1889	June 24, 25, 1889 July 22, 23, 1889
Angular value of telemeter threads when determined	17′ 01·0″ Nov. 17, 1884	16′ 39·3″ Jan. 19, 1887	14′ 00·7″ Jan. 19, 1887	18′ 58·5″ July 22, 1889	18′ 50·6″ July 22. 1889
Index corrections to metric rods †	mm E 62·4	mm C 60·0 61·2	mm D 59·8 61·2	mm L 0·0 —0·5	mm N 0·0 —0·5
(projection beyond zero line)	F 59·9	(60·6)	(60·5)	M 0·0 —0·5	O 0·0 —0·6
Temperature of rod when of standard length	16·7C 18·9C	20·3C	14·4C	21·7C 21·7	21·7C 21·7
Correction, when determined	June 5, 1885	Oct. 4, 1888 Jan. 5, 1889	Oct. 4, 1888 Jan. 5, 1889	July 23, 1889 Mar. 14, 1890	July 23, 1889 Mar. 14. 1890

*See Appendix No. 14, C. and G., Survey Report for 1887. † Previous to 1889 the graduated brass rod supporting the sliding target was fastened to the wooden rod only at its middle part, allowing expansion and contraction of the scale up as well as down; at the bottom of the rod there was sufficient space left to permit of the greatest expected expansion under high temperature. The index correction became thus large and variable. In July, 1889, this construction was changed, the brass scale and wooden rod were screwed fast at their bottom ends and the former was bent under the latter and in close connection with the contact knob which stands for the zero of the graduation; hence, there is no index correction to begin with, but wear will produce a small negative one.

Instruments Nos. 5 and 6 are of the same pattern as No. 1; see plate No. 46 in C. and G. Survey Report for 1880. Coefficient of expansion of brass scale of rods 0·000018 for centigrade scale.

A description of the permanent bench marks is appended to this report.

Computation and results.—Field computations were made by the respective observers; the second or independent office computation were made by the following computers under the general supervision of Mr. L. A. Bauer, viz: Section I by Subassistant J. Nelson, aided by A. Bonnot, J. B. Boutelle, and H. F. Flynn (October, 1892); Section II by F. A. Young, while temporarily attached to the computing Division, aided by J. Page; Section III by F. M. Little (September, 1888). Special tables of corrections for curvature, refraction, collar inequality, and temperature were constructed for each instrument to facilitate their application. The two computations were collated and the final results are as follows:

SECTION I.—FROM OKOLONA, MISS., TO GREENFIELD, TENN., OCTOBER, 1889, TO MARCH, 1890.

Date.	Benchmark. From—	Benchmark. To—	Distance. Between successive marks.	Distance. From initial mark.	Difference of height between marks. Direction of measure. Forward (northward).	Difference of height between marks. Direction of measure. Backward (southward).	Difference of height between marks. Mean.	Discrepancy. Partial. F–B	Discrepancy. Total. Accumulated.	Height of mark above average Gulf level at Biloxi, Miss.
1889–'90.			*km.*	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
Dec.		K		0·000						+94·6137‡
4	K	L	0·058	0·058	—1·5472	------	------	---	---	+93·0665‡
4 7	K	52	0·921	0·921	—5·9169	—5·9164	5·9166	—0·5	— 0·5	88·6971
4 7	52	53	1·074	1·995	—4·7158	—4·7107	—4·7132	—5·1	— 5·6	83·9839
4, 7, 6, 7	53	54	1·075	3·070	*—3·8072	*—3·8043	—3·8058	—2·9	— 8·5	80·1781
4 6	54	55	1·054	4·124	—1·7082	—1·7080	—1·7081	—0·2	— 8·7	78·4700
4 6	55	56	1·080	5·204	—0·7046	—0·7071	—0·7059	+2·5	— 6·2	77·7641
4 6	56	57	1·166	6·370	—0·4843	—0·4828	—0·4836	—1·5	— 7·7	77·2805
4, 5, 5, 6	57	58	1·347	7·717	+4·0848	+4·0890	+4·0869	+4·2	—11·9	81·3674
5 5	58	M	0·832	8·549	—5·3555	—5·3586	—5·3570	+3·1	— 8·8	76·0104
5 5	M	59	1·100	9·649	+3·5479	+3·5437	+3·5458	+4·2	— 4·6	79·5562
5, 9, 13, 12, 13	59	60	1·222	10·871	*—3·0834	*—3·0874	—3·0854	+4·0	— 0·6	76·4708
9, 13, 12, 13	60	61	1·270	12·141	*—3·7395	*—3·7444	—3·7420	+4·9	+ 4·3	72·7288
9, 13, 12, 13	61	62	0·801	12·942	*+1·8380	*+1·8340	+1·8360	+4·0	+ 8·3	74·5648
12 12	62	N	0·084	13·026	+2·5265	+2·5270	+2·5268	—0·5	+ 7·8	+77·0916
9 12	62	63	1·253	14·195	+2·2186	+2·2183	+2·2184	+0·3	+ 8·6	76·7832
9 12	63	64	1·238	15·433	—2·6445	—2·6433	—2·6439	—1·2	+ 7·4	74·1393
9 11	64	65	1·283	16·716	+2·4139	+2·4083	+2·4111	+5·6	+13·0	76·5504
9, 10, 14, 11, 14	65	66	1·287	18·003	*+3·4014	*+3·4008	+3·4011	+0·6	+13·6	79·9515
10, 14, 11, 14	66	67	1·290	19·293	*+4·6888	*+4·6816	+4·6852	+7·2	+20·8	84·6367

‡ Taken from Appendix No. 10, C. and G. Survey Report for 1888, p. 422.
* An asterisk attached to any number indicates that it is a mean value of two determinations; a double asterisk denotes a mean of three or more values.

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SECTION I.—FROM OKOLONA, MISS., TO GREENFIELD, TENN., OCTOBER, 1889, TO MARCH, 1890.

Date.		Benchmark. From—	Benchmark. To—	Distance. Between successive marks.	Distance. From initial mark.	Difference of height between marks. Direction of measure. Forward (northward).	Backward (southward).	Mean.	Discrepancy. Partial. F–B	Discrepancy. Total. Accumulated.	Height of mark above average Gulf level at Biloxi, Miss.
1889–'90. Dec.				*km.*	*km.*	*m.*	*m.*	*m*	*mm.*	*mm.*	*m.*
10	11	67	68	1·282	20·575	+1·6234	+1·6248	+1·6241	−1·4	+19·4	86·2608
10, 14	11, 14	68	69	1·104	21·679	*+5·8742	*+5·8796	+1·8769	−5·4	+14·0	92·1377
14	14	69	O	0·169	21·848	+3·9708	+3·9707	+3·9708	−0·1	+14·1	+96·1085
10	18	69	70	1·295	22·974	+4·7445	+4·7477	+4·7461	−3·2	+10·8	96·8838
10, 19	18, 19	70	71	1·289	24·263	*−7·3977	*−7·3967	−7·3972	−1·0	+ 9·8	89·4866
10, 19	18, 19	71	72	1·265	25·528	*−7·3938	*−7·3907	−7·3922	−3·1	+ 6·7	82·0944
10, 16, 19	18, 19	72	73	1·312	26·840	*−3·6463	*−3·6376	−3·6420	−8·7	− 2·0	78·4524
16	17	73	74	1·313	28·153	+2·0135	+2·0100	+2·0118	+3·5	+ 1·5	80·4642
16	17	74	75	0·765	28·918	+2·5369	+2·5380	+2·5374	−1·1	+ 0·4	83·0016
16	16	75	P	0·144	29·062	+1·9040	+1·9040	+1·9040	0·0	+ 0·4	+ 84·9056
16	17	75	76	1·196	30·114	−2·1084	−2·1057	−2·1070	−2·7	− 2·3	80·8946
16, 20	17, 20	76	77	1·204	31·318	*+0·4379	*+0·4416	+0·4398	−3·7	− 6·0	81·3344
16	17	77	78	1·068	32·386	−0·9050	−0·9030	−0·9040	−2·0	− 8·0	80·4304
16, 20	23	78	79	1·296	33·682	+1·3687	+1·3723	+1·3705	−3·6	−11·6	81·8009
20	23	79	80	1·017	34·699	*+0·6890	*+0·6929	+0·6910	−3·9	−15·5	82·4919
20	23	80	81	0·917	35·616	+0·6953	+0·6992	+0·6972	−3·9	−19·4	83·1891
20	23	81	82	0·923	36·539	+0·5114	+0·5175	+0·5144	−6·1	−25·5	83·7035
20	23	82	83	0·968	37·507	+1·9064	+1·8993	+1·9028	+7·1	−18·4	85·6063
20, 21	21	83	84	0·976	38·483	+1·5304	+1·5336	+1·5320	−3·2	−21·6	87·1383
21, 28	21, 27	84	85	1·014	39·497	*+1·0296	*+1·0258	+1·0277	+3·8	−17·8	88·1660

21	21	85	86	1·000	40·497	+1·9659	+1·9614	+1·9636	+4·5	—13·3	90·1296
21	27	86	87	0·944	41·441	+1·2753	+1·2745	+1·2749	+0·8	—12·5	91·4045
21	27	87	88	0·987	42·428	+3·5592	+3·5562	+3·5577	+3·0	— 9·5	94·9622
27	27	88	Q	0·072	42·500	+0·5042	+0·5042	+0·5042	0·0	— 9·5	+95·4664
24, 28	27, 28, 30	88	89	1·340	43·768	**+0·7279	**+0·7334	+0·7306	—5·5	—15·0	95·6928
24	26, 27	89	90	1·321	45·089	+2·1319	+2·1293	+2·1306	+2·6	—12·4	97·8234
24	26	90	91	1·229	46·318	+3·1698	+3·1686	+3·1692	+1·2	—11·2	100·9926
24	26	91	92	1·160	47·478	+5·2634	+5·2627	+5·2630	+0·7	—10·5	106·2556
24	26	92	93	1·123	48·601	+0·6343	+0·6371	+0·6357	—2·8	—13·3	106·8913
24	26	93	94	1·166	49·767	*+7·5600	*+7·5636	+7·5618	—3·6	—16·9	114·4531
24	24	94	R	0·259	50·026	+6·9860	+6·9848	+6·9854	+1·2	—15·7	+121·4385
30	Dec. 26) Jan. 4)	94	95	0·985	50·752	—0·7006	—0·6963	—0·6984	—4·3	—21·2	113·7547
30, J. 4,	Jan. 4	95	96	1·015	51·767	*—5·9547	*—5·9554	—5·9548	+1·2	—20·0	107·7999
30, 31, J. 4,	Jan. 3, 4	96	97	1·020	52·787	*—5·0448	*—5·0415	—5·0432	—3·3	—23·3	102.7567
31	3	97	98	1·012	53·799	—0·7431	—0·7449	—0·7440	+1·8	—21·5	102·0127
31	3	98	99	1·014	54·813	—1·0366	—1·0392	—1·0379	+2·6	—18·9	100·9748
31	3	99	100	1·032	55·845	+1·3272	+1·3230	+1·3251	+4·2	—14·7	102·2999
Dec.	Jan.										
31	3	100	101	1·042	56·887	+7·8915	+7·8930	+7·8922	—1·5	—16·2	110·1921
31	3	101	102	1·247	58·134	+3·4788	+3·4736	+3·4762	+5·2	—11·0	113·6683
J. 8	8	102	S	0·156	58·290	+0·5366	+0·5362	+0·5364	+0·4	—10·6	+114·2047
D. 31	3	102	103	1·005	59·139	—6·2119	—6·2150	—6·2134	+3·1	— 7·9	107·4549
D. 31	2	103	104	0·996	60·135	—3·8562	—3·8564	—3·8563	+0·2	— 7·7	103·5986
D. 31	7	104	105	1·001	*61·136	+0·7974	+0·7998	+0·7986	—2·4	—10·1	104·3972
D. 31, J. 7	6	105	106	1·725	62·861	—0·5222	—0·5227	—0·5224	+0·5	— 9·6	103·8748
Jan. 7	6	106	107	1·230	64·091	+0·8983	+0·8981	+0·8982	+0·2	— 9·4	104·7730
7, 8	6, 8	107	108	1·101	65·192	*+0·3100	*+0·3146	+0·3123	—4·6	—14·0	105·0853
7	6	108	109	1·226	66·418	+3·6314	+3·6375	+3·6344	—6·1	—20·1	108·7197
7	6	109	110	0·781	67·199	—0·3184	—0·3160	—0·3172	—2·4	—22·5	108·4025

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Date.	Benchmark. From—	Benchmark. To—	Distance. Between successive marks	Distance. From initial mark.	Difference of height between marks. Direction of measure. Forward (northward).	Difference of height between marks. Direction of measure. Backward (southward).	Difference of height between marks. Mean.	Discrepancy. Partial. F–B	Discrepancy. Total, Accumulated.	Height of mark above average Gulf level at Biloxi, Miss.
1889–'90. Jan.			*km.*	*km.*	*m.*	*m*	*m.*	*mm.*	*mm.*	*m.*
7 10	110	111	1·272	68·471	+2·5165	+2·5171	+2·5168	—0·6	— 23·1	110·9193
8 10	111	112	1·220	69·691	+1·5695	+1·5746	+1·5720	—5·1	— 28·2	112·4913
8, 16 10, 16	112	113	1·047	70·738	*+5·3678	*+5·3742	+5·3710	—6·4	— 34·6	117·8623
8, 14 10, 14	113	114	1·040	71·778	*+4·3813	*+4·3836	+4·3824	—2·3	— 36·9	122·2447
8 10	114	115	1·280	73·058	+9·3433	+9·3463	+9·3448	—3·0	— 39·9	131·5895
9 9	115	116	1·200	74·258	+9·3547	+9·3550	+9·3548	—0·3	— 40·2	140·9443
9, 16 9, 16	116	117	1·098	75·356	*+8·3130	*+8·3150	+8·3140	—2·0	— 42·2	149·2583
9, 16 9, 14, 16	117	118	0·886	76·242	*+6·2470	*+6·2522	+6·2496	—5·2	— 47·4	155·5079
11 11	118	T	0·142	76·384	+6·9047	+6·9055	+6·9051	—0·8	— 48·2	+162·4130
11 14	118	119	0·890	77·132	—7·0490	—7·0484	—7·0487	—0·6	— 48·0	148·4592
11 14	119	120	0·977	78·109	+1·0110	+1·0141	+1·0126	—3·1	— 51·1	149·4718
11 14	120	121	1·305	79·414	+0·9798	+0·9824	+0·9811	—2·6	— 53·7	150·4529
11, 17 13, 14, 17	121	122	1·279	80·693	*—7·0708	*—7·0726	—7·0717	+1·8	— 51·9	143·3812
11 13	122	123	1·301	81·994	—4·3645	—4·3612	—4·3628	—3·3	— 55·2	139·0184
11, 17 13, 17	123	124	0·970	82·964	*+4·9489	*+4·9473	+4·9481	+1·6	— 53·6	143·9665
11 13	124	125	1·172	84·136	—3·9849	—3·9878	—3·9864	+2·9	— 50·7	139·9801
11 13	125	126	1·145	85·281	—4·1770	—4·1751	—4·1760	—1·9	— 52·6	135·8041
11, 17 23	126	127	0·986	86·267	—3·7575	—3·7540	—3·7558	—3·5	— 56·1	132·0483
17 22, 23	127	128	0·983	87·250	—0·0649	—0·0624	—0·0636	—2·5	— 58·6	131·9847
17 22	128	129	0·981	88·231	+1·7627	+1·7633	+1·7630	—0·6	— 59·2	133·7477

17	22	129	130	1·169	89·400	−0·9090	−0·9105	−0·9098	+ 1·5	— 57·7	132·8379
23	23	130	U	0·254	89·654	+5·7252	+5·7284	+5·7268	−3·2	— 60·9	+138·5647
17, 18	22	130	131	0·973	90·373	−3·1784	−3·1839	−3·1812	+5·5	— 52·2	129·6567
18	22	131	132	1·000	91·373	−1·0759	−1·0756	−1·0758	−0·3	— 52·5	128·5809
18	22	132	133	1·033	92·406	−0·9033	−0·9087	−0·9060	+5·4	— 47·1	127·6749
18	21	133	134	0·973	93·379	+0·1574	+0·1579	+0·1576	−0·5	— 47·6	127·8325
18	21	134	135	0·984	94·363	+0·0889	+0·0874	+0·0882	+1·5	— 46·1	127·9207
18	21	135	136	0·990	95·353	−1·1058	−1·1088	−1·1073	+3·0	— 43·1	126·8134
18	21	136	137	1·105	96·458	−0·0470	−0·0423	−0·0446	−4·7	— 47·8	126.7688
18, 23	21, 23	137	138	0·717	97·175	*+0·0040	*+0·0028	+0·0034	+ 1·2	— 46·6	126·7722
18	20	138	139	0·954	98·129	−0·1358	−0·1386	−0·1372	+ 2·8	— 43·8	126·6350
23	20	139	140	1·110	99·239	+1·3019	+1·3017	+1·3018	+0·2	— 43·6	127·9368
24	20	140	141	1·288	100·527	+2·9675	+2·9705	+2·9690	−3·0	— 46·6	130·9058
24	27	141	142	1·130	101·657	+2·7087	+2·7133	+2·7110	−4·6	— 51·2	133·6168
24	25, 27	142	143	1·073	102·730	+3·8868	+3·8914	+3·8891	−4·6	— 55·8	137·5059
24	25	143	144	1·081	103·811	+1·8698	+1·8751	+1·8724	−5·3	— 61·1	139·3783
24	25	144	145	1·182	104·993	−6·8448	−6·8457	−6·8452	+0·9	— 60·2	132·5331
24	25	145	146	0·999	105·992	−3·6295	−3·6303	−3·6299	+0·8	— 59·4	128·9032
24	25	146	147	0·996	106·988	+0·9565	+0·9526	+0·9546	+3·9	— 55·5	129·8578
24, 27	25	147	148	1·079	108·067	+0·3503	+0·3497	+0·3500	+0·6	— 54·9	130·2078
27	25	148	149	1·005	109·072	+2·0512	+2·0550	+2·0531	−3·8	— 58·7	132·2609
31	31	149	V	0·384	109·456	+4·9679	+4·9664	+4·9672	+ 1·5	— 57·2	+137·2281
31	31	V	W	0·041	109·497	+0·0568	+0·0567	+0·0568	+0·1	— 57·1	+137·2849
27	25, 31	149	150	0·968	110·040	+1·9026	+1·9078	+1·9052	−5·2	— 63·9	134·1661
27, 28	31	150	151	0·990	111·030	+2·1269	+2·1323	+2·1296	−5·4	— 69·3	136·2957
28, 31	Feb. 1, 3	151	152	0·994	112·024	*+1·8222	**+1·8268	+1·8245	−4·6	— 73·9	138·1202
28, 31	Jan. 30, 31	152	153	0·976	113·000	*+2·4148	*+2·4192	+2·4170	−4·4	— 78·3	140·5372
31	31	153	154	0·973	113·973	+4·1856	+4·1878	+4·1867	−2·2	— 80·5	144·7239
28	30	154	155	1·260	115·233	+6·5890	+6·5941	+6·5916	−5·1	— 85·6	151·3155
28	30	155	156	1·265	116·498	−1·3296	−1·3304	−1·3300	+0·8	— 84·8	149·9855
28, F. 1	30, F. 1	156	157	1·272	117·770	*−7·4133	*−7·4098	−7·4116	−3·5	— 88·3	142·5739
28, F. 1	30, F. 1	157	158	0·985	118·755	*−5·4414	*−5·4346	−5·4380	−6·8	— 95·1	137·1359

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					Direction of measure.		Mean.	Partial.	Total.	
	From—	To—	Between successive marks.	From initial mark.	Forward (northward).	Backward (southward).		F–B	Accumulated.	
1889–'90. Jan.			*km.*	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
28 29	158	159	1·164	119·919	−3·1102	−3·1112	−3·1107	+1·0	− 94·1	134·0252
28, F. 1 29, F. 1	159	160	1·182	121·101	* −2·0511	* −2·0458	−2·0484	−5·3	− 99·4	131·9768
F. 3 29, F. 5	160	161	1·153	122·254	+0·2026	+0·2052	+0·2039	−2·6	−102·0	132·1807
3 5	161	162	1·126	123·380	* −5·9732	* −5·9765	−5·9748	+3·3	− 98·7	126·2059
3 5	162	163	1·257	124·637	−1·5643	−1·5676	−1·5660	+3·3	− 95·4	124·6399
3 4	163	164	0·958	125 595	+0·4410	+0·4423	+0·4416	−1·3	− 96·7	125·0815
3 4	164	165	0·964	126·559	+0·9545	+0·9535	+0·9540	+1·0	− 95·7	126.0355
3 4	165	166	0·903	127·462	−1·2348	−1·2359	−1·2354	+1·1	− 94·6	124·8001
6 4	166	167	0·360	127·822	−0·0722	−0·0733	−0·0728	+1·1	− 93·5	124·7273
6 6	167	X	0·302	128·124	+1·2938	+1·2937	+1·2938	+0·1	− 93·4	+126·0211
6 4, 6	167	168	1·187	129·009	* −0·2015	* −0·1968	−0·1992	−4·7	− 98·2	124·5281
6, 8, 13 4, 8, 13	168	169	0·992	130·001	** −0·2102	** −0·2073	−0·2088	−2·9	−101·1	124·3193
6, 8, 13 4, 8, 13	169	170	1·306	131·307	** +0·5630	** +0·5695	+0·5662	−6·5	−107·6	124·8855
6, 8, 13 12, 13	170	171	1·048	132·355	* +0·6272	* +0·6328	+0·6300	−5.6	− 113·2	125·5155
8 12	171	172	1·305	133·660	+2·2270	+2·2312	+2·2291	−4·2	−117·4	127·7446
8, 13 12, 13	172	173	1·143	134·803	* +0·7940	* +0·7888	+0·7914	+5·2	−112·2	128·5360
8, 14 12, 14	173	174	1·090	135·893	* +0·7816	* +0·7783	+0·7800	+3·3	−108·9	129·3160
8, 10 12	174	175	0·935	136·828	+0·5154	+0·5137	+0.5146	+1·7	−107·2	129·8306
14 14	175	Y	0·042	136·870	+1·4754	+1·4752	+1·4753	+0·2	−107·0	+131·3059

10	11, 12	175	176	0·971	137·799	+1·1858	+1·1898	+1·1878	−4·0	−111·2	131·0184
10, 14	11, 14	176	177	0·974	138·773	* +0·8127	* +0·8152	+0·8140	−2·5	−113·7	131·8324
10	11	177	178	1·341	140·114	+0·9444	+0·9422	+0·9433	+2·2	−111·5	132·7757
10	11	178	179	1·172	141·286	+0·3799	+0·3746	+0·3772	+5·3	−106·2	133·1529
10, 14	11, 14	179	180	1·061	142·347	* +1·9102	* +1·9155	+1·9128	−5·3	−111·5	135·0657
10	15	180	181	1·076	143·423	+0·8231	+0·8243	+0·8237	−1·2	−112·7	135·8894
10	15	181	182	1·138	144·561	+0·4572	+0·4524	+0·4548	+4·8	−107·9	136·3442
10	15	182	183	0·975	145·536	+0·2146	+0·2118	+0·2132	+2·8	−105·1	136·5574
10, 14	15	183	184	0·987	146·523	+0·0176	+0·0134	+0·0155	+4·2	−100·9	136·5729
14	15	184	185	0·926	147·449	+2·7822	+2·7802	+2·7812	+2·0	− 98·9	139·3541
20	20	185	Z	0·062	147·511	+2·9278	+2·9280	+2·9279	−0·2	− 99·1	+142·2820
14, 15, 20	20	185	186	0·991	148·440	* +1·3787	* +1·3785	+1·3786	+0·2	− 98·7	140·7327
15	20	186	187	0·961	149·401	+1·8633	+1·8601	+1·8617	+3·2	− 95·5	142·5944
15, 17	19, 20	187	188	0·957	150·358	+2·2057	+2·2065	+2·2061	−0·8	− 96·3	144·8005
17	19	188	189	0·970	151·328	+2·9576	+2·9576	+2·9576	0·0	− 96·3	147·7581
17, 21	19, 21	189	190	0·965	152·293	* +5·3928	* +5·3976	+5·3952	−4·8	−101·1	153·1533
17	19	190	191	0·957	153·250	−2·1143	−2·1107	−2·1125	−3·6	−104·7	151·0408
17	19	191	192	0·948	154·198	−5·3693	−5·3664	−5·3678	−2·9	−107·6	145·6730
17, 21, 26	19, 21, 26	192	193	1·190	155·388	** −6·7367	** −6·7314	−6·7340	−5·3	−112·9	138·9390
17, 21, 26	19, 21, 26	193	194	1·572	156·960	** −1·6523	** −1·6452	−1·6488	−7·1	−120·0	137·2902
18	18	194	XXVII	0·105	157·065	+1·7863	+1·7866	+1·7864	−0·3	−120·3	+139·0766
17	18	194	195	1·291	158·251	−3·1713	−3·1683	−3·1698	−3·0	−123·0	134·1204
17	18	195	196	0·967	159·218	−1·2685	−1·2646	−1·2666	−3·9	−126·9	132·8538
17, 22 / 25, 26	18, 25	196	197	0·971	160·189	* −1·3256	* −1·3196	−1·3226	−6·0	−132·9	131·5312
22, 26	18, 25, 26	197	198	0·995	161·184	** −1·1027	** −1·0954	−1·0990	−7·3	−140·2	130·4322
22	18	198	199	1·115	162·299	+0·1508	+0·1553	+0·1530	−4·5	−144·7	130·5852
22	18	199	200	1·008	163·307	−1·4508	−1·4505	−1·4506	−0·3	−145·0	129·1346
22	25	200	201	1·158	164·465	−0·8289	−0·8304	−0·8296	+1·5	−143·5	128·3050
22	25	201	202	1·159	165·624	−1·8641	−1·8633	−1·8637	−0·8	−144·3	126·4413
22	24	202	203	1·150	166·774	−0·9547	−0·9539	−0·9543	−0·8	−145·1	125·4870
22	24	203	204	1·156	167·930	−0·0736	−0·0751	−0·0744	+1·5	−143·6	125·4126
22, 28	24, 28	204	205	1·140	169·070	* −2·4561	* −2·4604	−2·4582	+4·3	−139·3	122·9544
22	24	205	206	1·143	170·213	−1·1269	−1·1318	−1·1294	+4·9	−134·4	121·8250

* An asterisk attached to any number indicates that it is a mean value of two denominations; a double asterisk denotes a mean of three or more values.

Results of geodetic spirit leveling in Mississippi, Tennessee, Kentucky, and Illinois, 1885–'90.

SECTION I.—FROM OKOLONA, MISS., TO GREENFIELD, TENN., OCTOBER, 1889, TO MARCH, 1890.

Date.	Benchmark		Distance		Difference of height between marks.			Discrepancy.		Height of mark above average Gulf level at Biloxi, Miss.
	From—	To—	Between successive marks.	From initial mark.	Direction of measure. Forward (northward).	Backward (southward).	Mean.	Partial. F–B	Total. Accumulated.	
1889–'90.			*km.*	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
Jan.										
22 24	206	207	1·166	171·379	−0·7595	−0·7583	−0·7589	−1·2	−135·6	121·0661
22, 28 24	207	208	1·014	172·393	−0·0244	−0·0239	−0·0242	−0·5	−136·1	121·0419
28, M. 3 24, M. 3	208	209	1·026	173·419	* +7·5460	* +7·5496	+7·5478	−3·6	−139·7	128·5897
M. 1 M. 1	209	XXVI	0·055	173·474	+2·3755	+2·3756	+2·3756	−0·1	−139·8	+130·9653
28, M. 3 24, M. 3	209	210	0·761	174·180	* +4·0981	* +4·1014	+4·0998	−3·3	−143·0	132·6895
28 M. 1	210	211	0·928	175·108	−5·2792	−5·2756	−5·2774	−3·6	−146·6	127·4121
28, M. 3 M. 1, 3	211	212	0·941	176·049	* −3·8215	* −3·8272	−3·8244	+5·7	−140·9	123·5877
28, M. 3 M. 1, 3	212	213	0·944	176·993	* −2·8371	* −2·8424	−2·8398	+5·3	−135·6	120·7479
28, M. 3 M. 1, 3	213	214	1·130	178·123	* −2·2343	* −2·2349	−2·2346	+0·6	−135·0	118·5133
28, M. 4 M. 1	214	215	0·804	178·927	−3·0059	−3·0089	−3·0074	+3·0	−132·0	115·5059
Mar.										
4, 7 7	215	216	0·965	179·892	* −0·0145	* −0·0160	−0·0152	+1·5	−130·5	115·4907
4 7	216	217	1·270	181·162	+1·9980	+1·9928	+1·9954	+5·2	−125·3	117·4861
4 7	217	218	1·272	182·434	−1·6141	−1·6094	−1·6118	−4·7	−130·0	115·8743
4 7	218	219	0·392	182·826	−0·2619	−0·2619	−0·2619	0·0	−130·0	115·6124
5 5	219	XXV	0·065	182·891	+1·8802	+1·8805	+1·8804	−0·3	−130·3	+117·4928
4 6	219	220	1·290	184·116	−0·9765	−0·9815	−0·9790	+5·0	−125·0	114·6334
4 6	220	221	0·950	185·066	+1·1314	+1·1358	+1·1336	−4·4	−129·4	115·7670
4 6	221	222	1·130	186·196	− 2·2181	− 2·2189	− 2·2185	+0·8	−128·6	113·5485
4 6	222	223	1·116	187·312	− 0.2805	− 0·2809	− 0.2807	+0·4	−128·2	113·2678

4	6	223	224	1·130	188·442	— 0·9523	— 0·9577	— 0·9550	+5·4	—122·8	112·3128
8	6, 11	224	225	1·146	189·588	+ 0·6578	+ 0·6584	+ 0·6581	—0·6	—123·4	112·9709
8, 12	11, 12	225	226	1·156	190·744	*— 0·9324	*— 0·9266	— 0·9295	—5·8	—129·2	112·0414
8, 12	11, 12	226	227	1·143	191·887	*+ 0·1016	*+ 0·1090	+ 0·1053	—7·4	—136·6	112.1467
8	11	227	228	1·159	193·046	— 0·7416	— 0·7419	— 0·7418	+0·3	—136·3	111·4049
8	10	228	229	1·271	194·317	— 3·2715	— 3·2732	— 3·2724	+1·7	—134·6	108·1325
8	10	229	230	0·956	195·273	— 0·8452	— 0·8456	— 0·8454	+0·4	—134·2	107·2871
8, 12	10	230	231	0·960	196·233	— 0·3909	— 0·3953	— 0·3931	+4·4	—129·8	106·8940
12, 13	10, 13	231	XXIV	1·041	197·274	*+ 0·2519	*+ 0·2556	+ 0·2538	—3·7	—133·5	107·1478
12	10	XXIV	232	0·831	198·105	— 0·3639	— 0·3617	— 0·3628	—2·2	—135·7	106·7850
13	10	232	233	1·045	199·150	+ 6·6199	+ 6.6250	+ 6·6224	—5·1	—140·8	113·4074
13, 14	10, 13, 14	233	234	1·056	200·206	**+ 9·3970	**+ 9·3997	+ 9·3984	—2·7	—143·5	122·8058
13	10	234	XXIII	0·776	200·982	— 2·5679	— 2·5675	— 2·5677	—0·4	—143·9	120·2381
1889.	Nov.										
27	27	XXIII	XXII	0·342	201·324	— 0·9770	— 0·9772	— 0·9771	+0·2	—143·7	119·2610
27	27	XXII	51	1·478	202·802	+14·6219	+14·6234	+14·6226	—1·5	—145·2	133·8836
27, 29	26, 27	51	50	1·541	204·343	+ 9·2887	+ 9·2853	+ 9·2870	+3·4	—141·8	143·1706
29	26	50	49	1·208	205·551	+ 7·0615	+ 7·0678	+ 7·0646	—6·3	—148·1	150·2352
29	26	49	48	1·325	206·876	— 4·3386	— 4·3365	— 4·3376	—2·1	—150·2	145·8976
30	26	48	47	1·548	208·424	— 3·5671	— 3·5678	— 3·5674	+0·7	—149·5	142·3302
30	26	47	46	1·259	209·683	—10·7991	—10·7956	—10·7974	—3·5	—153·0	131·5328
30	26	46	45	1·222	210·905	— 6·5313	— 6·5289	— 6·5301	—2·4	—155·4	125·0027
30, D. 2	25, 26	45	44	1·294	212·199	+ 3·9502	+ 3·9479	+ 3·9490	+2·3	—153·1	128·9517
D. 2	25	44	43	1·220	213·419	+ 5·0773	+ 5·0799	+ 5·0786	—2·6	—155·7	134·0303
	Nov.										
25	25	43	XXI	0.038	213·457	+ 0·3940	+ 0·3938	+ 0·3939	+0·2	—155·5	+134·4242
20	25	43	42	1·172	214·591	— 7·0808	— 7·0835	— 7·0822	+2·7	—153·0	126·9481
20	25	42	41	1·107	215·698	—11·8862	—11·8844	—11·8853	—1·8	—154·8	115·0628
20	19	41	40	1·418	217·116	— 3·5437	— 3·5466	— 3·5452	+2·9	—151·9	111·5176
21	19	40	39	1·313	218·429	+ 6·1414	+ 6·1359	+ 6·1386	+5·5	—146·4	117·6562
21, 22, 23	19, 23	39	38	1·351	219·780	**+11·9723	**+11·9723	+11·9723	0·0	—146·4	129·6285
21, 22, 23	19, 23	38	37	1·334	221·114	*+11·9760	*+11·9736	+11·9748	+2·4	—144·0	141·6033
22	19	37	36	1·355	222·469	— 1·4279	— 1·4307	— 1·4293	+2·8	—141·2	140·1740
22	19	36	35	1·349	223·818	+ 9·4356	+ 9·4336	+ 9·4346	+2·0	—139·2	149·6086
23	23	35	XX	0·261	224·079	+ 3·7447	+ 3·7454	+ 3·7450	—0·7	—139·9	+153·3536

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Results of geodetic spirit leveling in Mississippi, Tennessee, Kentucky, and Illinois, 1885–'90.

SECTION I.—FROM OKOLONA, MISS., TO GREENFIELD, TENN., OCTOBER, 1889, TO MARCH, 1890.

Date.		Benchmark.		Distance.		Difference of height between marks.			Discrepancy.		Height of mark above average Gulf level at Biloxi, Miss.
		From—	To—	Between successive marks.	From initial mark.	Direction of measure.		Mean.	Partial.	Total.	
						Forward (northward).	Backward (southward).		F–B	Accumulated.	
1889–'90. Nov.				*km.*	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
16, 23	15, 18	35	34	1·308	225·126	— 0·9261	— 0·9299	— 0·9280	+3·8	—135·4	148·6806
18	15	34	33	1·289	226·415	+10·9223	+10·9183	+10·9203	+4·0	—131·4	159·6009
18	15	33	32	1·104	227·519	— 4·5693	— 4·5726	— 4·5710	+3·3	—128·1	155·0299
18	11, 15	32	31	1·155	228·674	— 5·2154	— 5·2142	— 5·2148	—1·2	—129·3	149·8151
12, 15	11, 15	31	30	1·262	229·936	*— 6·2438	*— 6·2464	— 6·2451	+2·6	—126·7	143·5700
12	11	30	29	1·170	231·106	— 2·1280	— 2·1293	— 2·1286	+1·3	—125·4	141·4414
12	11	29	28	1·054	232·160	— 4·2961	— 4·3001	— 4·2981	+4·0	—121·4	137·1433
12	11	28	27	1·050	233·210	— 0·3322	— 0·3331	— 0·3326	+0·9	—120·5	136·8107
13	11	27	26	1·132	234·342	+ 1·3711	+ 1·3702	+ 1·3706	+0·9	—119·6	138·1813
13	11	26	25	1·190	235·532	— 1·5239	— 1·5242	— 1·5240	+0·3	—119·3	136·6573
13, 14	9, 14	25	24	1·082	236·614	*+ 1·1199	*+ 1·1178	+ 1·1189	+2·1	—117·2	137·7762
13	9	24	23	1·161	237·775	—10·2227	—10·2236	—10·2232	+0·9	—116·3	127·5530
14	14	23	XIX	0·519	238·294	+ 3·0949	+ 3·0953	+ 3·0950	—0·7	—117·0	+130·6480
7	7	23	XVIII	0·161	237·936	+ 1·2976	+ 1·2963	+ 1·2970	+1·3	—115·0	+128·8500
8, 13	7	23	22	1·450	239·225	+ 0·7772	+ 0·7803	+ 0·7788	—3·1	—119·4	128·3318
8	6, 7	22	21	1·063	240·288	— 6·6335	— 6·6326	— 6·6330	—0·9	—120·3	121·6988
8	6	21	20	1·132	241·420	+ 1·0800	+ 1·0788	+ 1·0794	+1·2	—119·1	122·7782
8, 14	6, 14	20	19	1·387	242·807	*— 1·3666	*— 1·3714	— 1·3690	+4·8	—114·3	121·4092
8, 9	1, 6	19	18	1·111	243·918	—10·3079	—10·3103	—10·3091	+2·4	—111·9	111·1001
9	1	18	17	0·961	244·879	— 1·0912	— 1·0887	— 1·0900	—2·5	—114·4	110·0101
9	1	17	16	1·291	246·170	— 3·6419	— 3·6381	— 3·6400	—3·8	—118·2	106·3701
2	1	16	15	1·315	247·485	+ 1·4194	+ 1·4249	+ 1·4222	—5·5	—123·7	107·7923

2, 6	1, 6	15	14	1·294	248·779	*+ 5·9091	*+ 5·9128	+ 5·9110	—3·7	—127·4	113·7033
2	1	14	13	1·286	250·065	+ 2·7948	+ 2·7996	+ 2·7972	—4·8	—132·2	116·5005
2	1, O. 29	13	12	1·440	251·505	— 5·9967	— 5·9933	— 5·9950	—3·4	—135·6	110·5055
O. 30	O. 29	12	11	1·295	252·800	+ 7·7842	+ 7·7798	+ 7·7820	+4·4	—131·2	118·2875
O. 30, N. 5	O. 29, N. 5	11	10	1·111	253·911	*+11·0664	*+11·0588	+11·0626	+7·6	— 123·6	129·3501
O. 30	O. 29	10	9	1·088	254·999	—10·6850	—10·6838	—10·6844	—1·2	—124·8	118·6657
O. 30	O. 29	9	8	1·146	256·145	— 8·8715	— 8·8746	— 8·8730	+3·1	—121·7	109·7927
N. 5	N. 5	8	XVII	0.083	256·228	+ 1·7363	+ 1·7356	+ 1·7360	+0·7	—121·0	+111·5287
Oct.											
30, 31	29	8	7	1·184	257·329	— 5·7929	— 5·7898	— 5·7914	—3·1	—124·8	104·0013
31	28	7	6	1·265	258·594	— 2·5663	— 2·5656	— 2·5660	—0·7	—125·5	101·4353
31	28	6	5	1·185	259·779	— 4·0804	— 4·0798	— 4·0801	—0·6	—126·1	97·3552
31, N. 4	28, N. 4	5	4	0·998	260·777	*— 3·1184	*— 3·1226	— 3·1205	+4·2	—121·9	94·2347
N. 4, 5	28, N. 4	4	3	1·084	261·861	*+ 6·7903	*+ 6·7843	+ 6·7873	+6·0	—115·9	101·0220
26	25	3	2	1·050	262·911	+10·3394	+10·3386	+10·3390	+0·8	—115·1	111·3610
28, 26	25, 28	2	1	1·278	264·189	*+12·7624	*+12·7645	12·7634	—2·1	—117·2	124·1244
26	25	1	XV	1·163	265·352	+ 8·6540	+ 8·6513	+ 8·6526	+2·7	—114·5	+132·7770

* An asterisk attached to any number indicates that it is a mean value of two determinations.

Supplementary heights (approximate) of points along the Mobile and Ohio and Illinois Central Railroad.

Location.	Description.		Height above average Gulf level.
			Metres.
Shannon, Miss.,	County road crossing, 2 miles south of,	ground	80·11
	Opposite Mobile and Ohio R. R. station,	do.	73·95
	County road crossing, at Clarke's sawmill, middle of track,	ground	74·73
Verona, Miss.,	Opposite Mobile and Ohio R. R. station,	middle of track, ground	91·92
Tupelo, Miss.,	Crossing of Mobile and Ohio and Kansas City Belt R. R.,	ground	82·41
	In front of Mobile and Ohio R. R. freight depot,	ground	83·66
Saltillo, Miss.,	County road crossing, 1½ miles south of,	middle of track, ground	90.06
	Opposite Mobile and Ohio R. R. station,	do.	95·24
	County road crossing, 3 miles north of,	middle of track, ground	106·34
Guntown, Miss.,	Opposite Mobile and Ohio R. R. station,	middle of track, ground	116·10
Baldwyn, Miss.,	do.,	do.	114·04
Wheeler, Miss.,	County road crossing, middle of track,	do.	109·25
Boonesville, Miss.,	County road crossing, 3 miles south of,	middle of track, ground	118·14
	Ground on line with north end of Mobile and Ohio R. R. station,		153·60
Rienzi, Miss.,	County road crossing, 2½ miles south of,	ground	136·68
	Opposite north end of Mobile and Ohio R. R. station,	ground	132·73
Corinth, Miss.,	County road crossing, 5½ miles south of,	do.	131·39
	County road crossing, 4 miles south of,	do.	140·36
	Crossing Mobile and Ohio and Massillon and Cleveland R. R.,	ground	132·88
	County road crossing, 3 miles north of,	do.	146·31
State line,	Crossing Mobile and Ohio R. R. and State line,	ground	154·01
Corinth, Miss.,	County road crossing, 5 miles north of,	do.	142·23
Ramer, Tenn.,	County road crossing, 1 mile south of,	do.	126·42
	Opposite Mobile and Ohio R. R. station,	do.	125·56
Falcon, Tenn.,	County road crossing, 3 miles south of,	do.	125·59
	Opposite Mobile and Ohio R. R. station,	do.	130·32
Selma, Tenn.,	County road crossing,	do.	132·95
Bethel Springs, Tenn.,	Opposite Mobile and Ohio R. R. station,	do.	139·76
	County road crossing, 1½ miles north of,	do.	144·30
	County road crossing, 4 miles north of,	do.	155·68
McNairy, Tenn.,	Opposite Mobile and Ohio R. R. station,	do.	136·85
	County road crossing, 3½ miles north of,	do.	129·65
Henderson, Tenn.,	Opposite Mobile and Ohio R. R. station,	do.	128·49
Pinson, Tenn.,	County road crossing, 2 miles south of,	do.	116·02
	County road crossing, 1 mile south of,	do.	117·65
	Opposite Mobile and Ohio R. R. station,	do.	115·96
Jackson, Tenn.,	Crossing Illinois Central and Mobile and Ohio R. R.	ground	129·57
Oakfield, Tenn.,	County road crossing, Illinois Central R. R., 1½ miles south of,	ground	126·76
	Opposite mail stand, Illinois Central R. R.,	do.	134·31
	County road crossing, ½ mile north of,	do.	131·04
Medina, Tenn.,	County road crossing, 4 miles south of,	do.	110·16
	County road crossing, 1½ miles south of,	do.	143·47
	Opposite Illinois Central R. R. station,	do.	150·09

Supplementary heights (approximate) of points along the Mobile and Ohio and Illinois Central Railroad.

Location.	Description.		Height above average Gulf level.
			Metres.
Medina, Tenn.,	County road crossing, 2 miles north of,	ground	162·09
Milan, Tenn.,	County road crossing, 5 miles south of,	do.	149·03
	County road crossing, 3½ miles south of,	do.	143·03
	County road crossing, 2½ miles south of,	do.	137·90
	County road crossing, 1½ miles south of,	do.	137·50
	Crossing of Illinois Central and Louisville and Nashville R. R.,	ground	128·44
	County road crossing, 2 miles north of,	do.	120·87
	County road crossing, 3 miles north of,	do.	120·68
Cadis Mill, Tenn.,	County road crossing,	do.	112·11
Idlewild, Tenn.,	County road crossing, 2 miles south of,	do.	108·46
	County road crossing, 1 mile south of,	do.	119·29
	County road crossing, just north of,	do.	111·98
	Opposite Illinois Central R. R. mail stand,	do.	112·52
Bradford, Tenn.,	County road crossing, 2 miles south of,	do.	119·64
	County road crossing, 1 mile south of,	do.	126·98
	In front of Illinois Central R. R. station,	do.	110·17
	County road crossing, 2 miles north of,	do.	99·93

Results of geodetic spirit leveling in Mississippi, Tennessee, Kentucky, and Illinois, 1885–'90.

SECTION II.—FROM GREENFIELD, TENN., TO VILLA RIDGE, ILL., OCTOBER, 1888, TO JANUARY, 1889.

Date.		Benchmark		Distance		Difference of height between marks.			Discrepancy.		Height of mark above average Gulf level at Biloxi, Miss.
		From—	To—	Between successive marks.	From initial mark, K.	Direction of measure.		Mean.	Partial.	Total.	
						Forward (northward).	Backward (southward).		F—B	Accumulated.	
1888–'89.				*km.*	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
Dec.		----	XV	-----	265·352	------	------	------	----	—114.5	+132·7770
31	31	XV	102	0·094	265·446	— 2·0392	— 2·0396	— 2·0394	+0·4	—114·1	+130·7376
29	28	102	101	0·870	266·316	— 0·1617	— 0·1583	— 0·1600	—3·4	—117·5	+130·5776
29	28	101	100	1·225	267·541	— 9·2622	— 9·2588	— 9·2605	—3·4	—120·9	+121·3171
29	28	100	99	1·199	268·740	—12·2485	—12·2435	—12·2460	—5·0	—125·9	+109·0711
29 J. 3, 4	29 J. 3, 4	99	98	1·259	269·999	**—10·9921	**—10·9889	—10·9905	—3·2	—129·1	+ 98·0806
29	28	98	97	1·526	271·525	— 0·0452	— 0.0423	— 0·0438	—2·9	—132.0	+ 98·0368
29	27	97	96	1·054	272·579	+10·5991	+10.6004	+10·5998	—1·3	—133·3	+108·6366
29	27	96	95	1·061	273·640	+10·7461	+10·7447	+10·7454	+1·4	—131·9	+119·3820
31	27	95	94	1·064	274·704	+ 5·9380	+ 5·9373	+ 5·9376	+0·7	—131·2	+125·3196
31	31	94	XIV	0·096	274·800	+ 1·3966	+ 1·3970	+ 1.3968	—0·4	—131·6	+126·7164
31, J. 2	27, J. 2	94	93	1·399	276.103	*— 3·7581	*— 3·7537	— 3·7559	—4·4	—135·6	+121·5637
31	27	93	92	1.292	277·395	—11·8435	—11·8383	—11·8409	—5·2	—140·8	+109·7228
21	20, 27	92	91	0·982	278·377	— 7·6315	— 7·6290	— 7·6302	—2·5	—143·3	+102·0926
21	20	91	90	0·608	278·985	+ 5·2189	+ 5·2147	+ 5·2168	+4·2	—139·1	+107·3094
21	20	90	89	0·726	279·711	+ 7·6193	+ 7·6180	+ 7·6186	+1·3	—137·8	+114·9280
21	19	89	88	1·210	280·921	+ 1·9326	+ 1·9337	+ 1·9332	—1·1	—138·9	+116·8612
21	19	88	87	1·285	282·206	+ 2·7597	+ 2·7573	+ 2·7585	+2·4	—136·5	+119·6197
22	19	87	86	1·252	283.458	+ 0·9764	+ 0·9807	+ 0·9786	—4·3	—140·8	+120·5983
22	19	86	85	1·244	284·702	— 6.0779	— 6·0752	— 6·0766	—2·7	—143.5	+114·5217
22	18	85	84	1·218	285·920	+ 2·7758	+ 2·7715	+ 2·7736	+4·3	—139·2	+117·2953

24	18	84	83	1.258	287.178	+ 7·0842	+ 7.0837	+ 7·0840	+0·5	−138·7	+124·3793
17	17	83	XIII	0·056	287·234	+ 2·2669	+ 2·2669	+ 2·2669	0·0	−138·7	+126·6462
17	18	83	82	0·729	287·907	− 3·5476	− 3·5462	− 3·5469	−1·4	−140·1	+120·8324
8	17	82	81	1·204	289·111	+ 0·2447	+ 0·2443	+ 0·2445	+0·4	−139·7	+121·0769
8	12	81	80	1.272	290·383	+ 2·0620	+ 2·0628	+ 2·0624	−0·8	−140·5	+123.1393
8, 17	12, 17	80	79	1·024	291·407	*−10·8002	*−10·8022	−10·8012	+2·0	−138·5	+112·3381
8	12	79	78	1·287	292·604	− 6·7615	− 6·7583	− 6·7599	−3·2	−141·7	+105·5782
8	12	78	77	1·114	293·808	− 6·1164	− 6·1108	− 6·1136	−5·6	−147·3	+ 99·4646
10	11, 12	77	76	1·315	295·123	+ 2·0876	+ 2·0879	+ 2·0878	−0·3	−147·6	+101·5524
10	11	76	XII	0·990	296·113	+ 6·6337	+ 6·6375	+ 6·6356	−3·8	−151·4	+108·1880
10, 14	11, 14	XII	75	0·990	297·103	*− 5·0130	*− 5·0100	− 5·0115	−3.0	−154·4	+103·1765
10	11	75	74	1·291	298·394	+11·0746	+11·0760	+11·0753	−1.4	−155.8	+114.2518
10, 14	5, 11, 14	74	73	1·629	300·023	*+ 5·7382	*+ 5·7538	+ 5·7410	−5·6	−161.4	+119·9928
6	5	73	72	1.228	301·251	+ 2·6030	+ 2·6047	+ 2·6038	−1·7	−163·1	+122·5966
6	5	72	71	1·247	302·498	+ 3·1957	+ 3·1978	+ 3·1968	−2·1	−165·2	+125·7934
6	5	71	70	1·115	303·613	− 4·1472	− 4·1443	− 4·1458	−2·9	−168·1	+121·6476
6	5	70	69	1·093	304·706	−10·8645	−10·8647	−10·8646	+0·2	−167·9	+110·7830
6	5	69	68	1·067	305·773	− 2·5208	− 2·5184	− 2·5196	−2·4	−170·3	+108.2634
4	4	68	XI	0·088	305·861	+ 0·9090	+ 0·9096	+ 0·9093	−0·6	−170·9	+109·1727
6	4	68	67	0·937	306·710	+ 9·8413	+ 9·8381	+ 9·8397	+3·2	−167.1	+118·1031
7, 15	4, 15	67	66	1·026	307·736	*+ 4·3100	*+ 4·3147	+ 4·3124	−4.7	−171.8	+122·4155
7	4	66	65	0·900	308·636	+ 6·4161	+ 6·4154	+ 6·4158	+0·7	−171·1	+128·8313
7	3	65	64	1.046	309·682	− 0·1425	− 0·1382	− 0·1404	−4·3	−175·4	+128·6909
7	3	64	63	1·232	310·914	+ 1·6754	+ 1·6783	+ 1·6768	−2.9	−178.3	+130·3677
7	3	63	62	1·248	312·162	− 3·7772	− 3·7753	− 3·7762	−1·9	−180·2	+126·5915
7, 18	3, 18	62	61	1·248	313.410	* − 9·0300	* − 9·0256	− 9·0278	−4·4	−184·6	+117·5637
7, 13	3	61	60	1·230	314·640	− 5·9950	− 5·9918	− 5·9934	−3·2	−187·8	+111·5703
13	3, N. 28	60	X	1·146	315·786	+ 0·6530	+ 0·6551	+ 0·6540	−2·1	−189·9	+112·2243
13	N. 28	X	59	1·468	317·254	− 9·2722	− 9·2743	− 9·2732	+2·1	−187·8	+102·9511
	Nov.										
30	28	59	58	1·178	318·432	+ 1·6766	+ 1·6797	+ 1·6782	−3·1	−190·9	+104·6293
30	28	58	57	1·254	319·686	− 3·8157	− 3·8156	− 3·8156	−0·1	−191·0	+100·8137
30	27	57	56	1·221	320·907	− 3·9769	− 3·9752	− 3·9760	−1·7	−192·7	+ 96·8377
30	27	56	55	1·084	321·991	− 0·5039	− 0·4998	− 0·5018	−4·1	−196·8	+ 96·3359

* An asterisk attached to any number indicates that it is a mean value of two determinations; a double asterisk denotes a mean of three or more values.

Results of geodetic spirit leveling in Mississippi, Tennessee, Kentucky, and Illinois, 1885–'90.

SECTION II.—FROM GREENFIELD, TENN., TO VILLA RIDGE, ILL., OCTOBER, 1888, TO JANUARY, 1889.

Date.	Benchmark		Distance		Difference of height between marks.			Discrepancy.		Height of mark above average Gulf level at Biloxi, Miss.
	From—	To—	Between successive marks.	From initial mark, K.	Direction of measure.		Mean.	Partial.	Total.	
					Forward (northward).	Backward (southward).		F—R.	Accumulated.	
1888 '89. Nov.			*km.*	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
30 27	55	54	1·036	323·027	+ 1·6990	+ 1·6971	+ 1·6980	+1·9	—194·9	+ 98·0339
D. 1 27	54	53	1·040	324·067	+ 8·5400	+ 8·5446	+ 8·5423	—4·6	—199·5	+106·5762
D. 1 27	53	52	1·180	325·247	+12·0295	+12·0340	+12·0318	—4·5	—204·0	+118·6080
D. 1 26	52	51	1·114	326·361	+ 4·4968	+ 4·5017	+ 4·4992	—4·9	—208·9	+123·1072
D. 1, 15 26, D. 15	51	50	1·125	327·486	*—11·3033	*—11·2999	—11·3016	—3·4	—212·3	+111·8056
17 19	50	IX	0·339	327·825	+ 6·7436	+ 6·7462	+ 6·7449	— 2·6	—214·9	+118·5505
19 17	50	49	1·266	328·752	— 6·6867	— 6·6839	— 6·6853	— 2·8	—215·1	+105·1203
20 17	49	48	1·286	330·038	— 0·9042	— 0·8997	— 0·9020	— 4·5	—219·6	+104·2183
20 17	48	47	1·228	331·266	— 2·4216	— 2·4185	— 2·4200	— 3·1	—222·7	+101·7983
20 17	47	46	1·122	332·388	+11·2851	+11·2848	+11·2850	+ 0·3	—222·4	*+113·0833
20 16	46	45	1·150	333·538	+ 0·0061	+ 0·0018	+ 0·0040	+ 4·3	—218·1	*+113·0873
20, 24 16, 24	45	44	1·156	334·694	*— 9·2772	*— 9·2704	— 9·2738	— 6·8	—224·9	*+103·8135
21, 26 16, 24	44	43	1·211	335·905	*— 5·7004	*— 5·6958	— 5·6981	— 4·6	—229·5	+ 98·1154
21 16	43	42	1·168	337·073	— 0·6118	— 0·6097	— 0·6108	— 2·1	—231·6	+ 97·5046
21, 24 10, 16, 23, 24	42	41	0·942	338·015	*— 0·0566	*— 0·0522	— 0·0544	— 4·4	—236·0	+ 97·4502
12, 21 10, 23	41	40	1·352	339·367	*+ 3·2287	*+ 3·2334	+ 3·2310	— 4·7	—240·7	+100·6812
12, 21 10, 23	40	39	1·336	340·703	*+ 1·4845	*+ 1·4892	+ 1·4868	— 4·7	—245·4	+102·1680
12, 22 10, 23	39	VIII	1·030	341·733	*+ 8·5105	*+ 8·5148	+ 8·5126	— 4·3	—249·7	+110·6806
12, 22 9, 10, 23	VIII	38	1·176	342·909	*— 3·0522	*— 3·0462	— 3·0492	— 6·0	—255·7	+107·6314
12 9	38	37	1·270	344·179	+ 3·7275	+ 3·7263	+ 3·7269	+ 1·2	—254·5	+111·3583
13, 22 9, 22	37	36	1·280	345·459	*+ 5·4424	*+ 5·4444	+ 5·4434	— 2·0	—256·5	+116·8017

13	9	36	35	1·332	346·791	+13·6762	+13·6738	+13·6750	+ 2·4	—254·1	+130·4767
13	9	35	34	1·255	348·046	+ 2·2478	+ 2·2488	+ 2·2483	— 1·0	—255·1	+132·7250
13	3, 9	34	33	1·260	349·306	—12·7028	—12·7042	—12·7035	+ 1·4	—253·7	+120·0215
5	3	33	32	1·144	350·450	—10·2894	—10·2928	—10·2911	+ 3·4	—250·3	+109·7304
3	3	32	VII	0·342	350·792	+ 9·2417	+ 9·2417	+ 9·2417	0·0	—250·3	+118·9721
5	3	32	31	1·108	351·558	— 2·4061	— 2·4054	— 2·4058	— 0·7	—251·0	+107·3246
5	2, 3	31	30	1·134	352·692	— 2·8359	— 2·8314	— 2·8336	— 4·5	—255·5	+104·4910
5	2	30	29	1·243	353·935	+ 8·7552	+ 8·7590	+ 8·7571	— 3·8	—259·3	+113·2481
5	2	29	28	1·224	355·159	— 3·5202	— 3·5224	— 3·5213	+ 2·2	—257·1	+109·7268
8	2	28	27	1·264	356·423	—10·3898	—10·3919	—10·3908	+ 2·1	—255·0	+ 99·3360
8, O 30	2	27	26	1·250	357·673	— 2·5325	— 2·5293	— 2·5309	— 3·2	—258·2	+ 96·8051
Oct.											
30	29	26	VI	0·984	358·657	— 0·1615	— 0·1590	— 0·1602	— 2·5	—260·7	+ 96·6449
30	29	VI	25	1·466	360·123	— 0·2761	— 0·2714	— 0·2738	— 4·7	—265·4	+ 96·3711
30	29	25	24	1·040	361·163	+ 0·2333	+ 0·2365	+ 0·2349	— 3·2	—268·6	+ 96·6060
30	27	24	V	0·882	362·045	+ 1·2848	+ 1·2844	+ 1·2846	+ 0·4	—268·2	+ 97·8906
30, 31	27	V	23	1·304	363·349	+ 1·2002	+ 1·2045	+ 1·2024	— 4·3	—272·5	+ 99·0930
31	27	23	IV	0·998	364·347	+ 2·1063	+ 2·1113	+ 2·1088	— 5·0	—277·5	+101·2018
24	27	IV	22	0·830	365·177	— 2·9262	— 2·9227	— 2·9244	— 3·5	—281·0	+ 98·2774
24	23	22	21	0·436	365·613	— 5·5295	— 5·5302	— 5·5298	+ 0·7	—280·3	+ 92·7476
24, 31, N. 14	31, N. 14	21	20	1·366	366·979	**+ 5·5095	*+ 5·5124	+ 5·5110	— 2·9	—283·2	+ 98·2586
24, 25	23	20	19	1·226	368·205	+ 0·3019	+ 0·3015	+ 0·3017	+ 0·4	—282·8	+ 98·5603
25	23	19	18	1·191	369·396	— 0·2420	— 0·2401	— 0·2410	— 1·9	—284·7	+ 98·3193
25, N. 1, 14	20, N. 1	18	16	2·492	371·888	**— 6·6283	*— 6·6181	— 6·6232	—10·2	—294·9	+ 91·6961
25	25	16	III	0·296	372·184	+ 6·5645	+ 6·5642	+ 6·5644	+ 0·3	—294·6	+ 98·2605
10	10	16	15	0·856	372·744	**+ 0·7628	**+ 0·7602	+ 0·7615	+ 2·6	—292·3	+ 92·4576
10, 12	16, 19	15	†U. S. E. 3	1·422	374·166	+ 6·4818	+ 6·4834	+ 6·4826	— 1·6	—293·9	+ 98·9402
12, 15	16, 19	U. S. E. 3	U. S. E. 1	0·531	374·697	*— 2·8148	*— 2·8141	— 2·8144	— 0·7	—294·6	+ 96·1258
12	16	U. S. E. 1	U. S. E. 2	0·325	375·022	+ 0·4077	+ 0·4074	+ 0·4076	+ 0·3	—294·3	+ 96·5334
12	16	U. S. E. 2	14	1·030	376·052	— 2·5539	— 2·5505	— 2·5522	— 3·4	—297·7	+ 93·9812
12	16	14	13	1·164	377·216	+ 2·9716	+ 2·9714	+ 2·9715	+ 0·2	—297·5	+ 96·9527
12	16	13	12	1·151	378·367	— 0·0003	— 0·0017	— 0·0010	+ 1·4	—296·1	+ 96·9517
12	11, 16	12	11	0·890	379·257	+ 0·3086	+ 0·3113	+ 0·3100	— 2·7	—298·8	+ 97·2617

* An asterisk attached to any number indicates that it is a mean value of two determinations; a double asterisk denotes a mean of three or more values.
† Crossing the Ohio River; observation made simultaneously.

Results of geodetic spirit leveling in Mississippi, Tennessee, Kentucky, and Illinois, 1885–'90.

SECTION II.—FROM GREENFIELD, TENN., TO VILLA RIDGE, ILL., OCTOBER, 1888, TO JANUARY, 1889.

Date.		Benchmark From—	Benchmark To—	Distance between successive marks.	Distance from initial mark, K.	Difference of height between marks. Direction of measure. Forward (northward).	Direction of measure. Backward (southward).	Mean.	Discrepancy. Partial. F–B	Discrepancy. Total. Accumulated.	Height of mark above average Gulf level at Biloxi, Miss.
1888–'89. Oct.				*km.*	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
13	11	" 11	10	1·304	380·561	+ 0·0761	+ 0·0802	+ 0·0782	— 4·1	—302·9	+ 97·3399
13	11	10	9	1·318	381·879	+ 0·1317	+ 0·1336	+ 0·1326	— 1·9	—304·8	+ 97·4725
13, 18	11, 18	9	8	0·941	382·820	*— 3·1300	*— 3·1260	— 3·1280	— 4·0	—308·8	+ 94·3445
13	11	8	7	1·196	384·016	+ 2·8838	+ 2·8815	+ 2·8826	+ 2·3	—306·5	+ 97·2271
13	11	7	11	1·307	385·323	+ 1·6853	+ 1·6891	+ 1·6872	— 3·8	—310·3	+ 98·9143
15	9	11	6	0·996	386·319	— 1·2644	— 1·2667	— 1·2656	+ 2·3	—308·0	+ 97·6487
8, 15	6, 15	6	5	1·084	387·403	*— 0·0038	*— 0·0061	— 0·0050	+ 2·3	—305·7	+ 97·6437
9	9	5	I	0·298	387·701	— 0·2903	— 0·2914	— 0·2908	+ 1·1	—304·6	+ 97·3529
8	6	5	4	1·317	388·720	+ 3·7603	+ 3·7607	+ 3·7605	— 0·4	—306·1	+101·4042
8	6	4	3	1·326	390·046	+ 5·6361	+ 5·6386	+ 5·6374	— 2·5	—308·6	+107·0416
8, 17	6, 17	3	2	1·251	391·297	*+ 8·1297	*+ 8·1302	+ 8·1300	— 0·5	—309·1	+115·1716
9	5	2	I	1·210	392·507	+ 8·3399	+ 8·3407	+ 8·3403	— 0·8	—309·9	+123·5119
9, 17	5, 17	I	Z_3	0·989	393·496	*— 6·7200	*— 6·7203	— 6·7202	+ 0·3	—309·6	+116·7917
5	9	Z_3	Y_3	0·124	----------	— 1·6770	— 1·6778	— 1·6774	+ 0·8	----------	----------

* An asterisk attached to any number indicates that it is a mean value of two determinations; a double asterisk denotes a mean of three or more values.

SECTION III.—FROM VILLA RIDGE, ILL., TO ODIN, ILL., APRIL AND JUNE, 1885.

Date.	Bench mark From—	Bench mark To—	Distance between successive marks.	Distance from initial mark K.	Difference of height between marks. Line run with Rod E.	Line run with Rod F.	Mean.	Discrepancy. Partial. E–F.	Discrepancy. Total. Accumulated.	Height of mark above average Gulf level at Biloxi, Miss.
1885.			*km.*	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
June.		Z_3		393·496					† 0·0	+116·7917
4	Z_3	Y_3	0·130		— 1·6831	— 1·6830	— 1·6830	—0·1		---------
Mean.	Z_3	Y_3	0·127	393·623			— 1·6802			+115·1115
June.										
4	Y_3	146	1·322	394·945	— 6·9017	— 6·9014	— 6·9016	—0·3	— 0·3	108·2099
3	146	145	1·238	396·183	— 4·0054	— 4·0026	— 4·0040	—2·8	— 3·1	104·2059
4	145	XII_a	0·332	396·515	— 0·4499	— 0·4503	— 0·4501	+0·4	— 2·7	+103·7558
3	145	144	1·009	397·192	— 0·6936	— 0·6926	— 0·6931	—1·0	— 4·1	+103·5128
3	144	143	0·956	398·148	+ 1·4270	+ 1·4258	+ 1·4264	+1·2	— 2·9	+104·9392
2	143	142	1·162	399·310	— 1·7729	— 1·7732	— 1·7730	+0·3	— 2·6	+103·1662
2	142	141	0·948	400·258	— 1·0033	— 1·0036	— 1·0034	+0·3	— 2·3	+102·1628
2	141	140	0·796	401·054	— 0·4167	— 0·4155	— 0·4161	—1·2	— 3·5	+101·7467
1	140	139	0·182	401·236	— 0·2007	— 0·2010	— 0·2008	+0·3	— 3·2	+101·5459
1	139	138	0·960	402·196	+ 1·2218	+ 1·2237	+ 1·2228	—1·9	— 5·1	+102·7687
1	138	137	0·970	403·166	— 0·2911	— 0·2890	— 0·2900	—2·1	— 7·2	+102·4787
1	137	136	1·001	404·167	— 0·4939	— 0·4917	— 0·4928	—2·2	— 9·4	+101·9859
May.										
29	136	135	0·842	405·009	+ 0·5375	+ 0·5377	+ 0·5376	—0·2	— 9·6	+102·5235
29	135	134	1·012	406·021	— 1·5279	— 1·5315	— 1·5297	+3·6	— 6·0	+100·9938
29	134	X_3	0·910	406·931	+ 1·0339	+ 1·0327	+ 1·0333	+1·2	— 4·8	+102·0271
29	X_3	133	0·560	407·491	+ 0·5119	+ 0·5121	+ 0·5120	—0·2	— 5·0	+102·5391

† A new start occasioned by the change in method of observing.

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SECTION III.—FROM VILLA RIDGE, ILL., TO ODIN, ILL., APRIL AND JUNE, 1885.

Date.	Bench mark		Distance		Difference of height between marks.			Discrepancy,		Height of mark above average Gulf level at Biloxi, Miss.
					Line run with		Mean.	Partial.	Total.	
	From—	To—	Between successive marks.	From initial mark K.	Rod E.	Rod F.		E-F.	Accumulated.	
1885. May.			*km.*	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
29	133	132	0·791	408·282	— 0·4613	— 0·4574	— 0·4594	—3·9	— 8·9	+102·0797
28	132	131	1·323	409·605	+ 0·9060	+ 0·9042	+ 0·9051	+1·8	— 7·1	+102·9848
28	131	130	1·068	410·673	+ 0·6246	+ 0·6256	+ 0·6251	—1·0	— 8·1	+103·6099
28	130	129	1·102	411·775	+ 2·0537	+ 2·0526	+ 2·0532	+1·1	— 7·0	+105·6631
28	129	128	1·087	412·862	+ 1·5932	+ 1·5950	+ 1·5941	—1·8	— 8·8	+107·2572
27	128	127	1·102	413·964	+ 1·0328	+ 1·0333	+ 1·0330	—0·5	— 9·3	+108·2902
27	127	126	1·116	415·080	+ 2·6794	+ 2·6827	+ 2·6810	—3·3	—12·6	+110·9712
27	126	125	1·099	416·179	+ 3·0293	+ 3·0312	+ 3·0302	—1·9	—14·5	+114·0014
27	125	124	1·127	417·306	+ 2·4325	+ 2·4293	+ 2·4309	+3·2	—11·3	+116·4323
26	124	123	1·281	418·587	+ 3·7083	+ 3·7073	+ 3·7078	+1·0	—10·3	+120·1401
25	123	122	0·975	419·562	+ 7·4480	+ 7·4461	+ 7·4470	+1·9	— 8·4	+127·5871
25	122	121	0·968	420·530	+ 6·9255	+ 6·9268	+ 6·9262	—1·3	— 9·7	+134·5133
25	121	120	1·831	422·361	+14·3573	+14·3631	+14·3602	—5·8	—15·5	+148·8735
23	120	119	0·874	423·235	— 4·2688	— 4·2671	— 4·2680	—1·7	—17·2	+144·6055
23	119	118	1·011	424·246	— 4·3739	— 4·3765	— 4·3752	+2·6	—14·6	+140·2303
23	118	117	1·552	425·798	+10·5299	+10·5279	+10·5289	+2·0	—12·6	+150·7592
22	117	116	1·028	426·826	+ 6·9973	+ 6·9952	+ 6·9962	+2·1	—10·5	+157·7554
22	116	115	1·006	427·832	+ 7·3985	+ 7·3958	+ 7·3972	+2·7	— 7·8	+165·1526
22	115	114	0·978	428·810	+ 5·4829	+ 5·4859	+ 5·4844	—3·0	—10·8	+170·6370
22	114	113	0·966	429·776	+ 4·8503	+ 4·8517	+ 4·8510	—1·4	—12·2	+175·4880
22	113	112	1·029	430·805	+ 6·7166	+ 6·7124	+ 6·7145	+4.2	— 8·0	+182·2025
21	112	111	0·854	431·659	+ 5·9285	+ 5·9290	+ 5·9288	—0·5	— 8·5	+188·1313

21	111	110	1·016	432·675	+ 2·4296	+ 2·4312	+ 2·4304	—1·6	—10·1	+190·5617
26	110	W_3	0·175	432·850	+ 0·3688	+ 0·3689	+ 0·3688	—0·1	—10·2	+190·9305
21	110	109	0·956	433·631	— 2·0681	— 2·0709	— 2·0695	+2·8	— 7·3	+188·4922
21	109	108	0·962	434·593	+ 2·7438	+ 2·7422	+ 2·7430	+1·6	— 5·7	+191·2352
21	108	XI_a	1·068	435·661	— 0·1298	— 0·1305	— 0.1302	+0·7	— 5·0	+191·1050
13	XI_a	85	0·996	436·657	— 1·4454	— 1·4479	— 1·4466	+2·5	— 2·5	+189·6584
13	85	86	0·976	437·633	+ 1·9838	+ 1·9856	+ 1·9847	—1·8	— 4·3	+191·6431
13	86	87	0·972	438·605	— 1·3718	— 1·3723	— 1·3720	+0·5	— 3·8	+190·2711
13	87	88	1·523	440·128	— 8·4779	— 8·4730	— 8·4754	—4·9	— 8·7	+181·7957
14	88	89	0·978	441·106	— 4·9997	— 4·9985	— 4·9991	—1·2	— 9·9	+176·7966
14	89	90	0·763	441·869	+ 5·1913	+ 5·1918	+ 5·1916	—0·5	—10·4	+181·9882
14	90	91	0·963	442·832	— 5·8892	— 5·8880	— 5·8886	—1·2	—11·6	+176·0996
14	91	92	0·976	443·808	— 7·2045	— 7·2041	— 7·2043	—0·4	—12·0	+168·8953
14	92	93	1·038	444·846	— 6·9093	— 6·9056	— 6·9074	—3·7	—15·7	+161·9879
15	93	94	0·992	445·838	*— 6·8766	*— 6·8767	— 6·8767	+0·2	—15·5	+155·1112
15	94	95	1·426	447·264	— 9·7114	— 9·7130	— 9·7122	+1·6	—13·9	+145·3990
16	95	96	1·055	448·319	— 4·2316	— 4·2317	— 4·2316	+0·1	—13·8	+141·1674
16	96	97	0·978	449·297	— 1·8170	— 1·8196	— 1·8183	+2·6	—11·2	+139·3491
16	97	98	0·952	450·249	— 4·4107	— 4·4101	— 4·4104	—0·6	—11·8	+134·9387
16	98	99	0·940	451·189	— 2·2511	— 2·2535	— 2·2523	+2·4	— 9·4	+132·6864
16	99	100	1·010	452·199	+ 0·4421	+ 0·4396	+ 0·4408	+2·5	— 6·9	+133·1272
18	100	101	1·356	453·555	— 2·0944	— 2·0900	— 2·0922	4·4	—11·3	+131·0350
18	101	102	0·982	454·537	— 1·4565	— 1·4620	— 1·4592	+5·5	— 5·8	+129·5758
18, 20	102	V_3	0.402	454·939	**+ 0·9480	**+ 0·9476	+ 0·9478	+0·4	— 5·4	+130·5236
18, 20	V_3	103	0·872	455·811	**— 2·8131	**— 2·8134	— 2·8132	+0·3	— 5·1	+127·7104
19	103	104	1·029	456·840	— 0·6453	— 0·6437	— 0·6445	—1·6	— 6·7	+127·0659
19	104	105	1·044	457·884	— 1·4617	— 1·4623	— 1·4620	+0·6	— 6·1	+125·6039
19	105	106	0·655	458·539	+ 0·3565	+ 0·3586	+ 0·3576	—2·1	— 8·2	+125·9615
20	106	107	0·944	459·483	— 2·6150	— 2·6136	— 2.6143	—1·4	— 9·6	+123·3472
20	107	84	1·002	460·485	+ 2·1777	+ 2·1750	+ 2·1764	+2·7	— 6·9	+125·5236
12	84	83	1·114	461·599	+ 6·1537	+ 6·1523	+ 6·1530	+1·4	— 5·5	+131·6766
12	83	82	1·145	462·744	+ 2·6632	+ 2·6666	+ 2·6649	—3·4	— 8·9	+134·3415
12	82	X_a	1·112	463·856	— 4·3952	— 4·3903	— 4·3928	—4·9	—13·8	+129·9487
6	X_a	63	1·112	464·968	*— 0·8606	*— 0·8602	— 0·8604	—0.4	—14·2	+129·0883
4, 6	63	64	0·652	465·620	*— 4·1992	*— 4·2000	— 4·1996	+0·8	—13·4	+124·8887
6	64	T_3	0·672	466·292	+ 0·8121	+ 0·8146	+ 0·8134	—2·5	—15·9	+125·7021

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SECTION III.—FROM VILLA RIDGE, ILL., TO ODIN, ILL., APRIL AND JUNE, 1885.

Date.	Bench mark		Distance		Difference of height between marks.			Discrepancy.		Height of mark above average Gulf level at Biloxi, Miss.
					Line run with		Mean.	Partial	Total.	
	From—	To—	Between successive marks.	From initial mark K.	Rod E.	Rod F.		E-F.	Accumulated.	
1885. May.			*km.*	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
6	T_3	65	1·136	467·428	— 6·3190	— 6·3192	— 6·3191	+0·2	—15·7	+119·3830
8	65	70	1·118	468·546	— 3·0084	— 3·0085	— 3·0084	+0·1	—15·6	+116·3746
8	70	71	1·127	469·673	+ 2·5467	+ 2·5481	+ 2·5474	—1·4	—17·0	+118·9220
8	71	72	0·962	470·635	— 1·6667	— 1·6660	— 1·6664	—0·7	—17·7	+117·2556
8	72	73	1·063	471·698	+ 3·1604	+ 3·1580	+ 3·1592	+2·4	—15·3	+120·4148
8	73	U_3	1·134	472·832	— 3·8644	— 3·8666	— 3·8655	+2·2	—13·1	+116·5493
11	U_3	81	1·324	474·156	— 0·7111	— 0·7089	— 0·7100	—2·2	—15·3	+115·8393
11	81	80	0·934	475·090	+ 2·3475	+ 2·3512	+ 2·3494	—3·7	—19·0	+118·1887
11	80	79	1·633	476·723	+ 0·8702	+ 0·8679	+ 0·8690	+2·3	—16·7	+119·0577
11	79	78	1·136	477·859	— 0·8876	— 0·8849	— 0·8862	—2·7	—19·4	+118·1715
9	78	77	0·940	478·799	+ 3·6725	+ 3·6716	+ 3·6720	+0·9	—18·5	+121·8435
9	77	76	1·138	479·937	+ 0·4596	+ 0·4619	+ 0·4608	—2·3	—20·8	+122·3043
9	76	75	1·010	480·947	— 1·5828	— 1·5836	— 1·5832	+0·8	—20·0	+120·7211
9	75	74	1·456	482·403	— 1·1611	— 1·1590	— 1·1600	—2·1	—22·1	+119·5611
9	74	69	1·179	483·582	+ 1·7745	+ 1·7769	+ 1·7757	—2·4	—24·5	+121·3368
7	69	68	1·406	484·988	+ 0·8582	+ 0·8569	+ 0·8576	+1·3	—23·2	+122·1944
7	68	67	1·116	486·104	— 1·6000	— 1·5975	— 1·5988	—2·5	—25·7	+120·5956
7	67	66	1·136	487·240	+ 0·7681	+ 0·7708	+ 0·7694	—2·7	—28·4	+121·3650
7	66	62	1·166	488·406	+ 0·9857	+ 0·9832	+ 0·9844	+2·5	—25·9	+122·3494
2	62	61	0·937	489·343	— 1·9184	— 1·9138	— 1·9161	—4·6	—30·5	+120·4333
2	61	60	1·137	490·480	+ 0·9144	+ 0·9133	+ 0·9138	+1·1	—29·4	+121·3471
2	60	59	1·081	491·561	+ 1·8457	+ 1·8479	+ 1·8468	—2·2	—31·6	+123·1939
2	59	58	1·149	492·710	+ 4·5999	+ 4·5952	+ 4·5976	+4·7	—26·9	+127·7915

2	58	57	1·169	493·879	−1·2550	−1·2544	−1·2547	−0·6	−27·5	+126·5368
1	57	56	1.114	494·993	+4·9279	+4·9250	+4·9264	+2·9	−24·6	+131·4632
1	56	55	1·106	496·099	+2·8403	+2·8422	+2·8412	−1·9	−26·5	+134·3044
1	55	54	0·888	496·987	+2·2198	+2·2195	+2·2196	+0·3	−26·2	+136·5240
1	54	R_3	1·083	498·070	+3·3809	+3·3807	+3·3808	+0·2	−26·0	+139·9048
May 1, Apr. 22	R_3	31	1·178	499·248	*−0·6763	* −0·6774	−0·6768	+1·1	−24·9	+139·2280
Apr. 22	31	32	1·909	501·157	−0·7163	−0·7172	−0·7168	+0·9	−24·0	+138·5112
23	32	33	1·239	502·396	−3·2802	−3·2790	−3·2796	−1·2	−25·2	+135·2316
23	33	34	1·390	503·786	+1·3167	+1·3132	+1·3150	+3·5	−21·7	+136·5466
23	34	35	1·171	504·957	+3·7370	+3·7355	+3·7362	+1·5	−20·2	+140·2828
23	35	36	1·171	506·128	+2·6436	+2·6455	+2·6446	−1·9	−22·1	+142·9274
23	36	37	1·244	507·372	+2·8710	+2·8695	+2·8702	+1·5	−20·6	+145·7976
24	37	38	1·220	508·592	+3·5859	+3·5852	+3·5856	+0·7	−19·9	+149·3832
24	38	39	1·140	509·732	+2·8122	+2·8147	+2·8134	−2·5	−22·4	+152·1966
24	39	40	1·060	510·792	−0·8240	−0·8239	−0·8240	−0·1	−22·5	+151·3726
25	40	41	1·475	512·267	+1·1311	+1·1308	+1·1310	+0·3	−22·2	+152·5036
25	41	42	1·220	513·487	−1·7863	−1·7865	−1·7864	+0·2	−22·0	+150·7172
25	42	43	1·248	514·735	+2·8939	+2·8920	+2·8930	+1·9	−20·1	+153·6102
25	43	44	1·883	516·618	+1·4003	+1·3987	+1·3995	+1·6	−18·5	+155·0097
25	44	45	1·173	517·791	−0·3384	−0·3329	−0·3356	−5·5	−24·0	+154·6741
27	45	46	1·176	518·967	* +1·3358	* +1·3357	+1·3358	+0·1	−23·9	+156·0099
27	46	47	1·149	520·116	* +1·0702	* +1·0668	+1·0685	+3·4	−20·5	+157·0784
27, 29, 28	47	48	0·914	521·030	** +2·2300	** +2·2278	+2·2289	+2·2	−18·3	+159·3073
28	48	49	1·209	522·239	−1·5375	−1·5370	−1·5372	−0·5	−18·8	+157·7701
28	49	50	1·163	523·402	−5·6879	−5·6892	−5·6886	+1·3	−17·5	+152·0815
28	50	51	1·198	524·600	+1·8255	+1·8211	+1·8233	+4·4	−13·1	+153·9048
28	51	S_3	0·618	525·218	−2·6792	−2·6793	−2·6792	+0·1	−13·0	+151·2256
28	S_3	52	1·154	526·372	+3·3989	+3·3967	+3·3978	+2·2	−10·8	+154·6234
29	52	53	1·660	528·032	+2·5075	+2·5075	+2·5075	0.0	−10·8	+157·1309
29	53	IX_a	1·139	529·171	−0·1979	−0·1985	−0·1982	+0·6	−10·2	+156·9327
21	IX_a	30	1·628	530·799	+5·5707	+5·5745	+5·5726	−3·8	−14·0	+162·5053
21	30	29	1·180	531·979	−0·9260	−0·9256	−0·9258	−0·4	−14·4	+161·5795
21	29	28	1·181	533·160	+3·5621	+3·5617	+3·5619	+0·4	−14·0	+165·1414
20	28	27	1·387	534·547	+3·1002	+3·1003	+3·1002	−0·1	−14·1	+168·2416
21	27	Q_3	0·410	534·957	+0·9339	+0·9320	+0·9330	+1·9	−12·2	+169·1746

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					Line run with		Mean.	Partial.	Total,	
	From—	To—	Between successive marks.	From initial mark K.	Rod E.	Rod F.		E-F.	Accumulated.	
1885. Apr.			*km.*	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
20	27	26	1·189	535·736	−2·3288	−2·3258	−2·3273	−3·0	−17·1	+165·9143
20	26	25	1·182	536·918	+1·0936	+1·0950	+1·0943	−1·4	−18·5	+167·0086
20	25	24	1·203	538·121	−4·0192	−4·0187	−4·0190	−0·5	−19·0	+162·9896
18	24	23	1·888	540·009	−0·8823	−0·8827	−0.8825	+0·4	−18·6	+162·1071
18	23	P_3	1·172	541·181	+2·6108	+2·6145	+2·6126	−3·7	−22·3	+164·7197
18	P_3	22	1·136	542·317	+5·7136	+5·7174	+5·7155	−3·8	−26·1	+170·4352
18	22	21	1·214	543·531	−2·6605	−2·6643	−2·6624	+3·8	−22·3	+167·7728
17	21	20	1·662	545·193	−4·0904	−4·0920	−4·0912	+1·6	−20·7	+163·6816
17	20	19	1·143	546·336	−3·1655	−3·1671	−3·1663	+1·6	−19·1	+160·5153
17	19	18	1·155	547·491	−2.2395	−2·2392	−2·2394	−0·3	−19·4	+158·2759
17	18	17	0·922	548·413	−2·0408	−2·0384	−2·0396	−2·4	−21·8	+156·2363
17	17	16	1·149	549·562	−3·7282	−3·7293	−3·7288	+1·1	−20·7	+152·5075
15	16	15	0·665	550·227	−1·2598	−1·2570	−1·2584	−2·8	−23·5	+151·2491
15	15	14	1·346	551·573	−3·2658	−3·2618	−3·2638	−4·0	−27·5	+147·9853
15	14	13	1·573	553·146	+1·2283	+1·2294	+1·2288	−1·1	−28·6	+149·2141
15	13	12	0·921	554·067	+0·4924	+0·4929	+0·4926	−0·5	−29·1	+149·7067
14	12	11	1·109	555·176	+0·0118	+0·0162	+0·0140	−4·4	−33·5	+149·7207
14	11	N_3	1·160	556·336	−0·1511	−0·1503	−0·1507	−0·8	−34·3	+149·5700
13, 16, 17	N_3	7	0·989	557·325	** +0·8354	** +0·8401	+0·8377	−4·7	−39·0	+150·4077
13	7	8	1·071	558·396	−2·1482	−2·1465	−2·1474	−1·7	−40·7	+148·2603
13, 14	8	O_3	1·438	559·834	* −5·8040	* −5·8035	−5·8038	−0·5	−41·2	+142·4565
13	O_3	9	0·856	560·690	+2·5133	+2·5157	+2·5145	−2·4	−43·6	+144·9710
14	9	10	1·142	561·832	+2·4235	+2·4242	+2·4238	−0·7	−44·3	+147·3948
14	10	6	0·676	562·508	+0·8138	+0·8124	+0·8131	+1·4	−42·9	+148·2079

II	6	5	1·133	563·641	+1·0096	+1·0106	+1·0101	−1·0	−43·9	+12	49·
II	5	4	1·140	564·781	+0·9158	+0·9145	+0·9152	+1·3	−42·6		+150·1332
II	4	3	1·334	566·115	+2·9919	+2·9905	+2·9912	+1·4	−41·2		+153·1244
II	3	2	1·105	567·220	+3·0323	+3·0333	+3·0328	−1·0	−42·2		+156·1572
IO	2	1	1·324	568·544	+3·1463	+3·1462	+3·1462	+0·1	−42·1		+159·3034
IO	1	V	0·862	569·406	+0·3148	+0·3116	+0·3132	+3·2	−38·9		+159·6166
II	V	M_3	0·104	569·510	+0·3607	+0·3596	+0·3602	+1·1	−37·8		+159·9768

* An asterisk attached to any number indicates that it is a mean value of two determinations; a double asterisk denotes a mean of three or four values.

The probable errors of the resulting heights will have to depend on the discordances of the forward and backward measures for Sections I and II, and on the differences between results by rods E and F for the simultaneous lines of Section III. Results thus obtained do not take into account systematic or constant errors to which the observations are known to be liable; this latter class of uncertainty can only be recognized by the closing of loops of lines of levels. The phenomenon of accumulation of errors in the same direction is quite prominent in Section II, where, in a distance of 128$^{km.}$, and during about three months (October to January), the discrepancy rises from 114$^{mm.}$ to 310$^{mm.}$ in a surprisingly steady manner; the rate of divergence is 1·53$^{mm.}$ per kilometre (or in angular value 0·3″), and is apparently uninfluenced by the varying circumstances during the measure of the line.* The serious part of this behavior is the fact that we have no assurance that the mean result of the two measures converges to or diverges from the true value of the level. Throughout the line the temporary marks are about equally distributed and not far from a kilometre apart; hence we use the formula for the probable error per km. for double leveling $r_{,,}=0{\cdot}675\sqrt{\frac{[dd]}{4[s]}}$ and for the terminal point at the distance $S=[s]$ the value $r=0{\cdot}675\sqrt{\frac{[dd]}{4}}$

Table of probable errors of results.

Designation of line of levels.	$S=[s]$	$[dd]$	$n.$	$r_{,,}$	r
	km.			*mm.*	*mm.*
Biloxi, Miss. (Tidal mark), to Mobile, Ala. (A).	94·3	454	94	±0·74	∓ 7·2
Mobile, Ala. (A), to Meridian, Miss. (C).	219·2	1084*	178	±0·83	±11·8
Meridian, Miss. (C), to Okolona, Miss. (K).	204·2	875*	138	±0·87	±12·5
Okolona, Miss. (K), to Corinth, Miss. (V).	109·5	839	100	±0·93	± 9·8
Corinth, Miss. (V), to Greenfield, Tenn. (XV).	155·9	2157	135	±1·25	∓15·7
Greenfield, Tenn. (XV), to Villa Ridge, Ill. (Z_3).	128·2	1175	112	±1·02	±11·6
Villa Ridge, Ill. (Z_3), to Odin, Ill. (V)	176·0	809*	160	±0·91	±12·1

*Part of this distance consists of simultaneous parallel lines, *i. e.*, the part from Mobile to Citronelle, Ala., for which $[dd]=225$ and $[s]=53{\cdot}66$, here we substitute 1·6 $[dd]$ for $[dd]$, and we likewise multiply by 1·6 the $[dd]$ of the other two marked (*) lines in order to allow (empirically) for the fact that such lines do not develop the whole of the discrepancies. This amounts to increasing the probable error by 25 per cent in order to give a more correct value and one more comparable with independent lines. (See App. No. 10, Report for 1888, p. 423.)

Since the *mean* error m_1 of a single leveling of a kilometre is frequently used for purposes of comparison, it is given below, computed

* In the parallel simultaneous lines of Section III the discrepancy rises slowly but more rapidly towards the end and always with the same sign from 0 to 44$^{mm.}$ in 176$^{km.}$, evidently *part* of the difference being common to the two lines; hence it disappears in the comparison.

by $m_1 = \sqrt{\frac{[dd]}{2\,[s]}}$. For lines from Okolona to Greenfield $m = \pm 2{\cdot}38^{mm.}$, Greenfield to Villa Ridge $m_1 = \pm 2{\cdot}14^{mm.}$, and from Villa Ridge to Odin $m_1 = \pm 1{\cdot}90^{mm.}$, the last increased by one-fourth of its amount.

The total resulting *probable* error of leveling between Biloxi and Odin, a distance of $1\,087^{km.}$, sums up as follows:

$\sqrt{[r^2]} = \pm 31{\cdot}1^{mm.}$ for uncertainty in leveling, and $\pm 25{\cdot}0^{mm.}$ for uncertainty in the tidal level, hence by combination the height and probable error of the Odin bench mark V becomes . . . $159{\cdot}617^{m.} \pm 0{\cdot}040^{m.}$.

Closing of loops.—In the first place we have the grand loop:

Sandy Hook, N. J., Odin, Ill., and Biloxi, Miss., with Gulf and ocean.

Appendix No. 11, Report for 1882, p. 552, gives for height of bench mark V at Odin $161{\cdot}348^{m} \pm 0{\cdot}047^{m}$, hence the discrepancy $1{\cdot}731^{m.}$. The part of the line from Hagerstown, Md., to Odin has no checks, nor is the identity of the Gulf and Atlantic levels assured; the assigned probable errors evidently do not represent the true state of the case.

Second loop: Cairo, Odin, St. Louis, Cairo.

		m.	Distance. *km.*	
I.	Δh Cairo (1) to Odin (V),	+63·4908	194·7	
	Δh Odin (V) to St. Louis (J_3 or K_3),	—34·4398	103·6	
	Δh St. Louis (J_3 or K_3) to Cairo (1),	—29·2737	275·7	Report Miss. River Comm'n, 1883.
	Closing error,	—0·2227		

Third loop: Corinth, Cairo, Memphis, Corinth.

		m.	*km.*	
II.	Δh Corinth (V) to Cairo (1),	—41·1023	265·2	
	Δh Cairo (1) to Memphis (U. S. B. M.),	—16·3222	330·8	Report Miss. River Comm'n, 1881–'83.
	Δh Memphis (U. S. B. M.) to Corinth (V),	+57·2160 (?)	153·1 (?)	Field comp'n.*
	Closing error,	—0·2085		

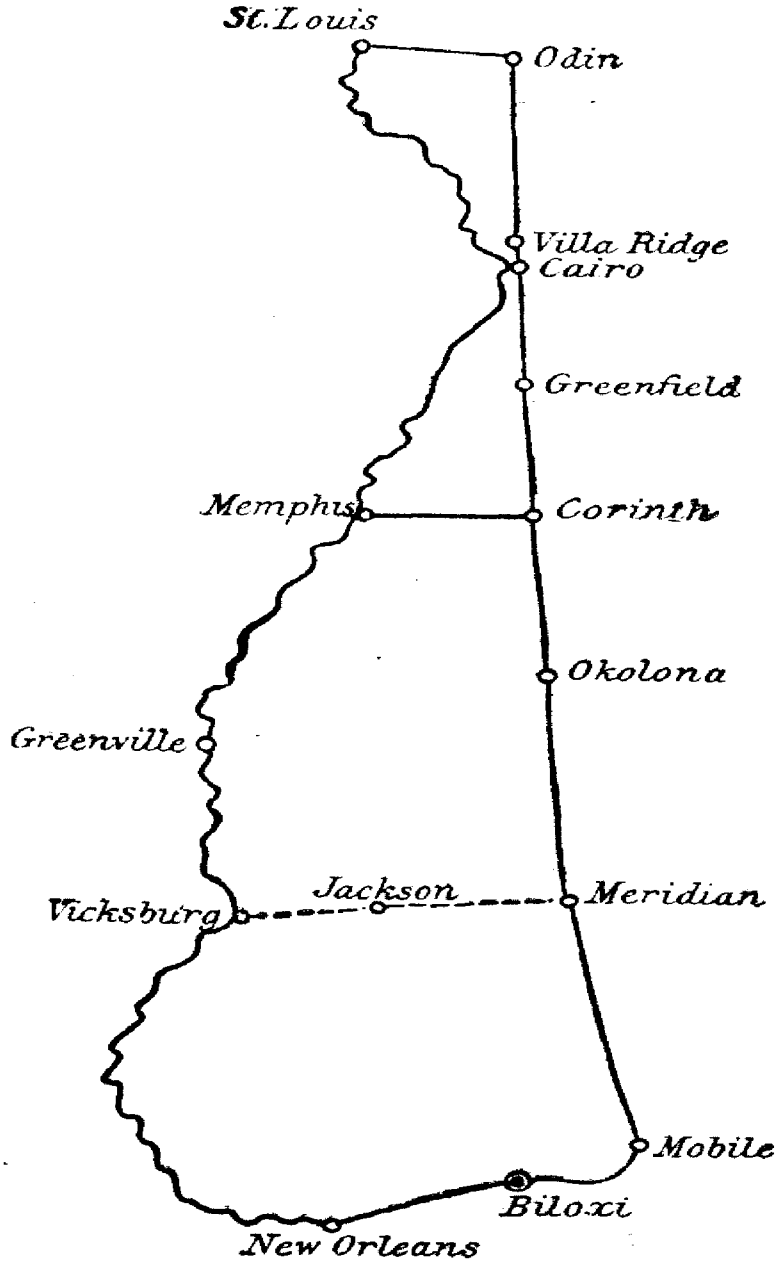
Fourth loop: Biloxi, Mobile, Corinth, Memphis, Vicksburg, New Orleans, Biloxi.

		m.	*km.*	
III.	Δh Biloxi (T. B. M.) to Corinth (V),	+137·2281	627·3	
	Δh Corinth (V) to Memphis (U. S. B. M.),	—57·2160 (?)	153·1 (?)	Field comp'n.*
	Δh Memphis (U. S. B. M.) to Biloxi (T. B. M.),	—80·3192	1053·8	Reports Miss. River Comm'n.
	Closing error,	—0·3071		

Making use of the conventional expression $5\sqrt{[s]}$ to indicate the greatest closing error that might be tolerated in a line, we have for the

*The office computation was nearly completed by Mr. Nelson when he was assigned to other duty.

above loops I, II, III, 0·120^{m}, 0·137^{m}, and 0·214^{m}; since the actual errors* are in excess of these numbers the above degree of standard accuracy is not attained, still the results may be accepted provided, as is highly desirable, that the tie line Vicksburg to Meridian (proposed in 1887) be sup-



plied, forming one more loop and sustaining the Mississippi River Commission loop Vicksburg, Shreveport, Fort Adams. Indeed, from a letter of Capt. J. H. Willard to the Chief of Engineers (November 8, 1890), it appears that Vicksburg and Meridian have been connected by ordi-

* They correspond to about 7 $\sqrt{[s]}$

nary railroad leveling, and are said to show a discrepancy of about 1 foot as compared with our result.

In consequence of the earth's rotation, the equipotential surfaces that may be passed through Biloxi and Odin will converge toward the north; the convergence will amount to $\triangle = -0{\cdot}00514\,h\,\triangle\varphi \sin 2\varphi$, where $\triangle\varphi$ = difference in latitude (expressed in parts of the radius). For latitudes 30° 24′ and 38° 38′ and $h = 126^m$ we have $\triangle = -0{\cdot}087^m$, a quantity of the same order of magnitude as the observing error. For *uniform* slope of profile one-half of this amount, or $0{\cdot}044^m$, would have to be subtracted from the elevation of the northern station. The expression for $\triangle$ depends on the relation $h_1 g_1 = h_2 g_2$, a constant, which must hold between any two adjacent equipotential surfaces, g representing gravity and h the corresponding distance between them, and this forms the basis of the so-called orthometric correction, now recognized as a necessity in refined leveling. This will be attended to, together with the adjustment of the loops, as soon as the needed data are all on hand.

I close this report with a statement of the several independent and unadjusted results for height of St. Louis, Mo. (B. M's. J_3 and K_3 at Bridge.)

	m.
1. By the Mississippi River route, Biloxi, New Orleans, Vicksburg, etc.	125·915
2. By the Meridianal route, Biloxi, Mobile, Corinth, Odin, etc.	125·177
(The difference of 1 and 2 equals the sum of the closing errors of the 3 loops.)	
3.* By the Great Lakes, Sandy Hook, Albany, Chicago, etc.	126·100
4. By the 39th parallel route, Sandy Hook, Cincinnati, etc.	126·908

For the present we may adopt the preliminary value 125·8 m. or 412·7 feet.

* Water levels, in part.

SECTION I.—*Location and description of permanent bench marks between Okolona, Miss., and Greenfield, Tenn.*

K.—At Okolona, Chickasaw County, Miss. Center of bolt leaded horizontally into a brick on the east front, and near the south end of cotton warehouse occupied by C. R. Smith. The brick is the thirteenth one south of the door and in the twenty-second course from the ground.

L.—At Okolona, Miss. The center of a copper bolt leaded into a brick on the southwest pier of the Mobile and Ohio Railroad station. The brick is the middle one on the west side of the pier, and is on the seventh course from the ground.

M.—On the Mobile and Ohio Railroad bridge over Tubby Lubby Creek. The bottom of a square cavity cut in the capstone of the pier on the north bank of the creek. It is west of the track and is roughly lettered:

U. S.
□
B. M.

N.—At Shannon, Lee County, Miss. The center of a cross cut on a brass bolt leaded horizontally into a brick in the east wall of R. B. Clark & Co.'s store. The brick is in the sixth row above the foundation, and is the sixth one from the north end of wall.

O.—At Verona, Lee County, Miss. The center of a cross cut on a brass bolt leaded horizontally into a brick in the south front of R. L. Trice & Son's store. The brick is in the sixth row above the foundation and is the third one from the west side.

P.—At Tupulo, Lee County, Miss. The center of a cross cut on a brass bolt leaded horizontally into a brick in the front wall of Fulton Hotel. The brick is in the twenty-first course above the ground, and is the second one from the southeast corner.

Q.—At Satillo, Lee County, Miss. The bottom of a square cavity cut in a limestone block set in the front yard of James R. Boling's house, near the northwest corner of the brick flower pit on the south side of the house. The block is 6 inches square, projects 6 inches above the ground, and is marked thus:

U. S.
□
B. M.

R.—At Guntown, Lee County, Miss. The center of a cross cut on a brass bolt leaded horizontally into a brick in the chimney on the north end of J. W. Kernodle's house. The brick is in the twelfth

course above the ground, and is the second one from the outer face of chimney on the side toward the front of the house.

S.—At Baldwyn, Lee County, Miss. The bottom of a square cavity cut in a limestone block set in the southwest corner of the yard to the Bucey Hotel. The block is 6 inches square, projects 6 inches from the ground, and is marked thus:

U. S.
□
B. M.

T.—At Booneville, Prentiss County, Miss. The center of a cross cut on a brass bolt leaded horizontally into a brick in the east front of the court-house. The brick is in the eighteenth course below the window sill, and is the second one from the northeast corner

U.—At Rienzi, Alcorn County, Miss. The center of a cross cut on a copper bolt leaded horizontally into a brick in the outer face of the north chimney on the east side of Taylor's Hotel. The brick is in the fifteenth course above the ground, and is the third one from the north side of chimney.

V.—At Corinth, Alcorn County, Miss. The center of a cross cut in a brass bolt leaded horizontally into a brick in the east wall of a store on Waldren street. The building is opposite the court-house, and is used as the city hall. The brick is in the third course above the sill of the window, near the north end of building, and is the fourth one south of this window.

W.—At Corinth, Miss. The center of a cross cut in a brass bolt leaded horizontally into a brick in the west wall of court-house. The brick is in the fourteenth row below the line of window sills, and in the fifth one south of the fifth cellar ventilator, counting from the northwest corner of the building.

X.—At Ramer, McNairy County, Tenn. The center of a cross cut in a brass bolt leaded horizontally into a brick in the pier under the southwest corner of the Mobile and Ohio Railroad station. The brick is in the fourth course from the top of pier, and is the one on the southeast corner. The end of the bolt is about one-fourth inch below the outer face of the brick.

Y.—At Falcon, McNairy County, Tenn. A brass bolt leaded horizontally into a brick in the chimney on the south end of a house, owned by the Mobile and Ohio Railroad Company, standing a short distance north of the railroad station, on the west side of track. The brick is the second one from the southeast corner of the chimney, in the sixth course above the foundation, on the side fronting the railroad track.

Z.—At Bethel Springs, McNairy County, Tenn. The surface inclosed between two file marks, about 1 inch apart, on one of the raised portions of the corrugated iron door sill of Stovall, Epps & Hendrix

Co.'s brick storehouse. The mark is 5½ inches from the front edge of the sill and 5½ inches from the wall forming the south side of the building.

XXVII.—At McNairy, McNairy County, Tenn. The bottom of a square cavity cut in the rock foundation of the chimney at the south end of E. W. Sheffield's house, near the southeast corner of the chimney. The house is on the west of and fronts the railroad, about one-fourth mile south of the station.

XXVI.—At Henderson, Chester County, Tenn. The center of a cross cut on a brass bolt leaded horizontally into a brick in the south wall of J. F. Hamlett's storehouse. The house is west and a short distance north of the railroad station, facing the railroad track. The brick is in the fifth course above the ground, and is the sixth one from the southwest corner of the building.

XXV.—At Pinson, Madison County, Tenn. The center of a cross cut in a brass bolt leaded horizontally into a brick in the front of M. M. Robin's store. The house is west and a short distance north of the railroad station, facing the track. The brick is in the eleventh course above the sidewalk, and is the second one from the northeast corner of the building.

XXIV.—About 2½ miles south of Jackson, Tenn. The bottom of a square cavity cut in the stone coping of the brick pier at the north end of the Mobile and Ohio Railroad bridge over Forked Deer River. The mark is west of the track near the lower chord of bridge, and about 13 inches from the north edge of stone.

XXIII.—At Jackson, Tenn. The bottom of a square cavity cut in the capstone of the retaining wall in front of one of the basement windows of the Federal Building, corner of Market and Baltimore streets. The window is just north of the Market-street entrance. The stone is marked thus:

U. S.
□
B. M.

XXII.—At Jackson, Tenn. The bottom of a square cavity cut in the capstone of the brick pier at the west end of the Illinois Central Railroad bridge over Market street. It is south of the track, and marked thus:

U. S.
□
B. M.

XXI.—At Oakfield, Madison County, Tenn. The bottom of a square cavity cut in a limestone block set in the southwest corner of the front yard of R. Daniel's house, which stands just north of the store and post-office. The block is 6 inches square, projects 6 inches from the ground, and is marked thus:

U. S.
□
B. M.

XX.—At Medina, Gibson County, Tenn. The bottom of a square cavity cut in a limestone block set in the northwest corner of W. A. House's front yard. The block is 6 inches square, projects 6 inches from the ground, and is marked thus:

U. S.
□
B. M.

XIX.—At Milan, Gibson County, Tenn. The bottom of a square cavity cut in a limestone block set in the front yard of Dr. M. D. L. Gordon's house, on Main street, near the public school. The stone is north of the house, near the back fence. It is 6 inches square, and marked thus:

U. S.
□
B. M.

XVIII.—At Milan, Tenn. The center of a cross cut on a brass bolt leaded horizontally into a brick, in the north wall of the building on the southwest corner of Front and Main streets. The brick is in the seventh course above the pavement, and is the sixth one east of the second doorway from the west end of the building.

XVII.—At Bradford, Gibson County, Tenn. The bottom of a square cavity cut in a granite block set in the southwest corner of the front yard of J. T. Cain's house, which stands southeast of the railroad station, fronting on the track. The stone is 6 inches square, projects 6 inches from the ground, and is marked thus:

U. S.
□
B. M.

XVI.—Near Greenfield, Tenn. Found unstable, abandoned.

XV.—At Greenfield, Tenn. A small hole in a brass bolt leaded horizontally into a brick in the front wall of Darling Jones' storehouse. The brick is in the eighth course above the pavement, and is the third one from the northwest corner of the building.

SECTION II.—*Location and description of permanent bench marks between Greenfield, Tenn., and Villa Ridge, Ill.*

XV.—At Greenfield, Tenn. Described in Section I.

XIV.—At Sharon, Weakly County, Tenn. The center of a cross cut in a brass bolt leaded horizontally into a brick in the chimney at the north end of Lloyd Dodd's house. The brick is in the twenty-fourth course above the ground, near the middle of the chimney. The mark is roughly lettered thus:

U. S.
•
B. M.

XIII.—At Martin, Weakly County, Tenn. A small hole in a brass bolt leaded horizontally into a brick in the west side of the bank building owned by J. R. Lovelace. The brick is in the tenth course above the ground, and is the fourth one from the northwest corner.

XII.—At McConnell, Obion County, Tenn. The bottom of a square hole cut in a limestone block set in the northeast corner of the yard of W. B. Smith, agent for the Illinois Central Railroad. The block is 6 inches square, projects 4 inches above the ground, and is lettered thus:

U. S.
□
B. M.

XI.—At Fulton, Fulton County, Ky. A hole in a brass bolt leaded horizontally into a brick in the east wall of E. C. Reed's livery stable. The brick is in the nineteenth course from the ground, and is the fifteenth one from the northeast corner of the stable. The wall is marked thus:

U. S.
•
B. M.

X.—At Alexander, Fulton County, Ky. The bottom of a square cavity cut in a limestone block set near the northwest corner of J. M. Cashon's frame house, on the east side of the railroad track, near the station. The block is 6 inches square, projects 4 inches from the ground, and is lettered thus:

U. S.
□
B. M.

IX.—At Clinton, Hickman County, Ky. The bottom of a square cavity cut in the stone doorsill at the south entrance of the courthouse. It is lettered thus:

U. S.
□
B. M.

VIII.—At Arlington, Carlisle County, Ky. The bottom of a square cavity cut in a limestone block set in the southwest corner of the back yard of Thos. G. Edward's house, near the railroad station. The block is 6 inches square, projects 4 inches from the ground, and is lettered thus:

U. S.
□
B. M.

VII.—At Bardwell, Carlisle County, Ky. The bottom of a square cavity cut in the south end of the stone doorsill at the west entrance to the court-house. It is marked thus:

U. S.
□
B. M.

VI.—On the Illinois Central Railroad bridge over Mayfield Creek, which is the boundary between Ballard and Carlisle counties, Ky. The bottom of a square cavity cut on the stone coping of the brick pier on the north bank of creek. It is west of the track, just outside of the bridge, and letterd thus:

U. S.
□
B. M.

V.—At Fort Jefferson, Ballard County, Ky. The bottom of a square cavity cut in a limestone block set in the northeast corner of J. M. Vance's yard, across the street from J. C. Depoyster's store. The block is 6 inches square, projects 4 inches from the ground, and is lettered thus:

U. S.
□
B. M.

IV.—At Wickliffe, Ballard County, Ky. The bottom of a square cavity cut in a limestone block set near the northwest corner of the Elliott House, close to the brick wall on the street. The block is 6 inches square, projects 3 inches from the ground, and is lettered thus:

U. S.
□
B. M.

III.—At East Cairo, Ballard County, Ky. The bottom of a square cavity cut in a limestone block set in the top of the levee protecting the Mobile and Ohio Railroad yard, just back of the yardmaster's office. The block is 6 inches square, projects 2 inches from the ground, and is lettered thus:

U. S.
C. □ S.
B. M.

U. S. E. 3.—At Cairo, Ill. Established by the U. S. Engineers, and described by the Miss. River Comm. as follows: "Is hole in copper bolt set in southeast end of Illinois Central Railroad freight depot, at junction of Fourteenth street and Ohio River. It is 0·49 metres southwest of the east corner and is marked with the letters U. S. B. M. cut in stone near bolt." When visited in 1888 some of the lead had been picked from around the bolt, but the bolt was still firm. The arches of the building were cracked, as if it had settled.

U. S. E. 2.—At Cairo, Ill. Established by the U. S. Engineers and described by the Miss. River Comm. as follows: "Is hole in copper bolt set horizontally in east side of building containing offices of the trustees of Cairo City public property. It is 4·44 metres from southeast corner, and 1·12 metres below lower surface of water table. Marked with the letters U. S. B. M. cut in stone near bolt. The building fronts on Washington avenue and is between Eighteenth and Nineteenth streets." The mark was found undisturbed in 1888.

U. S. E. 1.—At Cairo, Ill. Established by the U. S. Engineers and described by the Miss. River Comm. as follows: "Is small hole in center of copper bolt set horizontally in northwest side of custom-house. It is 7·18 metres from northwest corner and 0·37 metres below the junction of the sandstone and limestone, and 0·93 metres above surface of cement walk. Marked with the letters U. S. B. M. cut in stone near bolt. The custom-house is in the square bounded by Fourteenth and Fifteenth and Poplar streets and Washington avenue." It was found undisturbed in 1888.

II.—On one of the abutments of the Illinois Central Railroad bridge over Cash creek. The bottom of a square cavity cut in the abutment at the south bank of the creek, on the east side of the track, and marked thus:

U. S.
□
B. M.

I.—At Mound City Junction, Alexander County, Ill. The bottom of a square cavity cut in a limestone block set in the yard of the Illinois Central Railroad section house. It is near the front fence just north of the gate, near lone tree. The stone is 6 inches square, projects 4 inches from the ground and is lettered thus:

U. S.
□
B. M.

Z_3.—At Villa Ridge, Pulaski County, Ill. The center of a brass bolt leaded horizontally into a brick in the chimney at the east end of the Stoddard House. The brick is in the twenty-second course above the ground and is the fifth one from the south end of chimney.

Y_3.—At Villa Ridge, Ill. Top of a railroad spike driven in the root on the east side of a large beech tree which stands east of the railroad track and nearly opposite the 12-mile post. It is lettered thus:

(U. S. B. M. "Y")

SECTION III.—*Location and description of permanent bench marks between Villa Ridge and Odin, Ill.*

Z_3.—At Villa Ridge, Pulaski County, Ill., described in Section II.

Y_3.—At Villa Ridge, Ill., described in Section II.

XII_a.—South of the 14-mile post on the Illinois Central Railroad. The bottom of a square cavity cut near the center of south abutment of small railroad bridge No. 8, marked thus:

□

139.—About 523 metres south of the 17-mile post on the Illinois Central Railroad. The bottom of a square cavity cut in the capstone of the middle pier of small railroad bridge, No. 12. It is east of the track and is marked thus:

(□)

X_3.—Near Ullin Station, Pulaski County, Ill. The bottom of a square cavity cut on the northeast corner of the north abutment of bridge No. 15, over the Cache River. It is marked thus:

□
U. S.
B. M.

W_3.—At Anna, Union County, Ill. The bottom of a square cavity cut in the stone sill of window on southwest front of Otrich's drug store, under the Otrich House. It is about 0·10^m from west end of iron jamb and 0.06^m from south edge of sill. It is marked thus:

(□)

V_3.—About 1 mile north of Makanda, Jackson County, Ill., and 50 metres south of the 50-mile post on the Illinois Central Railroad. The bottom of a square cavity cut in the southeast capstone of south abutment of iron bridge No. 87, over Drury Creek. It is marked thus:

(□)

T_3.—At Carbondale, Jackson County, Ill. The bottom of a square cavity cut in the stone sill under the east front window of Jacob Beard's brick store. The store is on the west side of the railroad track at the north end of public park. The cavity is marked thus:

(□)

U_3.—In Jackson County, Ill., about 2 miles south of De Soto. The bottom of a square cavity cut in the west end of the south abutment of Illinois Central Railroad bridge over Big Muddy Creek. It is about one metre west of track and 0·3^m from north edge of abutment, and is marked thus:

(□)

R_3.—At Duquoin, Perry County, Ill. The bottom of a square cavity cut in corner of the stone sill at the main door of the Du Quoin bank building, opposite the Illinois Central Railroad depot. The cavity is marked thus:

(□)

S_3.—About 2 miles south of Radom Station, Washington County, Ill. The bottom of a square cavity cut in the northeast corner of the north abutment of the stone-arch bridge, over Little Muddy Creek, on the Illinois Central Railroad. It is marked thus:

(□)

Q_3.—Near Ashley, Washington County, Ill. The bottom of a square cavity cut in the capstone of the southeast wing wall of small culvert, No. 212, on the Illinois Central Railroad, about 175 metres south of the 99-mile post. The cavity is about 2 metres east of the track, and is marked thus:

(□)

P_3.—About 1 mile north of Richview Station, Washington County, Ill. The bottom of a square cavity cut in the southeast corner of the capstone on the east end of a stone culvert on the Illinois Central Railroad, 409 metres south of the 103-mile post. The cavity is marked thus:

(□)

N_3.—At Centralia, Marion County, Ill. The bottom of a square cavity cut in the stone sill of the south window of J. J. Pfaff & Co.'s drug store, marked thus:

(□)

O_3.—About 2 miles north of Centralia, Ill. The bottom of a square cavity cut in the capstone at the east end of the north abutment of the Illinois Central Railroad bridge over Crooked Creek. The cavity is about 3 metres from the iron foot-plate of bridge, and is marked thus:

(□)

M_3.—At Odin, Marion County, Ill. The bottom of a square cavity cut in a rock in the top layer of the foundation, at the southeast corner of Craig's brick store. It is about 1 foot above the ground, and is marked thus:

(□)

V.—At Odin, Ill. The bottom of a square cavity cut in the southeast end of a long arched culvert on the Ohio and Mississippi Railroad, about 45 metres east of the junction of that road and the Illinois Central Railroad.* This bench is the intersection of the transcontinental line of levels and the line from the Gulf of Mexico. It is marked thus:

(B. M. "V." □)

* This bench mark V was established by Assistant A. Braid in 1882. See Appendix No. 11, U. S. C. and G. Survey Report for 1882.

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APPENDIX NO. 4—1892.

ON THE RESULTS OF SPIRIT LEVELING OF PRECISION BETWEEN CORINTH, MISS., AND MEMPHIS, TENN., FROM OBSERVATIONS MADE IN 1890 AND 1891 BY ISAAC WINSTON, SUBASSISTANT, AND F. A. YOUNG, AID.

A report by CHARLES A. SCHOTT, Assistant and Chief of the Computing Division.
Submitted for publication March 10, 1893.

The following report on the results of spirit leveling between Corinth, Miss., and Memphis, Tenn., is herewith respectfully submitted. Corinth is a station in the line of levels from Mobile to Odin, and Memphis is connected with the levels of the Mississippi River from New Orleans to St. Louis; the present line therefore forms a tie be tween these lines.

Route line.—The levels follow the track of the Memphis and Charleston Railroad. Distance between bench mark V at Corinth and bench mark of U. S. E. at Memphis is 151·3 km., or 94·0 statute miles. See illustration (inset) to the preceding appendix (No. 3 for 1892).

Observers and dates.—The observers were Subassistant I. Winston and Mr. F. A. Young. The work was accomplished between November 7, 1890, and January 23, 1891.

Instruments and methods of observing.—Two independent lines were run, one forward (westerly), the other backward (easterly). Mr. Winston ran the forward line from Corinth to La Grange, Tenn., and the backward line from Memphis to La Grange, using level No. 5. Mr. Young ran the backward line from La Grange to Corinth, and the forward line from La Grange to Memphis, using level No. 6. Metric rods N and O were employed.

Instrumental constants.—These are as follows:

Instrument.	No. 5.	No. 6.
Aperture of telescope,	2·9cm.	2·9cm.
Focal length of telescope,	34·8cm.	34·8cm.
Magnifying power,	28	28
Value of one division of striding level,	2″·11 and 2″·08	2″·44 and 2″·62
Mean adopted,	2″·10	2″·53
When determined (E. G. F. and I. W., observers),	Sept. 26, '88, Apr. 8, '90	May 23, '88, Apr. 8, '90
Inequality of collars { + indicates object end. larger,	−0″·48 Mar. 15, '90	+0″·07 Mar. 15, 90
Inequality of collars { − indicates object end. smaller,	−0″·89 Feb. 13, '91	−0″·01 Feb. 9, 13, '91

Weighted mean, the second value having double weight,	—0″.76	+0″.02
Value of 100 divisions (1 turn) of micrometer screw,	313″.1 and 311″.6	310″.6 and 312″.0
Value adopted,	312″.0	312″.0
When determined (A. B., I. W. and J. H. G., observers),	Sept. 19, 22, '88, July 22, 23, '89	June 24, 25, '89, July 22, 23, '89
Angular value of telemeter threads,	18′ 58″.5	18′ 50″.6
When determined (I. W., observers),	July 22, 1889	July 22, 1889

Index corrections to metric rods:	N at beginning,	—0.55mm. at 12° C.	Mar. 14, 1890
	N at end,	—0.35	Feb. 1891
	O at beginning,	—0.63 at 13° C.	Mar. 14, 1890
	O at end,	—0.40	Feb. 1891

Temperature when rod graduations are of standard length, 21.°7 C.

Computation and results.—The field computation was made by the observers; the office computation by Subassistant J. Nelson, temporarily attached to the Computing Division. The two computations were collated, and the results are given in the following tabular form:

Results of geodetic spirit leveling in Mississippi and Tennessee, 1890–'91, from Corinth, Miss., to Memphis, Tenn.

Date.	Bench mark		Distance		Difference of height between marks.			Discrepancy.		Height of mark above average Gulf level at Biloxi, Miss.
	From—	To—	Between successive marks.	From initial mark.	Direction of measure: Forward (westerly).	Direction of measure: Backward (easterly).	Mean.	Partial. F—B	Total. Accumulated.	
1890–'91.			*km.*	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
Jan.		V		0·000					0·0	+137·2281*
31 31	V	W	0·041		+ 0·0568	+ 0·0567				
Dec. Dec.										
20 20			0·040		+ 0·0567	+ 0·0566				
Mean			0·040	0·040	+ 0·0568	+ 0·0566	+ 0·0567	+0·2	+ 0·2	+137·2848*
20 26	W	70	0·343	0·383	— 4·4635	— 4·4613	— 4·4624	—2·2	— 2·0	132·8224
20 26	70	71	1·106	1·489	+ 3·9480	+ 3·9471	+ 3·9476	+0·9	— 1·1	136·7700
20 26	71	72	0·962	2·451	+ 2·4790	+ 2·4738	+ 2·4764	+5·2	+ 4·1	139·2464
22 24	72	73	0·990	3·441	+ 4·8421	+ 4·8404	+ 4·8412	+1·7	+ 5·8	144·0876
22 24	73	74	0·966	4·407	+ 8·6416	+ 8·6413	+ 8·6414	+0·3	+ 6·1	152·7290
22 23	74	75	1·280	5·687	—10·9292	-------				
24 24			1·283		—10·9368	—10·9313				
Mean	74	75	1·281	5·688	—10·9330	—10·9313	—10·9322	—1·7	+ 4·4	141·7968
22	75	× Road (1)	0·328		— 1·2119					140·5849
22 23	75	76	1·153	6·841	— 5·9312	— 5·9339	— 5·9325	+2·7	+ 7·1	135·8643
22 23	76	77	1·170		+ 9·1223	+ 9·1130				
26 26			1·168		+ 9·1183	+ 9·1173				
Mean	76	77	1·169	8·010	+ 9·1203	+ 9·1152	+ 9·1177	+5·1	+12·2	144·9820
22	77	× Road (2)	1·026		+ 4·7340					+149·7160

* See report of December 20, 1892. (App. No. 3, Report for 1892.)

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Date.	Bench mark		Distance		Difference of height between marks.			Discrepancy.		Height of mark above average Gulf level at Biloxi, Miss.
					Direction of measure.		Mean.	Partial.	Total.	
	From—	To—	Between successive marks.	From initial mark.	Forward (westerly).	Backward (easterly).		F-B	Accumulated.	
1890–'91. Jan.			*km.*	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
22 23	77	78	1·276	9·286	+ 4·5100	+ 4·5053	+ 4·5076	+4·7	+16·9	+149·4896
22 23	78	79	0·702	9·988	— 2·0810	— 2·0831	— 2·0820	+2·1	+19·0	147·4076
26 23	79	80	1·016	11·004	— 9·7686	— 9·7667	— 9·7677	—1·9	+17·1	137·6399
26, 27 30	80	81	0·984	11·988	— 5·7740	— 5·7723	— 5·7731	—1·7	+15·4	131·8668
27 30	81	82	0·960	12·948	— 0·5979	— 0·5999	— 0·5989	+2·0	+17·4	131·2679
27 30	82	83	0·986	13·934	— 5·0828	— 5·0847	— 5·0837	+1·9	+19·3	126·1842
27 30	83	84	1·294	15·228	— 2·1891	— 2·1854	— 2·1872	—3·7	+15·6	123·9970
27	84	× Road (3)	0·170		+ 0·3128					124·3098
30 30	84	L	0·092	15·320	+ 1·5460	+ 15·461	+ 1·5460	+0·1	+15·7	125·5430
27 30	84	85	1·234	16·462	— 2·4519	— 2·4465	— 2·4492	—5·4	+10·2	121·5478
27 30, 31	85	86	1·234	17·696	— 1·8310	— 1·8284	— 1·8297	—2·6	+ 7·6	119·7181
27 31	86	87	0·948	18·644	— 1·6550	— 1·6568	— 1·6559	+1·8	+ 9·4	118·0622
27 31	87	88	0·967	19·611	— 1·2996	— 1·2976	— 1·2986	—2·0	+ 7·4	116·7636
Jan. Dec.										
2 31	88	M	1·307	20·918	+ 0·0220	+ 0·0216	+ 0·0218	+0·4	+ 7·8	116·7854
2 31	M	89	1·225	22·143	— 0·0419	— 0·0390	— 0·0404	—2·9	+ 4·9	+116·7450
2 29, 31	89	90	1·037		+10·0377					
Jan.										
2 2			1·039		+10·0383	+10·0415				
2			1·039		+10·0405	+10·0401				

		Mean	89	90	1·038	23·181	+10·0388	+10·0408	+10.0398	−2·0	+ 2·9	+126·7848
	Jan.	Dec.										
S. Ex. 37—14	2, 3	29	90	91	0·971		− 5·9608	− 5·9692				
	3				0·965		− 5·9590	− 5·9566				
		Mean	90	91	0·968	24·149	− 5·9599	− 5·9629	− 5·9614	+3·0	+ 5·9	120·8234
	3	29	91	92	1·248	25·397	− 5·7634	− 5·7646	− 5·7640	+1·2	+ 7·1	115·0594
	3	29	92	93	1·055	26·452	− 0·8390	− 0·8377	− 0·8384	−1·3	+ 5·8	114·2210
	3	29	93	94	1·196	27·648	+ 0·9230	+ 0·9193	+ 0·9211	+3·7	+ 9·5	115·1421
		Jan.										
	3	12	94	95	0·949	28·597	+ 0·4307	+ 0·4283	+ 0·4295	+1·4	+10·9	115·5716
	3	12	95	96	1·105	29·702	− 0·9869	− 0·9823	− 0·9846	−4·6	+ 6·3	114·5870
	3	12	96	97	0·858	30·560	+ 5·1725	+ 5·1741	+ 5·1733	−1·6	+ 4·7	119·7603
	5		97	N	0·073	30·633	+ 1·9079	+ 1·9082	+ 1·9080	−0·3	+ 4·4	121·6683
	5		N	R.R.S. (4)	0·075		− 1·6330		− 1·6330			120·0353
	5	10	97	98	1·241		+ 6·9543	+ 6·9634				
	12				1·244		+ 6·9562	+ 6·9529				
		Mean	97	98	1·242	31·802	+ 6·9552	+ 6·9582	+ 6·9567	−3·0	+ 1·7	126·71707
	Jan.	Jan.										
	5	10	98	99	1·351	33·153	−11·5391	−11·5355	−11·5373	−3·6	− 1·9	115·1797
	5	10	99	100	1·264		+ 0·7862	+ 0·7960				
	13	13			1·259		+ 0·7927	+ 0·7924				
		Mean	99	100	1·262	34·415	+ 0·7894	+ 0·7942	+ 0·7918	−4·8	− 6·7	115·9715
	5	10	100	101	1·298	35·713	+ 6·4514	+ 6·4550	+ 6·4532	−3·6	−10·3	122·4247
	5	10	101	102	0·998		− 5·3672	− 5·3611				
	13				0·996		− 5·3607	− 5·3601				
		Mean	101	102	0·997	36·710	− 5·3639	− 5·3606	− 5·3622	−3·3	−13·6	117·0625
	5	10	102	103	1·180	37·890	+ 2·7697	+ 2·7745	+ 2·7721	−4·8	−18·4	+119·8346

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Date.	Bench mark		Distance		Difference of height between marks.			Discrepancy.		Height of mark above average Gulf level at Biloxi, Miss.
					Direction of measure.		Mean.	Partial.	Total.	
	From—	To—	Between successive marks.	From initial mark.	Forward (westerly).	Backward (easterly).		F-B	Accumulated.	
1890–'91.			*km.*	*km.*	*m.*	*m.*	*m.*	*mm*	*mm.*	*m.*
Jan. 6	103	R. R. S.(5)	0·937		+ 4·1489		+ 4·1489			+123·9835
Jan. 6 Jan. 9	103	104	1·030	38·920	+ 3·6806	+ 3·6836	+ 3·6821	—3·0	—21·4	123·5167
8	104	0	0·091	39·011	+ 1·3123	+ 1·3126	+ 1·3124	—0·3	—21·7	124·8291
6 8	104	105	0·980	39·900	+ 8·4975	+ 8·4967	+ 8·4971	+0·8	—20·6	132·0138
6 8	105	106	1·267	41·167	+ 2·6365	+ 2·6394	+ 2·6380	—2·9	—23·5	134·6518
6 7, 8	106	107	1·282	42·449	— 5·6788	— 5·6782	— 5·6785	—0·6	—24·1	128·9733
6 7	107	108	1·322	43·771	+ 5·7190	+ 5·7247	+ 5·7218	—5·7	—29·8	134·6951
6 7	108	109	0·996		+ 7·1268	+ 7·1353				
13			0·992		+ 7·1206	+ 7·1266				
Mean.	108	109	0·994	44·765	+ 7·1237	+ 7·1310	+ 7·1274	—7·3	—37·1	141·8225
6 7	109	110	0·994		— 2·8901	— 2·8811				
13			0·992		— 2·8894	— 2·8870				
Mean.	109	110	0·993	45·758	— 2·8898	— 2·8840	— 2·8869	—5·8	—42·9	138·9356
6 and 14 7	110	111	1·126	46·884	+ 4·4670	+ 4·4685	+ 4·4678	—1·5	—44·4	143·4034
14	111	x Road (6)	0·183		+ 0·5318		+ 0·5318			+143·9352
14 7	111	112	1·081		+ 8·7684	+ 8·7749				
19			1·085		+ 8·7726	+ 8·7634				

		Mean.	111	112	1·083	47·967	+ 8·7705	+ 8·7692	+ 8·7698	+1·3	−43·1	+152·1732
14		19	112	113	0·944	48·911	+ 8·1018	+ 8·1038	+ 8·1028	−2·0	−45·1	160·2760
14		19	113	114	1·280	50·191	− 3·7915	− 3·7942	− 3·7928	+2·7	−42·4	156·4832
14		19	114	115	1·278	51·469	− 2·0418	− 2·0382	− 2·0400	−3·6	−46·0	154·4432
14		17, 19	115	116	0·932	51·401	+ 0·2744	+ 0·2767	+ 0·2756	−2·3	−48·3	154·7188
14		17	116	117	0·978	53·379	+ 9·0213	+ 9·0191	+ 9·0202	+2·2	+46·1	163·7390
14, 15		17	117	118	1·043	54·422	+ 9·1038	+ 9·1009	+ 9·1023	+2·9	−43·2	172·8413
15		17	118	119	1·142		− 9·3487	− 9·3558				
	20				1·140		− 9·3582	− 9·3602				
		Mean.	118	119	1·141	55·563	− 9·3534	− 9·3580	− 9·3557	+4·6	−38·6	163·4856
15		17	119	120	1·048	56·611	− 9·0299	− 9·0239	− 9·0269	−6·0	−44·6	154·4587
	15		120	R.R.S.(7)	0·953		+ 8·2111		+ 8·2111			162·6698
15		17	120	121	1·026	57·637	+ 9·2172	+ 9·2175	+ 9·2173	−0·3	−44·9	163·6760
	20		121	P	0·378	58·015	+ 1·3896	+ 1·3889	+ 1·3892	+0·7	−44·2	165·0652
15		16, 17	121	122	0·997	58·634	+ 9·3888	+ 9·3881	+ 9·3884	+0·7	−44·2	173·0644
15		16	122	123	0·963	59·597	+ 6·8406	+ 6·8456	+ 6·8431	−5·0	−49·2	179·9075
20		16	123	124	0·874		+ 0·0069	+ 0·0027				
	23				0·863		+ 0·0024	+ 0·0041				
		Mean.	123	124	0·868	60·465	+ 0·0046	+ 0·0034	+ 0·0040	+1·2	−48·0	179·9115
20		16	124	125	1·032	61·497	− 0·5980	− 0·5956	− 0·5968	−2·4	−50·4	179·3147
	20		125	x Road (8)	0·233		+ 0·4664		+ 0·4664			179·7811
20		16	125	126	1·224	62·721	− 4·4270	− 4·4252	− 4·4261	−1·8	−52·2	174·8886
20		16	126	127	1·007		− 9·4517	− 9·4398				
	22				1·011		− 9·4435	− 9·4454				
		Mean.	126	127	1·009	63·730	− 9·4476	− 9·4426	− 9·4451	−5·0	−57·2	+165·4435

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						Direction of measure			Partial.	Total.	
		From—	To—	Between successive marks.	From initial mark.	Forward (westerly).	Backward (easterly).	Mean.	F–B	Accumulated.	
1890–'91.				*km.*	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
Jan.	Jan. Jan.										
20	16	127	128	1·060	64·790	+ 6·5350	+ 6·5320	+ 6·5335	+3·0	—54·2	+171·9770
20, 21	22	128	129	0·993	65·783	+ 0·3981	+ 0·3973	+ 0·3977	+0·8	—53·4	172·3747
21	22	129	130	0·732	66·515	+ 2·4904	+ 2·4859	+ 2·4882	+4·5	—48·9	174·8629
	21	130	R	0·033	66·548	+ 2·0967	+ 2·0967	+ 2·0967	0·0	—48·9	176·9596
	21	130	R. R. x (9)	0·159	66·674	— 0·0149		— 0·0149			174·8480
21	22	130	131	1·112	67·627	— 3·4512	— 3·4437				
	23			1·116		— 3·4449	— 3·4383				
	Mean	130	131	1·114	67·629	— 3·4480	— 3·4410	— 3·4445	—7·0	—55·9	171·4184
21	22	131	132	1·090	68·719	+ 9·9455	+ 9·9434	+ 9·9444	+2·1	—53·8	181·3628
21	22	132	133	1·040	69·759	— 8·6040	— 8·6036	— 8·6038	—0·4	—54·2	172·7590
21	22	133	134	0·950	70·709	— 6·5675	— 6·5639	— 6·5657	—3·6	—57·8	166·1933
Dec. 21	19	134	I	1·002	71·711	— 4·0401	— 4·0368	— 4·0384	—3·3	—61·1	162·1549
19	19	I	K	0·052	71·763	+ 0·0321	+ 0·0321	— 0·0321	0·0	—61·1	162·1870
16	16	I	69	0·538	72·249	— 0·1246	— 0·1275	— 0·1260	+2·9	—58·2	162·0289
	16	69	68	1·016	73·265	— 6·9467	— 6·9461	— 6·9464	—0·6	—58·8	155·0825
17	16	68	67	0·978	74·243	+ 1·3614	+ 1·3600	— 1·3607	+1·4	—57·4	156·4432
17	16	67	66	0·996	75·239	— 4·1521	— 4·1533	+ 4·1527	+1·2	—56·2	152·2995
17	16	66	65	1·004	76·243	+ 1·0775	+ 1·0781	+ 1·0778	—0·6	—56·8	+153·3683
17	15, 16	65	64	0·998		— 4·3699	— 4·3685				

19				1·000		— 4·3554	— 4·3619				
19				0·998		— 4·3614					
	Mean	65	64	0·998	77·241	— 4·3592	— 4·3652	— 4·3622	+6·0	—50·8	+149·0061
17	15	64	63	1·008	78·249	+ 0·3464	— 0·3450	— 0·3457	—1·4	—52·2	148·6604
17	15	63	62	1·253	79·502	— 2·4429	— 2·4422	— 2·4426	—0·7	—52·9	146·2178
17, 18	15	62	61	1·259	80·761	— 7·2864	— 7·2900	— 7·2882	+3·6	—49·3	138·9296
	15	61	x Road (10)	0·176		+ 0·8151		+ 0·8151			139·7447
18	15	61	60	1·188	81·949	— 8·1255	— 8·1244	— 8·1250	—1·1	—50·4	130·8046
18	15	60	59	1·176	83·125	—10·0920	—10·0933	—10·0926	+1·3	—49·1	120·7120
Dec. 18	Dec. 15	59	58	1·060	84·185	— 8·0044	— 7·9979	— 8·0012	—6·5	— 55·6	112·7108
18	15	58	57	1·126	85·311	— 5·1168	— 5·1107	— 5·1138	—6·1	— 61·7	107·5970
18	13	57	56	0·956	86·267	+ 1·7642	+ 1·7658	+ 1·7650	—1·6	— 63·3	109·3620
18	13	56	H	1·168	87·435	— 1·9477	+ 1·9445	— 1·9461	—3·2	— 66·5	107·4159
5		H	G	1·190	88·625	— 4·8566	— 4·8593	— 4·8580	+2·7	— 63·8	102·5579
5		G	55	0·454	89·079	+ 0·2748	+ 0·2724	— 0·2736	+2·4	— 61·4	102·8315
6	5	55	54	0·979	90·058	— 1·9955	— 1·9948	— 1·9952	—0·7	— 62·1	100·8363
6	4, 5	54	53	1·055	91·113	+ 0·1366	+ 0·1355	+ 0·1360	+1·1	— 61·0	100·9723
6	4	53	52	1·115		+ 2·9207	+ 2·9269				
8				1·108		+ 2·9255	+ 2·9245				
	Mean	53	52	1·112	92·225	+ 2·9231	+ 2·9257	+ 2·9244	—2·6	— 63·6	103·8967
6	4	52	51	1·249		— 1·8694	— 1·8615				
8				1·253		— 1·8687	— 1·8676				
	Mean	52	51	1·251	93·476	1·8690	— 1·8646	— 1·8668	—4·4	— 68·0	102·0299
4		51	R.R.S. (11)	0·338		1·4186		+ 1·4186			103·4485
8	4	51	50	1·286		— 2·3477	— 2·3381				
8				1·291		— 2·3458	— 2·3457				
	Mean	51	50	1·288	94·764	— 2·3468	— 2·3418	— 2·3443	—5·0	— 73·0	+99·6856

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					Direction of measure.			Partial.	Total.	
	From—	To—	Between successive marks.	From initial mark.	Forward (westerly).	Backward (easterly).	Mean.	F-B	Accumulated.	
1890–'91.			*km.*	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
Dec. Dec. Dec.										
6,9 4	50	49	0·919	95·683	— 1·6297	— 1·6262	— 1·6280	—3·5	— 76·5	+98·0576
9 4	49	48	0·972		— 1·3892	— 1·3805				
12			0·972		— 1·3907	— 1·3865				
13			0·972		— 1·3869	— 1·3838				
Mean	49	48	0·972	96·655	— 1·3889	— 1·3836	— 1·3862	—5·3	— 81·8	96·6714
9 4	48	47	1·043	97·698	— 0·4312	— 0·4275	— 0·4294	—3·7	— 85·5	96·2420
9 4	47	46	1·134		— 1·8048	— 1·7987				
12			1·138		— 1·8012	— 1·7953				
13			1·134		— 1·8035	— 1·7999				
Mean	47	46	1·135	98·833	— 1·8032	— 1·7980	— 1·8006	—5·2	— 90·7	94·4414
9 4	46	45	1·228	100·061	— 0·1258	— 0·1253	— 0·1256	—0·5	— 91·2	94·3158
3	45	F	0·252	100·313	+ 0·6906	+ 0·6905	+ 0·6906	+0·1	— 91·1	94·0064
9 3	45	44	1·387	101·448	— 0·4077	— 0·4117	— 0·4097	+4·0	— 87·2	93·9061
3	44	R.R.S.(12)	1·068		+ 1·0321		+ 1·0321			+94·9382
9,10 3	44	43	1·286		+ 0·0129	+ 0·0213				
12			1·286		+ 0·0188	+ 0·0208				

		Mean	44	43	1·286	102·734	+ 0·0158	+ 0·0210	+ 0·0184	—5·2	— 92·4	+93·9245
10		3	43	42	1·276	104·010	— 0·6659	— 0·6641	— 0·6650	—1·8	— 94·2	93·2595
10		3	42	41	1·302	105·312	+ 5·4004	+ 5·4020	+ 5·4012	—1·6	— 95·8	98·6607
10		3	41	40	1·061	106·373	+ 3·9880	+ 3·9869	+ 3·9874	+1·1	— 94·7	102·6481
10		3	40	39	1·140	107·513	+ 6·7072	+ 6·7048	+ 6·7060	+2·4	— 92·3	109·3541
10		2, 3	39	38	1·038	108·551	— 3·7185	— 3·7205	— 3·7195	+2·0	— 90·3	105·6346
	2		38	X Road(13)	0·161		+ 0·3593		+ 0·3593			105·9939
10, 11		2	38	37	1·000	109·551	— 4·0632	— 4·0624	— 4·0628	—0·8	— 91.1	101·5718
		Nov.										
11		17	37	36	1·144	110·695	+ 4·7456	+ 4·7440	+ 4·7448	+1·6	— 89·5	106·3166
11		17	36	35	0·952	111·647	+ 7·7233	+ 7·7242	+ 7·7238	—0·9	— 90·4	114·0404
	Nov.											
	15		35	E	0·060	111·707	+ 3·3665	+ 3·3665	+ 3·3665	0·0	— 90·4	117·4069
Nov.												
15		17	35	34	0·978	112·625	— 3·5693	— 3·5712	— 3·5702	+1·9	— 88·5	110·4702
	17		34	R.R.S.(14)	0·866		+ 3·7767		+ 3·7767			114·2469
15		17	34	33	0·982	113·607	— 3·0443	— 3·0459	— 3·0451	+1·6	— 86·9	107·4251
21		17	33	32	1·148	114·755	+ 0·2086	+ 0·2056	+ 0·2071	+3·0	— 83·9	107·6322
22		21	32	31	1·518		— 4·7011	— 4·7124				
	Dec.											
	2				1·512		— 4·6967	— 4·7077				
	11				1·524		— 4·7095	— 4·7055				
		Mean	32	31	1·518	116·273	— 4·7024	— 4·7085	— 4·7054	+6·1	— 77·8	102·9268
Nov.		Nov.										
22		21	31	30	1·252		+ 3·3520	+ 3·3572				
	Dec.											
	2				1·258		+ 3·3553	+ 3·3571				
		Mean	31	30	1·255	117·528	+ 3·3536	+ 3·3572	+ 3·3554	—3·6	— 81·4	+106·2822

Results of geodetic spirit leveling in Mississippi and Tennessee, 1890–'91, from Corinth, Miss., to Memphis, Tenn.

Date.	Bench mark		Distance		Difference of height between marks.			Discrepancy.		Height of mark above average Gulf level at Biloxi, Miss.
	From—	To—	Between successive marks.	From initial mark.	Direction of measure. Forward (westerly).	Backward (easterly).	Mean.	Partial. F-B	Total. Accumulated.	
1890–'91.			*km.*	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
Nov. 22	30	D	0·089	117·617	+ 2·1867	+ 2·1866	+ 2·1866	+0·1	— 81·3	+108·4688
Nov. 22 Nov. 21	30	29	0·557	118·085	+ 1·0289	+1·0297	+ 1·0293	—0·8	— 82·2	107·3115
21	29	R.R.S.(15)	0·511		— 0·6646		— 0·6646			106·6469
22 21	29	28	1·259	119·344	+ 2·9755	+ 2·9783	+ 2·9769	—2·8	— 85·0	110·2884
22 21	28	27	0·934	120·278	+ 1·5384	+ 1·5360	+ 1·5372	+2·4	— 82·6	111·8256
22 21	27	26	0·955	121·233	— 2·3511	— 2·3508	— 2·3510	—0·3	— 82·9	109·4746
21	26	X Road(16)	0·371		+ 1·0476		+ 1·0476			110·5222
24 20	26	25	0·924	122·157	— 0·2714	— 0·2689	— 0·2702	—2·5	— 85·4	109·2044
24 20	25	24	0·942	123·099	— 1·4447	— 1·4447	— 1·4447	0·0	— 85·4	107·7597
24 20	24	23	1·236	124·335	+ 4·8628	+ 4·8668	+ 4·8648	—4·0	— 89·4	112·6245
24 20	23	22	1·231	125·566	+ 4·0760	+ 4·0791	+ 4·0776	—3·1	— 92·5	116·7021
24 20	22	C	1·233	126·799	— 1·6107	— 1·6151	— 1·6129	+4·4	— 88·1	115·0892
20	C	R.R.S.(17)	0·204		— 0·5511		— 0·5511			114·5381
24 20	C	21	0·500	127·299	— 4·4361	+ 4·4358	— 4·4360	—0·3	— 88·4	110·6532
24, 25 20	21	20	1·288	128·587	— 5·7824	— 5·7814	— 5·7819	—1·0	— 89·4	104·8713
25 19	20	19	0·937	129·524	— 3·7895	— 3·7870	— 3·7882	—2·5	— 91·9	+101·0831
25 19	19	18	1·038		+ 0·5208	+ 0·5325				

Dec. 1				1·034		+ 0·5265	+ 0·5226				
	Mean	19	18	1·036	130·560	+ 0·5236	+ 0·5276	+ 0·5256	−4·0	− 95·9	+101·6087
Nov.	Nov.										
25	19	18	17	1·324	131·884	− 2·2307	− 2·2261	− 2·2284	−4·6	−100·5	99·3803
25	19	17	16	1·253		+ 3·9835	+ 3·9900				
Dec. 1				1·248		+ 3·9930	+ 3·9948				
	Mean	17	16	1·250	133·134	+ 3·9882	+ 3·9924	+ 3·9903	−4·2	−104·7	103·3706
Nov.	Nov.										
25, 26	19	16	15	1·240	134·374	− 2·5543	− 2·5497	− 2·5520	−4·6	−109·3	100·8186
26	19	15	B	0·958		− 5·4651	− 5·4595				
29				0·959		− 5·4634	− 5·4640				
	Mean	15	B	0·958	135·332	− 5·4642	− 5·4618	− 5·4630	−2·4	−111·7	95·3556
26	19	B	14	1·276		+ 1·5580	+ 1·5641				
29				1·280		+ 1·5613	+ 1·5624				
	Mean	B	14	1·278	136·610	+ 1·5596	+ 1·5632	+ 1·5614	−3·6	−115·3	96·9170
26	18	14	13	1·034		− 7·5718	− 7·5847				
29	28			1·038		− 7·5814					
Dec. 1	29			1·036		− 7·5798	− 7·5834				
Dec. 1				1·033			− 7·5818				
	Mean	14	13	1·035	137·645	− 7·5777	− 7·5833	− 7·5805	+5·6	−109·7	89·3365
Nov.	Nov.										
26	18	13	12	1·139		− 1·2852	− 1·2926				
28				1·141		− 1·2882	− 1·2882				
	Mean	13	12	1·140	138·785	− 1·2867	− 1·2904	− 1·2886	+3·7	−106·0	+88·0479

Results of geodetic spirit leveling in Mississippi and Tennessee, 1890–'91, from Corinth, Miss., to Memphis, Tenn.

Date.			Bench mark.		Distance		Difference of height between marks.			Discrepancy.		Height of mark above average Gulf level at Biloxi, Miss.
			From—	To—	Between successive marks.	From initial mark.	Direction of measure: Forward (westerly).	Direction of measure: Backward (easterly).	Mean.	Partial. F-B	Total, Accumulated.	
1890–'91.					*km.*	*km.*	*m.*	*m.*	*m.*	*mm.*	*mm.*	*m.*
Nov.	Dec.	Nov.										
26		18	12	11	1·200	139·985	+ 3·7370	+ 3·7324	+ 3·7347	+4·6	—101·4	+91·7826
	18		11	x Road (18)	0·158		— 0·0278		— 0·0278			91·7548
11, 26		18	11	10	1·188	141·173	— 2·0136	— 2·0128	— 2·0132	—0·8	—102·2	89·7694
	18		10	A	0·076	141·249	+ 1·3684	+ 1·3683	+ 1·3684	+0·1	—102·1	91·1378
11		10	10	9	0·856	142·029	— 1·7642	— 1·7608	— 1·7625	—3·4	—105·6	88·0069
11		10	9	8	1·693		+ 2·8728	+ 2·8638				
14		18			1·698		+ 2·8747	+ 2·8771				
		Mean	9	8	1·696	143·725	+ 2·8738	+ 2·8704	+ 2·8721	+3·4	+102·2	90·8790
11		10	8	7	1·112		— 2·5017	— 2·5031				
	14				1·118		— 2·4979	— 2·5054				
	28				1·114		— 2·5054	— 2·5062				
		Mean	8	7	1·115	144·840	— 2·5017	— 2·5049	— 2·5033	+3·2	— 99·0	88·3757
11, 12		10	7	6	1·184	146·024	+ 5·6152	+ 5·6143	+ 5·6148	+0·9	— 98·1	93·9905
	10		6	x Road (19)	1·080		— 5·5554		— 5·5554			88·4351
12		10	6	5	1·052	147·076	+ 1·2252	+ 1·2258	+ 1·2255	—0·6	— 98·7	95·2160
12		8	5	4	1·290	148·366	— 8·0234	— 8·0251	— 8·0242	+1·7	— 97·0	+87·1918

12		8	4	3	1·277		−10·2550	−10·2416				
	13				1·280			−10·2547				
13		13, 14			1·279		−10·2573	−10·2590				
		Mean	4	3	1·279	149·645	−10·2562	−10·2518	−10·2540	−4·4	−101·4	+76·9378
12, 13		8	3	I	1·677	151·322	+ 5·2234	+ 5·2215	+ 5·2224	+1·9	− 99·5	82·1602
	7		I	U. S. E.	0·038	151·360	− 2·1142	− 2·1142	− 2·1142	0·0	− 99·5	+80·0460

The probable error of the resulting differences of heights depends on the discrepancies of the results as found by the forward and backward measures, and since the bench marks are about evenly distributed over the whole line at distances of about one kilometre we make use of the formula $r_{11} = 0{\cdot}675\sqrt{\frac{[d\,d]}{4\,[s]}}$, which gives the probable error per kilometre for double leveling; for the terminal point at the distance $S = [s]$ we have $r = 0{\cdot}675\sqrt{\frac{[d\,d]}{4}}$.

We have $[dd] = 1466$, hence $r_{11} = \pm 1{\cdot}05^{mm}$, also $r = \pm 12{\cdot}9^{mm}$. The column showing the accumulation of error exhibits the usual more or less systematic increase over the whole line, with a maximum value of 115^{mm}; this corresponds to an angle of $0''{\cdot}16$, or to a constant error of $0''{\cdot}08$ for each measure, supposed of the same sign.

Referring to my report of December 20, 1892 (Appendix No. 3, 1892), and to the closing of the two loops, II and III, we have now for $\triangle h$ Memphis to Corinth $+57{\cdot}1821^{m}$ in the place of the rough result (57·2160) given there, and the closing of the triangular figure Corinth, Cairo, Memphis, Corinth, becomes $-0{\cdot}2424^{m}$, and that of the larger southern loop, Biloxi, Mobile, Corinth, Memphis, Vicksburg, New Orleans, Biloxi, $-0{\cdot}2732^{m}$. The development in length of the first loop is 747.3^{km} and of the second $1832{\cdot}4^{km}$. A description of the bench marks concludes this report.

Description of bench marks on line of levels from Corinth, Miss., to Memphis, Tenn., 1890–'91.

V.—Corinth, Alcorn County, Miss. A brass bolt leaded in the east wall of the brick store on the northwest corner of Waldren and ——— streets. This building is opposite the court-house, and is used as the city hall. The bolt is in the third course of brick above the sill of the windows near the north end of the building and in the fourth brick south of this window. Two lines were cut in the end of the bolt, forming a + at its center, and the intersection of these lines is the B. M. The building is owned by Eli Mitchell.

W.—Corinth, Alcorn County, Miss. A brass bolt leaded in the west wall of the brick court-house. The bolt is between the fifth and sixth ventilators (small grated windows) to the cellar, counting from the northwest corner of the building, in the fifth brick south of fifth ventilator and in the fourteenth course below line of window sills. Two lines were cut in the end of the bolt, forming a cross at its center, and the intersection of these lines is the B. M.

L.—Chewalla, McNairy County, Tenn. A copper bolt leaded in the side face of brick chimney on east end of frame house occupied by F. S. McCallar, railroad agent, at Chewalla. The bolt is in the

fourteenth course above the ground and in the second brick from the end of the house, and in the side of the chimney towards the front of the house. The house is north of the track and east of the public road. Two lines were cut in the end of the bolt, forming a cross (+) at its center, and the intersection of these lines is the B. M.

M.—Cypress Creek, railroad bridge, Tenn. A square hole cut in the top of the stone pier of the iron railroad bridge over Cypress Creek. It is on the pier on the east bank of the creek and is south of the track outside of the superstructure of the bridge near the outer edge of the pier. The letters U. S. B. M. were roughly cut near it. The bottom of the square hole is the B. M.

N.—Pocahontas, Tenn. A copper bolt leaded in the front face of the "Brick Hotel." The bolt is in the twelfth course above the ground and in the third brick from the southwest corner of the building. The hotel is north of the track and nearly opposite the railroad station. It is kept by Mr. Seay, the railroad agent. Two lines were cut in the end of the bolt, forming a cross (+) at its center, and the intersection of these lines is the B. M.

O.—Middleton, Hardeman County, Tenn. A copper bolt leaded in the face of the west side wall to the brick store just west of the hotel. The building faces the railroad and is quite near it. The bolt is in the fourth course below the front door sill and in the second brick from the northwest corner of the building. Two lines were cut in the end of the bolt, forming a cross (+) at its center, and the intersection of these lines is the B. M.

P.—Saulsbury, Hardeman County, Tenn. A copper bolt leaded in the face of the west side wall to the brick store owned and occupied by R. M. Wright. The bolt is near the southwest corner, in the twenty-first course above the ground, and in the fifth brick from the southwest corner. Two lines were cut in the end of the bolt, forming a cross, and the intersection of these lines is the B. M.

R.—Grand Junction, Tenn. A copper bolt leaded into the face of the front wall to the brick store owned and occupied by W. R. Robinson. The store is north of the track and a short distance east of the railroad crossing. The bolt is near the southeast corner of the building, in the fourth course above the front porch, and in the second brick from the southeast corner. Two lines were cut in the end of the bolt, forming a cross, and the intersection of these lines is the B. M.

I.—La Grange, Fayette County, Tenn. A square hole cut in the top of the stone abutment to the railroad bridge over Main street. The abutment is on the east side of the street and is north of the track. The bottom of the hole is the B. M.

K.—La Grange, Fayette County, Tenn. A square hole cut in the top of the stone abutment to the railroad bridge over Main street.

The abutment is on the west side of the street, and is south of the track. The bottom of the hole is the B. M.

H.—Moscow, Fayette County, Tenn. A cavity was cut in the rough face of one of the stones in the foundation of the railroad water tank, and a square hole made in the bottom of this cavity for the B. M. This cavity is on the south face (next to the track) of the foundation, and 6·2 feet from the southwest corner. The bottom of the square hole is the B. M.

G.—Wolf River railroad bridge. A square hole was cut in the top of the stone pier to the iron railroad bridge over Wolf Creek. It is on the pier on the east bank of the creek, on the inner side of the raised portion on the north end of the pier, and 1·5 feet from its eastern face. It was roughly lettered $\overset{\text{U. S.}}{\square}$ B. M. The bottom of the hole is the B. M.

F.—Rossville, Fayette County, Tenn. A square hole about one-fourth inch deep, cut in the top of a ledge in the stone foundation of the railroad water tank. The hole is only a few inches above the ground, and is 2 feet north of the southeast corner of the foundation, and is on the east side of the tank. The bottom of the square hole is the B. M.

E.—Colliersville, Shelby County, Tenn. A copper bolt (five-eighths inch in diameter) leaded in the front of Duloney & Patrick's brick storehouse, which faces the public square and is on the east side, a short distance from the railroad. The bolt is in the eleventh course above the ground and in the third brick from the southwest corner of the building. Two lines were cut in the end of the bolt, forming a cross (+), and the intersection of these lines is the B. M.

D.—Bailey's Station, Shelby County, Tenn. A copper bolt (five-eighths inch in diameter) leaded in the brick chimney on the west end of J. W. Brown's residence. The bolt is in the corner brick on the south or front face of the chimney and in the twenty-second course above the ground. Two lines were cut in the end of the bolt, forming a cross, and the intersection of these lines is the B. M.

C.—Germantown, Shelby County, Tenn. A square hole, about one-fourth inch deep, cut in the northwest corner of the upper surface of the stone cap to the brick pier, under the northwest corner of the platform to the railroad station. The bottom of the square hole is the B. M.

B.—White's Station, Shelby County, Tenn. A square hole cut in the stone foundation to the railroad water tank. It is on the top of the offset, at the southwest corner of the tank, about 18 inches above the ground, and is marked as follows: U. S. $\blacksquare$, the letters being B. M. rudely cut in the limestone. The bottom of the square hole is the B. M.

A.—Buntyn, Shelby County, Tenn. This B. M. is the upper surface of the first brick offset above the ground on the west side of the entrance to the Methodist church. Three lines were cut in the brick, forming a square with the outer edge, and the bench mark is the upper surface of the brick within this square. It is on the third brick from the west side of entrance. This church is a substantial brick building.

U. S. P. B. M. "Memphis."—Memphis, Tenn. Is the center of a hole in head of copper bolt leaded horizontally in face of subsill of first basement window from southwest corner, on west side of new custom-house. It is marked U. S. $\odot$. This bench was established B. M. in 1878 (see Rep. of Miss. River Comm. for 1881, pp. 65, 66). When visited in November, 1890, this bench mark was found undisturbed and in good condition.

Crossroad No. 1.—Ground at center of track, at public road crossing, 3½ miles west of Corinth, Miss.

Crossroad No. 2.—Ground at center of track at public road crossing at Wenasoga.

Crossroad No. 3.—Ground at center of track at public road crossing at Cheewalla, Tenn.

Railroad station No. 4.—Ground at center of track opposite railroad station at Pocahontas.

Railroad station No. 5.—Ground at center of track opposite railroad station at Middletown.

Crossroad No. 6.—Ground at center of track, public road crossing at 64-mile post.

Railroad station No. 7.—Ground at center of track opposite railroad station at Saulsbury, Tenn.

Crossroad No. 8.—Ground at center of track at public road crossing 2½ miles west of Saulsbury, Tenn.

Crossroad No. 9.—Ground at center of track at crossing of Illinois Central Railroad, Grand Junction, Tenn.

Crossroad No. 10.—Ground at center of track at public road crossing 4 miles east of Moscow, Tenn.

Railroad station No. 11.—Ground at center of track opposite old railroad warehouse at Hays' Crossing.

Railroad station No. 12.—Ground at center of track opposite railroad station at Rossville.

Crossroad No. 13.—Ground at center of track, State line road crossing, 2½ miles east of Colliersville, Tenn.

Railroad crossing, station No. 15.—Ground at center of track opposite railroad station at Bailey's.

Crossroad No. 16.—Ground at center of track at public road crossing at Forest Hill.

Railroad station No. 17.—Ground at center of track opposite railroad station at Germantown.

Crossroad No. 18.—Ground at center of track at Grier's road crossing.

Crossroad No. 19.—Ground at center of track at Pigeon Roost road crossing.

[1]Railroad crossing, station No. 14.—Ground at center of track opposite railroad station at Colliersville, Tenn.

APPENDIX No. 5—1892.

ON THE TIDES AND CURRENTS IN THE HARBOR OF EDGARTOWN AND IN KATAMA BAY, MARTHAS VINEYARD.

A report by HENRY L. MARINDIN, Assistant.
Submitted for publication April 25, 1892.

The following report on the tides and currents in the harbor of Edgartown and Katama Bay, Massachusetts, based on observations made during the summer of 1891, is offered in advance of one to be made on the "comparisons of changes in the shores and bottom of the harbor since previous surveys."

In a report dated September, 1886, published as Appendix No. 9, Report for 1886, Assistant Henry L. Whiting says:

> The tidal currents through the inlet, through the narrows of Cotamy Bay, and through Edgartown Harbor, are now very strong. I am unable to state the exact velocity, but it has had the power to increase the depth of water along the front of the town and about the wharves. The currents are also effecting a change in the position and depth of the point of Chappaquiddick, opposite the town. They also affect the maneuvering of vessels, which now do not like to anchor above Chappaquiddick Point.
>
> The fishermen state that the scour of those currents is injuring the clam and quahaug grounds, which are of much local value. I have endeavored to ascertain whether the tidal currents through the harbor were stronger now than when the inlet of 1856 was open. From such facts as I could gather, however, they seem to have about the same force. * * *
>
> That these strong currents must produce changes in the physical condition of the harbor seems self-evident. The very peculiar tidal phenomena connected with it makes the study of the subject one of special interest. Comparing the time the present inlet has been open, about eight months, with the eighteen or twenty years of less effective action since the former inlet closed, the probabilities are that the harbor channels and shoals are still in a transition state.

At the time of Mr. Whiting's examination the barrier between Katama Bay and the ocean had just been broken through, creating the first opening in about twenty years. The statement about the strength of the currents in the report must have referred to the then condition

as compared to that with no outlet through Katama Beach, and it is to be regretted that no observations of the velocity of the currents were made either under the condition of no opening, as in 1871, or of one opening, as in 1886.

If the circulation through the harbor and bay induced strong currents with one opening, the case must be aggravated with three inlets.

It is believed that the conditions which now obtain throughout this locality have materially changed the régime of the tides and currents, and that, consequently, the present knowledge of these phenomena does not represent actual facts.

This change is attributed to the existence of three inlets through the south beach of Katama Bay, which admit such a volume of tide water from the ocean as to influence the tide throughout the bay and harbor as far as Vineyard Sound.

With no inlet from the ocean side, the tide in Katama Bay is of the type of the Sound tide; with three inlets, the ocean tide, of greater range and differing in time, being freely admitted is the cause of the interferences shown in the accompanying diagrams.

In order that one may follow the treatment of the tide and currents adopted in this report, there have been introduced much of the observed data in tabular form.

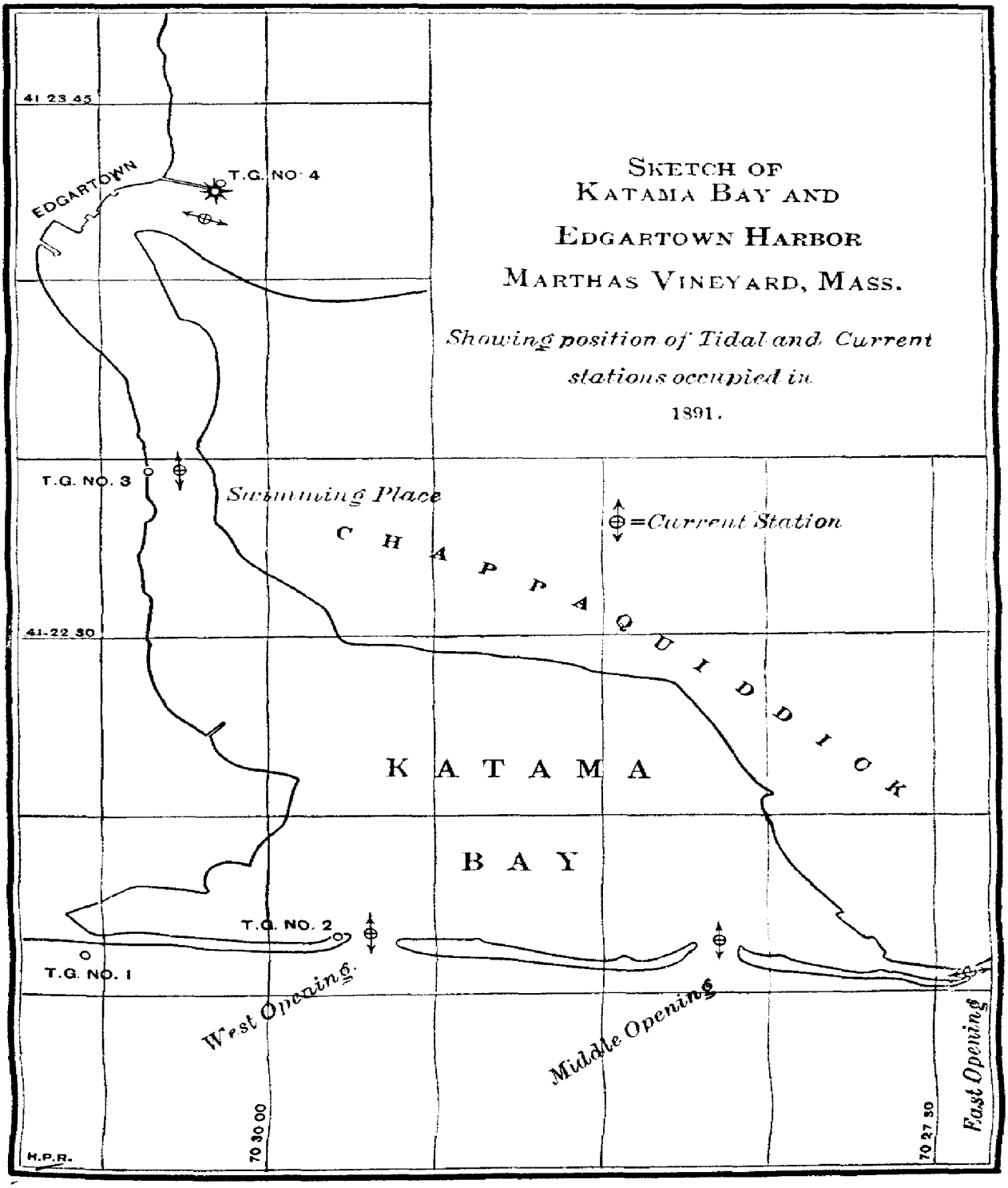
Illustration No. 20 shows the locality, with the location of the tide and current stations.

In Tables 1 to 4 will be found the observations of the tides at four stations situated along the path of the tide from the entrance into the harbor to the ocean at the south shore. These observations,* covering somewhat more than one lunar month, are here grouped according to lunar hours; from these we obtain a mean tide for each of the four stations.

Illustration No. 21 represents these mean tides, from which we can derive the synchronous differences in height of water surface at any time of the tide.

The times of maximum slopes and of restoration of level are graphically represented on Illustration No. 22. For greater practical use the intervals are referred to the time of high water at Edgartown lighthouse. It will be observed that maximum ebb slope occurs 13 minutes after high water; at this moment the current is setting from the Sound through Katama Bay to the ocean at nearly maximum strength. The mean difference in height between the ocean and the Sound is then 1·75 feet; of this difference 55 per cent is found between the ocean and Katama Bay, within a distance of one-eighth of a mile. It is safe to assume that under favorable conditions a slope 100 per cent greater than the above may be found within the same limits.

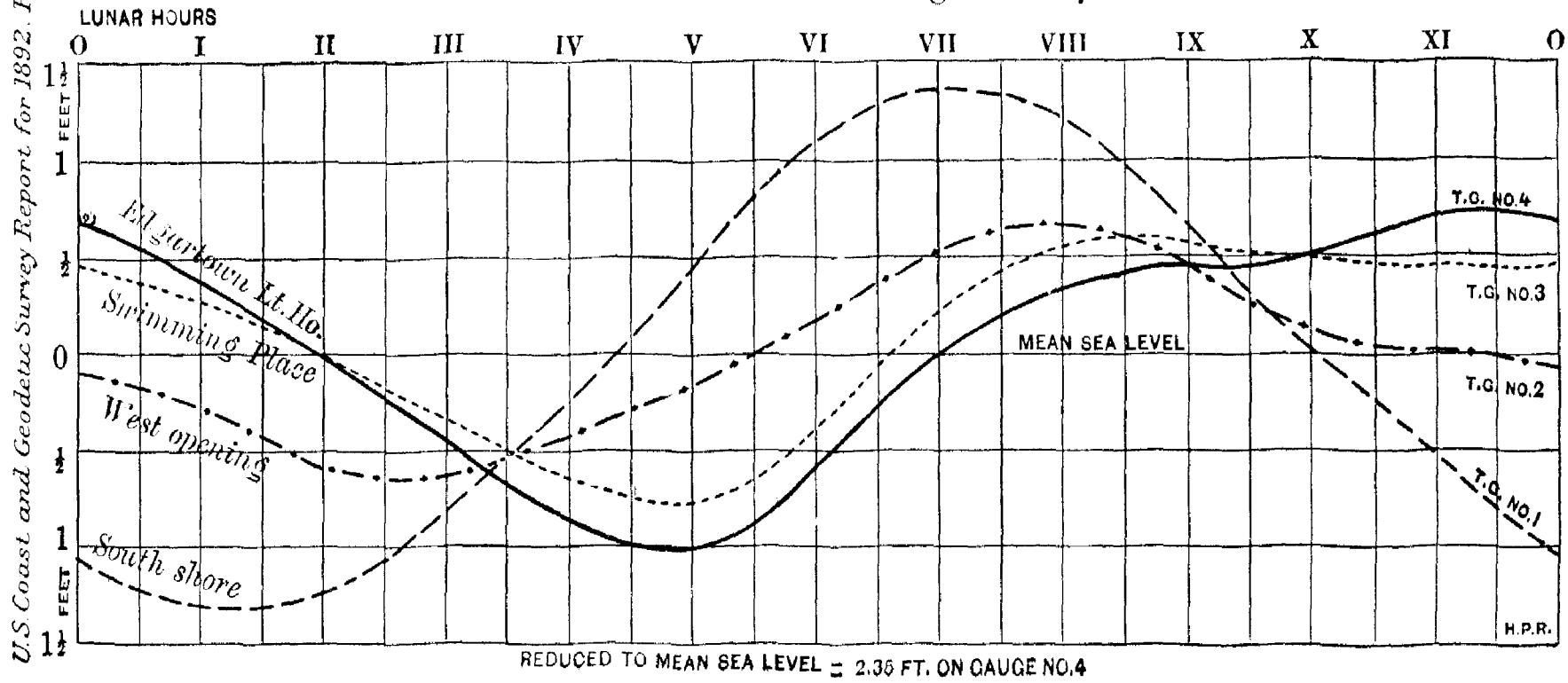
* Observations were made during the daytime only, as but one self-registering tide gauge was available.


41 23 45
EDGARTOWN
T.G. NO. 4
SKETCH OF
KATAMA BAY AND
EDGARTOWN HARBOR
MARTHAS VINEYARD, MASS.
Showing position of Tidal and Current stations occupied in
1891.
T.G. NO. 3
Swimming Place
=Current Station
CHAPPAQUIDDICK
41-22 30
KATAMA
BAY
T.G. NO. 2
T.G. NO. 1
West Opening
Middle Opening
East Opening
70 30 00
70 27 30
H.P.R.

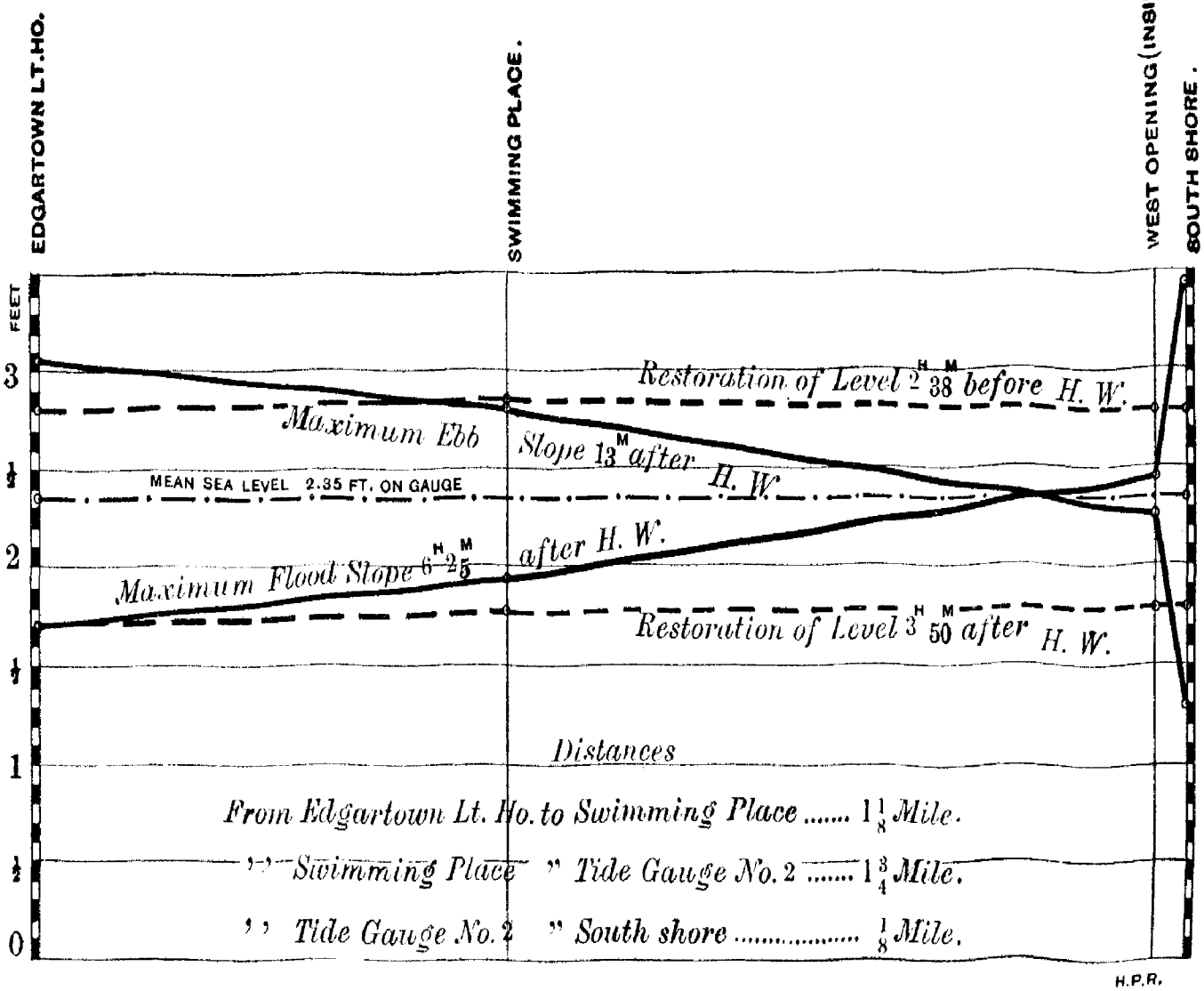
Tidal Curves

Marthas Vineyard, Mass.

From simultaneous observations, Aug. 24 to Sept.26 1891.



Intervals of Maximum Slopes
referred to High Water at Edgartown Lt. Ho.



On the tides and currents in the harbor of Edgartown and in Katama Bay.

Maximum flood slope occurs $6^h\ 25^m$ after high water; the current is now setting in the opposite direction, but nearly one hour past maximum strength. The difference in height between ocean and Sound is 1·72 feet, 57 per cent of which is located in the eighth of a mile between ocean and bay. The restoration of level on a rising tide* takes place $2^h\ 38^m$ before high water. This restoration of level is not quite synchronous, but the differences are so slight as to practically amount to a level plane. At this moment the current slacks and then turns from north to south throughout the bay.

The restoration of level on the falling tide occurs $3^h\ 50^m$ after high water, and at this moment the current slacks and the turning from south to north follows.

In Table 5 will be found the establishment of high and low water with that of the turning of the currents. It will be observed that the tide of the southern part of Katama Bay follows closely the tide of the south shore. At low declination of the moon this phase extends to the Swimming Place; we find then two distinct high waters, the first high water having the greater amplitude and occuring $VIII^h\ XXXIV^m$ after the moon's transit, while the second high water remains at the establishment ($XII^h\ XXIX^m$) given in the table as the mean for a lunar h.

TABLE 5.—*Establishment of high and low water, with turning of currents, from observations in 1891.*

Locality.	Time after moon's transit.									
	High water.		Low water.		Current turns.				Maximum strength (approximate).			
					East to west.		West to east.		Westerly current.		Easterly current.	
	h.	*m.*	*h.*	*m.*	*h.*	*m.*	*h.*	*m.*	*h.*	*m.*	*h.*	*m.*
Edgartown light-house.	XII	XI	V	VI	X	IV	III	XXI	XI	XXII	V	X
					North to south.		South to north.		Southerly.		Northerly.	
					h.	*m.*	*h.*	*m.*	*h.*	*m.*	*h.*	*m.*
Swimming Place.	XII	XXIX	V	XI	XI	XX	III	LVIII	XI	LIII	V	X
West opening (inside).	VIII	IX	II	LV	IX	XLI	III	XLI	O	OO	V	XLI
South shore (ocean).	VII	XXXIV	I	XXII	----------		----------		----------		----------	

* In speaking of rising and falling tide, reference is made to the tide in bay and harbor, and not to the tide on the south shore where the reverse would apply.

TIDAL CURRENTS.

Observations of the drift and set of the currents were made with two Ritchie-Haskell electric-current meters (Nos. 3 and 4) at the following stations: At the entrance to Edgartown inner harbor, at the Swimming Place, and in the inlets into Katama Bay. The observations in the inlets were not wholly satisfactory, owing to the great difficulty in stationing and holding a boat for any length of time in such exposed place. The observations are, however, sufficient to indicate the type of curve of velocity, but do not represent fairly the maximum strength. These curves of velocity are exhibited on Illustration No. 23, in connection with the tidal curves at the stations where observed.

Nowhere within this field have we recorded velocities exceeding 5 feet per second. We are convinced that velocities twice as great may be found and observed within these openings.

At the entrance to the inner harbor, off Edgartown light-house, the maximum strength observed for westerly current was 1·85 feet per second about 49 minutes before high water,* and for easterly set 1·35 feet per second, $5^h\ 23^m$ after high water.

At the Swimming Place, which is the narrowest part of the arm connecting Edgartown inner harbor with Katama Bay, the currents are decidedly stronger, the greatest strength was 4·2 feet per second for the southerly set 18 minutes before high water. During the northerly set, the maximum strength was found to be 2·75 feet per second $5^h\ 23^m$ after high water.

At the west opening, the greatest strength was not fully ascertained for reasons stated before. For southerly current, the maximum strength will occur near the transit of the moon, or about 13 minutes after high water at Edgartown light-house.

For northerly flow, the maximum strength will occur about $6^h\ 00^m$ after high water at Edgartown light-house.

In the middle opening, the period of strength of current follows closely that approximated for the western inlet.

At the eastern opening, which is on the threshold of closing up, the currents present anomalies which might develop in special features with a more extended series of observations than those now at hand.

The observed intervals of high and low water, and the turning of the currents after the transit of the moon, for four stations, will be found in Tables 6 to 9. These tables are appended more with the purpose of securing their preservation for future use, if needed, than for further discussion.

The data contained in them can be reënforced by supplementary observations at any future time.

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*Whenever high water is mentioned without specifying the place it refers to the high water at Edgartown light-house.

Current Velocity Curves observed at Marthas Vineyard, Mass. in 1891.

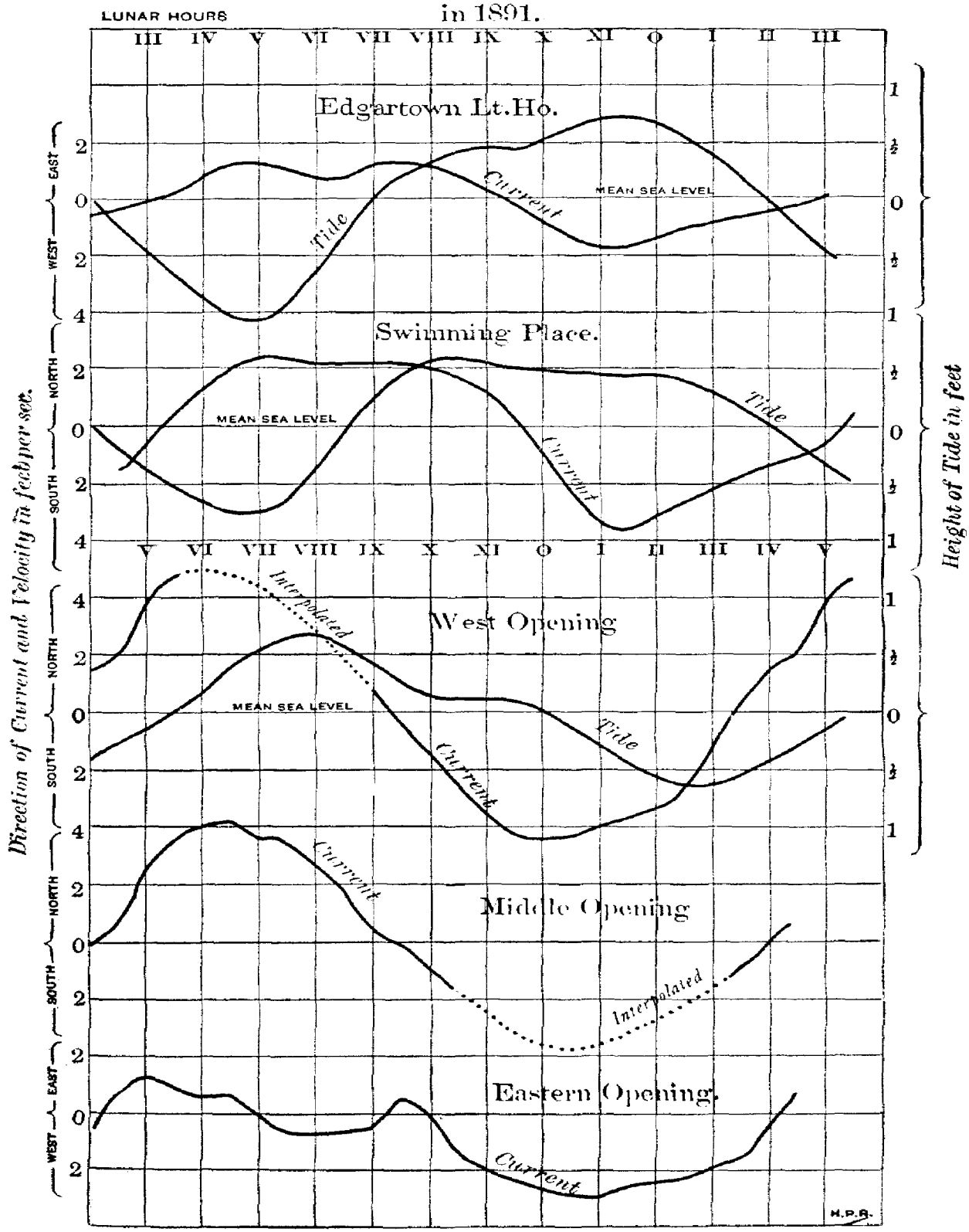

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TABLE 1.—*Tide*

SOUTH SHORE, MARTHAS

[Observed heights reduced to

Date.	O	¼	I	I½	II	III¼	III	III¾	IV	IV¼	V	V¼
1891.	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*
Aug. 28	----	----	----	----	----	----	----	----	----	----	----	----
29	1·29	1·17	1·10	1·10	1·12	1·23	1·35	1·52	1·75	1·96	2·30	2·60
30	1·00	0·90	1·00	1·00	1·01	1·12	1·21	1·35	1·55	1·80	2.10	2·40
31	1·18	1·10	1·10	1·09	1·10	1·18	1·36	1·68	1·80	2·10	2·30	2·60
Sept. 1	1·46	1·23	1·06	1·00	1·03	1·12	1·29	1·54	1·80	2·10	2·45	2·70
2	1·25	1·01	1·00	1·00	1·00	1·10	1·26	1·50	1·80	2·10	2·40	2·62
3	1·16	1·00	0·80	----	----	----	----	----	----	----	----	----
4	1·12	0·90	0·80	0·85	0·95	1·13	1·40	1·70	2·00	2·25	2·45	2·83 2·80
5	0·92	0·71	0·70	0·78	0·90	1·10	1·50	1·80	1·91	2·20	2·45 2·41	2·85
6	1·18	1·10	1·10	1·12	1·30	1·45	1·70	1·97	2·10 2·13	2·30	2·70	3·10
7	----	----	----	----	----	1·75	2·02	2·18	----	----	----	----
8	1·20	1·20	1·30	1·20	1·30	1·50	----	----	----	----	----	----
9	1·85	1·70	1·60	1·60	1·60	1·56	1·75	2·00	2·36	2·60	2·90	3·00
10	1·60	1·50	1·40	1·22	1·37	1·52	1·72	1·95	2·05	2·26	2·55	2·83
11	1·65	1·20 1·50	1·20	1·20	1·31	1·51	1·63	1·84	2·05	2·30	2·58	2·82
12	1·10	1·10	1·20	1·20	1·30	1·40	1·60	1·70	1·94	2·36	2·64	2·84
13	1·52	1·41	1·48	1·50	1·50	1·60	1·70	1·90	2·20	2·45	2·75	3·02
14	1·63	1·44	1·30	1·20	1·40	1·60	1·80	2·10	2·41	2·63	3·08	3·43
15	1·04	0·90	0·90	0·90	1·10	1·20	1·50	1·80	2·26	2·65	3·10	3·58
16	----	----	----	0·35	0·55	0·76	1·09	1·50	1·80	2·20	2·82	3·37
17	0·56	0·28	0·15	0·03	0·12	0·38	0·73	1·30	1·80	2·27	2·91	3·58
18	0·20	—0·05	—0·10	—0·10	0·00	0·36	0·75	1·26	1·80	2·38	3·00	3·52
19	----	----	----	0·10	0·25	0·55	0·91	1·45	1·94	2·46	3·10	----
20	0·83	0·62	0·50	0·50	0·60	0·88	1·20	1·61	1·90	2·82	3·56	4·10
21	0·93	0·77	0·63	0·66	0·85	0·97	1·30	1·88	2·40	2·81	3·35	3·93
22	1·47	1·36	1·25	1·30	1·40	1·52	1·80	2·18	2·68	3·11	3·64	3·96
23	1·73	1·53	1·40	1·42	1·40	1·65	2·00	2·20	2·45	2·78	3·20	3·55
24	2·18	1·97	1·60	1·65	1·87	2·05	2·25	2·40	2·60	2·86	3·17	3·66
25	1·30 1·91	1·28	1·20	1·30	1·31	1·44	1·60	1·80	2·10	2·37	2·65	2·95
26	1·40	1·40	1·40	1·40	1·40	1·50	1·60	1·81	2·05	2·33	2·57	2·86
Sums.	34·66	30·23	27·07	26·57	29·04	35·13	40·02	47·92	55·63	62·45	75·13	81·50
Means.	1·28	1·12	1·04	0·98	1·07	1·25	1·48	1·77	2·06	2·39	2·78	3·13

gauge No. 1.

VINEYARD, MASS.

Staff of Tide Gauge No. 4.]

VI	VI½	VII	VII½	VIII	VIII½	IX	IX½	X	X½	XI	XI½
ft.	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*
----	----	3·50	3·60	3·50	3·50	3·30	3·10	2·95	2·80	----	----
2·83	3·03	3·18	3·28	3·30	3·22	2·95	2·70	1·98	1·76	1·58	1·46
2·70	3·03	3·15	3·20	3·40	3·45	----	2·15	2·00	1·65	1·41	1·30
3·01	3·32	3·41	3·62	3·70	3·05	2·86	2·55	2·25	1·98	1·75	1·40
3·05	3·33	3·45	3·40 3·60	3·40	3·30	3·00	2·88	2·57	2·26	1·88	1·66
3·12	3·32	3·27	3·45	3·50	3·35	3·05	2·78	2·40	2·00	1·70	1·47
2·90	3·20	3·50	3·50	3·40	3·30	3·00	2·60	2·30	1·90	1·70	1·38
3·23	3·40	3·52	3·60	3·55	3·30	2·93	2·50	2·20	1·90	1·62	1·36
3·20	3·42	3·50	3·50	3·40	3·10	2·75	2·40	2·10	1·73	1·50	1·18
3·35	3·48	3·60	3·60	3·40	3·20	2·81	2·40	2·10	1·90	1·60	1·48
----	----	----	----	----	----	----	----	----	----	----	----
3·00	3·26	3·40	3·40	3·26	2·92	2·66	2·31	1·95	1·82	1·65	1·32
3·21	3·51	3·60	3·70	3·56	3·22	2·95	2·63	2·42	2·22	2·02	1·90
3·05	3·18	3·20	3·14	2·95	2·75	2·55	2·35	2·12	2·00	1·90	1·70
3·08	3·20	3·20	3·15	3·05	2·87	2·68	2·47	2·27	2·12	1·93	1·82
3·03	3·10	3·15	3·18	3·06	2·93	2·80	2·65	2·53	2·37	2·19	1·95
3·14	3·34	3·42	3·50	3·50	3·35	3·10	2·93	2·77	2·50	1·72	1·63
3·67	3·73	3·92	4·00	3·90	3·70	3·45	3·20	2·35	2·10	1·92	1·78
3·97	4·13	4·23	4·23	4·03	3·80	2·70	2·45	2·12	1·81	1·58	1·29
3·75	4·02	4·11	4·18	3·59	3·25	2·91	2·60	----	----	----	----
3·90	4·40	4·08 4·40	3·95	3·72	3·38	2·96	2·58	2·17	1·76	1·27	0·85
3·95 4·00	4·28	4·50	4·20	4·00	3·48	3·02	2·34	1·93	1·50	0·95	0·55
----	----	----	----	----	----	----	----	----	----	----	----
4·56	4·78	4·87	4·70	4·35	3·85	3·30	2·85	2·50	2·10	1·70	1·20
4·24	4·52	4·56	4·37	4·10	3·68	3·24	2·76	2·35	2·05	1·72	1·22
4·27	4·50	4·50	4·40	4·15	3·66	3·35	3·03	2·75	2·40	1·96	1·62
3·90	4·10	4·10	4·00	3·90	3·40	3·10	2·90	2·67	2·50	2·40	1·97
3·92	4·13	4·20	4·12	3·92	3·60	3·43	3·10	2·90	2·80	2·60	2·38
3·16	3·26	3·30	3·30	3·18	2·97	2·73	2·65	2·51	2·40	2·30	2·02
3·13	3·22	3·27	3·24	3·25	3·17	3·00	2·85	2·75	2·65	----	1·51
96·32	98·19	108·09	107·11	100·02	92·81	80·58	74·71	63·91	56·98	44·55	39·40
3·44	3·64	3·72	3·69	3·57	3·31	2·98	2·67	2·37	2·11	1·78	1·51

TABLE 2.—*Tide*

WEST OPENING INTO

[Observed heights reduced

Date.	O	¼	½	I½	II	III½	III	III¾	IV	IV½	V	V½
1891.	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*
Aug. 24	2·31	2·21	2·08	1·90	----	----	----	----	----	----	----	----
25	2·46	2·36	2·15	----	----	----	----	1·68	1·76	1·90	2·06	2·30
26	2·51	----	1·67	1·56	1·50	1·47	1·43	1·48	1·64	1·78	1·98	2·18
27	1·76	1·82	1·80	1·70	1·65	1·62	1·60	1.62	1·71	1·80	1·93	2·12
28	2·12	2.12	2·09	2·02	1.97	1·92	1.90	1·90	2·00	2·04	2·13	2·30
29	1·98	1·95	1·92	1·82	1·75	1·67	1·61	1·63	1.68	1·80	1·90	2·02
30	1·71	1.72	1·82	1.85	1·80	1·75	1·71	1·71	1·78	1·87	2·00	2·08
31	1·95	2·00	2·00	1·92	1·85	1·90	1·90	1·97	2·00	2·10	2·16	2·35
Sept. 1	2·10	2·11	2·08	2·00	1.90	1·85	1·83	1·83	1·90	2·05	2·20	2·32
2	1.88	1·90	1·90	1·88	1·80	1·77	1·80	1·85	1·90	2·05	2·10	2·27
3	1·82	1.85	1·80	1·72	1·62	1.60	1·68	1·80	1·92	2·05	2·15	2·27
4	1·92	1·93	1·91	1·86	1·77	1·73	1·78	1·90	1·96	2.06	2·10	2·21
5	2·00	2·00	1·92	1·80	1·67	1·62	1·74	1·85	1·98	2·05	2·08	2·20
6	2·21	2·27	2·17	2·10	2·07	2·03	2·04	2·10	2·05 2·08	2.05	2·16	2·30
7	----	----	----	----	----	----	----	2·20	2·36	2·50	2·60	----
8	2·17	2·15	2·14	2·02	1.92	1·86	1·75	1·80	1·95	2·15	2·30	2·30
9	2·54	2·50	2·40	2·30	----	1·95	2·00	2·07	2·10	2·20	2·25	2·25
10	2·36	2·35	2·23	1·85	1·82	1·85	1·85	1·84	1·85	1·88	1·98	2·08
11	2·45	1·82	1·80	1·80	1·78	1·77	1·75	1·75	1·80	1·87	1·93	2·10
12	1·75	1·75	1·73	1·73	1·68	1·68	1.66	1·68	1·75	1·88	2·00	2.17
13	2·01	2·02	2·03	1·96	1·86	1·84	1·77	1·75	1·86	1·98	2·10	2·21
14	2·41	2·38	2·28	2·12	2·05	1·98	1·97	2·05	2·18	2·27	2·40	2·52
15	2·17	2·14	2·03	1·80	1·72	1·65	1·65	1.70	1·87	2·00	2.20	2·46
16	2·15	2·09	1·95	1·72	1.55	1·43	1·47	1·57	1·78	1·96	2·18	2·41
17	2·23	2·15	1·94	1·70	1.50	1·27	1·25	1·32	1·66	1·85	2·18	2·52
18	2·08	2·00	1·83	1·55	1·30	1·10	1·10	1·35	1·60	1·90	2·20	2·40
19	2·46	2·33	2·10	1·85	1·60	1·40	1·25	1·40	1·65	1.90	----	2·40
20	2·75	2·60	2·37	2·10	1·83	1·60	1·35	1·42	----	2.10	2·40	----
21	2·78	2·60	2·40	2·19	1·92	1·65	----	1·77	1·91	----	----	----
22	2·95	2·80	2·62	2·42	----	----	2.02	----	----	2·31	2·47	2·78
23	3·03	2·90	2·77	----	1·98	----	----	----	2·15	2·21	2·35	2·50
24	3·08	3·05	2·51	2·45	----	----	2·30	2·32	2·37	2·40	2.50	2·68
25	1·98	1·93	1·85	1·80	1·76	1·75	1·74	1·70	1·78	1·90	2·00	2·15
26	----	----	----	----	2·05	2·01	1·94	1·88	1·91	2·01	2.08	2·22
Sums.	72·08	67·80	66·29	57·49	49·67	47·72	49·84	54·89	60·89	64·87	67·07	69·07
Means.	2·25	2·19	2·07	1·92	1·77	1·69	1·72	1·77	1·90	2·02	2·16	2·30

Gauge No. 2.

KATAMA BAY.

to Staff of Gauge No. 4.]

VI	VI½	VII	VII½	VIII	VIII½	IX	IX½	X	X½	XI	XI½
ft.	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*
----	---	----	----	----	----	----	----	----	----	2·32	2·34
2·50	2·65	2·80	2·92	2·91	2.81	2·67	2·45	2·31	2·32	2·45	2.48
2·30	2·52	2·68	2·80	2·83	2·72	2.60	2·46	2·40	2·45	2·52	2·55
2·22	2·45	2·65	2·78	2·85	2·80	2·73	2·62	2·60	2·65	2·68	----
2·46	2·60	2·80	3·00	3·02	3.05	3·00	2·94	2.92	2·93	1·96	2·08
2·18	2·30	2·40	2·58	2·69	2.70	2·68	2·60	1·98	1·93	1.90	1·92
2·30	2·50	2.50	----	----	----	----	1·92	1·90	1·70	1·66	1·70
2·47	2·60	2·78	2·98	----	----	----	----	----	----	----	2·00
2·44	2·60	2·80	----	----	----	----	----	2·28	2·17	2.07	2·10
2·35	2·60	----	----	----	----	----	----	2·16	1.97	1·86	1·87
2·30	2·40	2·55	2·73	2·87	2·66	2·55	2·30	2·10	1·88	1·77	1·78
2·35	2·50	2·68	2·80	2.85	2.80	2·60	2·35	2.14	1·98	1·91	1·88
2·35	2·50	2·68	2·80	2·87	2·72	2·55	2·32	2·12	2·00	1·98	1·93
2·48	2·60	2·80	2·90	2·88	2·80	2·66	2·50	2·25	2·12	2·13	2·23
----	----	----	----	----	----	----	----	----	----	----	----
2·33	2·50	2·65	2·71	2·75	2·70	2·55	2·33	2·15	2·10	2·18	2·18
2.40	2·50	2·75	3·04	3·06	3·04	2·95	2·70	2·62	2·60	2·59	2·56
2·20	2·32	2·48	2·52	2·55	2·50	2·44	2·35	2·25	2·28	2·37	2·37
2·26	2·40	2·55	2·67	2·70	2·65	2·53	2·40	2·35	2·42	2·48	2·47
2·30	2·42	2·56	2·67	2·68	2·65	2·64	2·60	2·55	2·70	2·78	----
2·35	2·50	2·70	2·82	2·92	2·93	2·88	2·83	2·85	----	1·97	2·00
2·75	2·94	3·17	3·32	3·35	3·35	3·33	----	2.30	2·32	2·35	2·38
2·73	3·00	3·25	3·38	3·48	----	2·40	2·28	2·20	2·12	2·12	2·17
2·65	2·93	3·18	----	3·00	2·90	2·68	2·50	2·30	2·25	2·23	2·23
2·73	----	3·07	3·12	3·10	2·94	2·75	2.50	2·25	2·17	2.17	2·22
2·67	2.90	----	----	----	3·00	2·66	2·31	2·06	1·98	1·95	2·00
----	----	----	3·52	3·48	3·30	2·92	2·56	2·43	2·45	2·50	2·52
----	----	----	3·71	3·68	3·50	3·10	2·85	2·75	2·72	2·78	2·88
2·91	3·20	3·41	3·47	3·46	3·38	3·10	2·80	2·71	2·78	2·82	2·81
3·08	3·28	3·51	3·67	3·61	3·45	3·30	3·14	3·08	3·06	3·05	3·00
2·70	2·98	3·20	3·30	3·30	3·20	3·01	3·02	3·01	3·05	3·13	3·11
2·65	3·10	3·33	3·41	3·46	3·35	3·26	3·10	3·10	3·17	3·15	3·13
2·30	2·41	2·56	2·70	2·71	2·75	2·74	2·67	2·65	2·70	2·76	2·80
2·35	2·50	2·70	2·80	2·87	2·90	2·88	2·90	2·90	----	----	----
74·06	76·70	79·19	81·12	81·93	79·55	78·16	72·30	75·67	68.97	72·59	69·69
2·46	2·64	2·83	3·00	3·03	2·94	2·79	2·58	2·44	2·37	2·34	2·32

SWIMMING-PLACE

TABLE 3.—*Tide*

[Observed heights reduced

Date.	O	¼	I	I½	II	II½	III	III½	IV	IV½	V	V½
1891.	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*
Aug. 24	2·98	2·88	2·70	2·54	----	----	----	----	----	----	----	----
25	3·02	2·91	2·75	2·32	2·12	1·96	1·75	1·53	1·35	1·17	1·18	1·39
26	3·08	----	2·27	2·16	2·06	1·88	1·70	1·50	1·36	1·15	1·10	1·21
27	2·47	2·45	2·35	2·26	2·18	2·10	1·92	1·70	1·57	1·36	1·30	1·45
28	2·68	2·64	2·61	2·50	2·40	2·31	2·18	2·00	1·95	1·80	1·71	1·74
29	2·48	2·49	2·49	2·42	2·25	2·12	1·97	1·82	1·74	1·60	1·50	1·50
30	2·30	2·35	2·37	2·35	2·30	2·22	2·16	2·10	1·95	1·83	1·77	1·70
31	2·50	2·50	2·49	2·45	2·36	2·35	2·30	2·20	2·10	2·08	2·04	1·97
Sept. 1	2·63	2·67	2·53	2·50	2·43	2·30	2·22	2·10	1·92	1·90	1·90	1·90
2	2·42	2·48	2·48	2·50	2·38	2·27	2·20	2·10	1·97	1·86	1·75	1·70
3	2·38	2·45	2·42	2·33	2·20	2·10	2·00	1·95	1·92	2·00	1·93	1·80
4	2·48	2·53	2·51	2·40	2·25	2·20	2·11	2·00	1·92	1·75	1·66	1·51
5	2·51	2·68	2·50	2·36	2·20	2·10	2·01	1·92	1·90	1·80	1·58	1·57
6	2·70	2·75	2·71	2·60	2·48	2·44	2·35	2·22	2·00 2·08	1·77	1·70	1·72
7	3·28	3·25	3·17	3·05	2·85	2·65	2·38	2·25	2·24	2·10	2·07	2·08
8	2·70	2·63	2·55	2·48	2·36	2·20	1·83	1·88	1·92	1·91	1·87	1·90
9	2·98	2·92	2·82	2·73	----	2·30	2·20	2·05	2·00	1·90	1·90	1·90
10	2·84	2·77	2·68	2·42	2·23	2·15	2·00	1·91	1·78	1·62	1·53	1·57
11	2·90	2·30	2·30	2·25	2·18	2·07	1·92	1·78	1·64	1·47	1·45	1·60
12	2·28	2·26	2·22	2·17	2·12	2·03	1·91	1·75	1·67	1·54	1·53	1·68
13	2·56	2·55	2·50	2·45	2·31	2·16	2·00	1·80	1·72	1·55	1·57	1·75
14	2·85	2·84	2·73	2·60	2·46	2·31	2·17	2·10	2·03	1·96	1·96	2·08
15	2·70	2·65	2·60	2·35	2·22	2·06	1·87	1·72	1·63	1·55	1·51	1·72
16	2·77	2·68	2·50	2·35	2·18	1·97	1·75	1·57	1·48	1·35	1·35	1·60
17	2·83	2·76	2·60	2·40	2·15	1·91	1·58	1·30	1·12	1·00	1·06	1·40
18	2·72	2·67	2·48	2·30	2·05	1·78	1·45	1·10	0·84	0·70	0·80	1·16
19	3·06	2·92	2·72	2·50	2·23	1·90	1·50	1·20	1·00	0·88	----	1·35
20	3·33	3·20	3·00	2·72	2·48	2·10	1·70	1·33	1·08	1·20	1·30	1·65
21	3·34	3·20	2·98	2·83	2·56	2·18	1·80	1·53	1·35	1·25	1·32	1·67
22	3·47	3·33	3·15	2·96	2·75	----	2·10	1·96	1·80	1·71	1·82	2·10
23	3·53	3·42	3·30	----	2·40	2·25	2·10	1·98	1·83	1·70	1·70	1·90
24	3·53	3·40	3·01	2·90	2·78	2·68	2·55	2·40	2·20	2·07	2·02	2·16
25	2·47	2·46	2·37	2·30	2·18	2·03	1·90	1·72	1·52	1·40	1·38	1·52
26	2·62	2·66	2·64	2·55	2·50	2·40	2·25	2·05	1·88	1·82	1·76	1·82
Sums.	95·39	90·65	89·50	82·00	74·60	69·48	65·83	60·52	58·46	52·75	51·02	55·77
Means.	2·80	2·74	2·63	2·48	2·33	2·17	2·00	1·83	1·72	1·60	1·59	1·69

EDGARTOWN HARBOR.

Gauge No. 3.

to Staff of T. G. No. 4.]

VI	VI½	VII	VII½	VIII	VIII½	IX	IX½	X	X½	XI	XI½
ft.	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*
----	----	----	----	----	----	----	----	----	----	2·94	3·00
1·75	2·12	2·49	2·70	2·80	2·83	2·80	2·68	2·70	2·83	2·98	3·02
1·54	2·06	2·40	2·62	2·73	2·76	2·77	2·70	2·72	2·87	3·00	3·06
1·67	2·00	2·40	2·60	2·72	2·86	2·85	2·83	2·92	3·03	3·14	----
1·93	2·24	2·57	2·78	2·92	3·05	3·14	3·20	3·25	3·30	2·42	2·54
1·60	1·83	2·10	2·40	2·52	2·61	2·70	2·80	2·12	2·18	2·27	2·37
1·80	2·10	2·35	2·60	2·87	3·12	----	1·95	1·92	1·92	2·02	2·20
2·02	2·15	2·50	2·80	----	2·45	2·48	2·47	2·45	2·38	2·38	2·47
2·00	2·20	2·49	2·32	2·48	2·55	2·62	2·60	2·48	2·42	2·46	2·52
1·85	2·10	1·93	2·22	2·40	2·50	2·51	2·50	2·40	2·26	2·27	2·32
1·47	1·70	2·00	2·30	2·50	2·53	2·50	2·42	2·30	2·21	2·22	2·26
1·60	1·90	2·20	2·43	2·62	2·68	2·66	2·55	2·40	2·30	2·35	2·42
1·70	1·90	2·21	2·51	2·65	2·68	2·61	2·52	2·45	2·35	2·35	2·47
1·85	2·10	2·42	2·62	2·72	2·75	2·73	2·63	2·50	2·50	2.58	2·61
2·25	2·53	2·83	3·02	3·13	3·05	2·93	3·02	3·05	3·20	3·25	3·25
2·02	2·21	2·40	2·55	2·70	2·80	2·68	2·50	2·46	2·56	2·63	2·70
2·08	2·30	2·55	2·82	2·98	3·02	3·00	2·93	2·90	2·92	2·93	2·94
1·72	2·00	2·25	2·42	2·53	2·58	2·58	2·55	2·55	2·66	2·75	2·85
1·85	2·13	2·40	2·56	2·68	2·70	2·78	2·70	2·70	2·80	2·87	2·90
1·90	2·20	2·40	2·58	2·68	2·75	2·80	2·84	2·96	3·10	3·20	----
1·90	2·24	2·53	2·70	2·80	2·93	3·02	3·10	3·18	----	2·31	2·47
2·35	2·67	3·00	3·20	3·32	3·40	3·48	----	2·60	2·60	2·70	2·80
2·13	2·60	2·95	3·20	3·40	----	2·48	2·47	2·42	2·43	2·54	2·62
2·10	2·37	2·82	----	2·81	2·85	2·82	2·75	2·65	2·65	2·72	2·78
2·05	2·48	2·56	2·80	2·93	2·92	2·81	2·72	2·66	2·63	2·72	2·81
1·60	2·10	2·66	2·90	2·93	3·00	2·87	2·65	2·48	2·48	2·57	2·70
1·83	2·35	2·88	3·25	3·35	3·35	3·15	3·00	2·88	2·90	3·03	3·10
2·18	2·75	3·30	3·52	3·60	3·53	3·40	3·23	3·16	3·20	3·30	3·38
2·10	2·65	3·05	3·30	3·38	3·35	3·24	3·20	3·15	3·25	3·37	3·40
2·45	2·85	3·30	3·50	3·61	3·60	3·55	3·48	3·48	3·50	3·55	3·55
2·28	2·66	2·93	3·18	3·30	3·30	3·30	3·29	3·40	3·51	3·56	3·60
2·40	2·77	3·13	3·35	3·46	3·48	3·48	3·48	3·47	3·47	3·54	3·54
1·80	2·10	2·35	2·52	2·62	2·70	2·75	2·80	2·90	3·06	3·17	3·33
2·02	2·30	2·51	2·70	2·82	2·93	3·00	3·12	3·21	3·33	----	2·50
63·79	74·66	84·86	88·97	92·96	93·61	92·41	89·68	90·86	88·80	92·09	90·48
1·93	2·26	2·57	2·78	2·90	2·93	2·89	2·80	2·84	2·77	2·79	2·74

TABLE 4.—*Tide*

EDGARTOWN

[Observed heights for each lunar

Date.	O	½	I	1½	II	II½	III	III½	IV	IV½	V	V½
1891.	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*
Aug. 24	3·25	3·10	2·85	2·64	----	----	----	----	----	----	----	----
25	3·32	3·17	2·85	2·36	2·10	1·88	1·60	1·30	1·08	0·92	0·92	1·18
26	3·38	----	2·35	2·20	2·15	1·82	1·60	1·36	1·15	0·95	0·90	1·09
27	2·66	2·56	2·43	2·31	2·20	2·03	1·82	1·58	1·32	1·16	1·10	1·18
28	2·85	2·78	2·70	2·56	2·45	2·33	2·12	1·90	1·72	1·54	1·46	1·54
29	2·63	2·63	2·54	2·42	2·28	2·10	1·92	1·75	1·52	1·38	1·30	1·32
30	2·38	2·50	2·51	2·40	2·37	2·30	2·08	1·95	1·78	1·52	1·42	1·44
31	2·67	2·62	2·52	2·49	2·43	2·38	2·25	2·14	2·00	1·73	1·80	1·70
Sept. 1	2·80	2·80	2·67	2·62	2·45	2·30	2·18	2·00	1·80	1·68	1·65	1·60
2	2·61	2·65	2·65	2·50	2·42	2·35	2·20	2·00	1·80	1·60	1·43	1·42
3	2·60	2·62	2·60	2·40	2·26	2·20	2·05	1·95	1·75	1·65	1·56	1·53
4	2·73	2·71	2·62	2·47	2·32	2·21	2·09	1·90	1·70	1·55	1·44	1·28
5	2·75	2·75	2·56	2·36	2·20	2·10	1·92	1·80	1·72	1·60	1·33	1·31
6	3·06	3·05	2·92	2·70	2·56	2·50	2·30	2·10	1·80 1·87	1·52	1·45	1·50
7	3·42	3·40	3·35	3·15	2·85	2·63	2·36	2·10	1·97	1·85	1·78	1·86
8	2·85	2·75	2·65	2·50	2·41	2·10	1·93	1·80	1·78	1·70	1·62	1·66
9	3·13	3·05	2·90	2·80	----	2·22	2·10	1·87	1·80	1·70	1·70	1·75
10	3·00	2·90	2·76	2·32	2·23	2·10	1·92	1·74	1·55	1·40	1·35	1·40
11	3·05	2·37	2·38	2·28	2·18	2·05	1·83	1·62	1·41	1·27	1·28	1·45
12	2·38	2·34	2·30	2·25	2·17	2·01	1·85	1·68	1·50	1·33	1·36	1·53
13	2·70	2·68	2·57	2·48	2·35	2·16	1·90	1·70	1·53	1·35	1·32	1·61
14	3·05	3·00	2·83	2·67	2·48	2·31	2·10	1·96	1·85	1·75	1·78	1·85
15	2·86	2·78	2·62	2·41	2·27	2·00	1·80	1·60	1·41	1·32	1·37	1·50
16	2·92	2·81	2·52	2·18	2·05	1·80	1·65	1·40	1·20	1·06	1·10	1·33
17	3·08	2·87	2·60	2·35	2·12	1·80	1·40	1·12	0·90	0·81	0·88	1·28
18	3·20	2·81	2·57	2·28	2·00	1·70	1·30	0·90	0·61	0·52	0·64	1·00
19	3·35	3·04	2·80	2·52	2·17	1·80	1·30	1·01	0·75	0·62	----	1·10
20	3·68	3·37	3·07	2·74	2·37	2·00	1·53	1·10	0·76	0·85	1·00	1·40
21	3·66	3·40	3·11	2·83	2·47	2·08	1·66	1·35	1·05	0·95	1·03	1·41
22	3·83	3·60	3·35	3·15	2·75	----	1·96	1·70	1·51	1·41	1·50	1·77
23	3·90	3·70	3·48	----	2·40	2·20	1·90	1·75	1·55	1·40	1·40	1·58
24	3·85	3·66	3·18	3·02	2·88	2·68	2·45	2·17	1·90	1·72	1·71	1·93
25	2·65	2·55	2·43	2·33	2·18	1·97	1·74	1·51	1·30	1·18	1·20	1·40
26	2·88	2·83	2·75	2·67	2·56	2·40	2·18	1·91	1·73	1·57	1·54	1·68
Sums.	103·13	95·85	92·99	83·36	75·08	68·51	62·99	55·72	51·07	44·56	43·32	48·58
Means.	3·03	2·90	2·73	2·52	2.34	2·14	1·91	1·69	1·50	1·35	1·35	1·47

Gauge No. 4.

LIGHT-HOUSE.

half hour reduced to staff.]

VI	VI½	VII	VII½	VIII	VIII½	IX	IX½	X	X½	XI	XI½
ft.	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*	*ft.*
----	----	----	2·58	2·67	2·77	2·78	2·85	2·92	3·09	3·30	3·33
1·53	1·90	2·20	2·42	2·60	2·70	2·68	2·70	2·84	3·10	3·34	3·36
1·42	1·87	2·22	2·38	2·53	2·62	2·72	2·78	2·90	3·00	3·22	3·41
1·45	1·81	2·18	2·42	2·52	2·70	2·78	2·84	3·05	3·20	3·45	----
1·80	2·08	2·40	2·56	2·74	2·86	3·00	3·10	3·30	3·47	2·55	2·78
1·42	1·67	1·94	2·18	2·35	2·43	2·58	2·80	2·10	2·20	2·38	2·51
1·51	1·78	2·13	2·40	2·67	2·92	----	1·80	1·90	2·08	2·17	2·35
1·72	1·92	2·33	2·60	----	2·14	2·22	2·30	2·40	2·40	2·42	2·56
1·73	2·00	2·30	2·05	2·20	2·30	2·36	2·45	2·42	2·42	2·57	2·68
1·60	1·90	1·70	2·00	2·15	2·27	2·35	2·37	2·40	2·32	2·35	2·50
1·25	1·50	1·80	2·10	2·20	2·31	2·38	2·32	2·30	2·25	2·38	2·45
1·36	1·70	2·00	2·25	2·37	2·48	2·46	2·42	2·41	2·40	2·47	2·62
1·43	1·70	2·02	2·30	2·40	2·48	2·48	2·45	2·46	2·44	2·50	2·72
1·55	1·92	2·20	2.42	2·48	2·53	2·58	2·60	2·61	2·67	2·79	2·88
2·03	2·32	2·62	2·76	2·90	3·05	3·18	3·12	3·15	3·31	3·50	3·50
1·90	2·03	2·12	2·30	2·51	2·70	2·66	2·55	2·50	2·73	2·85	2·90
1·90	2·12	2·42	2·61	2·80	2·92	3·00	3·00	3·02	3·08	3·13	3·20
1·57	1·90	2·10	2·28	2·41	2·50	2·55	2·58	2·68	2·82	2·95	3·07
1·70	2·00	2·25	2·42	2·54	2·60	2·68	2·75	2·80	2·97	3·10	3·14
1·80	2·07	2·26	2·42	2·54	2·65	2·78	2·98	3·13	3·35	3·48	----
1·85	2·09	2·30	2·50	2·65	2·80	2·98	3·17	3·35	----	2·47	2·63
2·15	2·46	2·80	2·98	3·10	3·21	3·38	----	2·60	2·70	2·88	3·03
1·98	2·33	2·63	2·92	3·12	----	2·31	2·41	2·49	2·60	2·78	2·90
1·74	2·23	2·61	----	2·55	2·65	2·68	2·70	2·75	2·83	2·95	2·98
1·66	2·22	2·34	2·45	2·62	2·70	2·75	2·68	2·75	2·85	3·04	3·14
1·45	1·93	2·30	2·55	2·68	2·78	2·76	2·75	2·60	2·66	2·90	3·12
1·51	2·12	2·56	2·85	3·02	3·07	3·03	3·00	3·02	3·22	3·46	3·45
2·00	2·40	2·80	3·04	3·26	3·32	3·30	3·28	3·38	3·56	3·78	3·80
1·85	2·35	2·63	2·92	3·03	3·20	3·20	3·24	3·38	3·60	3·82	3·83
2·18	2·55	2·90	3·13	3·32	3·45	3·55	3·50	3·70	3·90	4·00	3·98
2·02	2·40	2·68	2·90	3·10	3·20	3·30	3·35	3·60	3 87	4·01	4·00
2·20	2·58	2·90	3·10	3·26	3·35	3·43	3·48	3·60	3·70	3·90	3·90
1·70	1·95	2·18	2·36	2·50	2·60	2·70	2·82	3·01	3·25	3·48	3·57
1·86	2·12	2·35	2·52	2·70	2·80	2·92	3·12	3·32	3·56	----	2·75
56·82	67·92	77·17	83·67	88·49	91·06	92·51	92·26	96·84	97·60	100·37	99·04
1·72	2·06	2·33	2·53	2·68	2·76	2·80	2·80	2·85	2·96	3·04	3·09

TABLE 6.

EDGARTOWN LIGHT-HOUSE.

[Observed times of high and low water and turning of current referred to moon's transit.]

Date of observation.	Time interval after moon's transit.								Date of observation.	Time interval after moon's transit.							
	High water.		Low water.		Current turns from west to east.		Current turns from east to west.			High water.		Low water.		Current turns from west to east.		Current turns from east to west.	
1891.	h.	m.	h.	m.	h.	m.	h.	m.	1891.	h.	m.	h.	m.	h.	m.	h.	m.
Aug. 24	11	50	-----		-----		9	50	Sept. 12	12	34	4	52	3	37	10	07
25	11	29	4	59	2	29	9	29	13	12	37	5	08	3	08	10	08
26	12	06	5	06	2	36	9	36	14	12	12	4	42	3	37	10	07
27	-----		5	13	2	43	9	58	15	12	12	4	42	3	37	10	07
28	12	20	5	23	2	33	10	18	16	11	43	4	44	3	39	10	08
29	12	51	5	23	3	53	10	21	17	11	45	4	44	3	14	10	42
29	-----		-----		-----		9	53	18	11	47	4	20	2	50	10	17
30	13	27	5	15	4	00	10	27	19	11	30	-----		3	28	9	55
31	12	35	5	55	4	10	10	35	20	11	37	-----		3	06	9	32
Sept. 1	12	46	5	53	4	23	10	46	21	11	44	4	44	2	43	9	39
2	13	00	5	23	4	08	11	00	22	11	36	4	51	3	01	9	16
3	12	47	-----		3	56	10	47	23	11	50	4	55	2	50	9	20
4	12	36	-----		3	46	10	36	24	11	54	4	54	3	24	9	54
5	12	26	5	26	3	36	10	26	25	-----		4	59	2	59	9	29
6	12	46	5	16	3	27	10	16	26	12	31	5	05	3	05	10	05
7	11	37	5	07	3	37	9	22									
8	11	55	5	25	2	55	9	55	No. of obs.	32		29		33		35	
9	11	56	5	11	3	41	10	11	Sums.	389	40	148	01	110	37	352	18
10	11	54	5	24	3	24	9	44									
11	11	47	5	02	3	02	10	02	Means.	12	11	5	06	3	21	10	04

TABLE 7.

SWIMMING-PLACE, EDGARTOWN HARBOR.

[Observed times of high and low water and turnings of current referred to moon's transit.]

Date of observation.	Time interval after moon's transit.						Date of observation.	Time interval after moon's transit.					
	High water.		Low water.		Current turns *from* south to north.	Current turns *from* north to south.		High water.		Low water.		Current turns *from* south to north.	Current turns *from* north to south.
1891.	*h.*	*m.*	*h.*	*m.*	*h.* *m.*	*h.* *m.*	1891.	*h.*	*m.*	*h.*	*m.*	*h.* *m.*	*h.* *m.*
Aug. 25	12	04	5	04	-----	------	19	12	10	-----		3 28	9 55
26	12	14	5	14	-----	------	20	12	02	5	02	4 06	10 32
27	12	50	4	54	-----	------	21	11	39	4	39	3 39	11 08
28	12	29	5	32	-----	------	22	11	46	4	46	3 46	9 46
29	13	06	5	23	-----	------	23	11	50	4	50	3 50	10 20
30	13	27	5	45	4 30	10 27	24	11	54	4	54	3 39	10 24
31	13	05	5	40	5 10	12 05	25	------		4	59	3 29	------
Sept. 1	12	46	5	23	4 53	12 16	26	13	01	5	05	-----	------
2	13	00	5	23	4 08	12 30	Oct. 2	12	41	5	52	-----	------
3	12	57	5	26	4 26	12 17	3	13	02	-----		-----	------
4	13	02	-----		3 46	11 36	4	------		-----		-----	------
5	12	56	5	26	4 06	12 26	5	------		-----		-----	------
6	13	16	5	16	3 27	12 46	6	12	13	-----		-----	------
7	12	07	5	37	4 07	13 07	* 7	------		-----		-----	------
8	12	10	4	55	3 25	------	8	------		-----		-----	------
9	12	11	5	11	4 11	12 11	9	12	13	5	13	-----	------
10	12	09	5	24	3 24	11 24	10	11	48	5	18	-----	------
11	12	02	5	02	4 02	11 32	* 12	------		-----		-----	------
12	12	34	5	07	4 07	11 37	14	------		-----		-----	------
13	12	37	5	08	4 08	------							
14	12	37	5	07	4 07	11 07	No. of obs.	37		34		27	24
15	12	37	5	07	4 07	11 07	Sums.	461	42	176	25	107 14	272 10
16	12	08	5	09	4 09	10 38							
17	12	12	4	44	3 44	10 42	Means.	12	29	5	11	3 58	11 20
18	12	47	4	50	3 20	10 17							

*Gale.

TABLE 8.

WEST OPENING, KATAMA BAY.

[Observed times of high and low water and turnings of current referred to moon's transit.]

Date of observation.	Time interval after moon's transit.								Date of observation.	Time interval after moon's transit.							
	High water.		Low water.		Current turns from north to south.		Current turns from south to north.			High water.		Low water.		Current turns from north to south.		Current turns from south to north.	
1891.	h.	m.	h.	m.	h.	m.	h.	m.	1891.	h.	m.	h.	m.	h.	m.	h.	m.
Aug. 26	8	06	3	06	8	36	3	36	Sept. 14	8	04	3	04	10	22	3	37
27	8	13	2	58	9	16	3	46	15	8	10	3	04	9	52	3	37
28	8	48	3	18	8	48	3	48	16	-----		2	35	-----		3	09
29	8	38	2	55	9	53	4	23	17	7	38	2	40	9	42	3	14
30	-----		3	30	9	57	-----		18	8	43	2	46	9	32	3	05
31	-----		2	10	-----		-----		19	7	50	2	53	9	50	3	23
Sept. 1	-----		2	38	-----		4	08	20	7	42	3	01	9	27	3	31
2	-----		2	38	-----		3	53	21	8	03	2	52	9	34	3	34
3	8	14	2	23	10	14	3	53	22	8	10	-----		9	26	3	46
4	8	01	2	11	10	01	4	11	23	7	50	-----		9	20	3	50
5	7	56	2	36	9	56	3	36	24	8	09	3	24	9	38	3	09
6	7	46	2	42	10	16	3	16	25	8	29	3	29	9	59	3	44
7	-----		-----		-----		-----		26	-----		3	35	10	05	4	05
8	8	24	2	32	9	25	3	55	29	-----		2	39	-----		4	09
9	8	11	-----		9	41	3	41									
10	8	09	-----		9	54	3	54	No. of obs.	25		28		27		30	
11	8	02	3	17	9	32	3	32	Sums.	203	58	81	56	261	31	110	40
12	8	07	3	22	9	37	3	37									
13	8	35	3	38	9	38	3	38	Means.	8	09	2	55	9	41	3	41

TABLE 9.

SOUTH SHORE.

[Observed times of high and low water referred to the moon's transit.]

Date of observation.	Time interval after moon's transit.		Remarks.
	High water.	Low water.	
1891.	*h.* *m.*	*h.* *m.*	
Aug. 11	-----	1 04	
12	7 24	0 21	
12	7 21	2 49	
13	7 19	0 55	
13	7 55	1 41	
14	7 41	1 25	
14	7 45	2 18	
16	8 01	1 51	
17	7 51	-----	
19	7 19	1 09	
20	7 20	1 50	
20	7 22	0 52	
21	7 15	1 25	
21	-----	0 38	
28	7 38	1 41	
29	6 50	0 33	
29	8 13	2 17	
30	8 07	0 20	
30	8 10	-----	
31	8 25	-----	
31	8 40	1 40	
Sept. 1	7 46	2 16	
1	8 03	1 53	
2	8 10	2 20	
2	7 58	1 38	
3	7 47	1 57	
3	-----	1 06	
4	7 56	1 46	
4	7 36	1 16	
5	7 46	1 36	
5	7 26	0 56	
6	7 26	0 46	
6	7 47	0 47	
7	-----	-----	
8	7 15	1 06	
8	6 43	-----	
9	7 31	1 01	
9	7 17	1 37	
10	7 24	0 24	
10	-----	1 28	
11	7 08	0 32	
11	7 22	2 24	
12	7 54	0 37	
12	7 27	1 47	
13	7 17	1 58	
13	7 38	1 47	
14	6 37	0 27	
14	7 37	2 07	
15	7 17	1 27	
15	7 27	1 28	
16	7 28	1 19	
16	7 39	-----	
17	7 20	1 47	
17	7 29	1 34	

Date of observation.	Time interval after moon's transit.		Remarks.
	High water.	Low water.	
1891.	*h.* *m.*	*h.* *m.*	
Sept. 18	7 20	1 29	
18	7 10	1 15	
19	6 58	1 28	
20	7 04	0 56	
20	7 06	1 26	
21	7 14	0 51	
21	7 06	1 03	
22	6 58	0 56	
22	6 53	1 00	
23	7 15	0 50	
23	7 12	1 32	
24	7 14	1 04	
24	-----	1 56	
25	7 16	1 09	
25	7 29	1 41	
26	7 23	1 15	
26	7 45	1 33	
27	7 48	2 03	
27	8 13	2 08	
28	7 50	1 55	
28	8 20	2 01	
29	8 16	1 44	
29	8 19	1 50	
30	8 16	1 37	
30	8 07	-----	
Oct. 1	8 12	1 47	
1	7 57	1 27	
2	8 01	1 51	
2	7 47	1 27	
3	7 47	1 17	
3	7 37	1 27	
4	7 45	1 07	
4	7 42	1 17	
5	7 28	0 46	
5	7 10	1 07	
6	7 28	1 03	
6	7 15	0 45	
7	7 35	0 37	
7	7 32	0 47	
8	7 12	0 12	
8	7 18	0 42	
9	7 28	0 13	
9	7 14	1 21	
10	7 20	-----	
10	-----	-----	
13	7 33	1 20	
13	7 17	1 38	
14	7 28	-----	
No. of obs.	95	92	
Sums.	718 45	124 49	
Means.	7 34	1 22	

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APPENDIX NO. 6—1892.

ON THE CHANGES IN THE OCEAN SHORE LINES OF NANTUCKET ISLAND, MASSACHUSETTS, FROM A COMPARISON OF SURVEYS MADE IN THE YEARS 1846 TO 1887 AND IN 1891.

A report by HENRY L. MARINDIN, Assistant.

Submitted for publication April 7, 1893.

In comparing the surveys of the ocean shore lines of Nantucket Island, we have been obliged to limit the inquiry to an examination of the shore line as defined by the crest of the high-water line, without considering any shift of the submerged portion of the coast because of the insufficiency of the data afforded by the earlier hydrographic surveys.

Although the recent surveys of 1890–'91 contain numerous cross sections with soundings, they did not fall on a sufficient number of depths on the older surveys to permit of reliable results by comparison.

This comparison is based on measures taken from a point of origin, determined on each cross section by actual measurement in the field. The geographical position of this point of origin was afterward obtained by platting on the chart and measuring by scale to the nearest meridian and parallel of latitude.

The island of Nantucket is fast becoming of great importance as a summer resort, a statement fully warranted by the increasing number of hotels and cottages built there each year. The examination of the stability of its shores thus becomes of prime importance to this summer population as well as to the inhabitants engaged in its shore fisheries.

The absence of the knowledge which is brought out by just such a comparison as this, permitted the location, some years since, of a line of railroad and many valuable buildings upon a part of the shore where the changes are so great that in a few years more the ground on which they stand will have been washed into the sea and the capital involved lost beyond recovery.

The data upon which this comparison is based will be found in tabular form in Table No. 1; part of this data can again be utilized in future comparisons.

An inspection of the accompanying illustration (No. 24) reveals the fact that the ocean shore of the island of Nantucket is divided into two great exposures to wind and sea. The part from Great Point to Siasconsett having an eastern exposure, tempered by off-lying shoals, and the part from Siasconsett to Smiths Point, at the western end of the island, having a southerly exposure without mitigating conditions from off-shore shallows. The abrading force of the waves should therefore differ sensibly on these two portions of the shore, and we find this to be the case, as will be shown further on.

Before presenting the general results of the comparison as to these two divisions of exposure it will be interesting to note a few local changes since 1846.

THE STABILITY OF GREAT POINT.

(See illustration No. 24.)

The comparison discloses a remarkable stability of this exposed sandy point. The end of Great Point is sharp and well defined. The oscillation of the point as indicated by its position in 1846, and again in 1887–'90, is contained within the circumference of a circle with a radius of 15 metres. The position in 1846 and 1887 falls within a circle with a radius of only 5 metres.

It may be noted that during the period since 1846 these shores have been visited by very severe gales, notably that of 1851, known as the Minots Ledge gale. We have no survey showing the position of this point immediately preceding and following that gale, but whatever may have been the damage in 1851 it had regained its position in 1887 and maintains it to this day.

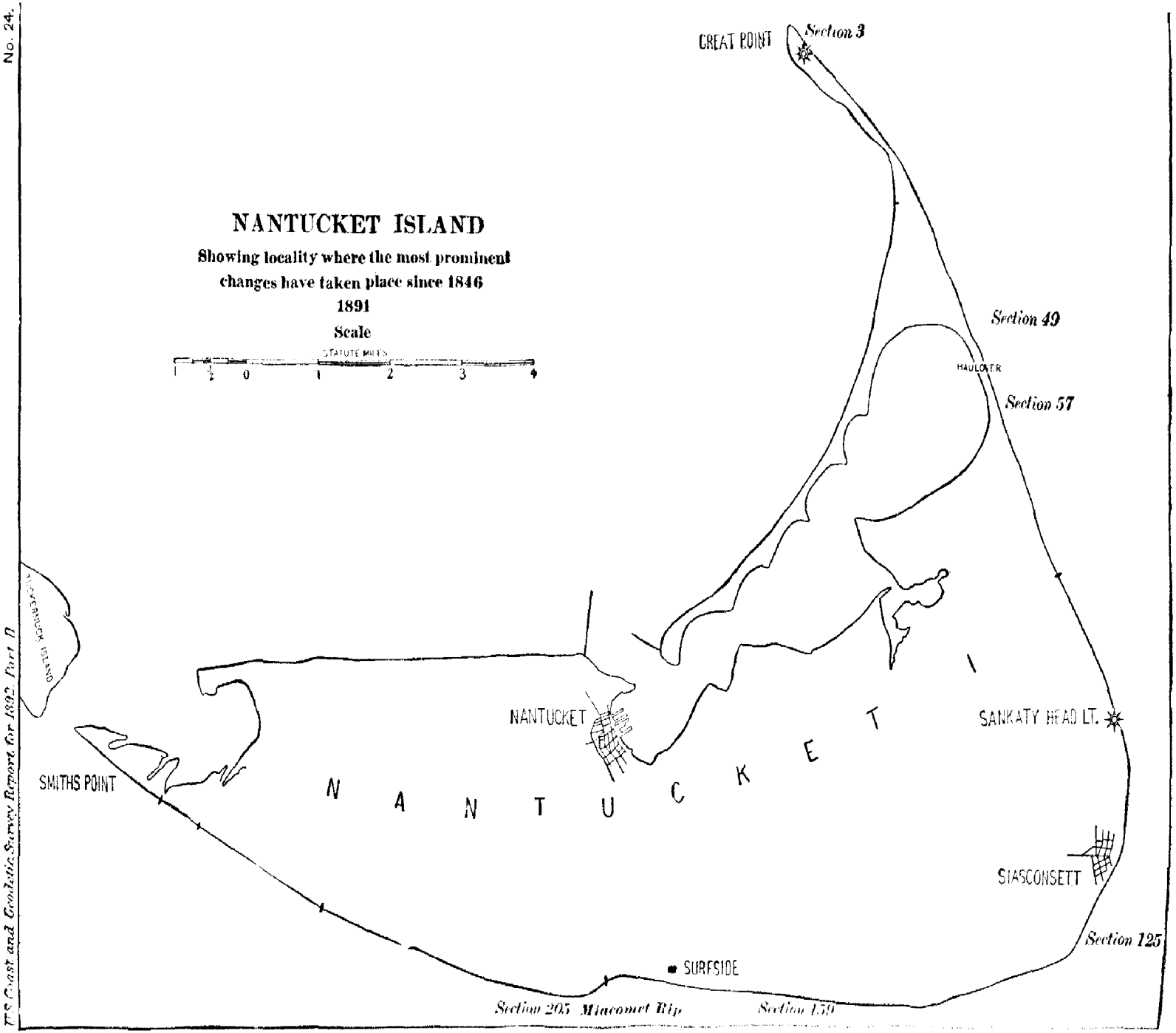
CHANGES AT THE HAULOVER.

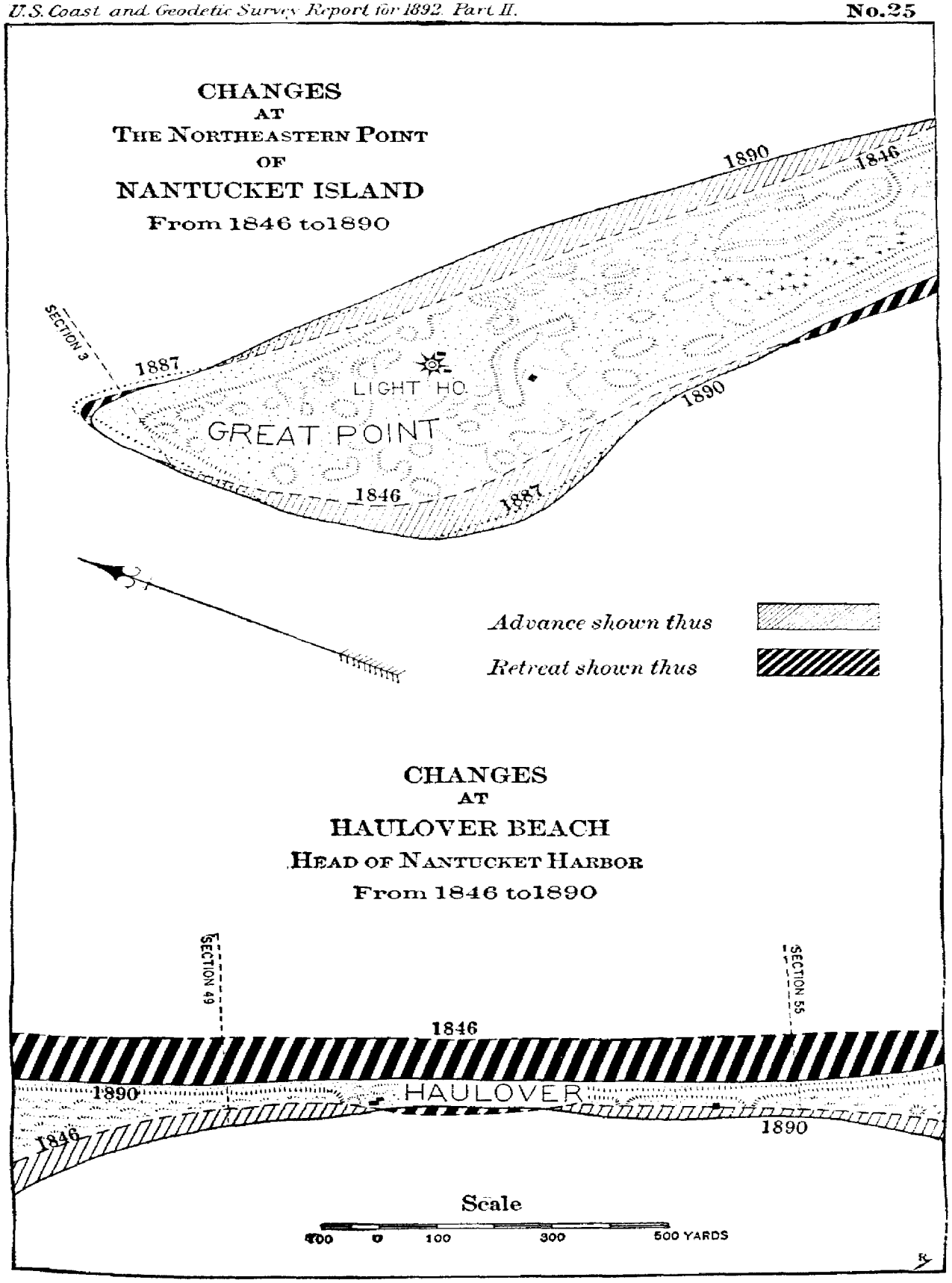
(See illustration No. 25.)

The name of Haulover is given to that part of the beach which separates the head of Nantucket inner harbor from the ocean. Some importance attaches to this locality from the fact that attempts have been made to induce the General Government to appropriate the means for cutting a passage through the beach.

An attempt of this kind was made in 1871 through a petition signed by the citizens of Nantucket. According to the petitioners, the object to be gained was twofold; first, that of creating an harbor of refuge in the bay called the head of the harbor, and, secondly, to increase the tidal scour over the bar at the entrance to Nantucket inner harbor.

The Coast Survey was asked at that time to make a resurvey of the locality, and the work was intrusted to Assistant Henry Mitchell.


NANTUCKET ISLAND
Showing locality where the most prominent changes have taken place since 1846
1891
Scale
STATUTE MILES
1 ½ 0 1 2 3 4
GREAT POINT
Section 3
Section 49
HAULOVER
Section 57
HOCKERBUCK ISLAND
NANTUCKET
SANKATY HEAD LT.
SMITHS POINT
N A N T U C K E T I
SIASCONSETT
Section 125
SURFSIDE
Section 205 Miacomet Rip
Section 159


CHANGES
AT
THE NORTHEASTERN POINT
OF
NANTUCKET ISLAND
From 1846 to1890
1890
1846
SECTION 3
1887
LIGHT HO.
GREAT POINT
1890
1846
1887
Advance shown thus
Retreat shown thus
CHANGES
AT
HAULOVER BEACH
HEAD OF NANTUCKET HARBOR
From 1846 to1890
SECTION 49
SECTION 55
1846
1890
HAULOVER
1890
1846
Scale
100 0 100 300 500 YARDS

On the changes in the ocean shore lines of Nantucket Island.

Mr. Mitchell reported favorably on the project, but nothing came of it. A few years later, in 1874, the United States Engineers again made surveys of the harbor and the Haulover, but their report, published in the report to the Chief of Engineers for 1875, Part II, p. 375, was not favorable to the project.

In 1874 the width of the Haulover beach was about 450 feet between low-water mark inside and the same datum outside.

In 1890 the narrowest width, on cross section No. 53, was found to be 290 feet between low-water marks. This shows a loss of width of 160 feet in sixteen years, or at the rate of 10 feet per year. This is, however, only a close approximation, because based on the assumption that the measurement taken in 1890 was made at the same place, or nearly so, as that of 1874.

If we compare the width in 1846 we find 459 feet as against 170 feet* in 1890, or a yearly erosion of 6½ feet. This is the more reliable statement, and if taken in connection with the preceding testimony we can only infer that the retreat of the shore may have been much greater between 1874 and 1890 than during the preceding period to 1846.

The erosion has obtained almost entirely on the ocean side, so that where the shore line was in 1846 we have in 1890 a depth of water of 18 feet. The shore line inside the beach has remained almost stationary. Usually where a beach is found with a low cross section and a body of water back of it, the storm waves, in making a breach over the beach, carry over a large volume of sand, which is deposited on the inner slope and maintains the width of the beach while beating it back. This does not seem to have been the case at the Haulover, although there are evidences of the wash of the waves through sloughs† into the harbor. As far as known there has never been an opening through the Haulover, and from a study of the conditions which exist here it is very doubtful if any opening could be maintained for any length of time.

The situation of the Haulover has been compared to that of Katama beach, Marthas Vineyard, but the conditions differ in these two cases. On the south shore of the Vineyard the seas are much more boisterous than at the Haulover, and these have repeatedly forced a passage through Katama beach, a wider and stronger barrier than the Haulover. There is also a greater difference of level of the water surfaces at Katama, giving a greater scouring force to the current than can be obtained at the Haulover.

A close approximation to the time when the natural forces shall have worn away the Haulover beach or forced a passage through into the

* These widths are measured between high-water marks, inside and outside, as measurements between low-water marks could not be obtained from the survey of 1846.

† The height of crest line within the sloughs is about 9½ feet above sea level; other parts of the beach range from 11 to 13 feet in height.

upper harbor may be made from the data quoted above. With the conditions remaining the same, it will take less than twenty-two years for the Haulover beach to be a thing of the past.

ADVANCE OF THE BEACH AT SIASCONSETT.

(See illustration No. 26.)

There has been a marked advance of the beach since 1846. This advance of the shore line begins at a point 428 metres north of Sankaty Head light-house, and extends for a distance of 3 miles to a point about one-half mile south and west of the village of Siasconsett.

The greatest advance (255 metres) occurred just west of the southern end of the village.

Directly opposite the center of the village, where the beach is used for bathing, the advance has been slightly less than 100 metres. This favorable condition has protected the foot of the bluffs in the vicinity from the wash of the seas so that they have suffered but slight erosion in years past.

The summer visitors to this quaint village continue to increase yearly, and the number of cottages built yearly is also on the increase. The fishing interest is represented by dory fishing for bluefish and mackerel in season.

POINT OF GREATEST WEAR OF THE SOUTH SHORE.

(See illustration No. 26.)

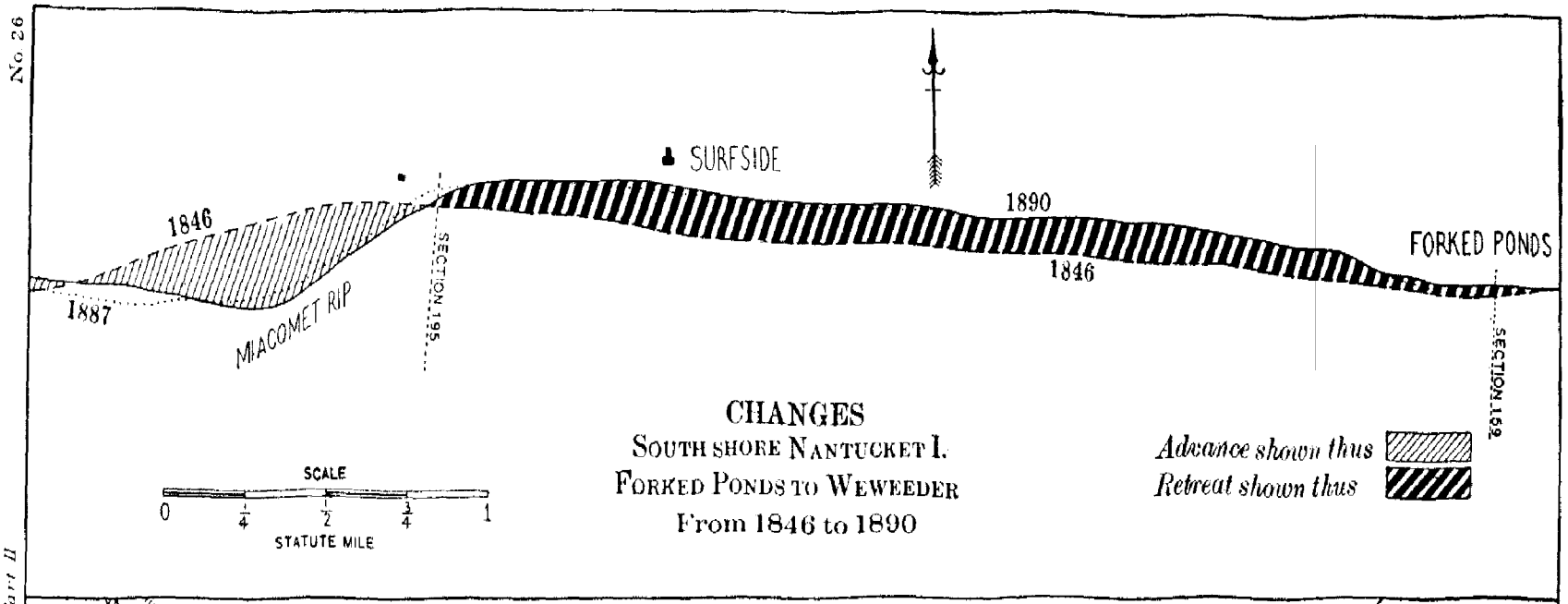
The greatest erosion of shore and bluff line since 1846 is found at Surfside, near Weeweeder. The cut into the bluffs extends from Forked Ponds to Weeweeder, a distance of 3½ miles. The height of the bluffs, within this distance, averages 22.4 feet above mean sea level, while the width of belt eroded varies from zero at Forked Ponds to 190 metres opposite Surfside Hotel. The mean width of the eroded belt is 389 feet, which gives an area of land of 165 acres worn away in forty-four years. The survey of 1846 shows the bluff line in the same relation to the high-water line as in 1890. We may therefore conclude that the volume lost at Surfside is about 6,000,000 cubic yards. Part of this material has gone westward and lodged on shore at Miacomet Rip.

CHANGE IN THE SHORE LINE AT MIACOMET RIP.

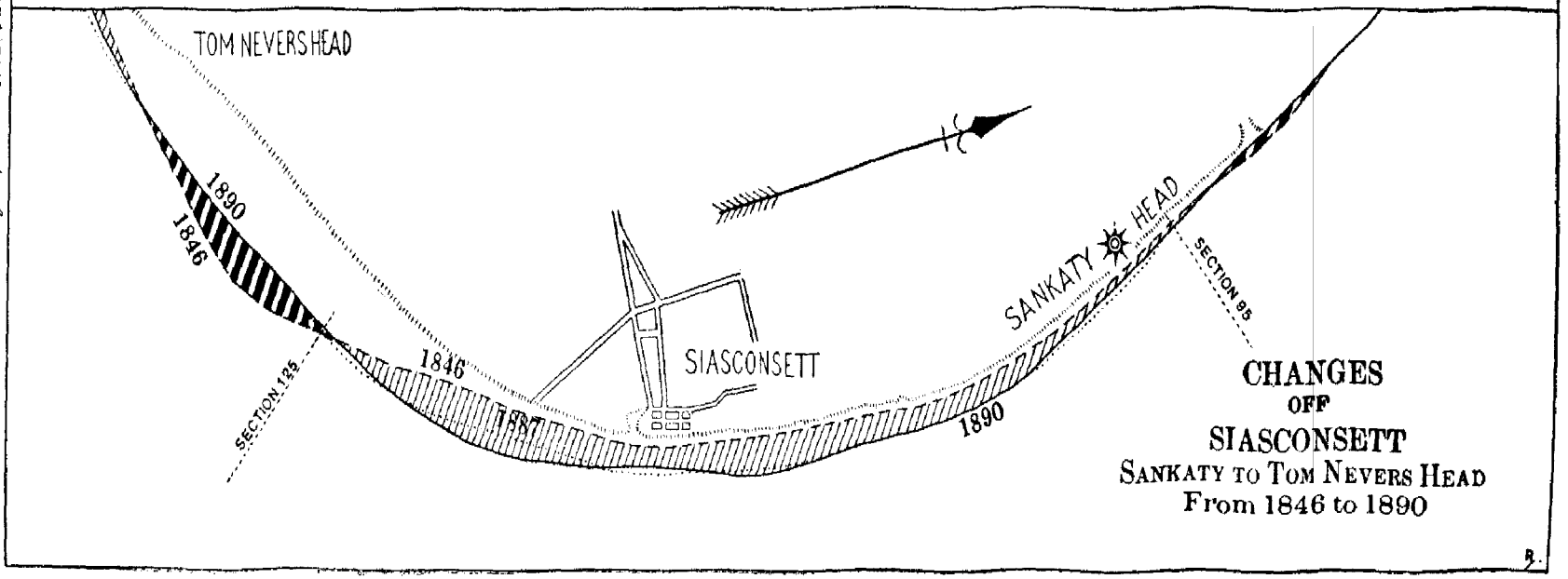
(See illustration No. 26.)

A sharp advance of the shore line is here noted. The configuration is changed from a concave shore in 1846 to a convex curve in 1890. In consequence of this change the landing place for the fishing fleet has been shifted eastward to Surfside.

The currents over the rip have not undergone any change as far as known. This is a noted fishing ground for bluefish and mackerel.



CHANGES
SOUTH SHORE NANTUCKET I.
FORKED PONDS TO WEWEEDER
From 1846 to 1890



CHANGES
OFF
SIASCONSETT
SANKATY TO TOM NEVERS HEAD
From 1846 to 1890

U.S. Coast and Geodetic Survey Report for 1892, Part II

On the changes in the ocean shore lines of Nantucket Island.

CHANGES AT SMITHS POINT.

(See illustration No. 27.)

The most western end of the island of Nantucket has been called Smiths Point. This point is most erratic in position. In 1846 it was situated at the end of a long and narrow sandy beach, almost directly west of the island of Tuckernuck.

Ten years later, in 1856, the point was about 1 mile to the eastward, and in 1887 the point to which the name of Smiths was applied was found to the southwest of Tuckernuck Island 3¼ miles east of its position in 1846.

These remarkable changes are exhibited in illustration No. 27, which shows the position in 1846–1856–1887 and 1891.

In 1846 the south shore extended westward past the island of Tuckernuck, leaving a waterway 250 metres wide between it and that island. In 1856 this waterway remained about the same, but between 1856 and 1887 the storms wrought a great change in this locality. The beach which was a part of Nantucket Island and which protected the south shore of Tuckernuck Island had been beaten back on the island, filling up the water space existing in 1856. A new inlet had also broken through this beach at a point southeast from Tuckernuck, which remained open at the last survey in 1891.

The survey of 1887 shows an extension of old Smiths Point beach to the westward as far as Muskeget Island since 1846, but under what condition this growth took place has not been ascertained and may not be.

It is probable that much of the material with which this point is maintained came from the south shore of Nantucket; the wear and tear of Tuckernuck Island shore and Muskeget Island shores may have furnished a small quota, but the bulk came from the south shore.

We may now consider the results of the comparison for the entire length of the exposed shore of Nantucket from its eastern end at Great Point to Smiths Point.

The comparison gives an average yearly erosion and consequent retreat of the shore line of 0·81 metres for the period 1846 to 1891. If, however, the seaward face of the island is divided into two great exposures, as previously alluded to, one from Great Point to the elbow at Siasconsett and the other from Siasconsett to Smiths Point, the first one, being an easterly exposure tempered by outlying shoals, should offer the minimum yearly wear and tear, and this is the case. Between Great Point and Siasconsett the average yearly retreat was 0·19 metres, and between Siasconsett and Smiths Point the average retreat was 1·42 metres, or 747 per cent of the rate for the easterly exposure.

Between 1846 and 1887 the yearly retreat was 1·29 metres for the southerly exposure and 0·18 metres for the easterly exposure.

The area of land lost to the sea on the south shore in the forty-five years since 1846 is 350 acres, or a yearly average of nearly 9 acres.

The conclusion from the foregoing evidence is that the sea is inevitably encroaching upon the southern and eastern shores of the island at a comparatively rapid rate. What is the result along the northern shore, bathed by the waters of the Sound, is not in evidence, but it is reasonable to say that the results are in the same direction, but on a smaller scale.

The comparison involved the examination of eleven original surveys, differing in scales, of which tracings had to be made. This will indicate to a certain extent the amount of work to be performed before arriving at results which may be expressed in a few figures.

The four diagrams accompanying the report were prepared by Mr. Homer P. Ritter.

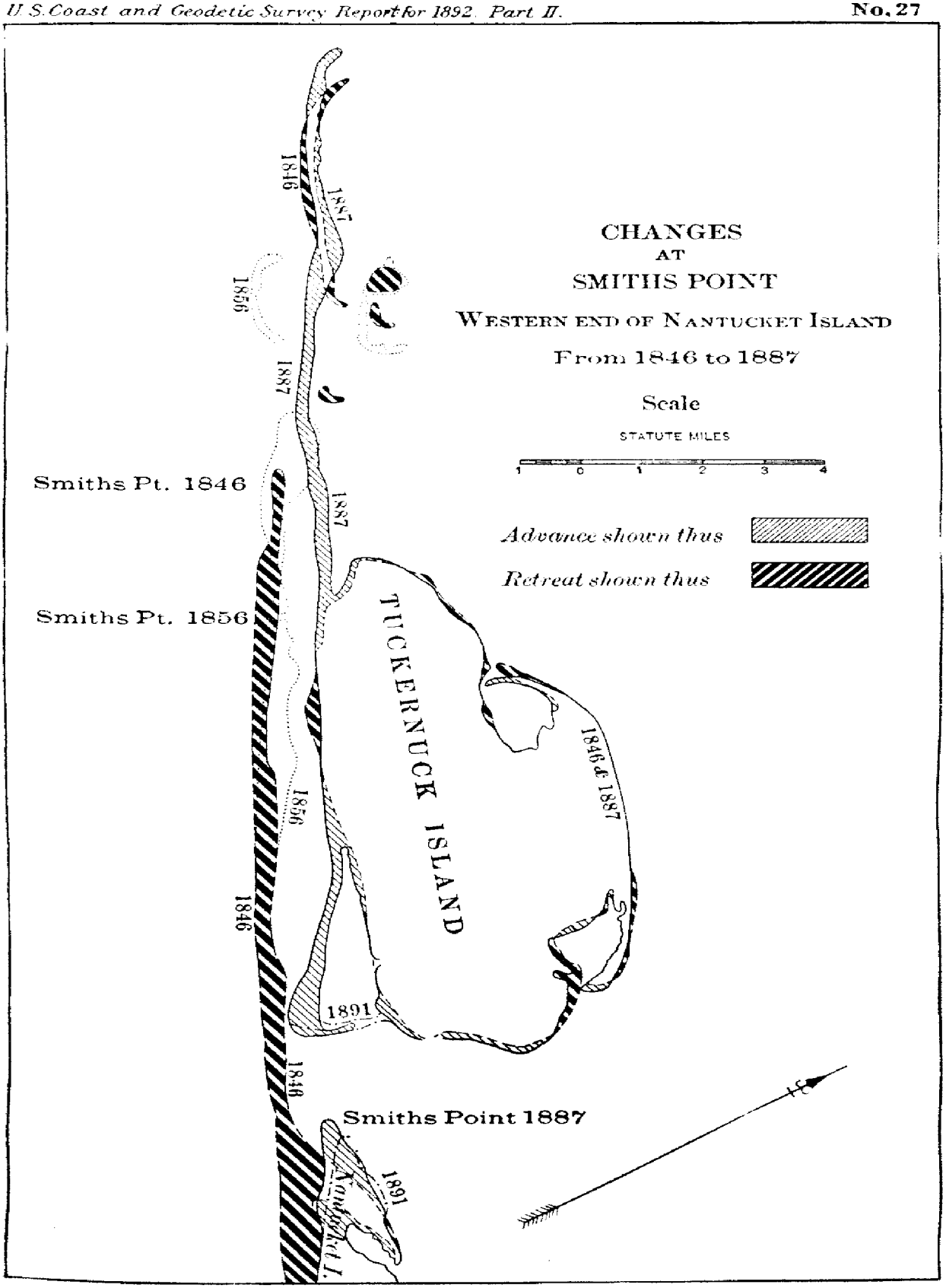

CHANGES
AT
SMITHS POINT
WESTERN END OF NANTUCKET ISLAND
From 1846 to 1887
Scale
STATUTE MILES
1 0 1 2 3 4
Advance shown thus
Retreat shown thus
Smiths Pt. 1846
Smiths Pt. 1856
TUCKERNUCK ISLAND
1846 & 1887
1846
1856
1887
1891
Smiths Point 1887
Nantucket I.

TABLE NO. 1.—*Changes in shore line of the eastern and southern shores of Nantucket Island from 1846 to 1891.*

Cross section.	Position of origin.			Distance to high-water line in—			Yearly change since 1846.		Remarks.
	Latitude.	Longitude.	Azimuth.	1846.	1887.	1890.	Advance.	Retreat.	
	° ′ ″	° ′ ″	° ′	*Metres.*	*Metres.*	*Metres.*	*Metres.*	*Metres.*	
No. 3	41 23 33·5	70 03 01·8	216 00	180	196	170	----	0·2	Near extremity of Great Point.
5	41 23 27·2	70 02 51·5	216 00	135	162	170	0·8	----	
7	41 23 21·2	70 02 41·2	218 00	110	170	170	1·4	----	
9	41 23 14·9	70 02 32·1	223 40	120	185	173	1·2	----	
11	41 23 07·7	70 02 23·8	226 10	155	200	180	0·6	----	
13	41 22 59·5	70 02 15·8	226 30	220	228	230	0·2	----	
15	41 22 54·0	70 02 05·4	229 00	185	172	178	----	0·2	
17	41 22 46·9	70 01 56·4	230 00	185	165	170	----	0·4	
19	41 22 39·2	70 01 47·8	230 00	193	160	160	----	0·8	
21	41 22 33·1	70 01 37·5	230 00	125	100	93	----	0·7	
23	41 22 26·2	70 01 28·4	235 00	95	60	60	----	0·8	
25	41 22 16·6	70 01 23·6	234 30	178	128	127	----	1·2	
27	41 22 06·0	70 01 21·5	235 00	320	260	260	----	1·4	
29	41 22 00·0	70 01 11·1	240 00	427	375	375	----	1·2	
31	41 21 50·2	70 01 06·4	240 00	270	208	212	----	1·3	
33	41 21 42·1	70 00 59·5	242 15	262	196	196	----	1·5	
35	41 21 33·8	70 00 52·9	244 10	239	168	167	----	1·6	
37	41 21 25·0	70 00 47·3	243 30	243	174	173	----	1·6	
39	41 21 17·2	70 00 38·3	243 30	170	90	91	----	1·8	
41	41 21 07·9	70 00 34·4	246 10	222	137	138	----	1·9	
43	41 20 58·6	70 00 29·2	244 30	224	140	138	----	2·0	
45	41 20 50·0	70 00 22·5	246 00	207	118	114	----	2·1	Off Haulover beach, northern end.
47	41 20 40·0	70 00 20·0	245 30	288	200	198	----	2·0	
49	41 20 32·7	70 00 10·7	245 30	183	90	90	----	2·1	
51	41 20 24·1	70 00 03·4	242 00	140	60	61	----	1·8	
53	41 20 15·3	69 59 58·1	242 00	140	53	52	----	2·0	
55	41 20 05·9	69 59 53·0	242 30	158	70	72	----	2·0	
57	41 19 55·6	69 59 50·8	242 30	240	180	180	----	1·4	Off Haulover beach, southern end.
59	41 19 47·0	69 59 45·4	244 00	240	181	183	----	1·3	
61	41 19 39·5	69 59 35·7	245 30	125	87	80	----	1·0	

TABLE NO. 1.—*Changes in shore line of the eastern and southern shores of Nantucket Island from 1846 to 1891.*

Cross section.	Position of origin.			Distance to high-water line in—			Yearly change since 1846.		Remarks.
	Latitude.	Longitude.	Azimuth.	1846.	1887.	1890.	Advance.	Retreat.	
	° ′ ″	° ′ ″	° ′	*Metres.*	*Metres.*	*Metres.*	*Metres.*	*Metres.*	
63	41 19 31·1	69 59 28·8	246 00	85	41	37	----	1·1	
65	41 19 21·2	69 59 26·5	247 50	154	115	110	----	1·0	
67	41 19 11·5	69 59 23·9	248 00	224	172	160	----	1·5	
69	41 19 01·9	69 59 20·2	247 40	260	203	184	----	1·7	
71	41 18 54·3	69 59 09·9	247 30	105	77	68	----	0·8	
73	41 18 42·3	69 59 12·8	244 00	285	270	261	----	0·6	
75	41 18 33·9	69 59 04·6	241 00	242	217	214	----	0·6	
77	41 18 26·2	69 58 55·7	240 00	188	170	170	----	0·4	
79	41 18 16·9	69 58 51·1	239 30	230	209	209	----	0·5	
81	41 18 09·9	69 58 40·9	239 30	152	130	115	----	0·8	
83	41 18 03·0	69 58 30·7	240 00	45	35	28	----	0·4	
85	41 17 52·7	69 58 28·6	242 30	122	122	126	0·1	----	
87	41 17 41·7	69 58 29·3	242 00	300	300	300	0·0	0·0	
89	41 17 34·6	69 58 20·0	244 30	206	200	205	0·0	0·0	
91	41 17 26·0	69 58 13·5	244 30	193	176	183	- --	0·2	
93	41 17 17·2	69 58 08·3	245 00	190	180	183	----	0·2	
95	41 17 08·2	69 58 02·4	246 30	152	160	156	0·1	----	
97	41 16 57·8	69 58 02·6	248 30	210	240	243	0·8	----	Off Sankaty Head light-house.
99	41 16 49·5	69 57 53·3	249 30	160	210	197	0·8	----	
101	41 16 40·6	69 57 43·9	251 30	122	212	210	2·0	----	
103	41 16 31·6	69 57 50·5	261 00	220	300	290	1·6	----	
105	41 16 22·8	69 57 50·9	269 00	233	325	322	2·0	----	
107	41 16 13·8	69 57 50·3	273 30	198	312	315	2·7	----	
109	41 16 04·9	69 57 51·9	277 00	190	322	331	3·2	----	
111	41 15 56·0	69 57 52·4	285 00	160	288	290	3·0	----	Off Siasconsett.
113	41 15 46·3	69 57 50·8	291 00	197	304	296	2·2	----	
115	41 15 38·2	69 57 57·6	293 00	145	247	251	2·4	----	
117	41 15 30·8	69 58 06·8	294 30	107	326	347	5·5	----	Off Siasconsett.
119	41 15 22·2	69 58 12·4	297 00	180	305	355	4·0	----	
121	41 15 16·0	69 58 19·8	304 30	222	354	375	3·5	----	

123	41 15 10·9	69 58 24·8	322 25	272	363	334	1·4	----	
125	41 15 04·6	69 58 33·9	324 00	355	348	325	----	0·7	
127	41 14 57·8	69 58 43·7	325 20	424	290	292	----	3·0	
129	41 14 53·5	69 58 54·4	328 20	445	322	323	----	2·8	
131	41 14 48·0	69 59 04·8	333 40	358	290	288	----	1·6	
133	41 14 45·6	69 59 16·1	338 05	338	338	330	----	0·2	
135	41 14 44·2	69 59 27·3	345 00	350	397	361	0·2	----	
137	41 14 38·5	69 59 38·6	345 40	233	295	270	0·8	----	
139	41 14 33·6	69 59 50·5	345 00	150	229	220	1·6	----	
141	41 14 33·3	70 00 04·1	345 10	217	312	290	1·7	----	
143	41 14 30·7	70 00 16·7	344 00	220	337	280	1·4	----	
145	41 14 28·3	70 00 29·5	345 00	243	310	295	1·2	----	
147	41 14 26·2	70 00 42·0	344 50	285	310	312	0.6	----	
149	41 14 22·5	70 00 52·4	357 00	208	223	212	0·1	----	
151	41 14 23·3	70 01 03·8	7 55	250	238	221	----	0·7	
153	41 14 24·3	70 01 17·2	4 00	258	247	230	----	0·6	
155	41 14 23·0	70 01 30·5	0 30	176	165	153	----	0·5	
157	41 14 25·8	70 01 42·8	4 05	247	260	255	0·2	----	
159	41 14 25·9	70 01 55·6	5 00	260	262	251	----	0·2	
161	41 14 26·6	70 02 08·3	8 30	262	258	242	----	0·5	
163	41 14 28·8	70 02 20·2	14 05	310	270	262	----	1·1	
165	41 14 30·5	70 02 33·4	13 10	315	222	219	----	2·2	
167	41 14 33·3	70 02 46·9	8 50	367	248	200	----	3·8	
169	41 14 29·5	70 03 01·2	8 20	214	106	100	----	2·6	
171	41 14 35·6	70 03 13·0	9 30	382	223	225	----	3·6	
173	41 14 37·0	70 03 26·2	5 00	395	233	230	----	3·8	
175	41 14 37·9	70 03 39·8	6 00	394	248	245	----	3·4	
177	41 14 37·0	70 03 52·3	5 00	330	187	181	----	3·4	
179	41 14 40·3	70 04 05·3	5 00	402	264	260	----	3·2	
181	41 14 38·0	70 04 18·8	5 00	304	172	179	----	2·8	
183	41 14 38·0	70 04 31·7	5 00	230	88	130	----	2·3	
185	41 14 41·3	70 04 44·2	6 25	382	210	196	----	4·2	
187	41 14 39·2	70 04 57·7	5 30	260	100	93	----	3·8	Off Surfside Hotel.
189	41 14 44·2	70 05 10·1	5 35	370	235	224	----	3·3	
191	41 14 44·4	70 05 23·1	7 05	335	220	220	----	2·6	
						1891.			
193	41 14 44·4	70 05 35·8	7 40	304	216	224	----	1·8	

TABLE NO. 1.—*Changes in shore line of the eastern and southern shores of Nantucket Island from 1846 to 1891.*

Cross section.	Position of origin.			Distance to high-water line in—			Yearly change since 1846.		Remarks.
	Latitude.	Longitude.	Azimuth.	1846.	1887.	1891.	Advance.	Retreat.	
	° ′ ″	° ′ ″	° ′	*Metres.*	*Metres.*	*Metres.*	*Metres.*	*Metres.*	
195	41 14 46·2	70 05 46·2	7 30	340	300	340	0·0	0·0	Off Weeweeder.
197	41 14 40·5	70 05 58·0	8 05	55	245	275	4·9	----	
199	41 14 39·8	70 06 08·7	8 00	137	365	400	5·8	----	
201	41 14 42 1	70 06 19·5	6 30	240	532	572	7·4	----	
203	41 14 37·3	70 06 29·9	7 00	123	406	424	6·7	----	
205	41 14 36·0	70 07 00·8	12 30	232	270	238	0·1	----	
206	41 14 38·4	70 07 26·8	12 55	220	207	191	----	0·6	
207	41 14 45·0	70 07 50·8	19 00	298	270	250	----	1·1	
208	41 14 50·5	70 08 16·2	20 05	334	222	190	----	3·2	
209	41 14 56·6	70 08 41·6	17 10	258	177	158	----	2·2	
210	41 15 05·3	70 09 05·1	15 30	330	278	169	----	3·6	
211	41 15 09·5	70 09 29·6	22 00	310	242	245	----	1·4	
212	41 15 05·3	70 09 56·3	19 35	248	163	155	----	2·1	
213	41 15 19·7	70 10 20·9	21 55	232	140	132	----	2·2	
214	41 15 29·6	70 10 43·3	22 40	292	218	202	----	2·0	
215	41 15 37·8	70 11 06·0	28 00	278	210	200	----	1·7	
216	41 15 48·7	70 11 28·4	29 35	340	252	250	----	2·0	
217	41 15 59·5	70 11 50·1	29 40	410	318	308	----	2·3	
218	41 16 04·1	70 12 16·8	29 00	255	125	100	----	3·4	
219	41 16 17·8	70 12 36·6	28 20	427	277	268	----	3·5	
220	41 16 23·5	70 13 02·1	31 00	318	143	127	----	4·2	
221	41 16 36·0	70 13 22·6	30 00	440	217	212	----	5·1	
222	41 16 46·0	70 13 44·7	33 05	453	221	204	----	5·5	
223	41 17 00·8	70 14 02·1	32 50	670	410	370	----	6·7	

APPENDIX NO. 7—1892.

RESULTS OF THE OBSERVATIONS RECORDED AT THE U. S. COAST AND GEODETIC SURVEY MAGNETIC OBSERVATORY AT LOS ANGELES, CAL., IN CHARGE SUCCESSIVELY OF MARCUS BAKER, ACTING ASSISTANT; CARLISLE TERRY, JR., SUB-ASSISTANT, AND RICHARD E. HALTER, ASSISTANT, BETWEEN THE YEARS 1882 AND 1889.

PART IV.—RESULTS OF THE DIFFERENTIAL MEASURES OF THE VERTICAL FORCE COMPONENT, AND OF THE VARIATIONS OF DIP AND TOTAL FORCE.

Discussion and report by CHARLES A. SCHOTT, Assistant.

Submitted for publication April 25, 1893.

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INTRODUCTORY REMARKS.

With the present or fourth report of the results of the magnetic observations made at Los Angeles, Cal., between 1882–1889, the series of reductions is concluded. Part I* contains the results of the absolute measures, parts II†, III‡, and IV, those of the differential measures of the declination, and of the horizontal and the vertical components of the magnetic force respectively; to the last part has also been added the resulting variation of the dip and of the total magnetic force.

The vertical-force magnetometer.—In consequence of the great sensitiveness of the adjustment of this instrument with respect to temperature, the scale of value of the balance magnetometer had to be frequently redetermined, nor could a uniform value be applied for the whole series.

The scale is exactly similar to the unifilar and the bifilar ivory scales, it being graduated to half millimetres. As mounted, the zero division is at the bottom end, the 500 or last division at the top. It is secured to a vertical brass arm attached to the slate slab, and its numbers appear erect and to increase downwards. Length of one division of scale, $0{\cdot}507^{mm}$.

The horizontal magnet, mounted on a knife edge, is about $16\frac{1}{3}^\circ$ out of the magnetic meridian, its north (seeking) end bearing magnetically N. $16\frac{1}{3}^\circ$ E.; thus the angle between the incident and reflected rays of light is $32\frac{2}{3}^\circ$. Increasing scale numbers correspond to a downward motion of the N. (seeking) end of the magnet or to increasing vertical force.

Angular value of one division of the scale.—We have the following dimensions: Front edge of upper end of scale to mirror, 40·89 inch ($103{\cdot}86^{cm}$); curvature, 0·065 inch ($0{\cdot}165^{cm}$); thickness of rim, 0·065 inch ($0{\cdot}165^{cm}$); two-thirds thickness of mirror, 0·067 inch ($0{\cdot}170^{cm}$); hence $r = 104{\cdot}36^{cm}$, angle of plane of scale with plane normal to center of mirror ($h = 0{\cdot}6$ inch), $\tan \alpha = \frac{1{\cdot}524}{104{\cdot}36}$, and $\alpha = 0^\circ\, 50'\, 12''$; hence $r \cos \alpha = 104{\cdot}35$ and $a = 3437{\cdot}75 \frac{l}{2r} = 0'{\cdot}8351$.

To determine the corresponding space on the vertical-force trace, we have distance from vertical-force cylinder to face of mirror, plus two-thirds thickness of mirror, $57{\cdot}135 + 0{\cdot}067$ inch (or $145{\cdot}293^{cm}$), and $\frac{0{\cdot}0507 \times 145{\cdot}293}{104{\cdot}35} = 0{\cdot}070\ 593^{cm}$. Now let $h' =$ the distance from the plane (or rather from the conical surface) of the ray which makes the luminous dot to the plane of the normal of the mirror, then by measure

* See Appendix No. 8—Report for 1890.
† See Appendix No. 9—Report for 1890.
‡ See Appendix No. 4—Report for 1891.

Results of the differential measures of the vertical component of the magnetic intensity at Los Angeles.

$h' = 11{\cdot}5$ inch ($29{\cdot}210^{cm}$) and distance $r = 104{\cdot}36^{cm}$, the angle i becomes $\sin^{-1}\frac{h'}{r} = 16^\circ\ 15'{\cdot}3$, and finally the value of one division of the scale as measured on the vertical-force cylinder becomes $0{\cdot}070\ 593 \cos i = 0{\cdot}067\ 77^{cm}$. Also 100 scale divisions equal $6{\cdot}78^{cm}$.

Scale value in terms of the vertical force.—Two methods are available for this purpose and both were employed.

First method; oscillations.—The change of vertical force V in terms of the force is given by the relation: $\frac{dV}{V} = \frac{T^2_h}{T^2_v} \cot \theta \,.\, \psi$. Where—

T_h = the time of one oscillation of the magnet with all its appendages in a *horizontal plane.*

T_v = the corresponding time of the same in a *vertical plane.*

θ = the dip, very nearly $59^\circ\ 30'$.

ψ = the angle, expressed in radians, through which the magnet turns for a change of position of the luminous spot corresponding to one scale division.

Observations were made September 5 and 25, 1882, by M. Baker as follows. Each set consists of 60 oscillations:

Sept. 5, 11 53 a. m.	$T_h = 6{\cdot}2362$ at 26·4 C.	Whence	$T^2_h = 38{\cdot}801$ at 28·6 C.
2 22 p. m.	157 29·7 C.		also 38·664 at 23·0 C.
2 50 p. m.	167 29·8 C.		

The temperature coefficient for Fahrenheit scale being 0·000 35 or 0·000 63 for C scale; torsion of suspension and rate of chronometer were allowed for.

The value of T_v was determined from twelve sets of oscillations on September 25, with a temperature of 23°·0 C, viz:

h.	m.		s.		
At 10	55	a. m.	$T_v = 7{\cdot}70$	weight	1
11	16		·40		2
11	24		·77		3
11	29		·52		4
11	35		·40		5
11	43		·53		5
11	50		·39		5
11	57	a. m.	·44		5
2	28	p. m.	·23		4
2	35		·32		5
2	46		·24		5
2	53		·23		5

Weighted mean $T_v = 7{\cdot}399$ s. and $T^2_v = 54{\cdot}7452$, hence $\frac{T^2_h}{T^2_v} \cot \Theta = \frac{38{\cdot}664}{54{\cdot}745} \cot 59^\circ\ 31' = 0{\cdot}416025$.

For the value of ψ we have for one centimetre of change in the luminous dot

$\frac{1}{2 \times 145{\cdot}293 \cos 16^\circ{\cdot}3} = 0{\cdot}003\ 5855$, hence $\frac{dV}{V} = 0{\cdot}001\ 491\ 6$ and 100 scale divisions were previously found to equal $6{\cdot}78^{cm}$, hence one centimetre corresponds to 14·749 scale divisions and $k = \frac{0{\cdot}001\ 4916}{14{\cdot}749} = 0{\cdot}000\ 1011$ which is the value of one scale division in terms of the vertical force.

Second method; deflections.—The method is as follows: First, a deflecting magnet is placed with its axis in the magnetic *prime vertical* at a distance r to the north, and also to the south, of the middle of the unifilar magnet, causing in the unifilar a deflection of u degrees (scale divisions times angular value); next, the same deflecting magnet is placed in a *perpendicular* position and in the plane of the axis of the vertical force magnet, at the same distance r to the north and also to the south of the middle of the vertical-force magnet, causing it to be deflected n scale divisions, then the value of one scale division or k in terms of the vertical force is found from the formula: $k = \frac{\tan u}{n} \cot \theta$.

On September 25, 1882, a set of deflections was made at a distance $r = 1$ foot 11 inches, or 58·42^{cm}, with the following result:

$h.$ $m.$		
At 3 17 p. m.	V-force magnet, at 23·9 C.,	deflected 55·82 scale divisions.
3 29 p. m.	U- magnet, at 23·9 C.,	deflected 36·68 scale divisions.
3 44 p. m.	V-force magnet, at 23·7 C.,	deflected 50·40 scale divisions.

Hence $u = 36{\cdot}68 \times 0'{\cdot}794 = 29'{\cdot}124$ and $n = 53{\cdot}11$

$$k = \frac{\tan 29'\ 07''\ 44}{53{\cdot}11} \cot 59°\ 30' = 0{\cdot}000\ 0940$$

Recapitulation of results, September 25, 1882:

Value of k by oscillations,	0·000 1011
deflections,	0·000 0940
Mean,	0·000 0976

It would take too much space and contribute nothing further to our knowledge than what we can get from the results themselves to present the record of the numerous observations for value of scale of the Lloyd balance magnetometer. The following abstract of the results is given in sufficient detail and with all needful explanation to show the basis of the adopted values for k during the seven years of use of the instrument:

Collection of results for scale coefficient $\frac{dV}{V} = k$ *for vertical-force magnetometer—from oscillations* (*O*), *from deflections* (*D*).

1882.	(O)	(D)	
Sept. 25,	$k = 0{\cdot}000\ 1011$	$k = 0{\cdot}000\ 0940$	
Oct. 31,	1839	1640,	change due to readjustment of mirror Oct. 13.
1883.			
Aug. 1,	0·000 1432	0·000 1019	

The vertical-force magnet was taken off its support, and rebalanced and readjusted August 3.

	(O)	(D)
Aug. 3, 1883.	$k = 0{\cdot}000\ 1395$	$k = 0{\cdot}000\ 1034$

On these values Mr. Baker remarks:

The results from the deflections on August 1 and August 3 are not satisfactory for the reason that the smaller the distance between the magnets the larger the resulting value of k, viz:

Results of the differential measures of the vertical component of the magnetic intensity at Los Angeles.

		Distance, 2·000 feet.	1·917 feet.	1·833 feet.		
Aug. 1	k =	0·000 924	0·000 1018	0·000 1114	Mean	0·000 1019
3	k =	1 005	1033	1064		1034

Secondly, for results from the oscillations it is apparent, particularly when the magnet oscillates in the vertical plane, that the time of one oscillation varies directly with the amplitude of the swing and not in accordance with the law for reduction to an infinitesimal arc; hence the greater the amplitude the smaller the value of k. If, therefore, we should make our amplitude of oscillation sufficiently large and our distance of deflection sufficiently small, we might get accordant results by the two methods, but we would not yet be sure that these accordant results accurately represent the degree of sensitiveness of our magnetometer.

The observer also points to friction as one of the causes of the discrepant results, and states his unfavorable opinion respecting the use of the balance magnetometer—an experience which he also shared with others.

January 2, 1884, the magnet was rebalanced, since the luminous dot, if unchecked, threatened to leave the cylinder; on the same date the scale value was redetermined.

	(O)		(D)	
Jan. 2, 1884,	k = 0·000 1339 before 1358 after 1253 after	rebalancing.	k = 0·000 0756 before 0·000 0842 after	rebalancing.

April 30, 1884. On account of the sudden change of reading, April 9, a redetermination of the scale value was made at the close of the month and with special care to discover the cause of the discrepancy previously noted. After rebalancing, new sets of observations were made on May 1, 1884.

1884.	(O)		(D)	
Apr. 30,	k = 0·000 1716	before.	k = 0·0000 971 before	rebalancing.
May 1,	1574			
May 1,	1297	after rebalancing.	0818 after	

The detail values from the deflections now show less variation due to change in deflecting distance, viz:

	2·000 feet.	1·833 feet.	1·667 feet.		
Apr. 30.	·000 0956	·000 0972	·000 0985	Mean	·000 0971
May 1.	0848	0834	0772		0818

The dependence of the time of one oscillation on the amplitude of the swing also is less pronounced than on former occasion.

May 31, 1884. Immediately after the rebalancing on May 1, the usual slowly progressive running off the scale set in and the reading had reached 400 and was still increasing on May 31; a rebalancing and a redetermination of the scale therefore became imperative, otherwise the luminous dot would have gone off the cylinder. The observations are by Mr. Baker, assisted by Messrs. Welker and Terry, and are pronounced by him unsatisfactory, viz:

(O) (D)

May 31. k = 0·000 1144. k = 0·000 0680 from three sets, showing but a trifling increase with distance.

1884.

Sept. 7. Values of k from deflecting distances 2·000, 1·833, and 1·667 feet, respectively. Observations by C. Terry, jr., 0·000 0782, 0·000 0794, 0·000 0773. Mean ·000 0783.

Nov. 30, k from oscillations 0·000 1335, results of one oscillation fairly accordant.

k from deflections 0·000 0882 three sets, fairly accordant, no dependence on distance.

Dec. 4, k from oscillations 0·000 1423, results fairly accordant.

k from deflections 0·000 0961 three sets, fairly accordant, no dependence on distance.

The remaining determinations have been omitted, since subsequently no use was made of the series 1885 to 1889.

Throughout the seven years' record, and for each observer, there appears a systematic difference in the values of k as derived from the oscillations and from the deflections, the latter giving a smaller value.

From a general scrutiny it would seem that the deflection results are more trustworthy than the oscillation results, at least the deviations of the values for different distances keep within tolerably narrow limits and with little dependence on the deflecting distance, whereas the oscillation results show greater discord due to the difficulty of getting a true value for the time of one oscillation when the magnet is made to swing on its knife-edge. In combining results for k (O) and k (D), I have given the former the weight ½ and the latter weight 1; we then have the following:

Recapitulation of resulting scale values of the vertical force magnetometer.

Date.	k in terms of V.	Remarks.
1882.		
Sept. 25 to Oct. 13.	0·000 096	Value to be used from beginning of series to Oct. 13.
Oct. 31.	171	To be used from Oct. 13 to 31.
1883.		
Aug. 1.	116	An apparent gradual decrease since Oct. 31, 1882.
Aug. 3.	115	Magnet rebalanced between Aug. 1 and 3.
1884.		
Jan. 2.	095	An apparent gradual decrease since Aug. 3, 1883.
Jan. 2.	100	After rebalancing.
Apr. 30, May 1.	117	The value of Apr. 30 from oscillations is rejected. These observations were made in consequence of a sudden change of readings on Apr. 9.
May 1.	098	After rebalancing.
May 31.	084	After May 4 a slow decrease in the scale readings was noted, hence the new determination after rebalancing.
Sept. 7.	078	From deflections only; making allowance for difference of (O) and (D) this value would be about 0·000 093.
Nov. 30.	103	
Dec. 4.	111	
1885.		
July 1.	132	An apparent increase since Dec. 4, 1884.
1886.		
Jan. 4.	071	Before rebalancing, from oscillations only. Values accepted though unusually small.

Recapitulation of resulting scale values of the vertical force magnetometer.

Date.	k in terms of V.	Remarks.
1886.		
Jan. 4.	098	After rebalancing, from oscillations only. Values accepted, though unusually small.
Feb. 11.	101	
Mar. 18.	073	Before rebalancing, from oscillations only.
Mar. 18.	094	After rebalancing.
June 25.	142	An apparent increase since Mar. 18.
1887.		
Jan. 4.	142	
June 24.	144	
Dec. 29.	113	
1888.		
Jan. 8.	090	Before rebalancing, from oscillations only. Value accepted.
Jan. 8.	147	After rebalancing.
June 26.	177	
Dec. 20.	173	
1889.		
Oct. 1.	182	An apparent increase in the value of k throughout the year 1888, and up to date.

The values of k actually adopted in the reduction of the differential observations are specially noted on the monthly records. The average value is 0.000 117.

It is obvious from the above results that the instrument was not performing satisfactorily; this is supposed due to the want of symmetry in the distribution of masses with respect to the plane of the knife-edge and to the fact that the instrument was badly balanced, turning almost about *one* point in the knife-edge nearest the magnet, instead of resting evenly along its entire edge. Thus the magnet had a tendency to place itself in the magnetic meridian. It is also supposed that the small dimensions of the magnet in comparison with the massive appendages were unfavorable to its efficiency, but the principal cause of the unsatisfactory working of the instrument was the unequal effect of changes of temperature on different materials *unsymmetrically* distributed with reference to the knife-edge.*

DETERMINATION OF THE TEMPERATURE OF THE VERTICAL-FORCE MAGNET AND OF ITS COEFFICIENT OF TEMPERATURE, AND METHOD OF REDUCTION OF OBSERVATIONS TO A UNIFORM TEMPERATURE.

The instrumental means by which the temperature of the vertical-force magnet has been ascertained are explained in detail in the preceding Part III of the discussion.

The thermograph record between February, 1883, and May, 1885, answers for both the bifilar and the Lloyd balance magnets, and the only difference in the treatment for the two magnets consists in substituting the thermometer readings under the bell glass of the vertical-force magnet for the readings of the thermometer of the bifilar

* These defects in the construction of the instrument were fully noticed and remedied in the spring of 1890 before remounting it at San Antonio, Tex.

magnet. The mean daily temperature of the Lloyd balance magnet as well as its hourly temperature were deduced by the formulæ given in Part III.

It has been noticed that the temperature of the vertical-force magnet is systematically lower by 0·7° C. than the temperature of the bifilar magnet. This is attributed by the observer to the unequal effect of the heat of the lamps on glass inclosures of different shape and volume, the bifilar bell glass being high and that of the Lloyd balance flat and low.

The diurnal range of the temperature throughout the year is close to 1½° C., and the range between the mean temperatures of the coldest and warmest month is about 9½° C.; the adopted standard temperature of the magnet is 23.6° C., as stated in Part III.

The temperature coefficient (q) of the balance magnet was determined directly by means of the deflections at high and low temperatures on September 8, 1882, and indirectly from the hourly readings of the photographic traces, the methods being similar to those explained in the case of the bifilar magnetometer.

Observations for temperature coefficient $q = \frac{an \cot u}{t - t_0}$ of the vertical-force magnet, September 8, 1882. Observers: Baker and Suess.

The magnet is placed in a copper box, south end westward and center of box placed on deflecting bar 2 feet east of unifilar magnet; the copper box is filled with broken ice, and to raise the temperature gradually warm water is added. Magnetometer No. 8 was mounted and read for changes of declination, but the readings proved unsatisfactory* to the observer, who substituted a normal diurnal change, as shown in the computation.

Local time, $8^h\ 55^m$.	No.	Readings of unifilar magnet's, 327·2.	Corr'n for diurnal change.	Corrected readings, U. M.	Temp. Fah't.	Combination.	U. M. reading mean.	Temp. Fah't.	n	$t - t_0$ Fah't.	
		d.	*d.*		°		*d.*	°	*d.*	°	
9 35	1	473·0	—2·7	470·3	33·1	1 and 11	473·65	32·90	—1·25	—26·72	
10 00	2	472·0	—2·1	469·9	45·0	2 10	473·10	45·65	—0·70	—13·97	
21	3	470·5	—1·1	469·4	56·0	3 9	472·75	55·10	—0·35	— 4·52	
45	4	469·8	+0·1	469·9	65·0	4 8	472·35	65·05	+0·05	+ 5·43	
11 10	5	469·0	+1·1	470·1	74·5	5 7	472·05	73·95	+0·35	+14·33	
28	6	468·8	+1·7	470·5	85·1	6	470·50	85·10	+1·90	+25·48	
13 11	7	470·9	+3·1	474·0	73·4						
35	8	472·0	+2·8	474·8	65·1	Means	472·40	59·62	Σ†	Σ†	
14 07	9	473·6	+2·5	476·1	54·2				4·60	90·45 =	50·25 C.
30	10	474·0	+2·3	476·3	46·3						
55	11	475·0	+2·0	477·0	32·7						
15 00	12	327·3				$n = 0'·794\ (-327·25 + 472·40) = 1°\ 55'·25$ $a = 0'·794\ /3437·75 = 0\ 000\ 231$ $\therefore q = 0·000\ 631$					

*Supposed to have been occasioned by development of torsion in the suspension.

†All values of n and of $t - t_0$ being taken positive.

Determination of the temperature of the vertical force magnet, etc.

With an average value of 1 division of scale, $k = 0{\cdot}000\ 117$ we have $\frac{q}{k} = 5{\cdot}4$ scale divisions of the vertical force instrument. The above observations were not satisfactory to the observer, and the method of deducing (q) from the differential observations themselves was adopted as preferable, as it takes in the *full* amount of temperature effect. It would appear that on account of the unsymmetrical distribution of masses attached to the magnet of the Lloyd balance, the direct effect of a change of temperature on the dimensions of these masses greatly influenced its balance, in consequence of which circumstance it was found necessary to reconstruct the instrument before remounting it in 1890 at the new station in Texas.

For the purpose of deducing a value of q from the hourly readings of the balance magnetometer, special selections were made of series of days with a large change of temperature and apparently unaffected by disturbances. For each series, the daily mean scale readings for days of low temperature were compared with the daily mean readings for days of high temperature; the differences of these means or $\triangle s$ and the corresponding $\triangle t$ gave a value for $q = \frac{\triangle s}{\triangle t}$. Twenty-seven selected series between February, 1883, and April, 1885, depending on more than 400 daily means produced the value $q = 7{\cdot}5$; ten of the best values gave $q = 6{\cdot}4$. These results are excessive, and were caused by large and irregular changes in the position of the zero of the scale, and all attempts to deduce any satisfactory value in this manner had to be abandoned. Further trials with smaller values led me finally to adopt $q = \frac{2}{3}$ of the value obtained from the direct observations or $q = 3{\cdot}6$ scale divisions. It will be noticed that even with this value the curve of the diurnal variation of the vertical force is considerably affected by the temperature correction; in fact the question whether the a. m. or p. m. extreme high value of the force is the greater of the two depends almost wholly on the amount of temperature correction applied.

We adopt for this correction $\triangle s = 3{\cdot}6\ (23^{\circ}{\cdot}6\ \text{C.} - t)$.

The following tables contain the hourly temperatures, deduced from monthly averages, of the vertical-force magnet under the bell glass, during the time when the thermograph was in operation:

Abstract of mean hourly temperature of the vertical force magnet in degrees centigrade.

1883.

Local hours.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
		°	°	°	°	°	°	°	°	°	°	°
1	----	17·35	20·62	20·29	22·52	26·12	28·01	28·16	29·36	24·56	22·43	21·30
2	----	17·20	20·53	20·16	22·44	26·01	27·90	28·03	29·26	24·48	22·29	21·17
3	----	17·06	20·47	20·07	22·37	25·92	27·83	27·91	29·15	24·41	22·19	21·07
4	----	16·96	20·38	20·00	22·33	25·83	27·75	27·82	29·07	24·36	22·12	20·96
5	----	16·87	20·33	19·91	22·28	25·77	27·68	27·74	29·00	24·33	22·03	20·87
6	----	16·78	20·26	19·83	22·26	25·70	27·64	27·65	28·93	24·30	21·95	20·78
7	----	16·71	20·22	19·77	22·25	25·69	27·61	27·61	28·89	24·30	21·89	20·68
8	----	16·67	20·21	19·79	22·34	25·79	27·62	27·66	28·87	24·34	21·85	20·59
9	----	16·72	20·26	19·92	22·47	25·98	27·70	27·80	28·95	24·45	21·85	20·56
10	----	16·99	20·43	20·16	22·69	26·33	27·93	28·12	29·11	24·62	22·05	20·66
11	----	17·15	20·49	20·26	22·77	26·42	28·04	28·30	29·23	24·71	22·19	20·80
Noon	----	17·44	20·58	20·41	22·88	26·53	28·24	28·55	29·42	24·86	22·40	21·04
13	----	17·69	20·72	20·60	22·98	26·69	28·48	28·78	29·69	25·01	22·68	21·30
14	----	17·94	20·90	20·77	23·08	26·85	28·69	29·04	29·90	25·14	22·97	21·53
15	----	18·14	21·04	20·87	23·12	26·94	28·81	29·21	30·05	25·24	23·18	21·75
16	----	18·29	21·14	20·97	23·18	27·01	28·90	29·29	30·18	25·30	23·35	21·91
17	----	18·40	21·19	21·03	23·20	27·08	28·96	29·36	30·28	25·35	23·49	22·05
18	----	18·45	21·22	21·09	23·22	27·18	28·99	29·41	30·35	25·44	23·54	22·09
19	----	18·41	21·18	21·08	23·21	27·22	29·01	29·41	30·32	25·36	23·45	22·03
20	----	18·28	21·10	21·00	23·12	27·14	28·92	29·28	30·17	25·21	23·29	21·89
21	----	18·14	20·99	20·87	23·02	26·98	28·71	29·06	29·94	25·03	23·11	21·74
22	----	17·96	20·88	20·72	22·88	26·79	28·49	28·81	29·72	24·85	22·91	21·59
23	----	17·77	20·76	20·59	22·75	26·61	28·29	28·62	29·55	24·72	22·70	21·42
Midn't	----	17·59	20·66	20·43	22·63	26·44	28·12	28·42	29·38	24·57	22·55	21·29
Means		17·54	20·69	20·44	22·75	26·46	28·26	28·50	29·53	24·79	22·60	21·29

1884.

Local hours.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
	°	°	°	°	°	°	°	°	°	°	°	°
1	19·49	19·89	20·17	20·39	22·34	24·96	27·54	28·51	26·37	24·06	22·54	20·35
2	19·35	19·80	20·07	20·30	22·27	24·87	27·41	28·41	26·24	23·96	22·43	20·27
3	19·22	19·77	19·97	20·25	22·23	24·79	27·34	28·30	26·13	23·86	22·35	20·20
4	19·13	19·69	19·91	20·17	22·19	24·72	27·23	28·25	26·04	23·76	22·27	20·13
5	19·07	19·62	19·84	20·10	22·14	24·67	27·14	28·14	25·98	23·66	22·20	20·04
6	19·00	19·57	19·79	20·06	22·12	24·61	27·05	28·06	25·94	23·55	22·12	19·97
7	18·93	19·50	19·72	20·01	22·11	24·58	27·00	28·00	25·89	23·47	22·06	19·90
8	18·86	19·44	19·73	20·05	22·15	24·64	27·04	28·03	25·90	23·42	22·03	19·84
9	18·87	19·45	19·76	20·32	22·44	25·00	27·34	28·35	26·06	23·58	22·14	19·85
10	19·00	19·54	19·98	20·79	22·58	25·02	27·43	28·41	26·18	23·69	22·26	19·90
11	19·08	19·63	20·06	20·79	22·60	25·10	27·54	28·52	26·34	23·86	22·45	19·98
Noon	19·29	19·77	20·17	20·81	22·65	25·20	27·72	28·68	26·52	24·09	22·70	20·14
13	19·55	19·95	20·27	20·90	22·75	25·28	27·88	28·85	26·69	24·30	23·00	20·32
14	19·82	20·15	20·38	20·97	22·86	25·34	28·05	29·02	26·84	24·49	23·32	20·52
15	20·05	20·33	20·44	21·05	22·93	25·39	28.19	29·15	26·96	24·64	23·54	20·68
16	20·27	20·50	20·51	21·09	23·01	25·44	28·28	29·26	27·08	24·77	23·73	20·84
17	20·47	20·60	20·55	21·17	23·09	25·51	28·38	29·31	27·18	24·88	23·84	20·94
18	20·60	20·71	20·63	21·33	23·18	25·61	28·51	29·38	27·25	24·96	23·93	21·06
19	20·54	20·68	20·62	21·26	23·13	25·60	28·48	29·34	27·23	24·92	23·80	21·01
20	20·37	20·55	20·56	21·14	23·04	25·55	28·38	29·26	27·12	24·77	23·55	20·86
21	20·15	20·39	20·47	20·97	22·92	25·46	28·22	29·15	26·97	24·60	23·29	20·70
22	19·97	20·27	20·35	20·82	22·80	25·34	28·04	28·99	26·79	24·42	23·04	20·55
23	19·82	20·13	20·23	20·68	22·68	25·22	27·87	28·84	26·61	24·26	22·86	20·41
Midn't	19·67	20·02	20·11	20·59	22·56	25·12	27 66	28·69	26·47	24·11	22·70	20·29
Means	**19·61**	**20·00**	**20·18**	**20·67**	**22·62**	**25·12**	**27·74**	**28·70**	**26·53**	**24·17**	**22·84**	**20·36**

1885.

Local hours.	Jan.	Feb.	Mar.	Apr.	Local hours.	Jan.	Feb.	Mar.	Apr.
	°	°	°	°		°	°	°	°
1	19·84	21·36	22·98	24·08	14	20·25	21·77	23·73	24·43
2	19·71	21·21	22·87	23·97	15	20·50	22·02	23·84	24·47
3	19·63	21·13	22·79	23·87	16	20·67	22·18	23·93	24·54
4	19·55	21·05	22·70	23·79	17	20·82	22·35	24·02	24·57
5	19·49	20·98	22·65	23·73	18	20·94	22·50	24·15	24·61
6	19·43	20·88	22·59	23·67	19	20·88	22·48	24·14	24·61
7	19·34	20·78	22·54	23·62	20	20·69	22·32	23·98	24·58
8	19·26	20·72	22·55	23·61	21	20·49	22·13	23·78	24·51
9	19·26	20·69	22·72	23·79	22	20·31	21·90	23·59	24·39
10	19·30	20·78	22·84	23·92	23	20·17	21·72	23·39	24·29
11	19·46	20·97	23·00	24·09	Midn't	20·01	21·55	23·23	24·19
Noon	19·68	21·22	23·24	24·21					
13	19·96	21·49	23·49	24·34	Means	19·98	81·51	23·28	24·16

Let m = value of one thermogram scale division in degrees centigrade and S'_0 = the mean temperature in scale divisions corresponding to the adopted standard temperature 23°·6 C., then, as in the case of the bifilar magnetometer (see p. 55, Part II, of Report for 1891).

Between

February, 1883, and July, 1884, inclusive, $m = -0{\cdot}68$ and $S'_0 = 17{\cdot}3$

August, 1884, to May 9, 1885, $m = +0{\cdot}69$ and $S'_0 = 12{\cdot}5$

hence for these periods

$\triangle s = -2{\cdot}47\,(17{\cdot}3 - s')$ and $= +2{\cdot}47\,(12{\cdot}5 - s')$, respectively, where s' = the thermograph reading. The temperature corrections for the hourly readings, as far as presented here, were applied by Mr. L. A. Bauer at intervals between June and October, 1892, who also supplied the daily mean values and the monthly averages for each hour during the same period (November, 1882, to October, 1884, inclusive).

Correction for shift of zero of scale.—A more serious difficulty than that of freeing the differential measures from the effects of change of temperature was encountered in the ever-changing adjustment for the zero of scale. After plotting the daily mean readings on suitable cross-ruled paper, it became apparent that these changes were of the most irregular character; the readings for short or long periods would sometimes decrease, sometimes increase. All that could be done was to trace a curve with a free hand and apply as a correction to each daily mean the corresponding ordinate between this trace and an adopted normal reading, viz, 100 divisions. This being done, the hourly values for any one day could likewise be reduced to the same standard of reference.

GENERAL REMARKS ON THE DIFFERENTIAL SERIES OF OBSERVATIONS OF THE VERTICAL FORCE COMPONENT.

The remaining uncertainty in the hourly readings of the balance magnetometer due to our imperfect knowledge of the reduction for changes of temperature, no less than the apparently fitful changes in the position of the zero of the scale due to the instrument not keeping in adjustment, has necessarily limited the useful record to the two years from November, 1882, to October, 1884. Even within this limit it was not found practicable to separate disturbed from normal values, since it was not known to what extent the above imperfections were mixed up with actual magnetic disturbances; more refined investigations, such as the lunar inequalities or the influence of solar rotation, were entirely out of the question.

The two-year series of hourly scale readings (corrected for changes of temperature and of scale) are appended to this paper; for each month there are given the daily means as well as the monthly means for each of the twenty-four hours, also the differences of each of the latter from the monthly mean value of all hours or days. These differences are collected in the following table; they constitute the solar diurnal variation for each month:

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Differential observations of the

[Monthly means of the diurnal variation of the

a $\left\{\begin{matrix}+\\-\end{matrix}\right.$ sign indicates $\left\{\begin{matrix}\text{greater}\\\text{less}\end{matrix}\right.$ force

Year and month.	Local mean time.											
	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1882.												
Nov.	+1·0	+0·9	+0·7	+0·5	+0·5	+0·6	+0·6	+1·1	+0·4	—2·8	—3·5	—3·8
Dec.	+2·0	+2·0	+2·0	+·19	+2·0	+1·9	+1·8	+2·0	+1·5	—1·3	—2·9	—3·7
1883.												
Jan.	+1·6	+1·4	+1·4	+1·1	+1·3	+1·2	+1·5	+1·4	+1·4	—0·1	—2·0	—3·1
Feb.	+0·8	+0·9	+0·5	+0·3	+0·1	+0·4	+0·7	+0·9	0·0	—1·4	—2·3	—2·6
Mar.	+1·0	+1·0	+0·7	+0·6	+0·4	+0·9	+1·1	+1·1	+0·1	—1·9	—3·1	—3·4
Apr.	+0·6	+0·5	+0·2	+0·3	+0·4	+0·8	+1·3	+1·4	—0·2	—1·8	—2·9	—2·6
May.	+0·9	+0·6	+0·6	+0·4	+0·7	+1·0	+1·2	+0·8	—0·3	—2·2	—2·5	—1·9
June.	+0·8	+0·8	+0·5	+0·9	+1·0	+1·3	+1·7	+1·3	+0·1	—1·9	—2·6	—2·2
July.	+1·0	+0·8	+0·7	+0·8	+1·3	+1·5	+2·0	+1·7	+0·7	—2·6	—3·1	—3·0
Aug.	+1·7	+2·0	+2·3	+2·7	+3·1	+3·7	+3·5	+3·2	+2·2	—0·1	—0·9	—1·8
Sept.	+1·8	+2·0	+2·0	+2·3	+2·6	+3·0	+3·2	+3·2	+1·5	—1·2	—2·0	—2·8
Oct.	+1·5	+1·6	+1·5	+1·9	+1·9	+2·3	+2·6	+2·5	+1·5	—0·6	—1·5	—2·0
Nov.	+1·9	+1·9	+2·2	+2·5	+2·7	+3·3	+4·0	+3·8	+2·6	—0·7	—2·1	—3·1
Dec.	+1·5	+1·8	+1·9	+2·4	+2·6	+3·0	+3·3	+3·6	+3·3	0·0	—1·7	—2·4
1884.												
Jan.	+1·0	+1·3	+1·2	+1·3	+1·5	+1·7	+2·2	+2·5	+1·4	—1·0	—3·8	—4·9
Feb.	+0·8	+0·9	+0·9	+0·8	+0·8	+1·2	+1·3	+1·9	+1·2	+0·5	—1·2	—2·5
Mar.	+1·5	+1·4	+1·3	+1·2	+1·4	+2·0	+2·5	+2·6	+0·9	—1·9	—3·9	—4·8
Apr.	+1·7	+1·7	+1·7	+1·5	+1·9	+2·2	+1·9	+1·4	—0·4	—2·2	—2·9	—3·6
May.	+0·9	+0·9	+0·8	+0·9	+1·3	+1·7	+1·7	+1·1	—0·6	—1·9	—2·6	—2·7
June.	+1·6	+0·9	+0·5	+0·2	0·0	+0·4	+0·4	—0·2	—1·9	—4·7	—5·3	4·7
July.	+2·8	+1·9	+1·1	+0·5	+0·3	+0·2	+0·4	—0·3	—3·3	—8·4	—8·4	7·8
Aug.	+2·6	+1·8	+1·2	+0·7	+0·7	+0·9	+0·9	—0·6	—4·4	—9·2	—9·1	—7·8
Sept.	+1·1	+0·7	+0·9	+0·5	+0·6	+1·2	+1·3	+0·8	—1·0	—5·5	—5·6	—5·4
Oct.	+1·5	+1·2	+1·0	+0·6	+0·4	+0·7	+0·7	+0·6	—0·7	—2·5	—3·4	—3·7

The zero correction necessarily precludes any investigation respecting the annual inequality and the annual change due to the secular variation. For the introduction of any absolute values we have to refer to Part I of the results of the Los Angeles observations,* which contains, in tabular form, the monthly mean values of the dip and of the vertical force throughout the series.

The solar-diurnal variation of the vertical force.—For the purpose of exhibiting the solar-diurnal variation of the vertical component of the magnetic intensity in a form quite suitable for comparison with the

*Appendix No. 8, Report for 1890.

vertical force at Los Angeles, Cal.

vertical force expressed in scale divisions.]

than the diurnal average.

Local mean time.												1 scale division in parts of V.
13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	
−3·3	−2·3	−1·4	−0·7	+0·4	+0·9	+1·5	+1·5	+2·1	+1·7	+2·0	+1·8	·000171
−4·1	−3·7	−2·9	−2·0	−1·0	−0·3	+0·1	+0·5	+1·0	+1·1	+1·1	+1·4	165
−3·0	−2·4	−1·9	−1·3	−0·9	−0·6	−0·2	−0·1	+0·3	+0·7	+0·9	+1·1	159
−2·9	−2·5	−1·1	−0·7	+0·1	+0·9	+0·9	+1·2	+1·6	+1·4	+1·7	+1·5	153
−3·2	−2·1	−0·9	+0·1	+0·9	+1·4	+1·5	+1·3	+1·1	+0·8	+0·5	+0·5	147
−2·4	−1·7	−0·8	+0·3	+1·0	+1·6	+1·6	+1·1	+0·8	+0·6	+0·2	+0·1	141
−1·6	−1·2	−0·5	+0·1	+0·4	+0·6	+1·0	+0·8	+0·6	+0·7	+0·5	+0·3	135
−1·8	−1·6	−1·0	−0·2	+0·2	+0·7	+0·8	+0·5	+0·2	+0·4	+0·2	+0·4	129
−2·6	−2·1	−1·4	−0·8	−0·3	+0·1	+1·5	+0·9	+0·7	+0·7	+0·9	+0·8	123
−2·5	−2·9	−3·0	−3·2	−2·9	−2·9	−2·6	−1·9	−1·1	−0·3	+0·5	+1·1	116
−3·1	−2·7	−2·4	−2·5	−2·4	−2·1	−0·9	−0·8	−0·4	+0·1	+0·6	+1·2	112
−2·1	−2·0	−2·1	−2·1	−2·1	−1·5	−1·4	−0·9	−0·6	−0·1	+0·3	+0·7	108
−3·5	−3·7	−3·6	−3·5	−3·1	−1·7	−1·3	−0·9	−0·5	+0·1	+0·8	+1·2	104
−3·4	−3·5	−3·5	−3·2	−2·5	−1·5	−1·4	−0·9	−0·6	0·0	+0·5	+0·8	100
−4·5	−3·6	−2·4	−1·2	−0·2	+1·3	+1·0	+0·7	+0·8	+0·9	+0·9	+1·3	100
−3·0	−3·2	−2·7	−1·7	−1·1	+1·0	+0·8	+0·7	+0·6	+0·7	+0·6	+0·7	104
−4·5	−3·5	−2·4	−1·4	−0·8	+0·8	+1·3	+1·1	+1·1	+1·3	+1·2	+1·4	108
−3·5	−3·4	−2·5	−1·9	−1·9	+0·6	+0·8	+0·7	+1·0	+1·3	+1·4	+1·5	112
−2·8	−2·5	−2·0	−1·1	−0·6	+0·8	+0·8	+0·8	+1·1	+1·2	+1·5	+1·6	091
−4·0	−3·3	−2·0	−0·5	+0·8	+3·4	+4·1	+4·0	+3·2	+2·8	+2·2	+1·3	089
−6·4	−4·9	−2·9	−0·8	+1·8	+5·7	+6·3	+6·0	+5·1	+4·3	+3·9	+2·6	086
−6·2	−4·4	−2·1	+0·2	+3·2	+5·9	+5·8	+5·1	+4·6	+4·0	+3·3	+2·6	082
−4·5	−2·8	−1·4	−0·2	+0·7	+3·0	+3·2	+3·2	+2·8	+2·3	+2·3	+1·8	080
−3·2	−2·1	−1·1	−0·4	+0·6	+2·0	+1·4	+1·4	+1·4	+1·4	+1·2	+1·1	095

like variation at other stations, the scale divisions were converted into values expressed in parts of the force as shown in the following table, which also contains mean values for each hour, separately for the first and for the second year as well as for the mean of both years, and furthermore two sets of hourly mean values, one for the time (summer half year) when the sun is in north declination, the other for the time (winter half year) when in south declination. To the tabular values $\frac{\delta V}{V}$ is added a column containing the observed values of V, which latter will be needed to express the diurnal variation in absolute measure (or dynes).

Monthly means of the diurnal variation of

0'000 [fourth and fifth

[Local mean time.]

Year and month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1882.												
Nov.	+17	+15	+12	+09	+09	+10	+10	+19	+07	—48	—60	—65
Dec.	+33	+33	+33	+31	+33	+31	+30	+33	+25	—21	—48	—61
1883.												
Jan.	+25	+22	+22	+38	+21	+19	+24	+22	+22	—02	—32	—49
Feb.	+12	+14	+08	+05	+02	+06	+11	+14	00	—21	—35	—40
Mar.	+15	+15	+10	+09	+06	+13	+16	+16	+02	—28	—46	—50
Apr.	+08	+07	+03	+04	+06	+11	+18	+20	—03	—25	—41	—37
May.	+12	+08	+08	+05	+09	+14	+16	+11	—04	—30	—34	—26
June.	+10	+10	+06	+12	+13	+17	+22	+17	+01	—24	—34	—28
July.	+12	+10	+09	+10	+16	+18	+25	+21	+09	—32	—38	—37
Aug.	+20	+23	+27	+31	+36	+43	+41	+37	+26	—01	—10	—21
Sept.	+20	+22	+22	+26	+29	+34	+36	+36	+17	—13	—22	—31
Oct.	+16	+17	+16	+20	+20	+25	+28	+27	+16	—06	—16	—22
Nov.	+20	+20	+23	+26	+28	+34	+42	+40	+27	—07	—22	—32
Dec.	+15	+18	+19	+24	+26	+30	+33	+36	+33	—00	—17	—24
1884.												
Jan.	+10	+13	+12	+13	+15	+17	+22	+25	+14	—10	—38	—49
Feb.	+08	+09	+09	+03	+08	+12	+14	+20	+12	+05	—12	—26
Mar.	+16	+15	+14	+13	+15	+22	+27	+28	+10	—20	—42	—52
Apr.	+19	+19	+19	+17	+21	+25	+21	+16	—04	—25	—32	—40
May.	+08	+08	+07	+08	+12	+16	+16	+10	—06	—17	—24	—25
June.	+14	+08	+04	+02	+00	+04	+04	—02	—17	—42	—47	—42
July.	+24	+16	+10	+04	+03	+02	+03	—03	—28	—72	—72	—67
Aug.	+21	+15	+10	+06	+06	+07	+07	—05	—36	—75	—75	—64
Sept.	+09	+06	+07	+04	+05	+10	+10	+06	—08	—44	—45	—43
Oct.	+14	+11	+10	+06	+04	+07	+07	+06	—07	—24	—32	—35
First year, Nov., 1882, to Oct., '83, inclusive.	+168	+164	+147	+150	+166	+201	+230	+227	+097	—211	—346	—389
Second year, Nov., 1883, to Oct., '84, inclusive.	+149	+132	+120	+109	+119	+154	+171	+147	—008	—276	—382	—416
Mean of two years.	+158	+148	+133	+129	+142	+177	+200	+187	+044	—243	—364	—402
Sun north of equator Apr. to Sept.	+149	+127	+110	+107	+129	+166	+182	+137	—045	—235	—395	—384
Sun south of equator Oct. to Mar.	+168	+169	+157	+151	+155	+189	+219	+238	+134	—152	—333	—421

the vertical force, expressed in parts of V.

decimal places in table].

[Local mean time.]

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	V.
—56	—39	—24	—12	+07	+15	+26	+26	+36	+29	+34	+31	0·4629
—68	—61	—48	—33	—16	—05	+02	+08	+16	+18	+18	+23	38
—48	—38	—30	—21	—14	—10	—03	—02	+05	+11	+14	+18	35
—44	—38	—17	—11	+02	+14	+14	—18	+24	+21	+26	+23	34
—47	—31	—13	+02	+13	+21	+22	+19	+16	+12	—07	+07	42
—34	—24	—11	+04	+14	+23	+23	+16	—11	+08	+03	+01	36
—22	—16	—07	+01	+05	+08	+01	—11	+08	+09	—07	+04	37
—23	—21	—13	—03	+03	—09	—10	+06	+03	—05	—03	+05	40
—32	—26	—17	—10	—04	+01	—18	+11	—09	—09	—11	—10	45
—29	—34	—35	—37	—34	—34	—30	—22	—13	—04	—06	+13	41
—35	—30	—27	—28	—27	—24	—10	—09	—04	+01	—07	+13	36
—23	—22	—23	—23	—23	—16	—15	—10	—06	—01	+03	+08	38
—36	—38	—37	—36	—32	—18	—14	—09	—05	+01	—08	+12	37
—34	—35	—35	—32	—25	—15	—14	—09	—06	00	+05	—08	40
—45	—36	—24	—12	—02	+13	+10	+07	+08	+09	+09	+13	29
—31	—33	—28	—18	—11	+10	—08	+07	+06	—07	+06	—07	30
—49	—38	—26	—15	—09	+09	—14	+12	—12	—14	+13	+15	27
—39	—38	—28	—21	—11	+07	—09	+08	+11	—15	+16	+17	37
—26	—23	—18	—10	—06	+07	—07	+07	—10	—11	—14	+15	36
—36	—29	—18	—04	+07	+30	+36	+36	+28	—25	+20	+12	37
—55	—42	—25	—07	+16	+49	+54	+52	+44	+37	+34	+22	37
—51	—36	—17	+02	+26	+48	+48	+42	+38	+33	+27	+21	22
—36	—22	—11	—02	+06	+24	+26	+26	+22	+18	+18	+14	29
—30	—20	—10	—04	+06	+19	+13	+13	+13	+13	+11	+10	36
—383	—316	—220	—141	—062	+002	+048	+061	+087	+099	+116	+130	0·4638
—389	—326	—232	—133	—030	+153	+165	+159	+152	+153	+151	+140	0·4633
—386	—321	—226	—137	—046	+077	+106	+110	+120	+126	+133	+135	0·4635
—347	—284	—189	—095	—004	+125	+160	+152	+139	+140	+136	+123	0·4636
—426	—358	—263	—179	—088	+032	+052	+068	+100	+113	+130	+146	0·4635

Units of the 6th place of decimals.

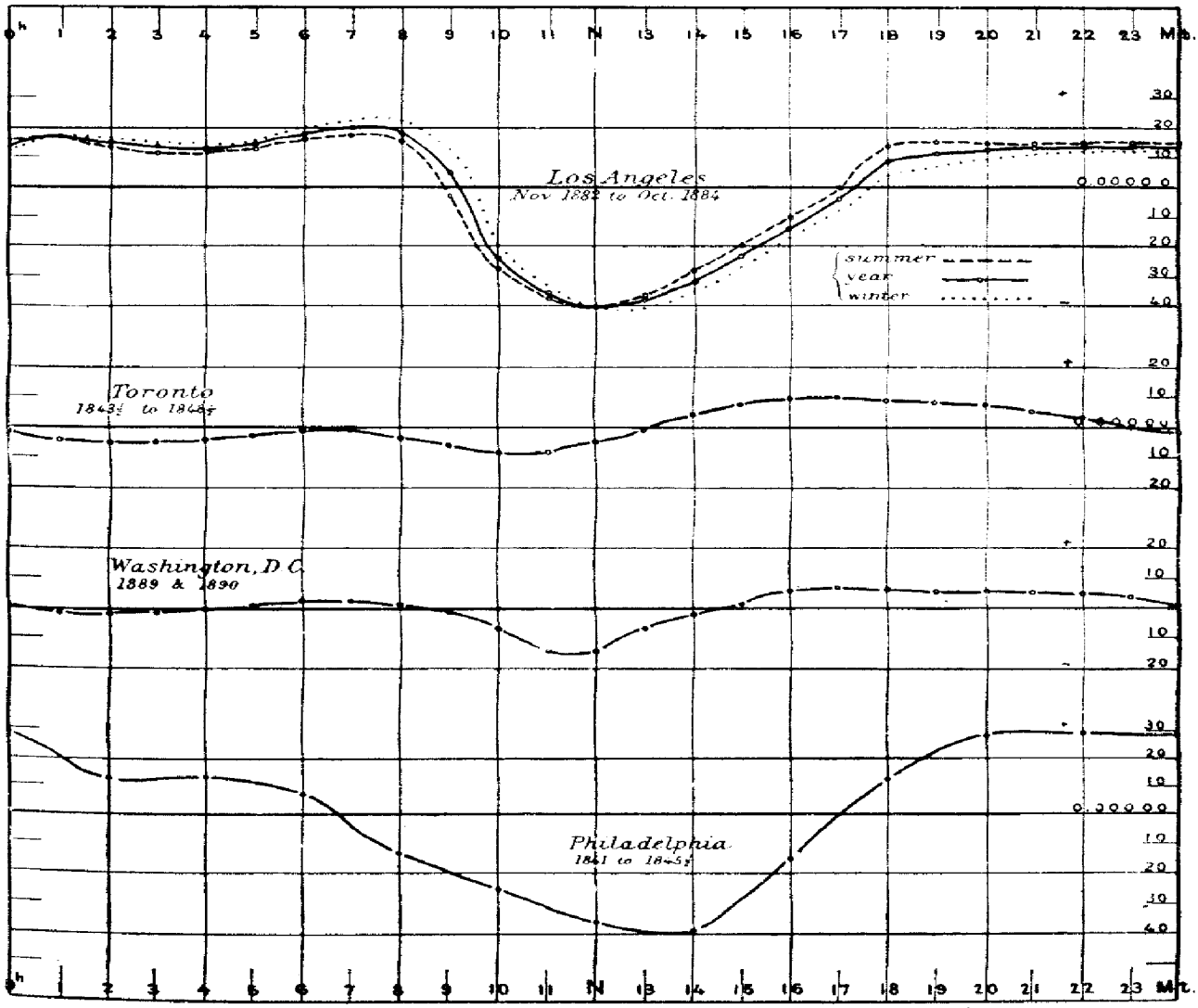
In general the diurnal variation of the magnetic force is characterized by two extreme values, one occurring at some morning hour, the other at some afternoon hour; these include the principal minimum, which takes place an hour or two from noon, either before or after. The result from two years of observations (November, 1882, to October, 1884, inclusive) shows that the maximum force is reached about 7 o'clock a. m., the minimum about noon, and a second extreme high value is reached about an hour after midnight, though the rapid rise of the intensity appears checked about 7 o'clock p. m. At other places the afternoon extreme is more intense than the morning extreme, but the two values differ little in intensity.

In the following table I have brought together for comparison the hourly values of $\frac{\delta V}{V}$ for the annual average at four North American stations. The values for Toronto, Canada, are those given by Sabine and refer to the five years from 1843½ to 1848½. The Philadelphia series is taken from the Coast Survey Report for 1863, appendices 19 and 20, changing, however, the sign of the variation which appeared to me demanded;* the values refer to the years 1841 to 1845½, and extend over four and one-half years; they are partly hourly, partly bi-hourly; only the latter are given here. The Washington series comprises the two years 1889–'90, and is taken from the results published by the U. S. Naval Observatory (Washington, 1891).

* When making some preliminary comparisons relating to the diurnal variation last July I noticed that at Philadelphia, when contrasted with other places, the occurrences of maxima and minima appeared to be interchanged or else that the signs were not correctly interpreted; this led me to a closer examination of the record and of the results. The observations were published in 1847 in four octavo volumes, as a public document, by order of the United States Senate; the reduction and discussion of these observations by Dr. A. D. Bache (and in which he was assisted by the writer) are given in the Smithsonian contributions to knowledge and reprinted in the Coast Survey Reports for the years 1859–'60–'62–'63–'64. In volume II, p. 1818, we find "*increasing* scale readings correspond to *decreasing* force," and the same information is engraved at the bottom of the plates of diagrams which exhibit the changes of the vertical force from January, 1841, to the close of the series, for which see volume IV (printed in 1845); further, the diagrams themselves show the force to be near its *maximum* about noon. In volume III, p. 3128, we again meet the statement that increasing scale readings for vertical-force magnetometer correspond to decrease of force; and the discussion is in accord with the record. Now it would appear that these statements in the original record are the reverse of what they should be, since about noon we ought to have the smaller values, *i. e.*, the minimum and not the maximum force should be indicated. This reversal by inadvertence, did not further obtrude itself on the attention of the computer at this early date, when hardly more than a single station existed with which to compare results, the non-accord of the variations deduced for Philadelphia and Toronto being ascribed to their different geographical positions.

With the signs reversed we obtain a good correspondence respecting the prevalence of a weaker vertical force about the time of midday than the average force throughout the twenty-four hours; this holds for Philadelphia, Toronto, Dublin, Washington, Los Angeles, and other places. The required change of sign also affects more or less the reductions and conclusions presented in the Coast Survey Report for 1863, appendices 19 and 20, and Report for 1864, appendices 16, 17, and 18.

Diurnal Variation of the Vertical Component of the magnetic force at Los Angeles, Cal. 1883 - 1884.



(N.B. The scale values are given in parts of the force)

Comparative table of the solar-diurnal variation of the vertical component of the magnetic force.

Values of $\frac{\delta V}{V}$ = 0·000 (with 4th, 5th, and 6th decimal places in table below).

Hour.	Los Angeles, Cal., Nov., 1882, to Oct., 1884.	Toronto, Canada, 1843 to 1845.	Washington, D. C., 1889 and 1890.	Philadelphia, Pa., 1841 to 1845.
Midnight	+135	—019	+007	+287
1	+158	—044	—010	----
2	+148	—054	—024	+115
3	+133	—050	—024	----
4	+129	—048	—014	+131
5	+142	—030	+007	----
6	+177	—012	+028	+068
7	+200	—014	+028	----
8	+187	—029	+017	—129
9	+044	—065	—021	----
10	—243	—091	—078	—257
11	—364	—087	—131	----
Noon	—402	—055	—131	—366
13	—386	—013	—072	----
14	—321	+040	—028	—377
15	—226	+076	+024	----
16	—137	+091	+059	—165
17	—046	+100	+069	----
18	+077	+092	+066	+121
19	+106	+084	+055	----
20	+110	+067	+053	+282
21	+120	+048	+052	----
22	+126	+028	+045	+290
23	+133	+005	+028	----
Midnight	+135	—019	+007	+287
V	0·4635	0·6201	0·5798	0·5912 of a dyne.
θ	59° 30′·2	75° 15′·1	71° 05′·2	71° 58′·7

N. B.—At Washington the observations were made eight minutes earlier and at Philadelphia twenty-one minutes later than the hour indicated in the table.

A + sign to a tabular number indicates excess, a — sign defect of intensity as compared with the average value.

Diurnal range at—

Los Angeles,	0·000 560 in parts of V.
Toronto,	191
Washington,	200
Philadelphia,	667

As partly explaining the relatively large amplitude at Los Angeles, it should be remembered that the epoch of observation at that place coincides with a sun-spot maximum (in 1883), whereas at Toronto it coincides with a sun-spot minimum (in 1843), and the same holds for Washington with minimum in 1889; yet we find Philadelphia, to be an exception, viz, a large amplitude though the sun-spot minimum occurred in 1843.*

The diurnal variation is shown graphically on the accompanying illustration, No. 28.

* Possibly the very heavy magnet employed at Girard College, Philadelphia, may furnish a clew to the large amplitude due to its inertia. The dimensions were: length, 2 feet 1 inch; width, 2 inches; thickness, ½ inch. The magnet at Los Angeles was but 5·4 inches in length, 0·8 broad, and 0·1 thick. The Washington and Toronto magnets also were small.

THE DIURNAL VARIATION OF THE INCLINATION AND TOTAL FORCE.

By a combination of the variations of the horizontal and vertical components of the magnetic force at Los Angeles for variations in the inclination and total force, we have the relations

$$\delta\theta = \sin\theta \cos\theta \left(\frac{\delta V}{V} - \frac{\delta H}{H}\right)$$

$$\frac{\delta F}{F} = \sin^2\theta \frac{\delta V}{V} + \cos^2\theta \frac{\delta H}{H}$$

From part III of the discussion of the observations at Los Angeles in Coast and Geodetic Survey Report for 1891, Part II, Appendix No. 4, pp. 68 to 73, we can get the values of δH in scale divisions, hence also the values $\frac{\delta H}{H}$, excepting for November, 1882, which had to be taken from the MS. computation. All disturbances are included, except for November, 1882, when the disturbances were so excessive as almost to obliterate the regular diurnal variation.

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Diurnal variation of the horizontal component of the magnetic intensity

November, 1882,

[·000 with fourth and fifth decimal places in table.

Year and month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1882.												
Nov.	+09	+13	+07	+14	+27	+24	+20	—14	—22	—56	— 57	—58
Dec.	—01	+02	+13	+14	+21	+26	+39	+43	+35	+02	— 39	—52
1883.												
Jan.	+10	+09	+17	+24	+26	+34	+37	+26	+13	—29	— 72	—63
Feb.	+17	+15	+24	+31	+37	+45	+50	+38	+27	—10	— 20	—23
Mar.	+07	+14	+21	+27	+29	+29	+16	+07	—04	—08	— 19	—27
Apr.	+37	+31	+25	+31	+27	+26	+22	—02	—26	—39	— 39	—33
May.	+14	+12	+12	+11	+19	+22	+11	—19	—20	—07	— 03	+10
June.	+20	+32	+15	+21	+21	+19	—03	—25	—26	00	+ 20	+25
July.	+26	+23	+19	+23	+27	+33	+17	+01	—14	—05	+ 03	+08
Aug.	+16	+25	+24	+24	+22	+26	—01	—40	—39	—25	— 21	—16
Sept.	+20	+22	+22	+34	+27	+37	+04	—32	—43	—39	— 38	—27
Oct.	+22	+27	+37	+39	+37	+29	+05	—39	—52	—54	— 52	—45
Nov.	+05	+13	+20	+38	+40	+34	+31	+07	—19	—41	— 46	—28
Dec.	+01	+08	+20	+22	+33	+41	+44	+31	+09	—33	— 63	—66
1884.												
Jan.	+08	+23	+27	+24	+35	+38	+40	+31	—02	—57	—104	—95
Feb.	+08	+12	+22	+28	+32	+37	+49	+60	+54	+28	— 16	—54
Mar.	+17	+34	+34	+39	+34	+37	+27	—02	—35	—39	— 41	—43
Apr.	+19	+20	+32	+34	+31	+37	+08	—08	—16	—15	— 35	—36
May.	+12	+13	+07	+05	+14	+14	—01	—15	—04	+11	+ 08	+05
June.	—01	+11	+07	+13	+15	+22	+08	00	+01	+13	+ 10	+20
July.	+04	+14	+13	+17	+24	+24	+07	—21	—16	00	+ 16	+27
Aug.	+28	+21	+27	+29	+27	+23	—05	—34	—25	—24	— 24	—13
Sept.	+21	+17	+28	+33	+35	+25	—03	—35	—44	—40	— 26	—13
Oct.	+13	+22	+26	+33	+34	+29	+20	—09	—25	—29	— 23	—17
Average, first year.	+16·4	+18·8	+19·7	+24·4	+26·7	+29·2	+18·1	—04·7	—14·2	—22·5	—28·1	—25·1
Average, second year.	+11·2	+17·3	+21·9	+26·2	+29·5	+30·1	+18·7	+00·4	—10·1	—18·8	—28·7	—26·1
Mean of 2 years.	+13·8	+18·1	+20·8	+25·3	+28·1	+29·6	+18·4	—02·1	— 12·2	—20·7	—28·4	—25·6
Apr. to Sept., inclusive. ⊙ north of equator.	+18·0	+20·1	+19·2	+22·9	+24·1	+25·7	+05·3	—19·2	—22·7	—14·2	—10·8	—03·6
Oct. to Mar., inclusive. ⊙ south of equator.	+09·7	+16·0	+22·3	+27·7	+32·1	+33·6	+31·5	+14·9	—01·8	—27·2	—46·0	—47·6

at Los Angeles, Cal., from two years of observation. Values of $\frac{\delta H}{H}$ *from to October, 1884.*

$\left\{\begin{matrix}+\\-\end{matrix}\right.$ when $\left\{\begin{matrix}>\\<\end{matrix}\right.$ than daily average.]

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Midnight.
—52	—37	—17	+02	+32	+39	+12	+36	+26	+14	00	+31
—49	—24	—08	—01	+09	+02	—02	—19	—12	—03	—05	+04
—46	—10	+04	+09	+04	+09	+10	+02	—04	—04	+05	+02
—37	—28	—17	—28	—27	—21	—25	—22	—17	—15	+04	+01
—29	—13	—03	+08	+01	—16	—13	—22	—12	—04	—02	+13
—28	—34	—15	—07	—15	—17	—13	+02	+11	+11	+17	+19
+11	+12	+03	—09	—21	—28	—21	—16	—14	—02	—01	+19
+13	+12	+01	—35	—46	—34	—36	—13	—10	+09	+11	+11
+09	+05	—21	—21	—34	—28	—25	—24	—16	—07	+02	+02
—13	—05	—03	—01	—15	—08	—07	+04	+10	+08	+16	+20
—09	+05	+19	+15	+01	—05	+04	+04	—08	—16	00	+02
—17	—04	—07	00	—04	+04	+11	+10	+15	+12	+09	+21
—13	—04	—01	—09	—03	+03	—08	—09	—09	—03	+03	—04
—43	—21	—02	+05	+10	+08	+05	+02	00	00	—05	+03
—64	—28	+05	+20	+15	+16	+17	+15	+08	+07	+08	+14
—59	—56	—41	—24	—19	—05	—04	—10	—21	—17	—12	—02
—34	—11	+01	—01	—07	—19	—14	—05	—07	+08	+04	+13
—23	—11	—01	—05	—21	—22	—15	—11	+05	+08	+03	+13
+11	+14	—05	—16	—29	—27	—21	—13	—03	—02	+08	+13
+21	+15	—01	—20	—33	—33	—21	—19	—12	—07	—03	+01
+16	+05	—05	—26	—41	—34	—21	—14	—11	—10	+09	+11
—11	—15	—15	—20	—19	+02	+04	+07	+10	+12	+12	+13
—03	+03	—05	—07	—08	—04	—10	—01	+08	—01	+10	+09
—10	—02	+03	+02	—04	—04	—15	—21	—17	—10	—08	+02
—20·6	—10·1	—05·3	—05·7	—09·6	—08·6	—08·8	—04·8	—02·6	+00·2	+04·7	+12·1
—17·7	—09·2	—05·6	—08·4	—13·2	—09·9	—08·6	—06·6	—04·1	—01·2	+02·4	+07·2
—19·1	—09·7	—05·4	—07·0	—11·4	—09·2	—08·7	—05·7	—03·4	—00·5	+03·6	+09·6
—00·5	+00·5	—04·0	—12·7	—23·4	—19·8	—15·2	—08·0	—02·5	+00·2	+07·0	+11·1
—37·7	—19·8	—06·9	—01·3	+00·6	+01·3	—02·2	—03·6	—04·2	—01·2	+00·1	+08·2

Diurnal variation of the inclination and of the total force at Los Angeles, Cal.

We have for the first year $\theta = 59° 30'.8$ and for the second year $\theta = 59° 29'.6$.

Hour.	$\frac{\delta V}{V}$	$\frac{\delta H}{H}$	$\delta\theta$	$\frac{\delta F}{F}$
			″	
1	+0.000158	+0.000139	+ 1.7	0.000153
2	+ 148	+ 182	— 3.1	157
3	+ 133	+ 209	— 6.9	+ 153
4	+ 129	+ 254	—11.3	+ 161
5	+ 142	+ 282	—12.6	+ 177
6	+ 177	+ 297	—10.8	+ 207
7	+ 200	+ 185	+ 1.4	+ 196
8	+ 187	— 020	+ 18.7	+ 134
9	+ 044	— 121	+ 14.9	+ 002
10	— 243	— 206	— 3.3	— 233
11	— 364	— 283	— 7.3	— 343
Noon	— 402	— 255	—13.3	— 364
13	— 386	— 190	—17.7	— 335
14	— 321	— 096	—20.3	— 263
15	— 226	— 053	—15.6	— 182
16	— 137	— 069	— 6.1	— 120
17	— 046	— 113	+ 6.0	— 063
18	+ 077	— 091	+15.2	— 034
19	+ 106	— 086	+17.3	+ 057
20	+ 110	— 056	+15.0	+ 068
21	+ 120	— 033	+13.8	+ 081
22	+ 126	— 004	+11.7	+ 093
23	+ 133	+ 037	+ 8.7	+ 109
Midnight	+ 135	+ 097	+ 3.4	+ 125

The extreme deviation of the dip from the mean value during the diurnal variation only reaches $\frac{1}{5}$ of a minute; hence, under ordinary circumstances, when observing the dip directly, no notice need be taken of this diurnal variation, and the same remark also refers to the ordinary observations for the horizontal (and total) intensity. In the latter case the extreme difference from the normal values is but one three-thousandth part of the respective force, a degree of accuracy which can not be assured in field work.

Comparative table of the solar-diurnal variation in the dip and total force at four North American stations.

Hour of local time.	Values of $\delta\theta$. Los Angeles, Nov., 1882, to Oct., 1884.	Toronto, 1843½ to 1848½.	Washington, 1889 and 1890.	Philadelphia, 1841 to 1845½.	Values of $\frac{\delta F}{F}$ Los Angeles (2 years).	Toronto (5 years)	Washington (2 years).	Philadelphia (4½ years).	Hour of local time.
	″	″	″	″					
0	+ 3·4	− 1·5	− 6	+15·8	+0·000125	−0·000017	+0·000016	+0·000262	0
1	+ 1·7	− 2·8	− 8	----	+ 153	− 040	+ 005	----	1
2	− 3·1	− 4·8	−10	+ 0·7	+ 157	− 048	− 008	+ 114	2
3	− 6·9	− 3·5	−11	----	+ 153	− 046	− 005	----	3
4	−11·3	− 7·5	−12	− 4·3	+ 161	− 039	+ 005	+ 137	4
5	−12·6	− 8·7	−14	----	+ 177	− 019	+ 031	----	5
6	−10·8	−10·3	−16	−14·7	+ 207	+ 001	+ 054	+ 090	6
7	+ 1·4	− 4·8	−10	----	+ 196	− 008	+ 044	----	7
8	+18·7	+ 9·7	+ 8	− 4·5	+ 134	− 041	+ 008	− 122	8
9	+14·9	+26·2	+30	----	+ 002	− 098	− 070	----	9
10	− 3·3	+39·5	+42	+12·1	− 233	− 142	− 145	− 276	10
11	− 7·3	+43·4	+38	----	− 343	− 143	− 194	----	11
Noon.	−13·3	+32·8	+25	− 1·7	− 364	− 097	− 173	− 363	Noon.
13	−17·7	+14·0	+ 8	----	− 335	− 031	− 086	----	13
14	−20·3	− 4·7	− 7	−29·0	− 263	+ 046	− 015	− 331	14
15	−15·6	−20·0	−12	----	− 182	+ 102	+ 044	----	15
16	− 6·1	−26·2	−12	−21·1	− 120	+ 125	+ 080	− 131	16
17	+ 6·0	−24·4	−10	----	− 063	+ 131	+ 085	----	17
18	+15·2	−17·7	− 4	+ 7·0	+ 034	+ 115	+ 072	+ 110	18
19	+17·3	−10·9	− 2	----	+ 057	+ 098	+ 059	----	19
20	+15·0	− 5·4	− 2	+19·9	+ 068	+ 074	+ 062	+ 211	20
21	+13·8	− 4·3	− 2	----	+ 081	+ 053	+ 055	----	21
22	+11·7	− 3·5	− 2	+19·3	+ 093	+ 032	+ 051	+ 259	22
23	+ 8·7	− 2·8	− 4	----	+ 100	+ 009	+ 036	----	23
Midnight.	+ 3·4	− 1·5	− 6	+15·8	+ 125	− 017	+ 016	+ 262	Midnight.
				F =	0·5379	0·6412	0·6129	0·6217	of a dyne.

N. B.—At Washington the tabular numbers refer to epochs 8^m earlier, at Philadelphia to epochs 21^m later, than the hours indicated.

It is evident from a perusal of the above table that a much greater number of stations and longer series will be needed before we can be justified in attempting any generalization for North America respecting the diurnal variations of the dip and of the total force.

Disturbances of the vertical component of the force.—The great magnetic storm* of November 17 to 20, 1882, falls within the limits of the better part of the vertical-force record and demands attention in order to complete our information of it as well as of the magnitude of the disturbances in the dip and in total magnetic force. The storm commenced suddenly a short time before 2½ o'clock a. m., November 17, and continued until about 7 o'clock p. m., November 20, thus lasting three days and sixteen and one-half hours. The agitation of the magnets was continuous during this time, but on each day it apparently abated somewhat in violence between the hours of 10 a. m. and 4 p. m. The storm set in according to the declination trace at $2^h\ 25^m$, *north* end of magnet deflected to the *west;* amount, 40′; according to the horizontal-force trace at $2^h\ 26^m$, force *increasing*, 26 divisions (or 0·00280 H) + an unknown distance, through the failing of the trace; according to the vertical-force trace at $2^h\ 26^m$, force *increasing*, 12 divisions, or 0·00205 parts of V. The extreme position of each magnet was reached before 3^h, after which the direction of the disturbance was reversed. During this time the dip was certainly affected as much as 1′ 8″ (less than the normal); how much more is not known for want of the horizontal-force trace. The total force had increased by 0·00224 F. and beyond. The disturbance was probably at its height November 17, between 18^h and 20^h, when the horizontal force was below and the vertical force above their normal values, while the north end of the declination magnet was deflected to the east.

This remarkable storm was accompanied by powerful and sympathetic (in time and energy) earth currents, which interfered with or interrupted telegraphic communication; besides, there was on the evening and night of November 17 an auroral display of unusual magnificence. On the same day there was visible to the (protected) naked eye an enormous sunspot,† which at the time of the first outbreak of the magnetic disturbance was yet to the *east* of the solar central meridian. As at Los Angeles, the outbreak of the storm was sudden at Greenwich‡ and happened at November 16, $22^h\ 15^m$ G. M. T., or at November 17, $10^h\ 15^m$ a. m.; subtracting the difference of longitude, 7^h

* See illustration No. 29, which is a copy of the photographic traces of November 17, 18, 19, and 20, 1882, but reduced (the same as the corresponding plate showing the disturbances in the horizontal force, part III) by photography in the proportion of 44 to 35·2.

† 194 seconds of arc in length and 134 seconds of arc in breadth; this spot is said to have been the largest yet photographed at Greenwich; it was near the central solar meridian on November 18. In connection with this it may be noted that the year 1883 was one of maximum spot production.

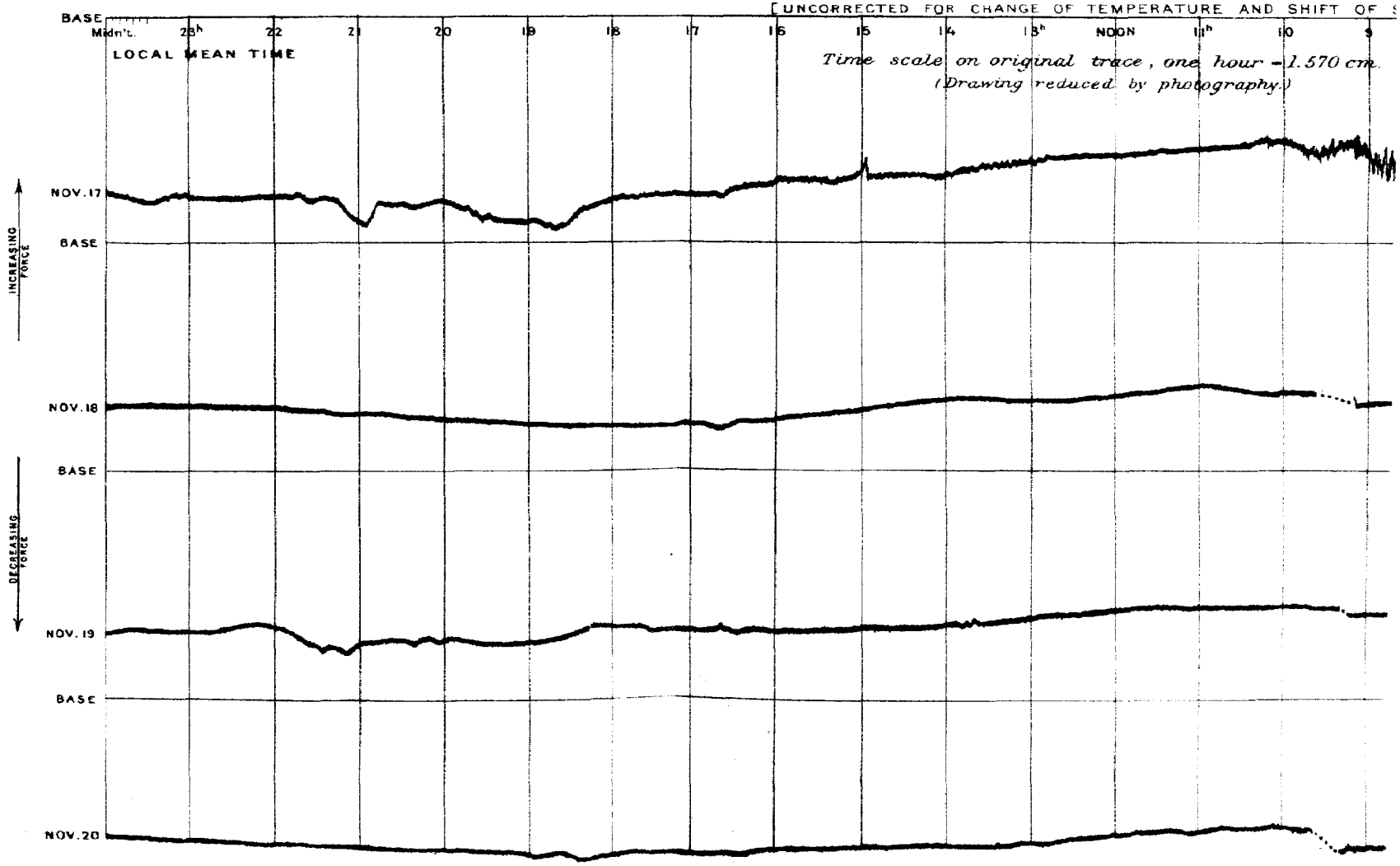
‡ Nature, vol. XXVII, p. 82 and fol.

U.S. COAST AND GEODETIC SURVEY.

MAGNETIC OBSERVATORY, LOS ANGELES.

VERTICAL FORCE TRACES, NOV. 17-2

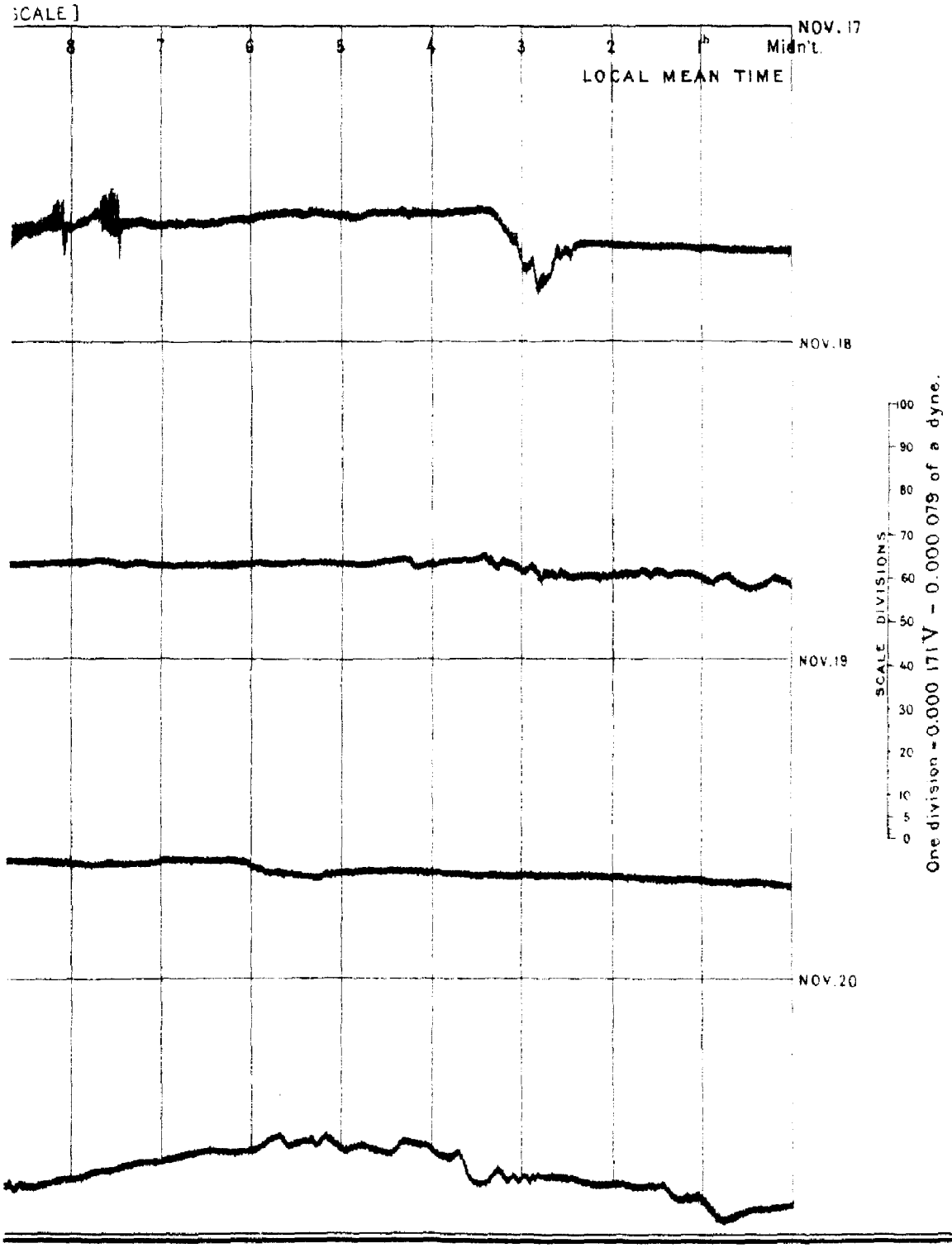
[UNCORRECTED FOR CHANGE OF TEMPERATURE AND SHIFT OF



Original drawing reduced in proportion of 44.0 to 35.3

;AL.

.0. 1882


SCALE]
8
7
6
5
4
3
2
h
Midn't.
LOCAL MEAN TIME
NOV. 17
NOV. 18
NOV. 19
NOV. 20
SCALE DIVISIONS
100
90
80
70
60
50
40
30
20
10
5
0
One division = 0.000 171 V - 0.000 079 of a dyne.

Disturbances of the vertical component of the magnetic intensity at Los Angeles.

53m, we have the corresponding Los Angeles time, 2h 22m, which is but 3m or 4m earlier than the time recorded on the Los Angeles traces.

A magnetic storm of but slightly less intensity occurred on June 24 and 25, 1885. It was characterized by two sudden outbreaks about 5¼ hours apart on June 24. Photographic traces of this remarkable storm were obtained at seventeen different observatories situated in North America, Europe, Asia, and Australia.* According to the Los Angeles traces, the outbreaks occurred as follows:

	First impulse (about 2½h p. m.).	Second impulse (about 7¾h p. m.).
	h. m.	*h. m.*
Declination trace	14 30, deflection west.	19 45, deflection west.
Horizontal-force trace	14 35, increasing force.	19 45, increasing force.
Vertical-force trace	14 35, decreasing force.	19 50, decreasing force.

The character of the disturbances at these epochs was the same, but the second one was the more intense of the two. I have appended the hourly trace readings of the vertical force (corrected as well as could be done for changes of temperature and reduced to 100 for the mean of the day) for the four days, June 23, 24, 25, 26, 1885.

The uncertainty in the measure of the times for the above two storms I estimate at about two minutes.

* Nature, July 9, 1881, p. 237.

DIFFERENTIAL MEASURES OF THE VERTICAL COMPONENT OF

Hourly trace readings of vertical-force magnetometer from
ences of temperature and

[Tabular values within brackets are interpolated, they follow in general the law of diurnal variation perfection in the monthly means

NOVEMBER, 1882.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	89	89	90	91	91	91	91	91	90	91	90	90
2	96	96	96	96	96	97	97	97	95	90	89	89
3	98	98	98	98	98	98	97	97	96	92	92	91
4	98	98	98	98	98	98	99	100	98	95	95	95
5	[90]	[90]	[90]	[90]	[91]	[91]	[92]	[91]	[90]	90	89	88
6	96	96	96	96	96	97	97	97	94	87	87	86
7	93	93	93	93	93	93	94	93	93	92	91	91
8	94	94	94	94	94	95	95	95	94	95	94	94
9	99	99	99	99	99	98	98	98	98	97	105	106
10	108	108	107	107	107	107	107	107	107	101	101	101
11	104	103	103	102	102	102	104	103	102	98	97	96
12	97	99	98	98	98	97	98	98	98	93	93	93
13	96	96	96	94	95	93	94	95	94	92	91	91
14	99	100	99	99	99	100	98	99	99	96	94	95
15	100	100	101	100	101	101	101	100	99	97	94	94
16	103	103	103	103	103	102	103	102	102	100	99	97
17	104	104	109	98	99	101	102	105	104	95	96	98
18	107	106	105	104	105	106	108	108	110	103	100	100
19	108	108	108	108	109	107	106	107	107	103	103	102
20	109	106	105	100	99	100	101	107	106	98	98	95
21	99	99	99	99	99	99	99	97	96	93	91	91
22	99	98	99	99	99	99	100	100	99	91	90	89
23	93	93	94	94	94	94	95	94	94	91	90	89
24	94	94	94	94	94	94	94	94	93	92	90	90
25	96	96	96	96	96	96	96	97	94	92	93	94
26	99	99	99	98	98	98	99	98	98	96	95	94
27	100	100	100	100	100	100	100	100	100	96	94	93
28	99	99	99	102	99	99	99	100	99	97	96	96
29	[104]	[104]	[104]	[105]	[105]	[106]	[106]	[106]	[105]	105	103	103
30	108	108	109	108	108	108	108	107	107	107	105	105
Means.	99·3	99·2	99·0	98·8	98·8	98·9	98·9	99·4	98·7	95·5	94·8	94·5
Diurnal variation.	+1·0	+0·9	+0·7	+0·5	+0·5	+0·6	+0·6	+1·1	+0·4	−2·8	−3·5	−3·8

THE MAGNETIC INTENSITY OBSERVED AT LOS ANGELES, CAL.

November 1, 1882, to October 31, 1884. Corrected for differ-
for shift of zero of scale.

and any then apparent progressive change; they were introduced in order to prevent any abrupt im-
no further use is to be made of them.]

NOVEMBER, 1882.

Value of 1 division of scale ·000 171 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
91	92	93	93	93	95	95	95	95	95	95	95	92·1
90	93	95	96	98	99	99	98	99	97	98	98	95·6
91	92	94	95	95	97	97	98	98	98	98	97	96·0
95	95	96	97	98	[98]	[98]	[98]	[99]	[99]	[99]	[99]	[97·5]
89	91	93	94	96	96	96	97	97	97	97	97	[92·6]
88	89	90	91	91	92	93	93	94	93	93	92	92·7
90	91	92	93	94	94	94	95	95	96	95	94	93·1
95	95	96	96	97	97	98	98	98	98	98	98	95·7
107	108	109	110	110	110	109	109	109	109	109	108	104·3
101	102	104	104	103	103	103	103	103	104	104	104	104·8
96	98	98	99	100	99	99	100	100	98	99	97	100·0
92	93	92	93	93	95	96	96	95	97	97	99	95·8
91	92	92	95	94	96	96	97	97	98	98	98	94·6
95	96	96	98	99	98	98	99	100	100	100	101	98·2
94	96	97	97	100	100	101	102	102	102	102	102	99·3
97	98	100	100	103	103	103	102	103	103	103	102	101·5
99	101	97	99	104	106	114	108	116	109	109	107	103·5
101	100	102	105	107	108	108	108	107	107	107	107	105·4
104	104	105	104	103	102	107	107	109	105	108	108	105·9
97	97	97	99	100	101	101	100	100	100	99	99	100·6
93	94	95	97	99	99	98	98	98	97	98	98	96·9
88	89	90	91	92	92	92	92	93	93	93	93	95·0
90	91	92	91	91	91	92	92	92	93	93	93	92·3
90	90	92	92	92	93	94	94	95	95	96	96	93·2
93	94	94	91	96	96	96	98	99	98	99	99	95·6
94	96	96	98	99	99	99	100	100	100	100	100	98·0
93	94	95	95	96	97	98	98	98	98	99	99	97·6
96	98	100	102	103	103	103	103	103	[103]	[103]	[103]	[100·2]
103	103	105	105	106	107	107	107	108	108	107	108	[105·4]
106	107	109	109	109	110	110	110	110	111	112	111	105·5
95·0	96·0	96·9	97·6	98·7	99·2	99·8	99·8	100·4	100·0	100·3	100·1	98·3
−3·3	−2·3	−1·4	−0·7	+0·4	+0·9	+1·5	+1·5	+2·1	+1·7	+2·0	+1·8	

DECEMBER, 1882.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	100	100	101	101	102	101	102	102	101	100	98	98
2	102	102	102	101	101	101	101	101	100	96	93	92
3	99	99	99	100	99	99	99	100	100	99	96	95
4	101	[102]	[102]	[103]	[103]	[104]	[104]	[105]	[106]	106	104	103
5	112	112	112	112	111	111	110	110	109	103	101	99
6	109	109	108	108	108	107	107	107	106	96	95	94
7	101	100	100	101	101	101	101	100	100	96	95	95
8	101	100	100	100	101	100	101	100	100	99	99	95
9	100	100	100	100	100	100	99	99	99	98	98	97
10	101	102	101	101	101	101	101	100	100	97	95	95
11	99	99	99	100	100	99	100	100	99	98	97	95
12	99	99	99	98	98	98	98	98	98	98	97	97
13	101	101	101	101	100	99	99	99	99	98	96	95
14	100	100	99	99	99	99	98	99	99	97	95	93
15	101	102	102	102	101	100	100	100	99	96	94	93
16	102	101	101	101	101	101	101	102	101	102	100	99
17	109	108	108	107	108	107	106	106	105	99	98	97
18	103	103	103	102	102	102	102	103	102	96	95	95
19	100	100	100	100	101	101	101	101	100	98	96	95
20	99	99	99	99	99	100	99	100	99	97	95	95
21	102	100	100	100	101	102	103	103	103	100	98	98
22	101	102	102	102	102	103	102	103	103	102	101	100
23	103	103	104	103	103	103	104	104	103	95	93	93
24	99	99	99	99	99	99	99	99	98	95	93	93
25	99	100	100	100	100	100	101	101	100	98	96	96
26	101	101	101	101	102	103	102	103	102	97	96	95
27	99	99	99	99	99	98	98	99	98	94	93	93
28	98	99	99	99	99	[99]	[98]	[98]	[98]	98	96	95
29	100	100	100	100	100	99	99	99	99	97	96	95
30	101	102	102	102	102	102	101	101	101	99	98	97
31	100	101	101	100	101	101	100	100	100	98	96	96
Means	101·4	101·4	101·4	101·3	101·4	101·3	101·2	101·4	100·9	98·1	96·5	95·7
Diurnal variation	+2·0	+2·0	+2·0	+1·9	+2·0	+1·9	+1·8	+2·0	+1·5	—1·3	—2·9	—3·7

DECEMBER, 1882.

Value of 1 division of scale, ·000165 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
98	98	99	98	100	101	101	101	102	102	102	102	100·4
92	94	95	96	97	97	98	99	99	99	98	99	98·1
95	95	96	97	98	99	99	100	100	99	99	100	98·4
104	104	106	107	110	110	111	111	111	111	111	111	[106·3]
100	102	105	107	110	110	110	111	111	111	110	109	108·3
94	95	97	99	101	101	102	101	101	101	100	100	101·9
93	94	95	95	97	98	100	100	100	99	100	101	98·5
95	95	96	96	97	98	98	99	99	99	100	100	98·7
97	97	97	98	99	99	99	100	100	100	100	100	99·0
95	95	95	96	97	97	98	98	99	99	99	99	98·4
95	95	95	94	95	95	96	96	97	97	97	98	97·3
96	96	96	96	98	98	99	99	99	99	100	100	98·0
94	95	96	98	98	99	99	100	100	100	100	101	98·7
94	95	96	97	98	99	99	100	101	101	101	101	98·3
92	95	96	100	99	100	101	102	102	103	103	104	99·5
100	102	103	105	105	106	106	106	106	106	107	107	103·0
96	98	99	100	101	102	101	101	102	102	102	103	102·7
95	95	97	97	99	99	99	98	99	102	99	99	99·4
94	94	96	97	99	99	99	99	99	99	99	99	98·6
94	93	93	95	96	100	101	101	102	101	102	101	98·3
98	97	98	97	98	99	99	100	101	100	101	101	100·0
98	99	100	100	101	101	101	102	102	102	102	103	101·4
93	93	93	94	94	96	96	97	97	98	98	98	98·3
92	93	92	94	94	95	96	96	97	98	98	98	96·4
94	94	94	95	94	96	96	97	98	99	99	99	97·8
93	92	92	95	95	94	95	95	96	97	97	97	97·6
93	92	92	94	94	94	95	96	97	97	98	97	96·1
95	95	96	97	97	97	97	97	97	98	98	99	[97·5]
95	94	94	94	95	97	96	98	98	100	99	100	97·7
96	96	96	96	97	98	98	100	100	100	100	100	99·4
95	96	96	96	97	98	98	98	99	98	98	99	98·4
95·3	95·7	96·5	97·4	98·4	99·1	99·5	99·9	100·4	100·5	100·5	100·8	99·4
—4·1	—3·7	—2·9	—2·0	—1·0	—0·3	+0·1	+0·5	+1·0	+1·1	+1·1	+1·4	

JANUARY, 1883.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	98	98	98	98	97	97	97	97	97	99	98	96
2	101	102	102	102	102	102	102	102	102	101	100	99
3	101	100	101	101	102	102	102	102	102	100	98	96
4	101	101	101	102	104	102	102	103	103	101	100	98
5	103	103	103	103	103	103	103	104	104	100	98	97
6	103	103	102	102	102	102	102	102	102	100	99	97
7	102	101	101	101	102	101	102	102	103	100	97	97
8	99	99	99	99	99	101	101	101	100	99	98	98
9	103	102	102	102	102	102	103	103	103	101	99	98
10	102	101	101	101	101	102	102	101	101	97	95	95
11	99	99	99	99	99	100	101	101	100	96	94	93
12	99	99	99	99	99	99	99	100	98	96	95	95
13	100	99	99	99	99	99	99	98	98	96	95	94
14	100	101	101	101	101	101	102	102	102	97	95	94
15	99	99	99	99	99	99	99	99	99	99	97	96
16	[103]	[104]	[104]	[104]	[107]	[105]	[106]	[105]	105	105	102	101
17	102	101	102	101	102	102	103	102	102	103	100	97
18	105	104	104	104	104	105	105	106	105	103	100	99
19	104	104	104	104	104	104	104	105	105	106	103	102
20	108	109	109	109	109	110	111	111	113	[111]	110	105
21	107	106	106	107	107	105	106	106	106	101	98	97
22	99	99	98	99	98	99	98	98	98	97	95	94
23	104	104	104	103	103	103	103	103	104	104	102	102
24	[110]	[110]	[109]	[107]	[107]	[107]	[107]	[107]	[107]	106	103	104
25	107	106	107	104	105	104	105	105	105	102	99	98
26	104	104	105	103	103	102	102	101	102	97	95	95
27	100	100	100	99	98	98	98	98	98	95	92	91
28	97	96	96	96	96	96	96	96	95	100	100	98
29	103	102	102	102	102	101	102	101	101	101	100	99
30	103	103	103	103	103	102	102	101	102	101	100	98
31	[103]	[102]	[102]	[101]	[101]	[101]	[101]	[100]	[100]	101	100	99
Means	102·2	102·0	102·0	101·7	101·9	101·8	102·1	102·0	102·0	100·5	98·6	97·5
Diurnal variation	+1·6	+1·4	+1·4	+1·1	+1·3	+1·2	+1·5	+1·4	+1·4	−0·1	−2·0	−3·1

JANUARY, 1883.

Value of 1 division of scale ·000 159 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
97	98	99	99	100	100	100	101	100	100	101	100	98·5
99	99	100	100	101	101	100	101	101	101	101	101	100·9
96	96	97	98	99	100	100	100	100	100	101	101	99·8
98	99	100	99	99	99	100	100	100	102	102	102	100·8
96	95	96	96	96	97	98	99	99	100	101	101	99·9
96	96	97	96	96	97	98	98	98	98	98	100	99·3
97	97	97	97	97	97	97	97	98	99	99	98	99·1
97	97	98	97	98	98	99	99	100	100	101	101	99·1
98	98	98	99	98	98	99	99	100	102	102	103	100·6
95	95	96	96	96	96	96	96	97	98	98	98	98·2
94	95	90	96	97	97	97	96	97	97	97	98	97·1
96	97	97	98	99	98	99	98	98	98	98	99	98·0
95	96	98	98	99	98	98	97	97	98	98	99	97·8
95	95	96	97	97	98	98	97	97	99	97	97	98·3
97	98	99	99	99	[101]	[101]	[102]	[102]	[103]	[101]	[102]	[99·5]
101	101	101	100	99	100	100	100	101	101	102	101	[102·4]
98	99	99	101	101	101	101	101	101	101	103	103	101·1
100	100	101	102	102	102	102	102	103	103	104	104	102·9
103	103	104	104	103	104	104	104	105	106	106	107	104·3
104	102	101	102	103	104	104	104	105	105	105	106	106·7
97	97	98	98	98	99	99	99	99	99	99	99	101·4
94	94	95	96	98	99	100	101	101	102	103	104	98·3
102	103	104	105	107	107	107	107	110	110	[110]	[110]	[105·0]
104	104	105	105	105	105	107	107	107	107	107	106	[106·4]
98	100	102	102	103	103	103	103	103	102	104	104	103·1
96	98	98	98	98	99	98	98	99	100	100	100	99·8
92	92	94	95	95	95	96	96	96	96	96	97	96·1
98	100	101	102	102	103	103	103	103	103	103	103	99·4
97	99	100	101	103	103	103	103	103	103	103	102	101·5
98	100	100	100	101	101	102	102	104	104	104	[103]	[101·7]
99	100	100	102	103	102	104	104	105	104	104	104	[101·8]
97·6	98·2	98·7	99·3	99·7	100·0	100·4	100·5	100·9	101·3	101·5	101·7	100·6
−3·0	−2·4	−1·9	−1·3	−0·9	−0·6	−0·2	−0·1	+0·3	+0·7	+0·9	+1·1	

FEBRUARY, 1883.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	103	103	104	103	103	103	102	101	101	97	98	97
2	101	101	101	101	100	100	101	101	100	97	96	97
3	100	99	100	100	100	99	101	102	100	97	98	98
4	98	99	100	99	100	100	101	102	102	101	102	102
5	105	105	105	105	106	106	107	108	107	107	108	108
6	110	110	109	111	111	111	113	112	111	102	102	101
7	103	103	102	102	101	104	104	105	104	93	91	89
8	95	94	94	95	93	94	95	95	94	95	94	94
9	99	99	99	99	98	98	99	100	99	98	97	97
10	100	100	99	98	99	98	99	100	100	100	98	97
11	100	101	100	100	100	100	101	101	99	96	95	95
12	101	101	100	100	100	100	101	102	101	100	98	98
13	102	102	103	102	102	102	103	103	102	100	98	98
14	103	103	103	103	103	103	104	104	103	99	98	97
15	100	101	100	100	101	102	102	101	100	98	98	97
16	100	100	99	99	100	100	100	100	100	100	99	99
17	99	100	99	99	99	100	99	98	94	93	92	90
18	97	97	96	95	95	96	95	96	95	93	91	90
19	100	101	101	99	99	99	99	100	101	101	101	101
20	104	100	101	101	101	100	99	99	100	101	99	99
21	100	99	99	97	98	98	99	99	99	97	96	95
22	96	96	95	95	95	95	94	94	94	97	96	95
23	100	99	99	98	98	98	98	99	97	95	95	94
24	96	95	95	94	94	95	95	94	94	91	89	88
25	92	92	91	91	90	91	91	92	91	98	97	98
26	102	102	101	101	100	101	100	99	98	97	98	98
27	106	105	104	104	104	103	104	103	101	102	98	99
28	102	107	106	105	104	104	104	104	104	105	104	105
Means	100·5	100·5	100·2	99·9	99·8	100·0	100·4	100·5	99.7	98·2	97·4	97·0
Diurnal variation	+0·8	+0·9	+0·5	+0·3	+0·1	+0·4	+0·7	+0·9	0·0	—1·4	—2·3	—2·6

FEBRUARY, 1883.

Value of 1 division of scale ·000 153 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
95	98	100	101	101	101	100	100	101	101	101	101	100·6
97	97	97	98	99	100	100	100	101	100	101	98	99·3
97	97	95	96	96	97	97	97	97	97	98	99	98·2
102	102	103	104	103	104	105	105	105	104	106	105	102·3
108	107	108	109	109	110	110	110	111	112	112	111	108·1
99	100	101	100	102	103	102	102	102	102	103	103	105·1
89	89	91	91	93	93	93	93	94	95	95	95	96·3
94	96	96	96	97	97	98	99	99	99	99	99	95·9
97	96	98	98	99	99	99	99	100	100	100	101	98·7
97	98	99	99	100	99	100	98	100	98	99	100	99·0
95	96	98	98	99	100	101	101	101	100	100	101	99·1
98	100	101	102	103	102	102	102	102	104	103	103	101·0
99	100	100	101	103	104	104	104	104	104	104	103	102·0
97	97	98	99	100	100	100	100	102	101	101	100	100·8
97	98	99	99	100	100	99	100	100	100	101	101	99·8
98	98	99	99	100	100	101	99	100	99	99	98	99·4
89	89	92	92	95	97	97	95	98	97	96	96	95·6
89	80	92	93	96	99	100	101	101	100	100	99	95·3
100	101	102	102	102	102	103	103	103	103	103	103	101·2
98	99	99	99	100	100	101	101	101	101	101	101	100·2
95	95	96	95	96	96	96	97	98	97	97	97	97·1
96	96	96	97	98	100	102	102	102	100	101	99	97·1
94	96	97	96	97	97	97	97	98	97	98	98	97·2
87	88	93	93	92	92	90	93	94	95	95	93	92·7
98	98	99	99	100	101	102	102	102	100	103	104	96·8
99	100	101	103	103	104	104	105	105	105	105	105	101·5
99	101	102	102	102	107	106	107	107	106	109	107	103·7
106	107	109	109	109	110	109	109	109	110	110	110	106·7
96·8	97·1	98·6	98·9	99·8	100·5	100·6	100·8	101·3	101·0	101·4	101·1	99·7
−2·9	−2·5	−1·1	−0·7	+0·1	+0·9	+0·9	+1·2	+1·6	+1·4	+1·7	+1·5	

MARCH, 1883.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	110	109	109	108	108	108	107	108	107	107	105	107
2	108	108	108	109	109	110	110	110	110	108	108	107
3	110	110	111	111	111	111	111	110	109	106	[106]	[106]
4	108	109	109	110	111	111	111	111	110	107	104	103
5	107	107	107	106	106	106	107	107	107	106	105	105
6	107	107	107	107	107	107	107	107	105	102	101	100
7	104	104	103	103	103	104	104	104	102	94	92	92
8	96	96	95	95	95	95	96	95	94	92	91	90
9	104	104	103	103	103	104	104	103	101	99	98	98
10	101	101	100	101	100	101	102	101	99	96	96	96
11	102	101	101	100	101	101	101	101	100	93	91	90
12	95	95	95	95	95	95	95	95	94	93	91	91
13	95	95	96	95	94	94	95	95	94	93	93	93
14	93	92	93	93	92	94	94	94	94	94	94	94
15	95	95	96	96	96	97	98	98	97	94	92	91
16	96	96	96	96	96	96	97	97	96	96	95	94
17	98	98	97	98	97	98	98	98	97	98	96	95
18	98	98	97	97	97	97	98	98	97	96	95	94
19	98	98	98	98	98	99	99	99	99	97	96	96
20	97	97	97	97	97	98	99	100	99	97	95	94
21	100	100	100	100	101	101	101	101	100	95	91	91
22	99	99	99	98	98	99	99	99	98	98	97	97
23	101	101	10[illegible]	101	102	102	102	103	102	93	92	92
24	99	98	97	96	96	97	98	98	97	96	95	94
25	98	98	97	96	96	96	97	97	97	94	92	92
26	102	101	102	100	99	99	99	99	99	98	97	96
27	102	103	102	101	101	101	101	102	101	100	101	101
28	104	104	104	104	104	104	104	103	103	103	102	102
29	108	108	107	107	106	106	107	106	105	106	104	103
30	105	105	105	105	105	105	105	105	105	103	102	[102]
31	105	105	105	104	103	103	103	103	102	101	101	101
Means	101·5	101·4	101·2	101·0	100·9	101·3	101·6	101·5	100·6	98·5	97·4	97·0
Diurnal variation	+1·0	+1·0	+0·7	+0·6	+0·4	+0·9	+1·1	+1·1	+0·1	−1·9	−3·1	−3·4

MARCH, 1883.

Value of 1 division of scale ·000 147 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
109	109	110	110	110	110	110	109	109	109	109	108	108·5
107	109	110	111	111	111	111	110	110	110	110	110	109·4
[106]	[106]	[107]	[108]	[109]	[110]	110	109	108	108	107	107	[108·6]
103	105	107	108	109	110	110	109	109	108	108	107	108·2
105	105	107	107	107	108	107	107	107	107	106	106	106·5
102	104	104	104	104	104	104	104	104	104	104	105	104·6
92	93	94	95	95	95	95	95	95	95	95	95	97·6
91	93	95	96	104	103	103	103	105	103	104	103	97·2
98	98	100	101	101	101	101	101	102	101	101	101	101·3
96	96	99	100	100	101	100	100	100	100	100	100	99·4
91	92	94	95	96	96	97	97	96	96	95	96	96·8
92	93	94	95	95	95	96	95	94	93	93	93	94·0
93	94	95	95	95	95	95	94	94	93	94	93	94·3
94	95	96	96	96	96	97	96	96	94	94	94	94 4
91	92	94	95	97	98	98	98	97	97	97	96	95·6
94	95	96	97	99	99	99	99	98	98	97	97	96·6
94	95	96	98	99	100	100	100	99	99	98	98	97·7
95	96	97	98	98	98	98	98	98	97	97	97	97·0
97	98	99	99	100	101	101	101	100	99	98	97	98·5
95	96	97	99	99	100	100	100	99	99	98	98	97·8
91	93	94	97	99	100	100	100	100	100	101	101	98·2
97	98	100	100	101	101	101	102	102	101	100	99	99·3
93	94	95	97	97	98	99	100	100	100	100	98	98·5
94	95	96	98	100	100	100	100	100	100	99	99	97·6
92	93	94	96	97	99	100	99	100	100	101	102	96·8
96	99	98	100	101	102	102	102	103	103	103	103	100.1
99	101	103	104	104	104	104	103	103	103	103	105	102·2
102	103	105	106	106	106	107	106	107	106	106	108	104·5
102	103	104	104	105	105	105	106	105	104	104	104	105·2
[103]	103	105	105	105	105	106	106	105	105	105	105	[104·6]
101	101	103	103	104	104	105	105	105	105	105	104	103·4
97·3	98·3	99·6	100·5	101·4	101·8	102·0	101·7	101·6	101·2	101·0	100·9	100·5
−3·2	−2·1	−0·9	+0·1	+0·9	+1·4	+1·5	+1·3	+1·1	+0·8	+0·5	+0·5	

APRIL, 1883.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	105	105	105	105	104	104	104	104	103	101	99	98
2	101	100	101	102	102	102	103	103	101	98	98	98
3	106	106	106	106	103	103	102	102	101	103	103	103
4	107	107	107	107	107	107	107	107	105	107	98	98
5	102	102	101	101	102	103	104	103	102	97	96	96
6	97	97	96	96	96	96	98	98	97	94	93	94
7	98	97	97	96	96	97	97	98	98	97	97	97
8	104	104	103	102	103	103	104	105	104	102	100	99
9	104	104	104	104	104	104	105	106	105	102	101	100
10	103	102	102	102	102	103	103	104	103	101	100	100
11	102	103	102	102	103	103	103	103	101	100	100	99
12	104	104	104	104	105	106	106	107	106	104	104	104
13	101	101	100	101	102	103	104	103	88	84	83	84
14	88	88	88	88	89	89	90	90	90	89	88	88
15	92	92	91	91	92	92	92	92	91	90	89	90
16	98	96	97	97	97	98	98	98	98	98	97	97
17	97	97	97	97	98	98	100	100	97	97	96	98
18	100	99	99	100	101	101	103	104	103	99	99	98
19	103	102	102	102	103	103	104	103	102	99	99	99
20	104	103	103	103	104	104	106	105	106	103	102	102
21	102	102	102	102	102	102	102	102	101	99	98	98
22	101	103	101	101	101	102	102	103	101	101	100	100
23	101	101	101	101	101	102	103	103	102	101	100	100
24	105	105	105	104	105	104	104	103	101	98	96	96
25	104	102	102	102	102	103	104	104	104	101	101	101
26	100	99	99	99	99	99	99	99	98	93	94	95
27	96	97	97	96	96	97	96	97	96	94	95	96
28	95	96	96	97	96	96	96	95	94	93	93	94
29	97	97	97	97	97	97	97	97	96	99	98	99
30	103	104	104	104	104	104	105	104	103	101	100	100
Means	100·7	100·5	100·3	100·3	100·5	100·8	101·4	101·4	99·9	98·2	97·2	97·4
Diurnal variation	+0·6	+0·5	+0·2	+0·3	+0·4	+0·8	+1·3	+1·4	—0·2	—1·8	—2·9	—2·6

APRIL, 1883.

Value of 1 division of scale, ·000141 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
98	98	101	102	103	104	104	103	103	102	101	100	102·3
98	100	101	102	103	104	106	105	105	104	104	104	101·9
103	102	105	106	108	109	109	108	108	108	109	109	105·3
99	99	100	102	104	104	104	103	103	102	102	102	103·7
97	97	99	101	102	101	100	100	100	99	99	97	100·0
95	97	98	99	99	100	100	99	99	99	98	98	97·2
98	98	99	100	101	104	105	104	103	103	102	102	99·3
99	101	102	102	104	104	105	104	104	104	104	104	102·9
101	101	103	104	105	104	104	103	103	103	102	102	103·3
101	101	102	103	103	105	105	104	104	103	103	102	102·5
101	101	102	103	104	103	103	102	103	103	103	102	102·1
104	103	104	104	103	102	101	100	100	100	100	100	103·3
86	86	86	88	90	90	89	89	89	89	89	89	92·3
88	88	89	90	92	92	92	92	92	92	91	91	89·8
89	90	92	94	95	97	97	97	96	96	97	97	93·0
98	98	99	99	99	100	101	100	100	99	98	98	98·3
98	100	101	102	103	103	102	102	101	100	100	99	99·3
98	99	101	102	104	105	105	103	104	101	101	103	101·3
99	99	101	101	103	103	103	104	104	103	103	103	102·0
101	101	101	101	101	101	102	101	102	102	102	102	102·6
97	97	98	100	100	101	102	102	101	101	101	101	100·5
100	100	100	102	103	103	102	102	102	102	101	101	101·4
100	100	101	102	103	103	103	103	103	103	104	104	101·9
95	95	98	99	102	104	105	104	104	105	104	104	101·9
101	101	100	102	101	101	101	100	100	100	98	98	101·4
95	95	97	97	97	97	97	96	96	96	95	95	96·9
97	97	98	97	97	96	96	96	95	95	95	95	96·1
95	95	95	95	96	97	97	96	96	96	95	95	95·4
100	101	102	103	104	105	105	104	103	103	103	103	100·2
101	102	103	105	105	107	107	106	105	105	105	104	103·8
97·7	98·3	99·3	100·3	101·1	101·6	101·7	101·1	100·9	100·6	100·3	100·1	100·1
−2·4	−1·7	−0·8	+0·3	+1·0	+1·6	+1·6	+1·1	+0·8	+0·6	+0·2	+0·1	

MAY, 1883.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	106	105	106	105	105	104	105	106	105	101	100	101
2	102	101	102	102	101	101	101	102	101	98	98	97
3	103	103	102	102	101	102	103	103	103	101	101	102
4	102	102	102	102	102	102	102	103	102	99	99	99
5	104	104	104	104	105	106	106	106	104	102	101	100
6	103	103	103	103	104	104	105	104	103	102	100	101
7	105	104	104	103	105	105	105	105	103	100	100	102
8	104	104	104	104	104	104	105	106	105	103	102	101
9	103	103	102	101	102	102	103	103	102	102	102	102
10	106	106	106	106	106	106	107	105	104	104	103	103
11	107	107	107	107	107	107	106	105	104	102	102	105
12	106	106	105	105	106	106	106	105	104	100	100	102
13	102	101	102	101	102	103	102	100	98	97	98	102
14	102	102	102	102	104	104	104	103	100	92	92	93
15	95	95	94	95	95	95	95	94	93	92	92	93
16	96	95	95	95	96	96	96	94	93	90	90	91
17	93	92	92	93	93	94	94	92	91	90	90	91
18	93	93	93	94	94	94	94	92	92	91	91	92
19	94	94	94	94	93	93	93	93	93	92	92	94
20	100	100	100	99	99	99	100	100	99	98	97	98
21	105	104	104	102	103	104	104	105	105	107	107	107
22	108	107	107	106	105	106	106	107	106	105	105	105
23	108	107	106	107	106	106	107	106	105	104	102	102
24	106	106	106	105	105	106	106	106	105	101	101	101
25	106	106	106	106	106	107	107	106	105	104	103	103
26	106	106	106	105	106	106	105	105	104	104	103	104
27	105	105	105	106	106	107	107	107	105	104	103	104
28	109	108	108	108	108	108	108	108	107	105	106	106
29	106	106	107	107	108	108	108	108	107	99	98	98
30	98	98	98	98	97	99	100	100	99	98	97	98
31	99	99	98	99	100	99	100	99	97	98	99	98
Means	102·6	102·3	102·3	102·1	102·4	102·7	102·9	102·5	101·4	99·5	99·2	99·8
Diurnal variation	+0·9	+0·6	+0·6	+0·4	+0·7	+1·0	+1·2	+0·8	−0·3	−2·2	−2·5	−1·9

MAY, 1883.

Value of 1 division of scale, ·000135 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
102	103	103	104	104	105	105	104	104	104	102	102	103·8
98	99	101	102	103	105	105	104	103	103	103	102	101·4
102	101	102	102	102	102	105	104	103	103	102	102	102·3
100	102	102	102	102	103	104	104	103	103	103	103	102·0
102	102	103	104	104	104	104	104	104	104	103	103	103·6
101	101	103	103	104	104	103	102	103	104	104	104	103·0
101	101	102	102	103	104	104	103	103	104	103	103	103·1
101	102	102	103	104	103	103	103	103	103	104	104	103·4
103	103	105	105	105	105	105	104	104	105	105	105	103·4
106	106	106	106	107	106	106	106	105	105	105	106	105·5
102	102	103	104	106	105	104	105	105	104	105	105	104·8
102	102	102	102	102	102	102	102	102	103	102	102	103·2
100	101	101	102	102	102	101	101	101	102	102	102	101·0
94	94	95	96	97	97	96	96	96	96	96	96	97·9
94	94	94	95	95	95	95	95	96	96	96	94	94·5
91	91	91	92	93	93	93	93	92	93	92	92	93·0
91	92	92	93	94	95	95	95	95	95	94	94	92·9
91	91	91	92	93	95	96	96	96	95	95	95	93·3
94	94	95	96	96	100	101	101	100	100	100	100	95·7
98	99	99	99	99	101	103	105	106	107	108	106	100·8
105	106	107	107	107	107	108	108	109	108	108	110	105·7
105	105	105	106	106	107	107	107	105	106	106	107	106·0
103	105	106	107	106	107	107	106	106	107	106	106	105·8
102	103	104	105	106	106	106	107	106	106	106	107	104·9
103	104	105	106	106	106	106	104	105	105	106	106	105·3
105	106	106	106	106	106	105	105	105	104	104	106	105·2
105	105	106	107	107	110	109	108	107	107	107	108	105·8
106	106	108	109	109	109	108	108	107	106	106	107	107·4
98	98	98	99	99	99	99	98	98	98	97	97	101·6
98	98	99	100	100	99	98	99	98	98	97	98	98·4
99	99	100	100	99	99	100	101	100	100	100	99	99·2
100·1	100·5	101·2	101·8	102·1	102·3	102·7	102·5	102·3	102·4	102·2	102·0	101·7
−1·6	−1·2	−0·5	+0·1	+0·4	+0·6	+1·0	+0·8	+0·6	+0·7	+0·5	+0·3	

JUNE, 1883.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon
1	98	99	99	99	99	100	100	100	99	97	95	95
2	99	99	99	101	100	100	100	99	97	96	97	97
3	99	99	98	100	101	101	100	99	98	98	98	100
4	99	99	99	99	99	100	102	100	100	98	98	98
5	102	103	102	103	104	104	105	105	103	101	101	102
6	106	106	104	105	105	105	105	105	104	103	102	102
7	107	107	107	107	107	107	108	107	106	101	102	104
8	105	104	104	104	104	104	105	105	103	102	100	100
9	102	102	102	103	103	102	102	102	100	98	98	100
10	102	102	102	103	103	103	103	102	102	98	98	98
11	99	99	99	99	100	100	99	99	97	94	94	94
12	95	95	95	95	96	97	97	97	95	93	93	93
13	96	95	95	96	97	98	98	97	97	94	93	93
14	96	97	97	97	98	99	99	98	97	96	94	95
15	97	97	99	98	99	100	100	100	98	98	97	98
16	99	99	99	100	101	101	102	101	99	96	95	95
17	101	101	101	100	100	100	100	100	100	99	100	98
18	101	101	101	101	101	102	103	103	103	100	99	98
19	100	101	100	101	101	100	101	100	100	99	99	99
20	100	100	100	100	100	100	101	100	98	98	98	98
21	100	99	99	99	99	99	99	99	98	95	93	94
22	101	100	100	100	100	101	101	101	101	95	95	96
23	100	99	98	98	98	98	98	97	96	94	93	92
24	99	99	99	99	99	98	99	99	97	94	94	94
25	101	101	101	101	101	102	103	103	102	99	97	98
26	104	103	102	102	103	103	103	103	102	101	99	99
27	104	104	102	102	102	101	102	103	102	95	96	96
28	104	104	104	104	103	104	106	105	104	103	102	101
29	105	103	104	104	104	104	104	103	102	103	101	101
30	104	104	104	104	104	104	105	105	103	102	102	102
Means	100·8	100·7	100·5	100·8	101·0	101·2	101·7	101·2	100·1	98·0	97·4	97·7
Diurnal variation	+0·8	+0·8	+0·5	+0·9	+1·0	+1·3	+1·7	+1·3	+0·1	—1·9	—2·6	—2·2

JUNE, 1883.

Value of 1 division of scale, ·000129 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
96	96	96	97	97	98	99	99	100	100	98	98	98·1
97	98	99	100	100	99	99	99	99	98	98	98	98·7
98	98	98	98	98	99	100	100	99	99	99	99	99·0
99	100	100	101	102	102	102	102	102	102	102	102	100·3
102	101	101	102	103	103	103	103	103	104	104	104	102·8
102	103	104	106	106	106	107	107	107	106	107	107	105·0
104	104	104	106	105	105	103	102	103	103	103	104	104·8
100	101	101	101	101	101	101	100	99	100	101	101	102·0
100	100	99	101	101	102	103	101	101	102	101	102	101·1
98	98	98	98	98	99	98	98	98	98	98	98	99·7
95	95	95	95	95	95	96	96	96	96	95	95	96·5
93	95	95	95	95	97	96	95	95	95	95	94	95·1
94	95	96	96	96	96	97	97	97	97	96	97	96·0
96	96	97	98	98	98	99	99	99	99	99	99	97·5
98	98	98	98	99	99	99	98	98	98	99	99	98·4
96	97	98	99	100	100	100	99	99	99	99	99	98·8
97	97	99	99	100	101	102	102	102	103	102	102	100·3
97	98	99	100	101	100	101	100	100	102	101	100	100·5
97	97	98	100	100	100	100	99	99	99	99	99	99·5
97	98	98	100	99	100	99	98	98	98	97	96	98·8
95	96	98	99	100	99	100	99	100	100	100	100	98·3
95	96	97	97	97	99	99	99	99	99	99	99	98·6
94	94	95	97	97	99	99	99	98	98	99	99	97·0
96	96	98	98	99	99	101	100	99	99	100	101	98·2
98	98	100	100	100	101	102	102	102	102	102	103	100·8
100	99	100	101	103	103	102	102	101	101	100	101	101·5
98	100	102	102	103	104	105	105	104	104	103	103	101·8
102	103	102	103	103	103	103	103	102	102	102	102	103·1
101	100	102	103	104	104	105	104	102	103	103	102	103·0
101	101	102	102	104	106	105	105	105	104	105	105	103·7
98·2	98·3	99·0	99·7	100·2	100·6	100·8	100·4	100·2	100·3	100·2	100·3	100·0
−1·8	−1·6	−1·0	−0·2	+0·2	+0·7	+0·8	+0·5	+0·2	+0·4	+0·2	+0·4	

JULY, 1883.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	104	104	104	104	106	106	106	107	105	102	101	100
2	103	103	103	103	104	106	107	107	105	103	103	103
3	105	105	104	105	105	106	106	106	105	103	103	102
4	102	102	102	103	103	103	103	104	102	101	102	102
5	103	101	102	103	103	103	104	103	102	101	101	102
6	103	103	103	103	103	105	105	103	102	99	100	100
7	103	103	103	103	103	104	103	103	102	98	95	95
8	99	98	98	99	99	99	99	100	98	96	96	95
9	100	99	99	99	100	100	101	100	99	96	96	98
10	101	101	100	101	102	103	103	103	102	99	99	99
11	102	101	101	101	101	102	101	101	101	100	100	101
12	103	103	103	103	104	104	106	105	103	98	97	97
13	103	103	103	103	104	105	105	105	105	100	99	98
14	102	102	102	101	102	100	100	99	100	97	96	96
15	100	100	99	100	101	101	102	101	99	97	97	97
16	101	101	100	100	101	101	102	101	100	95	94	94
17	100	100	99	100	101	101	103	102	101	98	97	97
18	103	102	100	101	102	101	102	102	101	98	97	98
19	103	103	103	103	104	104	106	105	104	102	100	100
20	106	106	106	105	106	106	106	105	106	94	92	92
21	99	98	98	98	98	99	99	99	97	95	96	95
22	99	99	99	99	99	99	99	100	99	95	93	93
23	103	102	101	101	101	101	102	103	101	96	95	96
24	99	99	99	98	98	99	100	100	98	95	95	96
25	102	102	102	102	102	102	102	101	101	95	95	96
26	100	100	100	100	100	100	101	100	99	99	99	100
27	103	103	103	103	103	103	102	102	101	99	98	98
28	102	102	102	102	103	103	104	104	102	95	93	93
29	95	96	97	97	97	98	98	98	99	98	97	97
30	101	101	103	103	101	101	101	102	100	96	95	95
31	104	105	105	106	106	106	107	105	104	104	105	106
Means	101·7	101·6	101·4	101·6	102·0	102·3	102·7	102·5	101·4	98·2	97·6	97·8
Diurnal variation.	+1·0	+0·8	+0·7	+0·8	+1·3	+1·5	+2·0	+1·7	+0·7	−2·6	−3·1	−3·0

JULY, 1883.

Value of 1 division of scale, ·000123 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
101	102	102	104	104	105	106	105	104	104	105	105	104·0
104	105	105	104	105	104	106	105	105	104	104	104	104·4
102	103	103	105	105	105	104	104	103	104	103	103	104·1
102	102	101	102	103	103	104	103	103	103	103	103	102·5
102	103	104	104	104	104	103	104	102	103	103	103	102·8
102	102	102	102	102	102	103	102	102	102	102	102	102·3
96	97	98	98	98	97	99	99	98	99	99	98	99·6
96	97	98	98	99	99	99	99	98	98	99	99	98·1
98	98	99	99	99	100	100	100	100	100	101	101	99·3
99	99	99	99	99	99	100	100	100	100	99	100	100·3
101	101	101	101	101	101	106	105	104	104	103	103	101·8
97	98	99	100	101	101	104	103	103	103	103	103	101·7
98	98	99	101	101	101	101	100	100	101	100	100	101·4
96	97	98	98	98	99	101	101	100	101	101	101	99·5
97	98	98	99	100	101	103	101	102	101	101	101	99·8
95	96	97	97	97	98	101	101	101	100	101	100	98·9
98	100	101	101	100	100	103	102	102	102	102	102	100·5
100	100	100	100	101	101	103	103	103	104	104	104	101·2
100	102	102	103	103	104	105	106	105	105	105	106	103·5
92	92	93	95	97	97	98	97	98	98	99	98	99·3
95	96	97	98	99	99	100	99	99	99	99	99	97·9
93	94	96	97	98	103	103	102	102	102	102	102	98·6
95	95	95	95	96	97	99	99	98	98	99	99	98·6
96	96	97	98	101	100	101	100	100	101	101	102	98·7
97	98	98	98	98	98	100	100	101	99	100	100	99·5
100	100	100	102	102	102	102	102	101	101	102	102	100·6
99	99	99	100	100	100	102	102	101	101	102	102	101·0
93	93	95	94	94	95	95	95	96	96	95	95	97·5
97	98	98	100	99	101	104	101	101	101	101	100	98·7
96	97	99	101	102	103	104	104	103	104	104	104	100·8
105	105	106	107	106	108	110	109	108	108	109	110	106·4
98·1	98·7	99·3	100·0	100·4	100·9	102·2	101·7	101·4	101·5	101·6	101·6	100·7
—2·6	—2·1	—1·4	—0·8	—0·3	+0·1	+1·5	+0·9	+0·7	+0·7	+0·9	+0·8	

AUGUST, 1883.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	105	109	108	109	110	109	111	110	106	105	108	[103]
2	107	107	107	107	109	111	112	113	112	94	94	94
3	107	108	108	108	109	111	112	111	109	97	96	[98]
4	98	98	98	98	99	100	100	100	98	95	94	94
5	98	98	98	99	99	100	99	99	97	95	94	92
6	101	101	102	101	102	103	102	102	100	99	98	96
7	101	102	102	102	103	103	103	102	102	99	99	98
8	104	104	105	105	106	106	105	105	104	103	103	102
9	107	108	108	108	108	109	109	108	106	106	105	103
10	108	108	109	109	109	109	108	108	107	105	104	103
11	109	109	110	110	110	110	111	110	109	108	107	106
12	108	107	107	107	107	107	106	106	105	104	103	103
13	106	106	106	107	107	108	108	108	107	104	103	102
14	104	104	104	105	105	106	106	106	105	104	102	102
15	104	105	106	107	107	108	108	107	106	104	103	102
16	105	106	106	[107]	[109]	[110]	[110]	[109]	[108]	107	105	104
17	106	106	107	107	107	108	107	107	106	104	103	103
18	105	105	105	106	106	106	105	105	105	103	102	102
19	105	106	106	107	107	108	108	108	107	105	104	103
20	103	104	104	105	105	106	105	105	105	104	103	103
21	108	108	108	108	107	107	107	106	106	104	103	102
22	105	104	104	105	105	105	105	105	105	104	103	102
23	105	105	106	106	106	107	106	106	106	105	104	102
24	108	109	109	109	109	110	109	110	109	107	106	104
25	107	107	107	108	108	108	107	107	106	104	103	102
26	105	105	106	107	107	108	108	108	107	105	104	103
27	102	101	101	101	102	102	101	101	100	99	98	98
28	99	100	100	100	100	100	100	99	98	98	98	97
29	99	99	99	100	101	101	101	101	100	100	99	98
30	100	100	101	102	102	103	103	103	102	102	102	100
31	103	103	104	105	106	106	106	106	106	104	104	103
Means	104·3	104·6	104·9	105·3	105·7	106·3	106·1	105·8	104·8	102·5	101·7	100·8
Diurnal variation	+1·7	+2·0	+2·3	+2·7	+3·1	+3·7	+3·5	+3·2	+2·2	—0·1	—0·9	—1·8

AUGUST, 1883.

Value of 1 division of scale, ·000116 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
[99]	94	[94]	95	106	106	104	105	105	106	106	106	[105·0]
96	98	99	102	104	104	106	107	107	108	107	108	104·7
[98]	[98]	[98]	[98]	98	96	98	98	97	97	98	98	[101·9]
94	94	95	95	95	96	96	96	97	97	97	98	96·8
92	93	94	94	94	95	97	97	98	99	100	101	96·8
95	95	96	97	98	98	99	100	100	101	102	101	99·5
99	99	100	101	101	101	100	102	102	103	104	104	101·3
103	103	104	104	104	103	103	104	105	105	106	107	104·3
103	103	103	102	102	102	103	104	105	106	107	108	105·5
102	102	102	102	102	102	103	105	106	107	109	109	105·8
104	104	104	103	103	103	103	104	105	106	106	107	106·7
102	101	101	101	100	100	101	101	103	104	104	105	103·9
102	100	100	100	100	100	99	100	101	101	102	103	103·3
100	100	99	98	98	98	98	98	100	101	102	103	102·0
101	101	100	99	100	100	100	101	101	103	103	104	103·3
102	102	101	101	100	100	101	102	103	104	105	105	[104·7]
102	102	102	102	102	102	102	103	103	104	105	105	104·4
101	100	100	99	99	99	100	102	102	104	105	106	103·0
102	101	100	99	99	100	100	101	101	102	103	103	103·6
103	103	103	103	102	102	102	102	104	105	106	107	103·9
101	100	100	99	99	99	100	101	102	103	104	105	103·6
102	102	102	101	101	101	101	102	102	103	104	104	103·2
101	101	100	100	100	100	102	102	103	104	105	107	103·7
103	102	102	101	100	100	101	101	103	104	105	106	105·3
102	101	101	100	100	100	99	100	101	102	103	103	103·6
102	101	100	99	99	98	98	98	99	99	100	100	102·8
97	97	96	96	95	95	95	95	96	97	98	99	98·4
96	96	96	96	96	96	95	96	97	97	98	98	97·8
98	97	97	97	97	97	96	97	98	98	99	99	98·7
99	99	98	98	98	98	98	99	100	101	102	102	100·5
102	101	101	100	100	100	99	100	101	101	102	103	102·8
100·1	99·7	99·6	99·4	99·7	99·7	100·0	100·7	101·5	102·3	103·1	103·7	102·6
−2·5	−2·9	−3·0	−3·2	−2·9	−2·9	−2·6	−1·9	−1·1	−0·3	+0·5	+1·1	

SEPTEMBER, 1883.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	103	104	103	103	104	104	104	104	103	102	101	100
2	101	102	102	102	103	103	103	103	102	101	100	98
3	99	100	100	101	102	102	102	102	101	89	89	89
4	99	98	99	99	100	101	101	100	98	91	91	92
5	99	99	99	99	100	101	101	109	96	96	95	94
6	103	104	104	104	104	105	105	103	101	96	96	96
7	105	105	105	105	106	107	108	107	105	100	99	97
8	105	105	105	105	106	107	107	106	104	100	99	98
9	106	106	107	107	108	108	110	109	103	100	99	98
10	104	103	103	104	104	104	104	104	102	99	98	97
11	103	103	103	103	103	104	103	103	102	96	95	94
12	99	100	100	100	101	101	101	101	100	97	96	95
13	100	99	99	99	99	99	98	98	97	96	95	94
14	100	100	100	100	101	102	102	102	101	98	98	97
15	100	100	100	100	100	100	100	99	98	98	98	97
16	104	105	103	105	104	105	106	107	105	104	102	102
17	109	109	109	109	109	109	109	109	108	104	102	101
18	105	104	104	104	104	104	105	104	102	98	95	94
19	99	99	99	98	98	98	99	99	97	96	95	94
20	101	101	101	101	101	101	100	100	99	97	96	94
21	94	94	94	94	93	93	94	93	94	95	95	95
22	95	96	96	96	95	95	95	96	97	100	100	100
23	100	100	100	101	101	100	100	100	100	100	99	97
24	100	100	101	102	102	102	103	103	102	101	99	99
25	103	103	104	103	103	103	103	103	103	102	102	102
26	104	104	104	104	105	106	106	106	104	100	99	98
27	100	100	100	101	101	101	100	100	98	98	98	98
28	103	104	103	106	106	107	109	108	107	102	101	100
29	104	104	105	105	106	107	108	108	107	102	101	100
30	101	102	102	103	103	104	105	105	104	101	100	100
Means	101·6	101·8	101·8	102·1	102·4	102·8	103·0	103·0	101·3	98·6	97·8	97·0
Diurnal variation	+1·8	+2·0	+2·0	+2·3	+2·6	+3·0	+3·2	+3·2	+1·5	—1·2	—2·0	—2·8

SEPTEMBER, 1883.

Value of 1 division of scale, '000118 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
99	98	98	98	98	97	97	97	99	99	100	101	100·7
97	97	96	96	96	96	95	95	96	97	98	99	99·1
91	91	93	93	93	93	96	96	96	96	96	96	96·1
93	94	94	94	93	93	97	96	96	96	97	97	96·2
95	97	98	98	98	98	99	99	99	101	102	103	99·0
95	96	97	97	97	97	102	102	102	103	104	105	100·8
97	98	99	98	99	100	100	101	101	102	102	104	102·1
99	99	101	101	102	102	103	104	105	105	106	107	103·4
98	99	100	100	100	100	102	102	102	102	102	102	102·9
97	98	98	98	98	98	100	100	101	101	101	101	100·7
94	94	96	96	96	95	95	96	96	96	98	98	98·4
94	94	94	94	95	95	95	96	97	98	98	99	97·5
95	95	96	96	96	97	97	98	98	99	99	99	97·4
97	97	97	97	97	97	98	98	99	99	99	99	99·0
96	96	97	96	96	97	100	101	100	99	102	103	98·9
102	103	104	103	103	103	106	105	106	106	107	108	104·5
99	99	99	100	100	100	100	101	101	102	102	103	103·9
93	94	94	94	94	95	97	97	98	98	98	99	98·9
94	94	95	97	97	97	98	98	99	99	100	101	97·5
94	94	93	93	93	93	94	93	94	95	94	95	96·5
93	94	93	93	94	94	95	94	95	95	95	96	94·1
100	101	100	101	101	101	101	100	100	100	100	101	98·6
97	97	98	97	97	97	98	98	97	98	99	100	98·8
98	98	98	98	97	101	101	101	102	103	102	102	100·6
101	101	101	100	100	100	101	101	102	103	103	104	102·1
98	98	98	98	98	98	98	98	98	99	100	100	100·9
98	98	98	98	98	99	100	101	102	102	103	103	99·8
99	100	99	99	99	100	100	101	101	102	103	103	102·6
98	97	97	97	97	98	99	99	99	100	101	101	101·7
100	101	100	100	100	100	102	102	102	102	102	101	101·8
96·7	97·1	97·4	97·3	97·4	97·7	98·9	99·0	99·4	99·9	100·4	101·0	99·8
−3·1	−2·7	−2·4	−2·5	−2·4	−2·1	−0·9	−0·8	−0·4	+0·1	+0·6	+1·2	

OCTOBER, 1883.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	102	102	101	102	102	102	103	102	101	100	99	99
2	103	104	104	104	104	105	105	105	104	100	99	99
3	97	97	97	97	97	97	97	97	97	96	96	96
4	101	102	102	102	102	103	104	104	103	99	98	98
5	102	103	103	104	104	106	107	106	106	103	103	102
6	102	102	102	102	101	101	101	101	101	101	101	101
7	104	104	104	104	104	104	104	104	104	100	100	99
8	99	101	101	102	103	103	104	104	103	100	98	97
9	97	97	97	97	97	98	98	98	97	97	96	96
10	100	101	101	102	102	103	104	104	104	100	99	98
11	103	104	103	103	103	104	105	104	103	102	100	100
12	106	106	105	105	105	105	106	106	104	102	101	100
13	102	102	102	102	101	102	102	101	100	98	98	96
14	104	104	103	103	103	102	103	102	101	100	100	100
15	104	104	103	103	103	103	103	103	101	99	98	99
16	101	99	99	101	101	102	102	102	101	97	96	96
17	100	99	99	100	99	99	100	100	99	97	96	97
18	99	99	99	99	99	99	99	99	98	97	96	97
19	99	101	101	102	102	102	103	103	101	99	98	97
20	101	101	101	101	101	101	101	102	100	97	95	94
21	97	97	97	98	98	98	99	99	97	94	94	93
22	100	100	101	101	102	102	102	102	101	98	97	96
23	99	98	98	98	98	98	97	97	96	98	98	97
24	102	102	102	103	103	103	104	104	103	103	102	101
25	105	105	105	105	105	105	105	105	104	104	104	103
26	106	106	106	107	108	108	108	108	108	106	105	105
27	108	108	108	109	109	109	109	110	109	103	102	102
28	105	105	106	106	106	107	108	107	106	101	98	97
29	102	102	102	101	101	102	102	101	100	96	94	93
30	95	96	95	96	96	96	96	96	95	95	94	93
31	100	100	101	101	101	101	101	102	100	98	97	96
Means	101·5	101·6	101·5	101·9	101·9	102·3	102·6	102·5	101·5	99·4	98·5	98·0
Diurnal variation	+1·5	+1·6	+1·5	+1·9	+1·9	+2·3	+2·6	+2·5	+1·5	−0·6	−1·5	−2·0

OCTOBER, 1883.

Value of 1 division of scale, ·000108 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
99	99	99	98	98	99	98	99	100	101	101	102	100·3
99	99	99	99	99	99	99	98	97	97	96	96	100·5
97	97	97	97	97	97	97	98	98	99	99	100	97·3
97	98	97	96	97	97	98	98	99	100	101	102	99·9
102	102	101	101	100	100	100	100	101	101	101	101	102·5
101	101	101	101	101	101	101	102	102	102	103	103	101·5
100	100	100	99	99	100	100	100	100	100	100	100	101·4
96	95	95	95	94	95	95	96	95	96	96	97	98·3
95	96	95	96	96	98	98	99	99	99	99	100	97·3
97	96	97	97	99	100	101	101	101	102	103	104	100·7
100	100	101	101	101	102	102	103	103	103	103	104	102·4
101	101	102	102	102	102	102	102	102	102	103	103	103·1
98	99	99	100	100	100	101	102	102	103	103	104	100·7
100	100	100	100	100	100	101	101	102	102	103	103	101·5
98	98	98	98	98	99	99	100	99	100	100	101	100·5
96	96	96	97	96	97	97	97	97	98	98	99	98·4
96	97	97	97	96	96	97	97	97	97	98	99	97·9
96	96	96	96	96	95	95	96	96	97	98	98	97·3
97	97	97	98	97	96	96	97	98	99	99	100	99·1
94	94	93	93	93	94	93	93	94	95	96	94	96·7
93	94	94	94	94	96	95	96	96	96	97	97	96·0
95	95	94	94	95	96	97	96	97	97	97	98	98·0
98	98	99	99	99	100	100	100	101	101	101	101	98·7
101	101	100	100	100	102	102	102	103	103	103	104	102·2
103	104	104	104	104	104	104	105	104	105	106	106	104·5
105	106	107	106	107	107	107	107	108	108	107	107	106·8
[101]	101	101	101	101	101	102	102	103	104	104	105	104·7
96	96	95	95	95	96	96	98	99	99	100	100	100·7
93	93	92	92	92	92	93	93	94	94	95	95	96·4
94	94	94	94	94	95	96	96	97	98	99	99	95·5
96	96	95	95	95	96	96	97	98	98	99	99	98·3
97·9	98·0	97·9	97·9	97·9	98·5	98·6	99·1	99·4	99·9	100·3	100·7	100·0
—2·1	—2·0	—2·1	—2·1	—2·1	—1·5	—1·4	—0·9	—0·6	—0·1	+0·3	+0·7	

NOVEMBER, 1883.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	99	99	98	99	99	98	100	99	98	98	97	97
2	106	105	105	105	105	106	107	106	105	104	103	102
3	107	107	107	107	107	108	108	108	108	109	108	108
4	111	111	111	111	112	113	113	113	111	103	103	102
5	108	108	109	109	110	111	112	113	112	105	102	101
6	101	101	101	100	100	100	102	101	101	103	103	102
7	109	109	109	109	109	110	112	111	108	105	104	103
8	107	107	107	107	107	108	107	106	104	94	93	91
9	97	97	97	98	98	98	99	98	96	93	91	89
10	96	95	96	97	97	98	99	99	98	95	94	93
11	96	96	96	96	97	98	99	99	98	93	91	90
12	95	95	95	95	95	96	97	97	96	97	96	95
13	104	104	104	104	104	105	106	106	105	105	104	104
14	110	110	110	110	111	112	114	112	111	104	102	102
15	110	110	111	111	111	113	114	114	112	105	103	102
16	107	107	106	107	107	107	107	106	105	104	103	102
17	113	113	113	113	113	114	115	114	113	107	106	104
18	114	115	116	116	116	117	117	117	116	109	107	106
19	110	110	110	111	111	112	112	113	112	110	108	108
20	111	112	113	114	114	115	116	116	116	103	101	99
21	99	99	100	100	100	100	100	100	100	98	97	97
22	105	105	106	106	106	105	106	106	106	102	101	100
23	106	107	107	107	107	108	108	109	107	105	104	103
24	105	106	106	107	108	108	109	109	108	106	103	102
25	104	104	104	104	104	105	105	105	104	99	98	97
26	98	98	99	99	99	100	100	100	99	97	96	95
27	98	98	99	100	100	100	101	101	99	98	96	95
28	96	97	97	98	98	98	99	99	97	96	94	92
29	98	98	99	100	100	101	101	101	100	99	97	96
30	102	101	102	101	101	101	102	102	100	98	97	97
Means	104·1	104·1	104·4	104·7	104·9	105·5	106·2	106·0	104·8	101·5	100·1	99·1
Diurnal variation	+1·9	+1·9	+2·2	+2·5	+2·7	+3·3	+4·0	+3·8	+2·6	—0·7	—2·1	—3·1

NOVEMBER, 1883.

Value of 1 division of scale, ·000104 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
97	96	96	97	97	101	103	104	104	104	105	105	99·6
102	102	103	103	103	104	104	105	106	107	109	110	104·9
107	107	108	108	108	108	107	107	108	108	109	109	107·8
102	103	104	104	105	107	107	106	106	107	107	107	107·5
100	100	100	100	100	100	102	102	101	101	101	101	104·5
103	102	103	103	103	104	105	106	107	107	108	109	103·1
103	103	103	103	103	103	105	105	106	106	107	108	106·4
91	90	90	90	90	91	92	93	93	94	95	96	97·6
88	88	87	87	88	92	92	93	93	93	94	94	93·3
92	92	92	92	91	91	93	94	94	95	95	95	94·7
90	89	89	89	90	92	92	92	93	93	94	94	93·6
95	95	95	95	98	101	101	102	102	103	103	103	97·6
103	103	102	102	103	104	106	107	107	109	110	110	105·0
101	101	102	102	103	105	105	106	106	107	108	108	106·8
102	102	101	102	102	104	104	104	105	105	105	106	106·6
102	102	102	103	104	105	105	107	108	109	110	110	105·6
104	104	104	104	105	110	110	111	111	112	113	113	110·0
105	105	106	106	106	106	106	106	107	107	108	109	110·1
107	106	106	106	106	107	107	107	108	108	110	111	109·0
98	99	98	99	99	100	100	99	99	99	99	99	104·9
97	98	98	98	100	100	100	101	102	103	102	103	99·7
100	99	99	98	98	100	100	101	102	103	103	104	102·5
103	102	101	101	101	101	101	102	102	103	103	104	104·3
101	100	100	100	101	101	101	101	102	102	103	103	103·8
97	96	96	96	95	96	96	96	96	97	97	98	99·5
94	93	93	93	93	93	94	94	94	95	96	96	96·2
93	92	92	91	91	91	91	91	92	93	95	95	95·5
91	90	90	90	91	95	95	96	96	96	97	97	95·2
95	96	97	98	99	101	102	101	101	102	101	102	99·4
98	99	100	100	100	101	101	101	101	102	103	103	100·5
98·7	98·5	98·6	98·7	99·1	100·5	100·9	101·3	101·7	102·3	103·0	103·4	102·2
−3·5	−3·7	−3·6	−3·5	−3·1	−1·7	−1·3	−0·9	−0·5	+0·1	+0·8	+1·2	

DECEMBER, 1883.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	103	104	103	103	103	104	104	104	102	98	96	97
2	101	101	100	100	101	101	101	101	95	91	89	88
3	96	97	97	97	97	97	98	98	97	92	92	92
4	97	97	98	98	98	98	99	99	99	93	94	93
5	101	102	103	104	104	105	106	106	104	101	100	98
6	103	103	104	104	105	105	107	107	106	103	101	99
7	102	102	102	103	103	104	104	104	103	102	98	96
8	103	103	104	105	105	105	105	105	105	104	101	97
9	102	102	102	102	102	102	102	103	103	101	99	98
10	102	103	103	103	104	104	105	106	105	100	99	98
11	101	102	103	103	104	104	104	104	103	101	98	96
12	102	102	103	103	104	104	105	105	104	102	101	100
13	102	102	102	102	103	104	104	105	105	103	101	100
14	102	103	103	103	103	104	104	105	105	103	101	100
15	97	98	97	96	96	95	95	95	93	93	92	90
16	95	95	94	94	93	94	92	92	93	94	92	93
17	103	100	100	101	100	100	99	99	99	97	96	95
18	108	109	108	109	108	108	108	109	107	99	97	97
19	106	106	106	106	106	107	107	107	109	97	96	94
20	96	97	96	96	96	96	96	96	97	95	94	93
21	101	102	102	103	103	103	103	103	102	103	101	100
22	106	106	107	107	108	108	108	109	110	103	101	101
23	106	107	108	108	109	109	110	111	111	103	100	98
24	94	94	94	95	96	96	96	96	97	95	93	91
25	96	96	97	98	99	100	101	102	103	93	93	91
26	93	92	93	93	93	94	93	95	96	94	103	99
27	98	98	98	101	101	101	101	101	101	101	100	98
28	102	102	102	103	103	103	104	104	104	101	99	98
29	102	103	102	104	104	105	105	106	106	103	101	100
30	100	100	101	102	102	103	104	105	105	105	102	101
31	102	103	103	104	104	104	106	106	107	104	103	101
Means	100·7	101·0	101·1	101·6	101·8	102·2	102·5	102·8	102·5	99·2	97·5	96·8
Diurnal variation	+1·5	+1·8	+1·9	+2·4	+2·6	+3·0	+3·3	+3·6	+3·3	0·0	—1·7	—2·4

DECEMBER, 1883.

Value of 1 division of scale, ·000100 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
97	98	97	97	97	100	99	100	100	100	100	100	100·3
88	88	91	90	92	93	94	94	94	94	96	96	95·0
93	94	94	94	94	94	95	96	96	96	97	97	95·4
93	93	94	95	95	96	97	98	98	98	100	101	96·7
97	97	98	97	97	99	100	100	100	100	102	102	101·0
98	99	99	98	98	99	99	99	100	101	101	101	101·6
95	94	94	95	96	100	100	100	101	101	102	103	100·2
97	98	98	98	98	99	99	100	100	101	102	102	101·4
98	98	97	98	99	100	100	101	101	101	101	101	100·5
98	97	95	95	98	99	98	99	99	100	101	101	100·5
95	95	94	95	95	96	97	98	99	100	101	101	99·5
97	98	97	98	98	99	99	99	100	101	100	102	101·0
99	98	98	98	98	99	100	101	101	101	101	102	101·2
97	99	99	99	100	97	97	98	97	96	97	96	100·3
90	91	90	92	92	94	94	94	95	95	94	94	93·8
93	93	95	96	97	99	98	99	99	100	100	100	95·4
94	95	98	100	101	105	106	106	107	108	108	109	101·1
97	97	97	98	99	103	103	103	103	103	104	104	103·3
93	93	93	93	95	94	94	94	95	95	95	95	99·0
93	93	93	95	96	100	100	100	100	101	101	101	96·7
101	101	102	102	104	105	104	104	105	105	105	105	102·9
101	101	101	101	103	103	102	103	103	104	104	105	104·4
97	96	95	97	97	94	93	93	93	93	94	94	100·7
90	89	89	88	89	89	90	91	92	93	94	94	92·7
90	88	88	88	88	89	88	89	90	90	90	91	93·3
97	96	95	95	95	95	95	96	96	98	98	98	95·5
96	96	97	97	97	99	99	100	100	101	102	102	99·4
99	98	97	98	98	99	99	99	99	100	100	101	100·5
98	97	95	95	95	96	96	96	97	99	99	99	100·1
98	98	98	97	98	98	99	99	99	100	101	101	100·7
101	99	98	97	98	98	98	98	99	100	101	102	101·5
95·8	95·7	95·7	96·0	96·7	97·7	97·8	98·3	98·6	99·2	99·7	100·0	99·2
−3·4	−3·5	−3·5	−3·2	−2·5	−1·5	−1·4	−0·9	−0·6	0·0	+0·5	+0·8	

JANUARY, 1884.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month	1h	2h	3h	4h	5	6h	7h	8h	9h	10h	11h	Noon.
1	103	104	105	106	108	109	110	111	110	107	103	102
2	104	104	104	104	104	104	105	105	105	101	99	[99]
3	97	98	97	97	98	97	97	98	97	98	94	93
4	99	99	98	98	98	98	97	98	97	95	93	92
5	105	105	103	103	103	103	102	103	102	103	98	97
6	103	103	103	104	104	104	104	103	102	99	97	96
7	107	107	106	106	106	106	106	107	106	104	99	97
8	107	107	107	107	107	107	109	109	109	103	101	99
9	104	105	104	105	105	105	106	106	105	99	95	92
10	98	98	98	98	97	97	99	99	99	95	92	91
11	98	98	99	100	100	101	101	103	102	102	99	99
12	102	102	102	101	102	101	102	102	101	100	98	97
13	107	107	107	107	106	107	107	107	106	101	97	97
14	104	104	104	103	103	104	104	104	104	105	104	103
15	104	105	105	105	105	105	105	106	105	101	100	99
16	106	106	107	107	107	107	108	108	107	104	100	100
17	106	106	107	106	106	107	108	107	106	102	99	98
18	101	102	102	102	103	103	103	104	101	99	97	97
19	102	102	102	102	103	103	103	103	103	101	98	98
20	103	104	104	104	104	104	105	105	104	100	96	94
21	101	102	102	102	103	103	103	105	104	104	102	100
22	105	106	106	107	107	107	107	107	105	88	84	83
23	94	94	93	94	94	94	94	95	93	94	92	90
24	103	103	103	103	104	104	105	106	105	102	99	97
25	106	106	106	106	106	108	108	109	108	96	92	92
26	95	96	97	96	96	97	98	98	97	93	90	91
27	95	95	95	95	95	96	96	96	96	95	93	92
28	93	93	93	93	93	94	94	94	94	95	93	91
29	94	94	93	94	94	94	95	95	95	95	93	92
30	96	96	96	96	96	96	97	96	96	93	91	88
31	93	93	93	93	92	92	93	93	93	97	96	94
Means	101·1	101·4	101·3	101·4	101·6	101·8	102·3	102·6	101·5	99·1	96·3	95·2
Diurnal variation	+1·0	+1·3	+1·2	+1·3	+1·5	+1·7	+2·2	+2·5	+1·4	−1·0	−3·8	−4·9

JANUARY, 1884.

Value of 1 division of scale, '00100, in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
102	102	103	102	101	102	103	103	103	103	103	103	104·5
[99]	[98]	[98]	[97]	[97]	[97]	[97]	[96]	[96]	[96]	96	97	[100·1]
94	96	97	99	102	101	101	100	100	100	99	99	97·9
92	95	99	103	105	106	106	105	105	105	105	104	99·7
98	100	102	105	107	107	106	105	105	105	103	104	103·1
96	98	99	102	103	109	109	109	108	108	107	108	103·3
98	100	103	104	105	107	106	105	104	103	102	108	104·3
98	99	100	103	102	103	104	105	105	105	104	105	104·4
93	94	96	97	97	99	100	99	99	99	99	99	100·0
91	92	94	95	95	96	96	97	97	97	98	98	96·1
97	99	99	99	100	101	101	102	103	103	103	103	100·5
98	99	100	101	101	104	105	106	107	107	107	107	102·2
99	100	101	101	101	104	104	104	104	104	104	104	103·6
104	104	104	104	103	104	103	103	103	104	105	106	103·9
100	100	101	101	102	104	104	104	104	105	105	106	103·4
100	101	102	103	104	104	104	104	104	104	105	105	104·5
98	99	99	100	100	101	100	100	100	100	101	102	102·4
97	98	100	100	100	99	100	100	101	101	101	101	100·5
98	99	100	101	101	104	103	103	103	103	103	103	101·7
95	96	98	100	101	102	101	100	100	101	101	101	101·0
100	101	103	105	106	108	107	105	105	105	105	106	103·6
84	85	87	90	102	95	95	95	95	95	94	94	96·8
90	91	92	95	98	106	105	105	104	105	105	105	96·8
98	100	101	102	103	105	105	105	106	106	106	106	103·2
93	95	95	94	95	98	97	96	96	97	98	98	99·8
93	93	93	94	94	96	95	95	95	95	95	95	94·9
92	92	93	94	94	94	94	94	94	94	94	94	94·3
92	92	93	93	94	95	94	94	94	94	94	94	93·5
92	93	94	94	95	96	95	95	96	96	96	96	94·4
89	90	91	92	94	96	95	94	94	94	94	94	93·9
93	92	93	95	95	99	98	98	98	98	98	99	94·9
95·6	96·5	97·7	98·9	99·9	101·4	101·1	100·8	100·9	101·0	101·0	101·4	100·1
−4·5	−3·6	−2·4	−1·2	−0·2	+1·3	+1·0	+0·7	+0·8	+0·9	+0·9	+1·3	

FEBRUARY, 1884.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	98	98	98	98	98	98	98	99	99	101	99	99
2	101	102	101	101	101	102	102	102	103	109	107	105
3	106	106	106	106	106	106	105	106	106	106	104	102
4	108	107	107	107	107	107	107	107	107	101	100	98
5	100	99	100	100	100	100	100	101	100	101	99	97
6	101	101	101	101	102	103	103	103	103	101	99	98
7	99	99	100	99	99	99	100	100	101	100	99	98
8	100	100	101	101	102	102	103	104	103	102	101	99
9	104	104	104	104	104	104	105	107	107	107	106	104
10	105	105	104	104	104	104	104	104	105	103	102	100
11	102	102	102	102	101	100	100	101	100	100	99	98
12	101	101	101	101	101	101	101	101	100	98	97	95
13	99	99	100	100	100	100	101	102	101	99	97	97
14	100	101	101	102	103	104	105	105	102	101	99	99
15	100	101	102	102	102	103	103	104	103	102	100	97
16	99	98	98	98	98	97	97	97	98	104	103	101
17	102	102	102	102	101	101	101	101	101	101	99	98
18	101	102	102	102	102	103	103	103	103	104	103	101
19	104	105	104	105	105	106	106	107	105	103	101	99
20	104	104	104	105	105	105	105	105	105	103	102	100
21	97	97	97	98	98	98	98	99	99	97	96	96
22	98	97	97	97	98	98	99	100	98	93	92	91
23	99	100	100	100	101	102	102	102	100	96	94	92
24	101	100	100	99	100	101	100	101	99	99	98	96
25	104	104	105	102	102	103	103	104	103	102	100	98
26	100	100	100	99	99	99	100	101	100	101	100	99
27	102	101	101	101	100	100	99	100	98	97	96	95
28	106	106	105	103	103	103	103	104	105	101	98	96
29	98	97	98	97	96	97	98	96	95	95	91	92
Means	101·3	101·3	101·4	101·2	101·3	101·6	101·8	102·3	101·7	100·9	99·3	97·9
Diurnal variation	+0·8	+0·9	+0·9	+0·8	+0·8	+1·2	+1·3	+1·9	+1·2	+0·5	—1·2	—2·5

FEBRUARY, 1884.

Value of 1 division of scale, ·000104 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
97	98	99	100	101	103	102	101	101	101	101	101	99·5
105	105	105	107	108	109	108	108	108	108	108	107	105·1
102	103	105	106	107	107	108	108	107	108	108	107	105·9
97	97	97	98	98	99	100	100	100	100	100	100	102·0
96	95	96	97	98	99	99	100	99	99	99	100	98·9
97	97	98	99	100	100	101	101	101	101	100	100	100·5
97	96	96	97	98	99	99	100	100	100	100	100	99·0
97	97	98	98	100	101	102	102	102	103	104	104	101·1
103	102	102	102	103	104	104	104	105	105	105	105	104·3
98	98	97	98	100	101	102	101	102	102	102	102	102·0
98	98	99	100	100	100	100	99	100	100	100	100	100·0
94	94	94	94	94	96	95	95	96	96	97	98	97·5
96	95	96	97	98	99	98	98	98	99	99	99	98·6
98	97	97	97	100	101	100	99	99	99	99	99	100·3
96	96	97	98	98	99	99	98	98	98	98	99	99·7
101	101	101	101	102	103	102	102	101	101	101	101	100·2
97	98	98	98	99	101	100	100	100	100	100	100	100·1
100	100	100	102	102	104	103	103	103	103	103	104	102·3
99	99	99	99	100	102	102	102	102	102	102	102	102·5
99	99	99	101	98	98	97	97	96	97	97	97	100·9
95	97	99	101	95	105	105	104	103	101	100	98	98·9
99	89	90	91	93	98	98	98	98	98	98	99	96·1
92	92	92	94	95	100	100	101	101	102	100	101	98·3
96	97	98	99	101	104	103	104	104	103	104	104	100·5
97	98	98	99	101	103	103	102	102	101	101	101	101·5
98	98	100	100	102	103	102	102	102	102	102	102	100·5
95	95	96	97	98	107	107	107	107	106	106	105	100·7
95	97	98	98	98	100	100	99	99	99	99	99	100·6
93	91	93	94	95	97	98	98	97	98	100	99	96·0
97·5	97·2	97·8	98·7	99·4	101·4	101·3	101·1	101·1	101·1	101·1	101·1	100·5
−3·0	−3·2	−2·7	−1·7	−1·1	+1·0	+0·8	+0·7	+0·6	+0·7	+0·6	+0·7	

MARCH, 1884.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	100	100	100	99	99	100	100	100	99	100	99	98
2	103	103	102	102	102	102	102	103	102	100	100	98
3	102	102	101	100	100	100	100	101	102	102	100	98
4	101	101	101	101	101	102	102	103	103	100	98	95
5	99	99	99	100	99	100	101	101	102	102	100	98
6	101	100	101	101	101	102	103	103	100	97	95	93
7	99	99	99	98	99	99	99	100	98	100	98	96
8	102	102	102	103	103	104	104	103	103	100	98	96
9	101	101	100	100	101	101	101	101	100	96	93	93
10	100	99	99	99	99	99	100	100	99	93	90	89
11	96	96	95	95	95	95	97	98	93	93	90	89
12	100	100	100	99	100	100	101	101	97	94	94	94
13	100	99	99	99	99	100	101	100	99	98	97	97
14	105	105	105	105	105	105	106	106	106	99	96	96
15	103	103	103	103	103	103	105	105	105	102	100	99
16	106	106	106	105	106	106	108	108	106	103	102	102
17	105	105	105	105	106	106	106	106	104	87	85	84
18	97	96	96	96	97	97	98	98	97	93	90	90
19	95	95	95	95	95	95	95	95	94	93	89	87
20	96	96	97	97	98	98	98	98	97	93	91	88
21	98	98	98	98	99	99	100	100	98	97	95	94
22	104	104	104	105	105	105	105	105	102	99	99	97
23	105	104	104	104	104	105	105	104	100	98	97	97
24	101	101	101	100	101	103	103	104	102	101	100	99
25	103	103	103	104	105	106	106	105	103	99	96	96
26	104	104	104	104	104	105	106	105	103	99	98	97
27	101	101	101	101	102	104	105	105	103	96	92	92
28	99	99	98	98	97	98	98	98	97	97	96	95
29	103	103	103	103	103	103	104	103	100	95	94	94
30	100	99	99	98	98	98	99	100	97	96	94	94
31	100	100	101	100	100	100	101	101	97	96	94	94
Means	100·9	100·7	100·7	100·5	100·8	101·3	101·9	101·9	100·3	97·4	95·5	94·5
Diurnal variation	+1.5	+1·4	+1·3	+1·2	+1·4	+2·0	+2·5	+2·6	+0·9	—1·9	—3·9	—4·8

MARCH, 1884.

Value of 1 division of scale, ·000108 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
96	97	98	99	101	104	103	104	104	103	103	103	100·4
98	98	98	99	100	101	101	101	101	103	102	102	101·0
98	98	99	100	100	100	101	100	100	100	101	101	100·3
95	95	97	98	99	100	100	99	99	99	99	99	99·5
97	98	99	100	102	101	101	101	101	101	100	101	100·1
94	96	97	99	99	99	99	98	97	98	98	98	98·7
98	100	102	103	103	104	104	104	104	104	104	104	100·8
97	98	99	100	101	101	101	101	101	101	101	101	100·9
94	95	96	97	98	98	99	99	99	99	99	100	98·4
90	92	94	94	94	96	96	96	96	95	96	96	95·9
91	93	93	95	94	96	97	98	99	99	99	99	95·2
96	96	96	96	96	98	100	100	100	100	99	100	98·2
98	98	100	101	102	102	104	103	104	104	105	105	100·6
98	98	100	101	101	101	102	101	102	103	103	103	102·2
99	100	101	102	104	104	104	104	104	105	105	105	103·0
101	103	104	105	105	106	106	105	105	105	104	105	104·9
86	88	91	92	94	98	97	97	97	97	97	97	97·3
90	91	93	94	95	96	96	95	95	95	95	95	94·8
87	88	90	92	93	95	97	95	96	95	95	95	93·4
87	90	93	94	94	98	100	99	100	99	98	98	95·7
94	95	95	96	97	101	102	102	102	102	102	103	98·5
97	97	98	97	98	102	103	103	103	103	104	103	101·8
97	98	99	99	99	99	100	100	100	100	100	100	100·8
98	98	98	99	100	102	104	103	103	103	103	102	101·2
97	98	98	99	101	101	101	102	102	103	103	103	101·5
98	100	100	101	101	101	101	101	101	101	101	101	101·7
92	93	94	94	94	96	97	96	97	99	97	98	97·9
95	95	96	98	99	101	101	103	102	102	103	103	98·7
95	96	98	99	99	100	99	100	99	99	99	100	99·6
94	94	95	96	96	100	101	100	101	101	100	99	97·9
95	94	95	96	98	102	104	103	102	102	103	103	99·2
94·9	95·8	97·0	97·9	98·6	100·1	100·7	100·4	100·5	100·6	100·6	100·7	99·4
−4·5	−3·5	−2·4	−1·4	−0·8	+0·8	+1·3	+1·1	+1·1	+1·3	+1·2	+1·4	

APRIL, 1884.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	103	103	103	103	103	104	104	104	103	99	96	94
2	103	103	103	102	103	103	102	103	101	97	95	95
3	102	101	101	101	102	103	103	103	101	97	95	93
4	102	102	101	101	101	101	102	102	99	96	95	92
5	101	102	102	102	103	104	104	102	100	98	98	98
6	104	104	103	103	104	104	104	104	101	98	98	97
7	106	106	106	106	105	106	106	104	102	97	96	96
8	103	103	103	103	103	103	103	103	100	97	96	95
9	104	103	[102]	[102]	[100]	[100]	[98]	[98]	[96]	[93]	95	96
10	100	101	100	100	101	101	100	99	95	93	93	93
11	99	100	100	100	101	101	102	102	100	102	102	101
12	105	105	104	105	105	106	104	104	101	101	101	101
13	103	103	103	103	103	104	104	103	102	100	100	98
14	100	100	100	100	100	101	101	100	98	99	98	98
15	103	103	103	103	103	103	101	100	99	99	99	99
16	102	101	101	101	101	101	100	100	98	96	96	97
17	100	101	101	100	101	101	100	99	96	95	94	94
18	99	100	100	100	100	100	100	101	98	97	96	95
19	94	94	96	96	96	96	96	96	94	92	90	90
20	95	95	95	96	96	97	97	97	95	97	96	95
21	98	98	98	98	99	100	99	99	97	96	94	94
22	100	100	100	100	101	102	101	100	98	[96]	[94]	[94]
23	[97]	[97]	[96]	[96]	[97]	[95]	[95]	[95]	[94]	93	91	91
24	101	101	101	101	101	101	101	100	98	96	94	94
25	104	103	103	102	103	103	103	103	101	101	101	101
26	104	104	103	103	103	104	104	104	103	101	99	98
27	104	104	104	104	104	105	104	103	101	101	101	101
28	106	107	107	107	108	109	108	107	107	106	105	104
29	99	99	100	100	100	100	101	100	100	99	99	98
30	102	102	102	102	102	103	102	102	100	97	96	95
Means	101·4	101·5	101·4	101·3	101·6	102·0	101·6	101·2	99·3	97·6	96·8	96·2
Diurnal variation	+1·7	+1·7	+1·7	+1·5	+1·9	+2·2	+1·9	+1·4	−0·4	−2·2	−2·9	−3·6

APRIL, 1884.

Value of 1 division of scale, ·000118 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
95	96	98	99	100	102	102	102	102	102	102	102	100·9
95	96	97	98	99	102	102	103	102	102	103	101	100·4
93	95	96	97	99	102	102	102	102	102	101	101	99·8
92	92	94	96	98	101	102	102	102	102	102	102	99·1
97	96	97	98	99	103	104	104	104	104	104	104	101·2
97	99	100	102	103	105	105	106	106	106	106	106	102·7
96	98	100	101	102	103	103	103	102	103	102	103	102·2
96	97	100	101	101	102	103	103	103	103	103	104	101·2
98	98	99	100	100	100	99	99	100	100	100	100	[99·2]
93	94	96	97	98	99	99	99	99	99	100	100	97·9
101	100	101	102	102	102	102	102	103	103	103	104	101·5
100	100	101	102	102	103	102	102	102	103	102	102	102·6
98	98	98	98	99	100	100	100	100	100	102	101	100·8
98	98	98	98	99	101	100	100	101	102	102	103	99·8
99	99	100	101	99	102	101	101	102	102	102	102	101·0
97	96	96	97	98	99	99	99	98	98	99	99	98·7
94	93	94	96	96	98	99	99	100	100	99	99	97·9
95	94	94	95	94	96	95	96	95	96	95	95	96·9
89	89	89	90	91	95	96	94	95	96	96	96	93·6
95	95	95	96	96	98	99	98	99	99	99	99	96·6
94	93	94	94	95	99	99	99	99	99	99	99	97·2
[94]	[94]	[94]	[94]	[95]	[95]	[96]	[97]	[98]	[98]	[98]	[98]	[97·4]
91	91	92	93	100	100	101	100	101	101	101	103	[96·3]
94	95	98	100	99	104	104	104	105	106	104	104	100·3
101	101	101	101	101	103	104	103	104	104	104	105	102·5
99	99	99	100	100	100	99	100	100	102	103	104	101·5
101	102	103	103	103	104	104	104	100	105	106	106	103·2
102	101	101	101	101	101	101	101	101	101	101	100	103·9
98	98	97	98	98	100	100	100	101	101	102	102	99·6
95	[95]	[94]	89	[94]	94	93	94	94	93	92	94	[96·9]
96·2	96·4	97·2	97·9	98·7	100·4	100·5	100·5	100·7	101·1	101·1	101·3	99·7
−3·5	−3·4	−2·5	−1·9	−1·0	+0·6	+0·8	+0·7	+1·0	+1·3	+1·4	+1·5	

MAY, 1884.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	91	92	92	93	94	95	95	94	92	92	[92]	[92]
2	97	98	98	98	99	100	101	100	99	93	93	92
3	98	97	97	97	97	97	97	95	93	94	93	94
4	99	99	99	98	99	100	100	99	98	97	97	97
5	104	104	104	105	106	106	[106]	[105]	[103]	102	100	100
6	102	102	101	101	102	103	103	102	100	99	99	98
7	103	103	102	103	103	104	104	103	102	108	108	107
8	112	112	111	111	112	112	112	112	111	108	108	107
9	112	111	111	112	112	113	113	112	110	104	103	101
10	104	104	104	104	104	104	104	104	102	96	96	95
11	101	102	101	101	101	101	101	101	91	87	86	87
12	96	96	96	96	96	96	95	95	94	95	94	94
13	97	97	97	97	96	97	97	97	96	88	87	88
14	93	93	93	93	93	93	93	92	90	88	88	88
15	92	91	91	92	92	92	92	91	89	87	85	86
16	91	91	91	92	92	92	92	91	91	91	90	90
17	92	92	92	92	92	92	92	92	91	91	91	90
18	94	94	94	94	94	94	93	93	91	91	92	92
19	95	95	95	95	95	95	94	93	92	93	93	93
20	94	94	94	95	95	95	95	93	93	93	93	93
21	95	95	95	95	96	96	97	97	96	96	95	94
22	97	97	97	96	96	97	97	96	95	95	94	94
23	101	101	101	101	102	102	102	102	101	101	99	99
24	102	102	102	102	102	103	103	102	101	101	100	99
25	102	102	102	102	102	103	103	102	101	99	99	100
26	101	101	101	101	101	102	102	102	101	100	98	98
27	102	102	103	103	103	105	105	105	103	98	97	97
28	102	101	101	101	102	103	103	102	100	99	98	98
29	102	102	102	102	103	103	103	102	101	100	100	99
30	101	101	100	100	101	101	101	101	100	99	98	98
31	101	101	101	101	102	102	102	102	100	100	99	99
Means	99·1	99·1	99·0	99·1	99·5	99·9	99·9	99·3	97·6	96·3	95·6	95·5
Diurnal variation	+0·9	+0·9	+0·8	+0·9	+1·3	+1·7	+1·7	+1·1	−0·6	−1·9	−2·6	−2·7

MAY, 1884.

Value of 1 division of scale, ·00009 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
[91]	[91]	[91]	98	[97]	96	95	95	96	96	97	98	[95·0]
92	92	93	94	95	97	97	98	98	98	99	99	96·7
94	94	94	95	96	99	99	99	100	100	100	100	96·6
97	97	98	98	99	103	103	103	104	104	105	105	99·9
98	99	99	99	100	102	102	103	103	104	104	103	[102·5]
98	98	99	99	102	103	103	103	103	103	104	104	101·3
106	105	106	108	108	110	110	110	110	111	111	112	106·5
108	109	110	111	111	112	111	111	112	112	112	112	110·8
101	102	103	105	106	105	105	105	105	105	105	105	106·9
94	95	97	99	100	102	102	102	102	102	103	103	100·9
87	88	87	87	88	94	95	94	96	95	95	96	94·2
94	94	95	97	95	98	97	97	97	97	97	97	95·8
88	89	89	90	91	92	92	92	92	92	92	93	92·8
88	89	89	89	90	92	91	92	92	92	93	93	91·1
87	87	87	88	89	90	90	89	90	90	90	91	89·5
89	89	90	90	91	92	91	91	91	91	92	92	91·0
89	88	88	90	91	92	93	93	93	93	93	94	91·5
92	92	92	92	93	94	94	94	94	94	95	96	93·2
94	93	93	93	94	94	94	94	94	94	95	94	93·9
94	94	94	95	95	97	97	96	96	96	96	96	94·7
94	93	94	95	96	97	97	97	97	97	96	96	95·7
94	94	94	96	96	100	100	101	102	102	102	102	97·2
99	99	101	101	101	101	101	101	101	102	102	102	101·0
100	101	101	101	101	102	102	102	102	103	103	103	101·7
99	100	100	101	100	100	101	101	101	101	101	101	101·0
98	99	100	100	100	102	102	102	103	103	103	103	101·0
98	98	99	99	99	100	100	101	101	100	101	101	100·8
98	99	99	99	100	101	101	101	102	102	102	101	100·6
100	101	101	101	101	101	102	101	101	101	102	101	101·3
98	99	100	100	100	101	101	101	101	101	101	101	101·2
99	99	[100]	[100]	[100]	[100]	[100]	[100]	100	101	100	99	[100·3]
95·4	95·7	96·2	97·1	97·6	99·0	99·0	99·0	99·3	99·4	99·7	99·8	98·2
−2·8	−2·5	−2·0	−1·1	−0·6	+0·8	+0·8	+0·8	+1·1	+1·2	+1·5	+1·6	

JUNE, 1884.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	98	98	97	98	100	100	100	99	98	96	95	96
2	94	94	93	94	95	96	96	96	94	88	87	90
3	92	90	89	90	90	90	90	90	89	87	87	87
4	91	90	89	88	88	89	89	88	87	86	86	87
5	95	95	94	94	93	95	95	94	93	90	88	87
6	99	98	98	97	97	98	98	97	96	94	93	92
7	98	98	98	98	97	98	99	98	96	92	93	93
8	101	100	100	99	100	100	101	101	100	96	96	96
9	117	116	116	117	117	117	117	117	116	111	111	113
10	115	114	113	113	113	114	114	114	113	112	111	110
11	116	115	115	114	114	115	114	113	112	109	107	106
12	109	108	108	108	108	108	108	107	105	104	104	103
13	104	103	103	102	102	102	102	101	96	92	91	92
14	94	93	92	92	91	91	91	91	90	87	86	86
15	91	91	90	90	89	89	89	88	88	85	85	85
16	95	95	94	94	94	94	94	91	90	88	88	92
17	104	104	104	103	104	105	104	103	99	91	89	91
18	102	102	102	102	102	102	102	100	98	95	92	91
19	103	102	103	103	103	104	103	102	100	98	96	97
20	104	104	104	104	104	104	105	104	103	100	101	101
21	106	106	105	105	106	107	106	106	104	105	105	105
22	109	109	109	109	109	109	109	109	108	105	105	104
23	107	106	103	106	104	104	103	104	103	100	99	98
24	106	105	105	105	103	103	103	103	103	101	102	103
25	112	111	110	110	110	109	110	110	110	109	109	110
26	118	116	114	112	111	110	111	111	102	96	96	96
27	106	105	104	104	104	105	105	105	101	99	96	95
28	106	105	103	103	103	104	103	103	99	92	91	93
29	102	100	99	100	100	99	100	100	99	96	95	97
30	101	101	99	99	98	99	98	98	98	103	105	110
Means	103·2	102·5	102·1	101·8	101·6	102·0	102·0	101·4	99·7	96·9	96·3	96·9
Diurnal variation	+1·6	+0·9	+0·5	+0·2	0·0	+0·4	+0·4	−0·2	−1·9	−4·7	−5·3	−4·7

JUNE, 1884.

Value of 1 division of scale, '000089, in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
96	96	97	98	98	98	100	100	99	98	97	96	97·8
90	90	91	92	93	96	95	95	95	94	92	92	93·0
87	87	87	88	90	91	91	92	91	91	91	91	90·0
86	87	88	90	91	95	95	95	95	95	95	94	90·2
87	87	88	90	92	97	98	99	99	99	98	98	93·5
90	91	93	98	99	102	101	100	100	99	99	98	97·0
94	96	98	99	101	101	100	100	100	100	100	100	97·8
96	96	98	101	103	103	115	114	114	115	115	115	103·2
114	113	114	115	116	118	117	117	117	116	116	115	115·5
111	112	114	114	115	117	117	116	115	116	116	115	113·9
107	107	108	109	110	110	110	109	108	108	108	108	110·5
103	104	104	105	105	105	105	104	102	102	102	102	105·1
91	91	92	93	93	95	95	95	94	94	93	93	96·2
86	86	88	88	86	92	92	92	92	91	91	90	90·0
86	87	89	91	92	93	96	97	95	95	95	94	90·4
93	95	97	99	101	106	106	106	105	104	104	102	97·0
92	95	97	99	100	104	104	104	102	101	101	101	100·0
93	95	99	101	102	104	105	105	104	105	103	102	100·3
99	100	102	103	104	106	106	106	105	104	104	103	102·3
103	103	104	104	105	106	105	105	105	105	105	104	103·8
106	107	108	108	108	109	110	110	109	110	110	109	107·1
104	104	104	106	107	111	112	112	111	111	111	109	108·2
99	99	101	102	103	106	106	106	106	106	106	106	103·5
104	103	104	107	108	115	115	115	114	114	113	112	106·9
111	111	112	114	116	125	125	125	124	122	121	117	114·3
97	98	98	100	102	108	109	110	109	107	106	106	106·0
98	102	104	107	109	114	116	115	114	113	110	109	105·8
97	99	99	100	103	109	108	108	107	104	101	102	101·7
97	98	99	100	103	107	108	107	106	105	104	101	100.9
112	111	111	113	116	108	108	109	108	107	106	104	105·1
97·6	98·3	99·6	101·1	102·4	105·0	105·7	105·6	104·8	104·4	103·8	102·9	101·6
−4·0	−3·3	−2·0	−0·5	+0·8	+3·4	+4·1	+4·0	+3·2	+2·8	+2·2	+1·3	

JULY, 1884.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	104	103	101	101	100	100	100	101	102	101	101	101
2	106	103	103	102	101	100	100	99	99	[98]	[99]	[100]
3	---	---	---	---	---	---	---	---	---	---	---	---
4	[104]	[103]	[102]	[101]	[101]	[101]	[101]	[100]	[99]	[97]	95	96
5	103	102	101	101	101	102	102	102	101	101	101	101
6	113	112	112	112	110	111	111	111	108	95	94	94
7	106	105	104	104	104	104	104	104	101	98	99	99
8	107	106	105	104	104	104	106	105	104	102	101	102
9	112	111	110	109	109	108	109	109	102	100	99	100
10	110	110	109	108	108	107	107	107	107	107	107	109
11	118	119	118	118	117	117	115	115	113	107	107	107
12	110	110	109	109	108	109	111	109	106	99	99	98
13	106	106	105	106	106	106	103	102	98	93	93	95
14	104	104	102	102	101	100	100	100	99	97	96	96
15	99	98	97	96	95	94	94	94	93	96	96	97
16	101	101	98	97	95	95	96	96	90	84	84	86
17	91	90	90	89	89	90	91	89	87	80	80	80
18	128	127	126	126	125	125	126	125	119	96	95	96
19	103	101	101	99	98	98	99	98	92	78	80	80
20	97	95	94	93	94	94	94	94	91	90	91	93
21	101	100	98	98	98	98	98	97	94	89	88	89
22	106	105	105	105	105	106	108	107	102	94	94	96
23	113	111	110	109	109	109	109	109	101	89	87	87
24	104	103	103	103	103	104	104	101	98	95	94	94
25	99	99	98	99	99	99	100	99	96	89	89	90
26	98	97	96	95	96	96	95	95	90	87	88	90
27	102	101	100	98	99	99	98	97	93	86	86	87
28	99	98	98	96	97	97	97	96	92	85	86	86
29	95	95	95	95	95	95	94	93	91	88	87	88
30	103	103	103	103	103	103	102	102	100	96	96	95
31	93	92	92	90	91	90	90	89	83	86	86	87
Means	104·5	103·7	102·8	102·3	102·0	102·0	102·1	101·5	98·4	93·4	93·3	94·0
Diurnal variation	+2·8	+1·9	+1·1	+0·5	+0·3	+0·2	+0·4	−0·3	−3·3	−8·4	−8·4	−7·8

JULY, 1884.

Value of 1 division of scale ·oooo86 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
103	105	105	107	108	116	115	115	114	112	110	107	105·5
[101]	[102]	[103]	[104]	[105]	[106]	[107]	[106]	[105]	[105]	[105]	[105]	[102·7]
96	98	99	101	102	104	105	105	105	103	103	102	[101·0]
103	104	106	110	112	118	118	117	116	115	115	115	107·0
96	99	101	104	105	110	109	108	108	106	106	105	105·8
101	103	105	108	109	111	111	111	111	110	109	107	105·3
103	105	107	109	110	113	113	114	114	114	114	112	107·4
101	102	104	105	109	112	111	112	112	111	111	110	107·4
110	110	112	114	115	120	120	120	119	120	119	118	112·2
108	109	111	111	112	113	114	113	112	112	112	112	112·9
99	99	101	108	109	109	108	107	104	104	104	105	105·6
96	97	98	98	100	102	108	109	106	105	106	103	102·0
96	97	99	100	102	103	102	102	101	100	99	99	100.0
98	98	99	100	103	106	106	105	104	102	100	100	98·8
88	91	93	93	95	97	98	98	97	95	93	92	93·9
81	82	84	86	113	134	135	134	133	132	130	128	100·8
97	99	100	102	105	109	109	109	106	105	104	103	110·9
82	85	88	90	91	98	101	102	101	102	100	99	94·4
95	96	97	101	102	106	106	106	105	104	103	101	97·6
90	94	97	99	101	111	112	112	111	111	109	107	100·1
98	99	101	107	109	116	115	114	114	114	115	113	106·2
89	92	94	96	99	101	103	103	103	103	103	103	101·3
95	96	98	99	100	101	102	100	101	100	101	101	100·0
91	92	94	96	97	101	101	102	100	100	99	98	97·0
90	92	94	98	100	105	106	106	105	104	103	102	97·0
88	88	92	95	97	101	102	102	100	99	99	98	96·1
87	89	91	93	93	96	96	96	96	96	95	94	93·7
91	94	98	100	102	106	105	105	104	104	104	102	96·9
95	97	99	102	102	102	102	101	99	97	97	94	99·8
90	92	94	95	98	98	99	99	99	99	99	98	92·9
95·3	96·9	98·8	101·0	103·5	107·5	108·0	107·8	106·8	106·1	105·6	104·4	101·7
−6·4	−4·9	−2·9	−0·8	+1·8	+5·7	+6·3	+6·0	+5·1	+4·3	+3·9	+2·6	

AUGUST, 1884.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	97	97	97	97	98	99	99	97	92	84	83	83
2	94	92	92	92	93	94	94	92	88	82	81	84
3	88	87	87	86	87	87	87	87	87	84	84	85
4	107	107	106	105	105	105	105	105	103	101	103	105
5	111	109	109	108	108	108	108	107	105	102	103	104
6	114	113	113	112	112	112	112	112	110	107	109	111
7	115	114	114	113	112	112	113	112	106	104	104	106
8	119	119	119	116	116	115	114	112	106	99	98	101
9	109	108	108	107	106	106	106	105	102	96	95	96
10	102	101	101	101	101	101	102	99	94	82	84	85
11	100	99	100	99	99	100	100	98	93	86	87	88
12	96	96	96	96	96	97	97	95	91	80	82	83
13	95	95	95	95	94	94	94	92	89	74	74	74
14	91	91	91	91	90	90	90	88	84	78	80	81
15	96	96	96	95	95	96	96	96	94	92	94	95
16	112	113	112	111	111	112	112	111	108	107	109	110
17	113	112	112	112	111	111	111	109	98	94	94	94
18	106	105	104	104	105	106	107	105	103	100	101	102
19	107	105	105	105	105	105	105	105	99	96	95	96
20	100	99	97	99	100	101	101	98	97	97	97	98
21	105	104	102	101	102	101	102	100	96	90	90	91
22	97	95	94	92	92	91	91	92	94	97	100	102
23	119	119	115	114	114	114	114	114	113	110	108	108
24	107	107	105	105	105	106	104	100	97	88	86	87
25	122	122	121	120	121	122	120	119	114	103	101	103
26	109	108	106	106	107	107	106	105	99	93	91	92
27	99	99	98	98	99	100	99	93	88	83	82	85
28	94	92	89	88	87	86	85	84	78	77	77	77
29	87	85	85	84	84	84	85	85	83	83	82	84
30	91	90	90	90	89	89	89	86	84	83	82	83
31	100	99	99	100	100	100	101	100	91	86	85	86
Means	103·3	102·5	101·9	101·4	101·4	101·6	101·6	100·1	96·3	91·5	91·6	92·9
Diurnal variation	+2·6	+1·8	+1·2	+0·7	+0·7	+0·9	+0·9	−0·6	−4·4	−9·2	−9·1	−7·8

AUGUST, 1884.

Value of 1 division of scale, ·000082 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
85	86	87	91	93	95	96	96	96	94	94	94	92·9
84	86	89	91	92	93	93	92	92	92	91	90	90·1
86	89	92	94	98	106	106	106	107	107	107	107	93·4
107	108	110	111	111	112	112	112	112	111	111	110	107·7
104	107	110	112	113	114	114	114	114	114	114	114	109·4
112	112	113	115	116	116	116	116	116	116	115	114	113·1
108	111	113	115	115	120	119	119	119	119	118	118	113·3
107	109	108	109	109	111	111	110	110	110	110	109	110·3
98	98	103	105	106	106	106	104	105	105	104	103	103·6
87	90	91	94	96	99	98	98	98	98	99	98	95·8
91	93	96	97	99	98	97	98	97	96	97	96	96·0
85	86	88	90	91	97	97	96	95	94	94	94	92·2
76	79	83	84	86	91	92	91	91	90	90	91	87·9
81	82	88	92	95	99	99	98	98	97	96	95	90·2
98	99	101	103	107	114	115	115	114	114	113	112	101·9
110	111	113	116	118	119	118	118	117	115	114	113	112·9
96	100	103	107	108	111	111	110	109	109	108	107	106·2
103	104	105	106	108	109	109	108	108	108	108	108	105·5
96	96	96	101	102	104	105	104	104	104	103	101	101·8
100	102	104	106	107	107	107	108	107	106	106	105	102·0
93	95	96	99	103	106	107	105	103	102	100	96	99·5
103	106	108	108	109	116	118	117	117	119	118	118	103·9
109	109	109	111	112	114	112	110	110	109	107	107	111·7
88	89	91	93	135	131	130	129	128	126	125	123	107·7
106	109	111	114	116	117	116	115	113	111	109	109	113·9
95	97	100	103	105	107	105	104	103	101	100	100	102·0
87	89	93	96	99	104	101	100	98	97	94	94	94·8
79	81	86	87	89	92	92	91	89	89	86	86	85·9
86	86	89	91	91	94	95	94	93	93	92	91	87·8
84	86	90	93	95	103	104	103	103	102	102	101	92·2
87	89	91	94	96	100	100	100	99	99	98	98	95·8
94·5	96·3	98·6	100·9	103·9	106·6	106·5	105·8	105·3	104·7	104·0	103·3	100·7
−6·2	−4·4	−2·1	+0·2	+3·2	+5·9	+5·8	+5·1	+4·6	+4·0	+3·3	+2·6	

SEPTEMBER, 1884.

[Increasing scale readings correspond to increasing force

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	98	98	99	98	100	101	102	101	99	93	93	94
2	107	107	107	107	108	108	109	108	105	102	102	103
3	106	106	107	106	107	108	107	107	100	95	95	96
4	97	97	98	98	98	99	100	98	93	89	91	92
5	100	100	101	100	101	102	102	102	101	98	97	98
6	106	105	105	105	105	105	105	104	102	87	88	88
7	91	89	89	89	88	88	88	89	83	78	78	79
8	93	92	91	90	90	91	91	93	101	97	95	95
9	96	96	96	95	94	94	94	94	92	84	84	84
10	95	93	94	93	93	95	94	94	92	91	92	92
11	103	103	103	104	104	104	104	104	104	104	103	103
12	111	111	110	110	111	111	111	110	109	108	109	109
13	116	114	115	115	114	113	113	111	110	104	104	104
14	109	109	110	109	109	109	109	106	103	97	97	97
15	101	101	102	101	101	100	100	100	98	95	96	98
16	100	99	99	100	99	100	99	98	98	98	98	98
17	100	99	100	100	100	99	99	97	94	89	90	89
18	98	97	97	96	97	98	97	95	91	85	84	85
19	94	93	92	92	92	93	92	91	90	86	86	87
20	93	93	93	92	92	93	93	93	92	91	91	91
21	104	104	104	104	103	104	105	105	103	102	102	102
22	105	105	105	105	105	106	107	107	107	105	104	103
23	116	115	115	114	115	116	115	113	110	99	98	98
24	101	100	100	100	100	102	102	101	99	96	94	95
25	98	98	98	98	98	98	98	97	95	92	92	91
26	97	97	97	97	96	97	97	96	97	94	94	95
27	104	105	104	103	104	105	105	107	104	92	92	92
28	99	100	101	100	100	100	100	101	98	86	86	86
29	94	94	95	95	95	96	98	98	97	95	94	93
30	98	97	97	97	97	99	99	101	101	99	99	98
Means	101·0	100·6	100·8	100·4	100·5	101·1	101·2	100·7	98·9	94·4	94·3	94·5
Diurnal variation	+1·1	+0·7	+0·9	+0·5	+0·6	+1·2	+1·3	+0·8	—1·0	—5·5	—5·6	—5·4

SEPTEMBER, 1884.

Value of 1 division of scale, '000080 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily means.
												d.
96	98	102	104	105	109	110	110	109	108	108	107	101·8
103	104	106	107	107	107	106	106	106	106	107	107	106·0
97	97	98	98	98	99	99	100	99	98	98	97	100·8
95	98	99	99	100	100	100	102	101	101	100	100	97·7
99	100	101	104	104	105	105	106	105	106	105	106	102·0
89	90	90	92	92	92	92	92	93	92	91	91	95·9
82	85	87	88	91	91	[91]	[92]	[92]	[92]	[93]	[93]	[87·8]
96	98	99	101	101	101	100	99	99	98	98	97	96·1
85	88	90	92	93	97	96	96	94	96	96	95	92·5
93	94	95	100	101	105	105	104	104	104	105	105	97·2
104	106	107	108	109	114	114	114	113	113	112	112	107·0
110	110	112	113	114	117	117	117	116	116	115	114	112·1
105	106	107	108	109	112	113	114	112	110	111	109	110·4
97	99	100	101	102	105	106	105	105	104	103	103	103·9
97	97	98	98	99	103	104	104	104	103	102	101	100·1
98	100	101	100	101	103	103	103	103	103	102	101	100·2
90	94	94	94	97	99	99	100	102	99	99	98	96·7
87	90	91	93	93	95	97	97	97	95	97	95	93·6
88	91	92	92	92	96	96	96	96	95	95	95	92·2
92	94	96	97	100	106	106	106	106	106	106	106	97·0
101	102	104	106	106	106	107	107	106	106	107	106	104·4
103	104	106	107	107	117	117	116	116	116	116	115	108·5
99	101	102	104	104	104	104	104	103	103	102	102	106·5
96	98	100	100	101	102	102	102	100	100	99	99	99·5
92	93	96	97	98	98	97	97	97	97	97	97	96·2
96	99	100	101	101	106	106	106	106	105	104	105	99·5
93	94	95	97	98	99	99	99	99	99	99	99	99·5
86	87	90	92	93	95	95	95	95	94	95	95	94·5
94	95	96	98	99	101	102	101	100	99	100	99	97·0
99	100	100	100	102	104	104	104	103	103	103	103	100·3
95·4	97·1	98·5	99·7	100·6	102·9	103·1	103·1	102·7	102·2	102·2	101·7	99·9
−4·5	−2·8	−1·4	−0·2	+0·7	+3·0	+3·2	+3·2	+2·8	+2·3	+2·3	+1·8	

OCTOBER, 1884.

[Increasing scale readings correspond to increasing force.

Local mean time.

Day of month.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	Noon.
1	103	103	102	102	102	102	101	101	100	100	100	100
2	105	104	103	104	103	101	101	98	98	98	99	98
3	102	101	101	100	100	100	100	100	98	95	94	94
4	96	95	95	94	93	93	94	94	94	91	90	91
5	95	95	93	93	93	93	92	93	93	94	93	92
6	97	96	95	95	94	94	94	94	94	93	92	92
7	100	99	98	97	96	95	95	96	95	95	94	95
8	101	101	100	100	100	100	100	101	102	102	101	101
9	110	110	110	109	109	109	109	109	108	105	104	104
10	107	107	106	106	106	106	106	106	105	101	100	99
11	106	105	105	105	105	105	104	105	104	101	99	99
12	104	104	104	104	104	105	104	104	103	101	100	100
13	101	102	102	103	103	103	103	101	100	96	95	94
14	96	94	95	95	95	96	96	95	93	92	92	90
15	97	96	95	96	95	96	96	96	94	92	91	91
16	93	93	92	92	92	93	94	94	92	90	89	88
17	94	94	94	93	92	92	91	92	90	89	88	87
18	94	94	93	93	93	93	93	93	91	87	86	86
19	91	91	91	91	90	91	91	92	89	89	88	88
20	92	92	92	91	91	92	93	92	89	88	87	87
21	92	91	91	91	91	91	91	91	91	90	89	90
22	95	95	94	94	93	95	96	96	97	100	100	99
23	107	108	107	108	108	107	107	107	106	108	108	107
24	112	111	110	110	110	110	110	111	109	108	107	107
25	113	113	113	113	113	113	113	112	111	108	106	106
26	112	112	112	111	111	112	113	113	111	108	107	107
27	110	110	108	108	108	110	109	108	107	104	103	103
28	107	107	116	107	107	108	108	107	104	99	98	98
29	103	103	103	103	103	103	104	104	101	96	95	95
30	101	100	99	99	99	100	100	101	99	95	94	92
31	99	100	100	100	100	100	100	109	97	95	93	94
Means	101·1	100·8	100·6	100·2	100·0	100·3	100·3	100·2	98·9	97·1	96·2	95·9
Diurnal variation	+1·5	+1·2	+1·0	+0·6	+0·4	+0·7	+0·7	+0·6	−0·7	−2·5	−3·4	−3·7

JUNE, 1885.

[Increasing scale readings correspond to increasing force.

Local mean time.

23	101	99	99	100	101	101	101	101	99	98	97	97
24	101	101	101	101	102	103	103	102	99	96	95	95
25	99	99	98	97	97	98	97	98	98	98	98	99
26	101	100	99	99	99	99	99	100	99	98	99	99

OCTOBER, 1884.

Value of 1 division of scale, ·000095 in parts of V.]

Local mean time.

13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	Mid-night.	Daily mean.
												d.
100	100	102	102	103	104	99	102	106	106	104	104	102·0
99	99	98	100	102	101	101	104	104	105	103	101	101·2
95	95	97	96	96	97	98	97	97	97	97	96	97·6
91	92	92	92	93	94	95	95	95	96	95	96	93·6
93	94	95	95	96	97	97	96	97	97	97	97	94·6
93	94	96	95	96	98	97	98	98	99	98	99	95·5
95	96	98	98	100	102	102	102	102	102	102	101	98·1
102	105	106	107	107	110	109	108	107	108	109	110	104·0
104	105	106	107	108	109	107	106	106	106	107	107	107·2
101	103	104	105	105	106	104	104	104	103	104	104	104·2
98	99	101	102	104	105	104	104	103	104	103	103	103·0
100	101	102	102	102	103	102	101	101	100	100	100	102·1
95	95	96	97	97	98	99	98	98	97	96	95	98·5
89	92	93	94	95	96	96	97	97	97	96	96	94·5
90	91	92	92	94	95	95	96	96	94	94	94	94·1
88	90	90	90	91	92	91	92	93	94	95	96	91·8
89	89	91	92	93	95	95	95	95	95	95	94	92·2
86	87	88	89	89	91	90	90	91	91	91	91	90·4
88	90	90	91	92	93	93	02	92	92	92	92	90·8
88	89	90	90	90	92	91	91	91	91	91	91	90·4
92	93	94	95	96	96	96	95	95	95	95	96	92·8
99	99	100	103	105	106	106	106	106	106	106	106	100·1
107	108	109	111	112	113	113	113	112	111	112	112	109·2
108	109	109	110	111	114	112	111	110	111	111	112	110·1
107	109	111	111	112	115	115	114	113	112	112	112	111·5
108	109	110	111	112	112	111	111	110	110	109	109	110·5
103	104	105	106	108	110	110	109	109	108	108	107	107·3
98	100	100	101	102	104	103	104	104	104	103	103	103·8
95	96	97	98	99	101	101	101	100	100	100	100	100·0
93	94	96	97	98	99	99	99	99	100	100	100	98·0
94	94	95	97	99	101	101	100	100	100	99	99	98·2
96·4	97·5	98·5	99·2	100·2	101·6	101·0	101·0	101·0	101·0	100·8	100·7	99·6
−3·2	−2·1	−1·1	−0·4	+0·6	+2·0	+1·4	+1·4	+1·4	+1·4	+1·2	+1·1	

JUNE, 1885.

Value of 1 division of scale, ·000 132 in parts of V.]

Local mean time.

97	99	100	100	101	102	102	102	102	101	101	101	100
95	96	99	100	100	103	103	103	103	103	102	102	
98	100	102	102	101	101	101	102	103	102	102	101	
99	99	99	100	101	101	101	102	102	102	101	100	

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APPENDIX NO. 8—1892.

ON THE MEASUREMENT OF THE HOLTON BASE, HOLTON, RIPLEY COUNTY, INDIANA, AND THE ST. ALBANS BASE, KANAWHA COUNTY, WEST VIRGINIA.

PREFATORY NOTE.

The two chains of the transcontinental triangulation near the thirty-ninth parallel, which had been carried westward from Chesapeake Bay and eastward from the Mississippi, having reached a junction upon a line in the State of Indiana in September, 1890, the measurement of a base of verification became desirable.

A reconnoissance made by Assistant A. T. Mosman under instructions from the Superintendent for the purpose of finding a suitable site for this measurement, resulted, after a few examinations, in the selection of a locality near the town of Holton, Ripley County, Indiana.

In the spring of 1891 preliminary arrangements for the measurement were begun by Mr. Mosman, to whom had been assigned the charge of the party and the general direction of its operations.

It was decided that this measurement should be conducted in such a way as to obtain most thorough tests of the relative merits of several forms of base apparatus. To this end the charge of the measurements with the metallic tapes and with the iced bar apparatus, and the experimental work with the 100-metre comparator was assigned to Assistant R. S. Woodward, while the conduct of the measurement with the new secondary contact-slide apparatus was committed to Assistant O. H. Tittmann.

The reports from these officers which follow elucidate fully the methods of measurement adopted and their results.

They are here published as prepared quite independently by Assistants Woodward and Tittmann. Each contains the details required to enable the reader to form an independent judgment of the accuracy of the method employed and to reach his own conclusions as to relative merits. It may be well, however, to invite attention to one or two inferences in relation to essential points, the correctness of which will be admitted, it is believed, by all who are familiar with the subject.

It is evident that either of the two principal methods of base measurement described, namely, the use of the tape and the so-called "secondary apparatus," will give an accuracy entirely sufficient for any demands likely to arise. In both cases it appears that the probable error derived from the range of different measures is less than one part in one million, and it may safely be assumed that, including errors from all sources, a result true to one part in half a million can be reached.

It is doubtless correct to say that no known method of base measurement aside from the two here considered is capable of giving as good results, when accuracy and expense are both included. The use of an iced bar, applied to the measurement of considerable distances for the first time in the Holton Base, is unquestionably the method of highest precision, and its cost is not believed to be greater than that of other methods in use in Europe, but it will not be found necessary to resort to it in ordinary practice, except for the purpose of standardization as here described. The metallic tape is not only capable of giving a result of great accuracy when in the hands of experts, but it is evidently the best device for rapid base measurement where no great precision is aimed at. As to the cost of the "tape-line method" and that of the "secondary apparatus," when the accuracy required is one part in from two hundred to five hundred thousand, there can be little difference.

Subsequent to the measurement of the Holton Base, a check base in West Virginia, known as the St. Albans Base, was measured by Assistant Woodward, the tape line being used and the conditions being such as to render it easy to determine quite accurately the cost of the work.

The use of the secondary apparatus at Holton having been continuous, when it was once begun, the circumstances were such that a very good measure of the cost of that work is also possible. When these results are compared, it is found that under ordinary conditions both time and cost of the two methods would be essentially the same. Local peculiarities will therefore usually determine which method shall be used.

T. C. MENDENHALL,
Superintendent.

MEASUREMENT OF THE HOLTON BASE.

I. EXTRACTS FROM THE RECORDS AND FROM THE REPORTS OF A. T. MOSMAN, ASSISTANT.

LOCATION OF BASE LINE, MARKINGS, ETC.

The primary base line in Ripley County, Ind., laid out by my party in October and November, 1890, was located on the high and nearly level table land between the towns of Holton and New Marion. Its length is 5,500 metres, and its direction is nearly north and south. In order to avoid the creek near its north end, it was swung to the westward from a true north and south line.

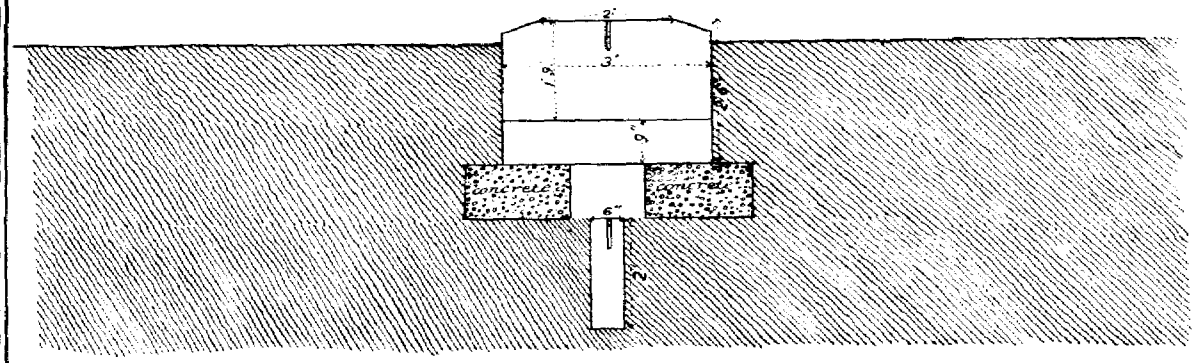
North base is in Otter Creek Township, about 1 mile east of Holton, on the Ohio and Mississippi Railroad. South base is in Centre Township, and about half a mile north of the village of New Marion.

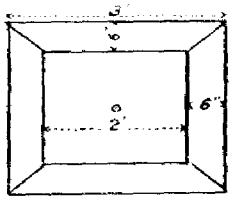
The underground and surface marks placed to secure the ends of the base after the completion of the measurements were as shown in the accompanying sketch (illustration No. 30). The sub-surface mark was a copper bolt, set in a limestone post 6 inches square and 2 feet long, which was sunk in the earth till its top was 3 feet below the surface. At the intersection of cross lines cut on the top of the bolt, a fine hole one-quarter of an inch deep was drilled to mark the station point. Above and around the post, except for a space of 1 foot square immediately over it, is a layer of Portland cement concrete 1 foot thick and 4 feet square. Upon this foundation rests a block of limestone 3 feet square and 30 inches high, having a copper bolt with cross and fine drill hole sunk into its top at the centre, this hole being the surface mark of the end of the base.

To protect these surface marks from injury and to indicate conspicuously the ends of the base, limestone shafts were placed in position over them. The one at North Base had inscriptions on three of its faces, as shown in the illustration, and the one at South Base was similarly marked.

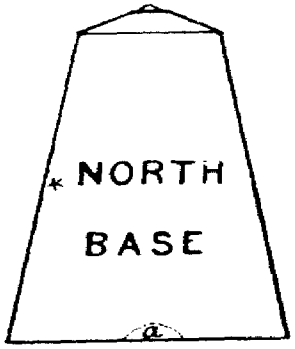
All of the preparations for the actual measurement were completed early in June, 1891. These included the clearing of the line, the fencing of the camp ground, the pitching of the tents twenty-two in number, the construction of a comparing house with railroad and track 110 metres long, the setting of posts for microscopes, and the determination of the height of North Base above sea level by levelings to the nearest bench mark of the transcontinental line of geodetic leveling.

Surface and Sub-surface Markings of the Ends of the Holton Base, and Sketch of Monument at North Base.

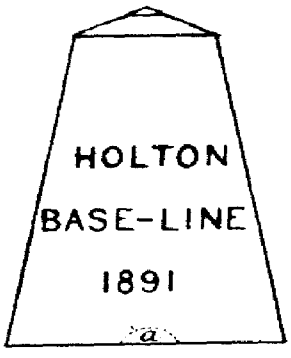




South Face.



East Face.



West Face.

Before the base apparatus arrived from Washington, section stones were set at 1,200, 2,100, 3,000, 4,000, and 5,000 metres from South Base and carefully aligned. Stones were also set at 3,900 and 4,900 metres to mark the ends of the kilometre intended to be measured with the standard bar encased in ice. These section stones were cubical blocks of limestone, 1 foot in each dimension, and set in beds of concrete 15 inches square and 15 inches deep. In the top of each stone was secured a copper bolt, 1 inch in diameter and 3 inches long, with a fine cross cut on its head. The levels run along the line from North Base to South Base made each of these section stones a bench mark well determined in height above sea level. The comparing house, with railroad and track 100 metres long, and posts for microscopes, was finished.

BEGINNING AND PROGRESS OF THE MEASUREMENTS.

On July 27, 1891, the instruments were unpacked and prepared for work, by Assistants Tittmann and Woodward, and on the 28th a measurement of the base was begun by Assistant Tittmann at △ South Base. His party consisted of 4 officers, beside himself, and 4 men, and they were employed for thirteen and one-half days in making two complete measures of the base, besides two additional measures of the kilometre, between stones at 3,900 and 4,900 metres from South Base, laid out to be measured by the standard bar in ice, the second measure having been finished on August 13.

During this time Assistant Woodward, assisted by Mr. Siebert and one or two hands, was engaged in experiments with iced bar and tapes in the comparing house.

On August 14 he began setting posts on the kilometre for supporting the portable track, and the microscopes, using all of the available force of laborers. Owing to continual rains, the ground was saturated with moisture and the work was much delayed. Stakes for supporting the tape during measurement were also set, at 10 metres apart, the whole length of the base line by Assistant Woodward, and some parts of the line were made ready for tape measurement, and on August 28, 29, and 31, and September 1 and 2, measurements were made by officers alone, and on September 8, 16, 17, 18, 23, and 28, and October 1, 2, 3, 7, and 8, measurements were made, usually at night, with a full force of officers and men.

The first measure of the kilometre, with standard bar in ice, was begun by Assistant Woodward on September 10 and continued on the 11th, 14th, 15th, 26th, 28th and 29th.

Experimental measurements were made by Assistant Tittmann with the secondary apparatus, on August 17, 18, 22, 25 and 26, and September 3, 7, and 8. He measured 100 metres of the kilometre with this apparatus on the 21st and 22d, and made measures of the whole kilometre with secondary apparatus on September 24 and 25, and October 5.

I must refer you to the reports of Assistants Woodward and Tittmann for the details of the work, and the results obtained by each method.

COST OF THE DIFFERENT OPERATIONS AND EXPERIMENTS.

A record, as nearly as possible, was kept of the number of days spent in each operation, with the number of officers and men employed each day, and an attempt is made in the table which follows to give the comparative cost, calling each officer or man employed one day one day's work, and the results are expressed in day's work of one officer or man.

In regard to the time spent in preparing the standard kilometre for measurement, and the time spent in setting stakes, there was necessarily some confusion, as frequently some of the party would be at work on each of these operations on the same day, but the result given is believed to be substantially correct.

The time spent in measures with the tape could not be absolutely fixed, as the measures were made sometimes for a few hours in the night with a detail of officers alone, sometimes with officers and men in the day or night, and no complete measurement of the whole base with the tape was ever made but only by parts, several times repeated.

Measurements were made on fourteen different days or nights, and assuming that each measurement took one-half day's time of each officer and man employed we would have sixty-nine and one-half days for one man spent in tape measure.

Distances of from 300 metres to 4,800 metres were measured at one time, mostly at night, and when men were employed we paid them for one-half a day's work.

Holton base line, 1891. Time spent on the different operations.

Kind of operation.	Day's work of one man.
Building ice-house	9
Building comparing house and laying track	67½
Opening lines and setting section stones	15
Leveling from Delaware to base line, two officers	11 = 22
Leveling from Osgood to △ Correct and △ Reizin, three officers	8 = 24
Measuring base twice and two extra measures of the standard kilometre with secondary apparatus	121½
Testing length of bars of secondary apparatus in comparing house	30½
Measuring kilometre with secondary apparatus, twice	18
Measuring 100 metres of kilometre with secondary apparatus	18
Experiments in comparing house with iced bar and tapes	65
Preparing kilometre for measurement with iced bar	114
Measurement of kilometre with iced bar	35
Setting stakes and preparing for tape measurement	53½
Measurements of parts of base with tapes fourteen different days or nights, or $\frac{139}{2}$ days	69½

SERVICES RENDERED.

Assistant R. S. Woodward had entire charge of all measurements with tape and standard bar and all experiments conducted with them in the comparing house and on the base line, including measurements with standard bar in ice of the kilometre. I refer you to his report for details of methods, description of the instruments used, and discussion of the results. Assistant Woodward joined the party on July 22 and left on October 12.

Assistant O. H. Tittmann joined the party on July 22 and left on October 8. He had full charge of all measurements and experiments made with the secondary apparatus on the base line, including kilometre, and in the comparing house, and he has presented to you a very full and complete report of all the work done by him.

Prof. J. H. Gore joined the party on June 20 and left on September 14. He rendered very efficient service while with the party in assisting in the measurements with the secondary apparatus; also with those made by Assistant Woodward with the tape. He had charge of the leveling party determining the heights of Versailles court-house and the △'s Reizin and Correct, being assisted by Messrs. Gjertsen, Pennington, and Cope, recorders.

Mr. John F. Hayford rendered very efficient service in measurements under Assistant Tittmann's charge and the computations pertaining thereto, also assisting in the measurements made with the steel tape and in the computation of the results. He joined the party on July 22 and left on October 12.

Mr. John S. Siebert, temporary aid, joined the party May 21 and assisted in the building of comparing house and railroad and in setting section stones along the base line. He had charge of the leveling from the B. M. at Delaware to △ North Base and △ Glasgow. He assisted in all the measurements and experiments made with tapes and standard bar in ice and in the computations. He remained till the close of work at the Holton base line, and performed all duties entrusted to him with accuracy, zeal, and intelligence.

Recorders Th. Gjertsen, Robert Pennington, and Frank B. Cope were attached to the party during their summer vacation. They rendered satisfactory service in the leveling operations, and also in measurements with the secondary apparatus.

Mr. E. E. Torrey, foreman, was attached to the party during the whole season, and, as usual, rendered most valuable service.

II. THE ICED BAR AND TAPE BASE APPARATUS AND RESULTS OF MEASURES MADE WITH THEM ON THE HOLTON AND ST. ALBANS BASES.

By R. S. WOODWARD, Assistant.

Submitted for publication June 26, 1893.

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LETTER OF TRANSMITTAL.

UNITED STATES COAST AND GEODETIC SURVEY,
Washington, D. C., June 26, 1893.

SIR: I have the honor to transmit herewith my complete report on the Iced Bar and Tape Base Apparatus of the Survey, and on the results obtained with them in the measurements of Holton and St. Albans bases.

Since the attempt to use such forms of apparatus constitutes what some geodesists would consider a bold if not doubtful experiment, I have thought it desirable to give a pretty full account of them and of the measures made with them. Accordingly, all of the results obtained in standardizing the apparatus and in measuring lines with them are given in detail.

Although the critical reader will thus be able to form his own estimate of the character of these forms of apparatus and of the work done with them, it may not be out of place here to state briefly what appear to me to be attainable limits of precision in measures made with each form.

With respect to the iced-bar apparatus, it seems practically certain from our experience that a probable error of one five-millionth part in the length of the measuring bar is attainable, and that an equal precision will result in the mean of a few measures of a line made with the apparatus. This degree of precision has not been reached in work already done with the apparatus, but its attainment, should it be desirable, depends only on the removal of two obstacles. These are, in the order of their importance, 1st, the uncertainty in the length of the Prototype Metres at 0° C., and, 2d, the uncertainty from personal equation, due in the case of the Survey bar to inequality in the widths of its terminal graduations.

With respect to the tape apparatus, it seems equally certain that a precision indicated by a probable error of one two-millionth part of a measured line is an attainable limit. A probable error of one one-millionth part appears to be easily and cheaply attainable with the long tapes after they are standardized. This would seem to be amply sufficient for the present purposes of geodesy, but the sole obstacle in the way of much higher precision, should it ever be deemed essential, is, I believe, the difficulty of measuring the tape's temperature.

Very respectfully yours,

R. S. WOODWARD,
Assistant, Coast and Geodetic Survey.

DR. T. C. MENDENHALL,
Superintendent Coast and Geodetic Survey.

CHAPTER I.

DESCRIPTION OF THE ICED BAR BASE APPARATUS AND ITS APPLICATION TO THE MEASUREMENT OF BASE LINES.

(1) *Historical Note.*—The use of ice in thermometry to furnish a standard temperature naturally suggests the availability of ice to fix the temperature of a standard of length when used in laboratory comparisons or in measuring base lines. It does not appear, however, that ice has been generally used even in laboratory work with standards of length,* and I am not aware that any attempt has been made hitherto to measure a base with a bar whose temperature is controlled by means of melting ice. The feasibility of using such an apparatus in base measurement has, nevertheless, been suggested and maintained by several persons. One of the first, if not the first, to outline a scheme for such an apparatus is, I believe, Mr. E. S. Wheeler, a former colleague on the U. S. Lake Survey. Mr. Wheeler's plan is advocated by Prof. T. W. Wright in his treatise on the Adjustment of Observations.† The late Capt. C. O. Boutelle, of the U. S. Coast and Geodetic Survey, also advocated the use of such apparatus.

Soon after joining the U. S. Coast and Geodetic Survey in July, 1890, I was requested by Dr. Mendenhall, Superintendent, to devise means of testing in the most thorough way practicable the efficiency of the various forms of base apparatus used by the Survey, and especially the efficiency of long steel tapes or wires. Accordingly, considerable study was given to this subject during the autumn of 1890 and the winter of 1890–'91, and the plans and specifications for the iced-bar apparatus considered in this paper were matured and approved early in the spring of 1891. It was constructed in Washington, partly by the machinists E. N. Gray & Co., and D. Ballauf, and partly by the Instrument Division of the Survey.

Before proceeding to a description of the apparatus I desire to acknowledge my indebtedness to colleagues of the Survey for valuable suggestions and criticism. I am specially indebted to Mr. John S. Sie-

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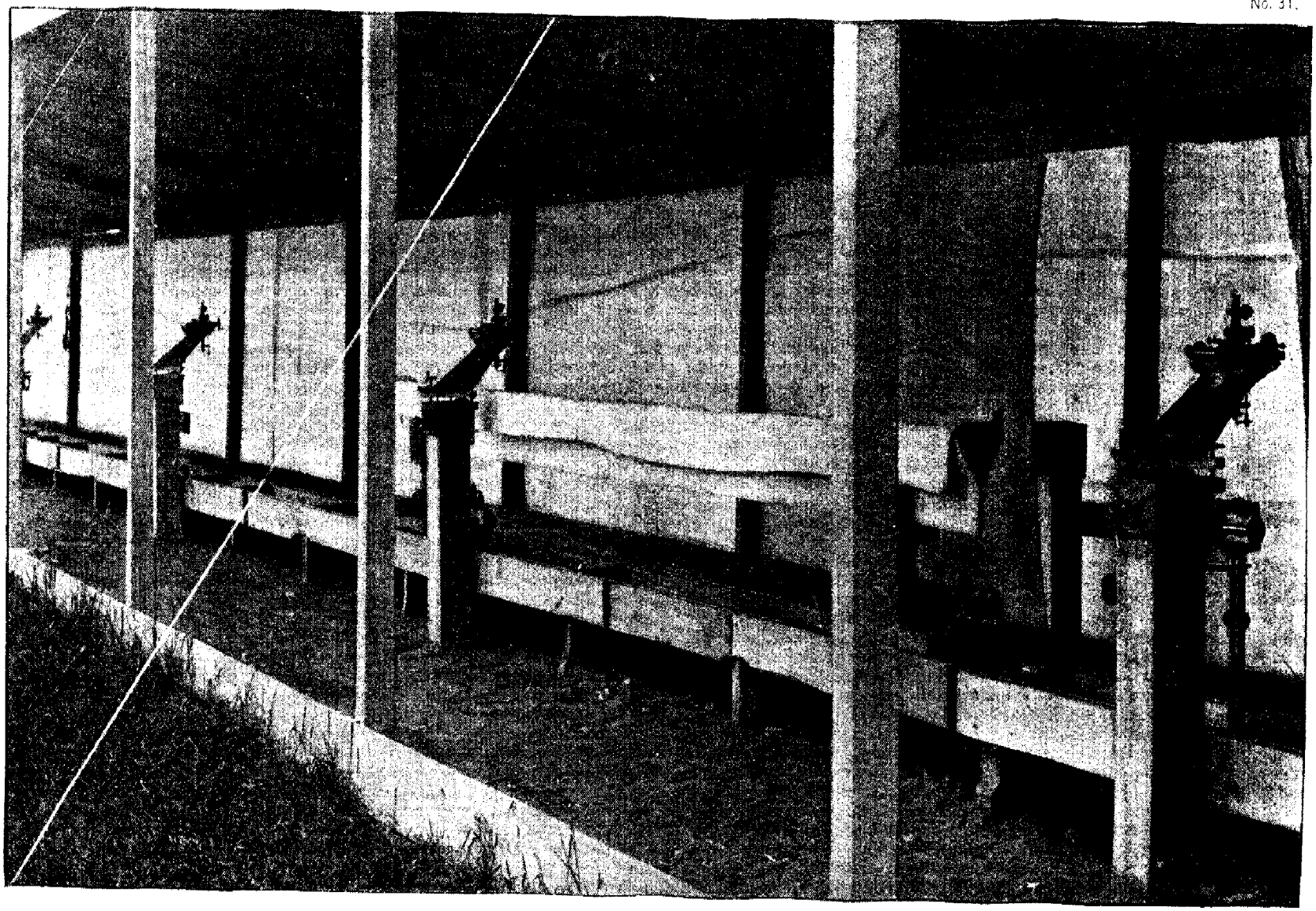
* From published accounts it would appear that the most extensive series of laboratory comparisons of standards, wherein ice was used, are those of the U. S. Lake Survey, conducted under the superintendence of Gen. C. B. Comstock, Corps Engineers, U. S. Army. In these comparisons ice was successfully used during several years. See Professional Papers, Corps Engineers, U. S. Army, No. 24.

† D. Van Nostrand, New York, 1884. See also Am. Jour. Science III, vol. XXVIII, p. 479.



ICED BAR APPARATUS AND WEST END OF 100^{m} COMPARATOR OF HOLTON BASE.

ICED BAR APPARATUS ON STANDARD KILOMETRE OF HOLTON BASE.

bert, who verified all of the preliminary calculations relative to the stability and efficiency of the apparatus, and who elaborated many of the designs and made most of the working drawings for its construction. I am particularly indebted also to Mr. E. G. Fischer, chief mechanician of the Survey, whose knowledge of and skill in mechanical appliances were frequently appealed to. Finally, it affords me pleasure to state that my friend, Mr. E. S. Wheeler, who has had extensive experience with base apparatus, happened to visit Washington about the time the plans for this apparatus were completed, and gave me the benefit of his advice and criticism.

(2) *General features of apparatus.*—The iced-bar apparatus belongs to that type in which a single rigid bar is used as the element of length, along with micrometer microscopes to mark its successive positions. An idea of the nature of the apparatus may be gained from illustrations 31 and 32. The first of these shows the apparatus as used on the 100^{m} comparator of Holton base, described at length in section 2, Chapter III. The other plate shows the apparatus as used on the standard kilometre of the same base. It will be seen that the measuring bar is carried in a trough, where it can be kept surrounded by melting ice. This trough is mounted on two cars, which move on tracks, stationary or portable, as the case may be. The microscopes, as shown in the illustrations, are mounted on wooden posts, which are ranged out and set firmly in the ground beforehand. This sort of support for the microscopes, though not essential to the apparatus, is convenient and economical when a series of measures of the same line is to be made. The microscopes are easily clamped to and detached from the posts, and are moved forward as the measure of a line progresses. The trough carrying the bar and ice load is likewise easily rolled forward on the cars along the tracks.

The apparatus is 5^{m} long, so that the microscope posts are set 5^{m} apart. The supports for the car tracks are also set 5^{m} apart, but are placed half way between the microscope posts, so as to avoid transmitting disturbance to the latter through the ground.

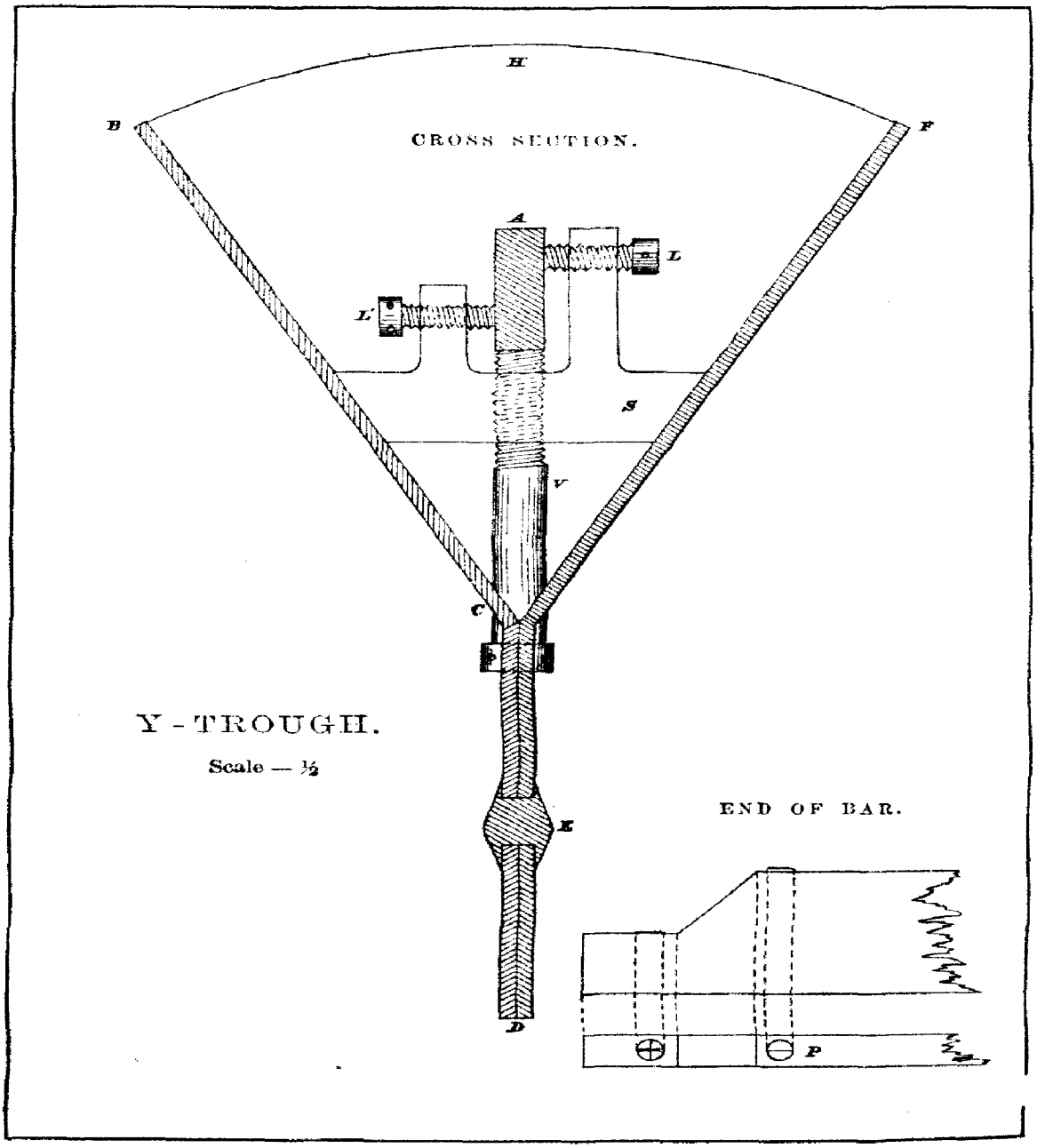
As shown in illustration No. 31, the apparatus on the 100^{m} comparator was protected from the direct rays of the sun by a shed. For field-work, on the other hand, the microscopes were shielded by means of large umbrellas, as shown in illustration No. 32.

The general features of the apparatus are thus apparent, and we may proceed to describe its parts in detail.

(3) *The measuring bar.*—The measuring bar of this apparatus is a rectangular bar of tire steel. It was rolled in the steel works at Lancaster, Pa. It is 5·02^{m} long, 8^{mm} thick, and 32^{mm} deep. A cross section is shown at A in the accompanying illustration, p. 340.

The upper half of the bar is cut away for about 2^{cm} at either end to receive the graduation plugs of platinum-iridium, which are inserted so that their upper surfaces lie in the neutral surface of the bar. Three

lines are ruled on each of these plugs, two in the direction of and one transverse to the length of the bar. These lines were ruled by Mr. Louis A. Fischer, adjuster in the Office of Standard Weights and Measures. The longitudinal lines, which serve to limit the parts of the transverse lines used, are $0{\cdot}2^{mm}$ apart. Although great pains were taken to have



these lines of the same width, the transverse lines differ widely, the narrower one being $16{\cdot}3''$ and the broader one $36{\cdot}2''$ wide.

This bar is known in the Survey records as No. 17. It is designated in all the work done with it thus far as B_{17}. The end having the narrower transverse graduation mark is called the A end.

Description of Iced Bar Base Apparatus.

To secure alignment of the bar, eleven German silver plugs of 5^{mm} diameter are inserted at intervals of 495^{mm} along the bar, so that they project about 1^{mm} above its top surface. The upper surfaces of these plugs are all the same distance, within a few hundredths of a millimetre, from the neutral surface of the bar. On the top of each plug is ruled a fine line in the direction of the bar, as shown at P in the illustration. The length of the bar as regards alignment is defined to be the distance between the transverse graduation marks when the upper surfaces of the alignment plugs are all in one plane and when the lines on these plugs are in one straight line. The means of securing these two adjustments are described below.*

(4) *The Y-Trough.*—The most important and distinctive part of this apparatus is the trough which supports the bar, keeps it aligned, and carries the ice load essential to control the bar's temperature. This trough is called the Y-trough by reason of the resemblance of its cross section to the letter Y. The drawing shows a cross section of this trough. It is made of two steel plates $5{\cdot}14^{m}$ long, $25{\cdot}5^{cm}$ wide, and 3^{mm} thick. They are bent to the angle B C D of the figure, and are riveted together as shown at E, thus making the angle of the trough B C F= 60°. The bar, shown in cross section at A, is supported at every half metre of its length by saddles, one of which is shown in the figure. These saddles are rigidly attached to the sides of the trough by screws at S, S. Each saddle carries one vertical and two lateral adjusting screws as shown at V, L, L'. These screws serve to fix the alignment of the bar. The lateral adjusting screws of the saddles at the ends of the bar are of the same height, which is equal to that of the lower screw L' of the diagram. The lateral adjusting screws of the intermediate saddles on either side of the bar are alternately high and low. The object of this disposition is two-fold, to wit: 1st, to prevent pinching the bar, which might more readily occur if the lateral screws were all opposite to one another; 2d, to afford means of rotating the bar slightly about its longitudinal axis, so that for a fixed and nearly vertical position of the trough the graduated surfaces of the bar may be made horizontal. The vertical adjusting screws of the saddles project, as shown in the diagram, below the vertex of the trough, and their capstan heads are accessible through slots cut in the web of the trough. These slots serve also as drainage ways for the melted ice. To prevent circulation of air through them they are stuffed with cotton batting, through which the water percolates freely. The ends of the trough are closed with wooden V-shaped blocks.

The trough is very rigid in all directions and especially so with

*The form of bar described is evidently not the best form. Theory and experience indicate that a bar having a Y-shaped cross section with metric subdivisions on its neutral surface would best meet the requirements. However, the question which presented itself in planning the apparatus was not what is the best form of bar, but what is the most economical form possessing the requisite properties.

respect to vertical stresses. It weighs 82 kilogrammes, exclusive of the bar and ice load. The whole trough is covered by a closely-fitting jacket of heavy white cotton felt, which protects the trough and ice load alike from direct radiation.

For measuring grade angles a sector reading by two opposite verniers to 10″ is attached to one side of the trough near its middle point. Thus arranged this sector has great stability.

(5) *The ice load and ice crusher.*—When the apparatus is in use the Y-trough is completely filled with pulverized ice, the upper surface of which is rounded to about the height shown by the curve B H F in the diagram. The amount of ice required for this purpose is about 40 kilogrammes, or 8 kilogrammes per metre of the bar's length. The ice, by reason of its weight and the sloping sides of the trough, is kept in close contact with the bar. This is especially the case when the apparatus is in use, for it is then trundled along on its cars with sufficient jarring to overcome any tendency of the ice to pack. For covering the ends of the bar a small quantity of ice is cut with a jack plane. Ice thus cut, like wet snow, packs well and permits making a small conical hole through it to the graduation plugs.

A very essential auxiliary to the use of the apparatus is an ice crusher to pulverize the ice. The machine used is a modification of the Creasey ice breaker manufactured at Philadelphia, Pa. It is a small light hand machine which, as modified, does its work very satisfactorily. With it 40 kilogrammes of ice may be pulverized in ten minutes or less. The particles of crushed ice vary in size from the smallest visible up to the bulk of a cubic centimetre; and this gradation in size appears to be advantageous as compared with uniformly finer particles like those of snow, since there is less liability of regelation and packing.

(6) *The cars and portable track.*—The Y-trough is mounted on two cars, the saddles or bolsters of which are attached to the trough 40^{cm} from either end. Each saddle is attached rigidly to the trough above and to a jackscrew below. The jackscrew is attached to a slide rest which is connected rigidly with the base of the car. The slide rests are provided with screws to give slow motions in the direction of the trough's length and transverse to its length.

The jackscrew cylinders have right and left handed threads at their respective ends and are turned by a short capstan bar. They give thus the rapid vertical motion to the trough essential in bringing the bar quickly to focus under the microscopes which define its position.

The cars have each three wheels and run on a portable track whose width is 30^{cm}. Three sections of this track, each 5^{m} long, are provided; and each section is carried forward as the cars are rolled along during the measurement of a line. It thus appears that instead of lifting up and carrying forward the measuring bar as with most forms of appa-

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ratus, this rather delicate and difficult operation is supplanted by that of moving the portable tracks.

(7) *The micrometer microscopes.*—To define the successive positions of the bar in measuring a line, micrometer microscopes are used. Through the courtesy of Gen. Casey, Chief of Engineers, U. S. Army, the Survey was enabled to borrow the four microscopes and the cut-off cylinder of the Repsold base apparatus* used on the U. S. Lake Survey. These are especially well adapted for use with any line measure apparatus. As designed by the Repsolds and as used on the Lake Survey the microscopes were mounted on iron tripods. These latter having been destroyed by fire while stored at the Engineer depot at Willetts Point, N. Y., it was essential to replace them by some equivalent device. In view of the economic and other features of the special work contemplated with the iced-bar apparatus, it was decided to mount the microscopes on wooden posts set firmly in the ground. To connect the microscope with the post a cast-iron post cap is provided. It fits like a box cover on the end of the post and is clamped rigidly to it by means of a screw.

The microscopes are provided with levels and leveling screws, so that their axes may be made vertical. They are mounted on slide rests which give a motion of 2^{cm} in the direction of the line measured or transverse to it. To secure additional displacement in the direction of the line a small rotary motion is provided for in the connection of the microscope with the post cap. The micrometer heads of the microscope are divided to read microns directly, one revolution of the screws corresponding to $0{\cdot}1^{mm}$. When used in the field the microscopes are shaded from the sun by large umbrellas.

With this method of mounting the microscopes it is advantageous if not essential to set the microscope posts and those supporting the portable track before beginning measurement.

(8) *End marks and method of reference thereto.*—The method of marking the end of a line is essentially that of the Repsolds and fully described in the Lake Survey Report referred to above. It consists in the use of a metallic bolt terminating in a spherical head, the bolt being embedded in a stone or other stable mass set in the ground. The center of the bolt head is the fiducial point. To refer to this point a cylinder called a cut-off cylinder is used. It terminates at one end with a conical hole which fits over the spherical head. The other end is provided with a transverse level and graduated scale. The scale is brought by a rack and pinion motion to focus under the microscope whose position relative to the fiducial point is sought. The scale and level, which are parallel to each other, are placed parallel to the line measured. With the cylinder thus disposed, readings of the micrometer on the

* Fully described in General Comstock's report referred to in section (1).

scale and of the position of the level bubble are made. The cylinder is then turned 180° in azimuth and the scale and level readings are again observed. From these observations and the height of the scale above the bolt head, the horizontal distance (in the direction of the line) between the micrometer zero and the fiducial point may be accurately determined.

(9) *Adjustments of apparatus.*—The most important adjustment of the apparatus is the alignment of the bar in the Y-trough. This adjustment is made when the ice load is in the trough and after the latter has had time to assume a stable shape. This time does not exceed 15 minutes.

As already stated, the alignment of the bar requires that the upper surfaces of the alignment plugs be in one plane, and that the lines on these plugs be in the same straight line. The former requisite is secured by a striding level whose feet are 99^{cm} apart, so that they reach from any plug to the second adjacent plug. Beginning at one end of the bar the plugs are numbered 1, 2, 3,—11. By placing the level feet in succession on plugs 1 and 3, 3 and 5, etc., plugs 1, 3, 5,—11 are brought into the same plane by means of the corresponding vertical adjusting screws, the screws under plugs 2, 4,—10 being loosened if need be to secure this end. Having thus adjusted plugs 1, 3, 5,—11, the level is placed on plugs 2 and 4, 4 and 6, etc., and the vertical screws are brought up to contact with the bar, but not raised enough to disturb the previous adjustment of 1, 3, 5—11, which are the principal defining plugs in this adjustment.

To place the lines on the plugs in the same straight line a sharp-pointed plumb bob suspended from a fine brass wire stretched over the trough was originally used. This device, with the aid of the lateral adjusting screws of the saddles, permits placing the lines in proper position within $0{\cdot}1^{mm}$ when the trough is fully loaded with ice. Experience with the apparatus, however, showed that the simpler method of stretching the wire, or, better still, a fine thread, close over the plugs when the trough is about four-fifths loaded secures equally good results.

It was feared before using the apparatus that the daily temperature range might produce an appreciable effect on the length of the bar through change in curvature of the trough. Hence the accurate method of measuring such change by the striding level was provided. But experience shows that the change in shape of the trough gives rise to quite insignificant changes in length of the bar. Indeed, the alignment of the bar may be maintained so perfectly that the correction for its curvature will not exceed a few tenths of a micron.

The grade sector of the apparatus is adjusted to zero when the graduated surfaces of the bar are in the same horizontal plane. To secure the latter condition an engineer's level is used; and with appropriate care the difference in height of the ends of the bar can be made zero with a probable error not exceeding $\pm 0{\cdot}1^{mm}$.

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The microscopes are provided with fixed levels, which, when once adjusted, enable the operator to make the axes of the microscopes vertical. They are also provided with clamp screws, so that they may be rigidly held in proper position.

When posts are used to support the microscopes, as has been the case with this apparatus thus far, they must be set in their proper positions within a centimetre or two. It is easy and convenient, however, to adjust their sides facing the line to be measured with much greater precision. When firmly set, a line parallel to the base may be deliberately ranged out with a theodolite of high magnifying power, and this line may be defined by suitable marks on each post. Then by simply noting the distance of the axis of the bar during measurement from this reference line an accurate correction for deviation of the bar from parallelism with the base may be obtained. This adjustment of the posts, though not essential to the use of the apparatus, has been followed.

Another convenient adjustment which the use of posts permits is that of making the tops of several or many posts conform to one grade. By this means, since the four microscopes used are closely alike, the grade angles for several or many bar lengths are nearly the same, a condition favorable to precision in determining grade corrections. As an additional precaution in the use of this apparatus, the relative heights of the alternate post tops have been determined with an engineer's level.

(10) *Method of measurement.*—To conduct the measurement of a line with this apparatus eight men are required, to wit: Three observers; one recorder; one man to move the microscopes; and three men to move the car tracks, the microscope shades, and the ice and ice-crusher.

The operation of measurement proceeds as follows: The position of the microscope relatively to the fiducial point at the end of the line having been observed as explained above, the rear end of the bar is brought to focus under that microscope by the rear end observer. By means of a lever which grips into the track and hinges on the car, the latter observer holds the bar near to bisection under the microscope, while the front end observer brings his microscope into position over the front end of the bar; to do which he can make use of the lateral motion of the trough, of the microscope, or both. When the bar is adjusted at both ends the rear observer brings the rear end graduation accurately to bisection between the micrometer wires by use of his lever without turning the micrometer screw. Simultaneously he gives the signal "read" to the front observer, who brings his micrometer wires to bisect the front end graduation mark by moving the microscope, the micrometer wires, or both. The observers then read their micrometers and the recorder notes them down in his book, after which the rear observer turns his micrometer screw a half revolution or less backwards. The observers then exchange positions. The rear observer carrying with him his lever applies it to the front car and brings the

front end graduation to bisection without disturbing the micrometer threads from their previous position; while at the signal "read" the front observer bisects the rear end graduation by moving the threads with the micrometer screw. They then announce the readings as before and the recorder jots them down, notifying the observers at the time if the screw revolutions differ from their previous values. This process eliminates the personal equation of the observers and checks any blunders of whole revolutions in reading the microscopes, each of them being read four times, and the four readings being the same within a few microns. The probable error of a bisection is less than $\pm 1^{\mu}$.

While the bar is in position under the microscopes, the third observer measures the distance of the front end (and the rear end at starting) of the axis of the bar from the reference line, and adjusts the sector-level bubble to center, taking care at the same time to keep away from the microscope posts when the bar is observed. The grade sector reading is then made and recorded, and the bar is rolled rapidly forward to a new position.

As soon as the rear end of the bar is brought safely to position under a microscope the one previously at the rear end is taken up and carried forward by the microscope porter, who clamps and adjusts it on a new post. Likewise as soon as a section of track is passed over it is carried forward to a new position.

The observers stand on platforms which rest at their ends on the ground at a distance of about 1 metre on either side of a microscope post.

At intervals of twenty to forty minutes fresh ice is supplied, the trough being run to the rear of or ahead of the two microscopes which were last used. The trough is completely uncovered in this operation and the ice stirred up and supplemented by the amount requisite to replace the waste. This amount is usually 3 to 5 kilogrammes.

The speed of measurement has varied somewhat with circumstances. It has usually been about 100^{m} per hour; 750^{m} were measured in seven hours on two different dates; and a kilometre would not be an excessive day's work.

(11) *Specimen of record and computations.*—Tables I and II following give a copy of the records of a measure with the iced bar of the interval of the 100^{m} comparator of Holton Base described in section 2, Chapter III. In Table I the first column gives the number of the bar length applied in the measure, and the second the time of application. The third column gives the number of the microscope used at the left-hand end of the bar, the left and right-hand ends of the bar being to the left or right, respectively, of the observer as he stands facing the bar while reading the microscopes. The fourth column gives the microscope readings on the left-hand end of the bar, the readings being expressed in screw revolutions of $0{\cdot}1^{mm}$ each, so that $0{\cdot}01^{r} = 1^{\mu}$. The fifth and

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sixth columns give the corresponding data for the right-hand end of the bar. The letters S and W preceding the first two readings of microscopes 1 and 2 show the order in which the observations on the bar were made. Thus observer S read with microscope No. 2 on the right-hand end of the bar, while observer W read with microscope No. 1 on the left-hand end of the bar. The observers then exchanged positions, S reading with No. 1 and W with No. 2; and this order of reading was followed throughout the work. The seventh column gives the differences in microns between the mean readings of any microscope at the two ends of the bar in the order left minus right or $(l-r)$. Thus the first value in this column comes from the readings of microscope No. 2, namely, $\frac{1}{2}(21{\cdot}33^r + 21{\cdot}26^r) - \frac{1}{2}(21{\cdot}33^r + 21{\cdot}33^r) = -3^\mu$, neglecting tenths of microns. The last value in this column is derived in the same manner from the readings of microscope No. 1 at the beginning and end of the measure. The last column gives the height of the left-hand end of the bar at starting, and the heights of the right-hand end in its several positions, above the horizontal plane of reference of the comparator. This method of determining grade corrections was used in place of reading the grade sector. The measurement in this case proceeded from left to right.

Table II gives the records of the cut-off measures at starting and stopping. The first column gives the time of day. The second column designates the orientation of the cut-off scale, the terms right and left being used in the same sense as explained above. The third column gives the number of the graduation line of the scale observed on with the microscope. When the microscope is to the right of the zero of the cut-off scale the images of the graduation numbers appear inverted as seen through the microscope. Hence the letters I (inverted) and E (erect) serve to show the position of the microscope with respect to the axis of the cut-off cylinder. The fourth column gives the microscope readings on the cut-off scale; the fifth gives the height of the scale above the center of the cut-off sphere; and the last two columns give the readings of the level, l and r denoting the left and right hand positions respectively of the bubble.

TABLE I.—*Specimen of record. Measure of 100m Comparator interval, August 7, 1891.*

No. bar lengths.	Time, p. m.	Left-hand end of bar.		Right-hand end of bar.		L.—R.	Height of bar.
		No. microscope.	Microscope reading.	No. microscope.	Microscope reading.		
			Rev.		*Rev.*		*mm.*
1	3·54	1	W. 20·83	2	S. 21·33		491
			S. 20·83		W. 21·33	— 3μ	504
2	·56	2	21·33	3	20·73		
			21·26		20·73	— 3	500
3	4·01	3	20·74	4	21·26		
			20·66		21·26	— 2	509
4	·03	4	20·26	1	21·22		
			20·21		21·22	— 4	502
5	·05	1	21·22	2	22·03		
			21·13		22·03	— 0	506
6	·08	2	22·03	3	21·02		
			22·03		21·02	— 4	505
7	·11	3	21·02	4	21·57		
			20·95		21·57	— 5	507
8	·12	4	21·57	1	20·60		
			21·47		20·60	— 4	505
9	·15	1	20·60	2	23·10		
			20·52		23·10	— 5	509
10	·17	2	23·10	3	21·30		
			23·00		21·30	— 2	514
11	·20	3	21·30	4	21·66		
			21·26		21·66	— 8	520
12	·25	4	21·66	1	20·81		
			21·50		20·81	— 5	516
13	·28	1	20·81	2	21·91		
			20·72		21·91	+ 1	522
14	·31	2	21·91	3	20·97		
			21·94		20·97	— 7	524
15	·34	3	20·97	4	21·74		
			20·84		21·74	+ 2	523
16	·36	4	21·74	1	21·49		
			21·77		21·50	+ 1	515
17	·40	1	21·50	2	22·01		
			21·52		22·01	— 5	523
18	·44	2	22·01	3	20·70		
			21·91		20·70	— 2	520
19	·48	3	20·70	4	21·21		
			20·66		21·21	+ 3	530
20	·49	4	21·21	1	21·80		
			21·28		21·80	—97	524

TABLE II.—*Cut-off measures. August 7, 1891.*

AT WEST (LEFT-HAND) END OF COMPARATOR.

[Repsold cut-off cylinder.]

Time, p. m.	End of cut-off scale, right.	Scale division observed.	Microscope reading.	Height of scale.	Level readings.	
					l.	*r.*
			Rev.	*mm.*	*d.*	*d.*
3·46	A	29 1	18·55	840	5·5	9·0
	B	29 1	20·83		4·5	10·0

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AT EAST (RIGHT-HAND) END OF COMPARATOR.

[U. S. Coast and Geodetic Survey cut-off cylinder.]

4·46	A B	10 E 11 E	25·70 18·46	838	3·0 5·0	3·0 1·0

Value of 1 division of level on Repsold cylinder = 1″·6
Value of 1 division of level on U. S. Coast and Geodetic Survey cylinder = 6″·0

The computation of the measured distance proceeds by means of the following formulas:

Calling R and L the mean readings of any microscope on the right and left hand ends respectively of the bar, the correction due to these readings (supposing them to increase from right to left as with the micrometers used) is

$$\Sigma\,(L-R). \qquad (1)$$

Similarly, if R_c and L_c are the mean microscope readings on the cut-off scales at the right and left hand ends, respectively, of the line, they give a correction equal to

$$-(L_c-R_c). \qquad (2)$$

Denote the reading on the cut-off scale at the right-hand end of the line by S_r, and that at the left-hand end by S_l. Then they give a correction of

$$S_l-S_r, \qquad (3)$$

where S_l and S_r are plus or minus according as the image of the graduation-mark number appears inverted or erect.

Let I_r and I_l denote the inclinations of the cut-off cylinders at the two ends of the line, and H_r and H_l the corresponding heights of the cut-off scales. Then, since the inclinations are always small, they give a correction of

$$H_l\,I_l-H_r\,I_r; \qquad (4)$$

where the inclinations are expressed in arc and both are supposed to be toward the right from the vertical. If l_1, l_2, r_1, r_2 are the left and right hand readings of the level bubble in any case, and v is the value of one division of the level in seconds of arc, the corresponding value of I is

$$I=\tfrac{1}{4}\,(l_1+l_2-r_1-r_2)\,\frac{v}{\rho''},$$

where $\rho''=206264''{\cdot}8$, or the number of seconds in the radius. Using the values given in the above table we have

$$\tfrac{1}{4}\,H_l\frac{v}{\rho''}=1{\cdot}63^{\mu},$$

$$\tfrac{1}{4}\,H_r\frac{v}{\rho''}=6{\cdot}07^{\mu}.$$

The correction for grade or slope of any bar length is expressed with sufficient accuracy by the formula $-\frac{(\Delta h)^2}{2s}$, where Δh is the difference in

height of the two ends of the bar and s its length. For Δh in millimetres, this gives a correction of $-0{\cdot}1\ (\Delta h)^2$ microns per bar length, since $s=5^m$ very nearly. The correction for grades is then in microns

$$-0{\cdot}1\ \Sigma(\Delta h \text{ in millimetres})^2. \qquad (5)$$

The correction for alignment was computed by the same formula (5) in the case of the field measures. For the measures on the comparator here considered the correction for alignment was made nil by always placing the bar in the vertical plane of the comparator. When grades are observed by means of a sector, as in field work, it is most convenient to make use of a table giving the corrections in terms of the grade angles.

Finally, denoting the number of bar lengths measured by N, the length of the bar by B_{17}, and the distance between the terminal spheres by D, we have

$$D=NB_{17}+(1)+(2)+(3)+(4)+(5).$$

Using the data given in the above tables we have the following values of the several corrections:

	mm.
(1)	$-$ 0·149
(2)	$+$ 0·239
(3)	$+$39·500
(4)	$-$ 0·039
(5)	$-$ 0·075
Sum	$+$39·476

Hence, the length of the comparator interval resulting from this measure is

	mm.
20 B_{17}	$+$39·476

CHAPTER II.

LENGTH OF ICED BAR B_{17}.

DIRECT DETERMINATION ON OFFICE COMPARATOR.

(1) *Description of apparatus and method.*—The most important operation attending the use of the iced-bar apparatus is the determination of the length of the measuring bar in terms of one of the International Prototype Metres. The general plan which has been followed in the execution of this operation is that of direct comparison of the bar and Prototype Metre, each being packed in melting ice. According to this plan the use of thermometers is dispensed with entirely. It suffices simply to measure a distance of five times the length of the metre when packed in melting ice, and then transfer this distance to the 5^m bar packed in melting ice.

The means available for such work at the time the apparatus was constructed were the office comparator and comparing room. The latter is an underground room situated between the front of the Survey building and the adjacent street. The comparator consists of an iron **I**-beam about $6{\cdot}5^m$ long, supported at its ends on brick piers. Between these piers micrometer microscopes can be attached to the beam at any desired interval up to 6 metres. For supporting the bars under comparison, a car moving at right angles to the **I**-beam is provided. The car rails rest on piers well removed from those supporting the beam.

The comparing room is lighted by electric lights which can be placed in any convenient position for illuminating or reading the microscopes.

For the special comparisons in question six micrometer microscopes were attached to the **I**-beam at intervals of 1 metre. The means of attachment were cast-iron brackets which could be rigidly clamped to the beam, while the microscopes themselves were fastened to the shelves of the brackets. The brackets are adjustable on the beam; so that the focal planes of the microscopes can be brought into the same horizontal plane, very nearly, and the axes of the microscopes made vertical within narrow limits. The microscopes in turn are provided with lateral adjustments on the shelves of the brackets. Hence the optical axes of the microscopes can be brought into one vertical plane, very nearly, and the distances between consecutive axes can be made closely equal to 1 metre.

The microscopes used in all determinations of the length of the iced bar No. 17 are Nos. 5 and 6 of the Survey and Nos. 1, 2, 3, and 4 of the Repsold base apparatus loaned to the Survey by the Corps of Engineers, U. S. Army. The magnifying power of the two former is 50 diameters; of the four latter 27 diameters. The Survey microscopes Nos. 5 and 6 were mounted at the ends of the comparator, while the Repsold

microscopes were placed in the intermediate positions. These latter microscopes have each about 2^{cm} lateral motion by means of a fine screw, so that their axes could be easily put in line with the axes of the end microscope at any time. This adjustment was accomplished by bringing a fine thread well stretched to focus under the terminal microscopes and then moving the intermediate microscope till their axes were coincident with the thread. The error of this adjustment has never exceeded $0{\cdot}2^{mm}$. The field illumination of the microscopes was secured by glass prisms attached in front of the microscope objectives. Light was thrown into the prism and thence to the object observed by means of lenses in front of the illuminating lamps.

In making comparisons the 5^{m} bar B_{17} was mounted in its Y-trough, iced, and aligned in the manner already described. The trough was supported on its lifting jacks, which rested on the comparator car. The metre used was Prototype No. 21. It was mounted in a wooden box whose interior depth, breadth, and length are $11{\cdot}0^{cm}$, $12{\cdot}5^{cm}$, and $115{\cdot}0^{cm}$, respectively. It was supported by light brass suspension hooks attached to wooden crosspieces which rested on the sides of the box. The hooks were placed at distances of 20^{cm} from the ends of the metre. In packing the metre for comparisons it was completely surrounded to a depth of 5^{cm} to 8^{cm} by a mixture of ice pulverized in the ice crusher and by the snow-like particles cut with a jack plane. These finer particles could be crowded into the grooves of the bar (which has an X cross section) so that the ice would be in actual contact with the surface of the bar throughout its length. After thus covering the metre completely small holes were made through the snow packing at the ends of the box down to the graduated surfaces, which were the only parts of the metre exposed to the external air. To remove condensed moisture, which is sometimes troublesome, from these polished surfaces, a dry camel's hair brush was found very effective. The upper surface of the ice pack and the sides of the box were protected by a heavy felt wrapping, through which small holes were cut over the graduation marks. The box was provided with 3-foot screws which stood on plates of glass resting on the platform of the comparator car. These foot screws served to bring the ends of the metre to focus under the microscopes, the requisite lateral and longitudinal adjustments being secured by sliding the box on its foot screws on the smooth glass.

To pack the 5^{m} bar 40 to 45 kilogrammes of ice were used, and 10 to 12 kilogrammes to pack the metre. The ice was renewed at intervals of twenty to forty minutes, but the melting was generally very slight during such intervals.

Assuming constancy of temperature of the bars under comparison, the precision of the method just outlined evidently depends on the stability of the microscopes used. The first series of comparisons, made in July, 1891, indicated marked instability of the microscopes; and several series made during February to May, 1892, rendered it certain

that the office comparator described above was quite inadequate to the work in question. It is to be remarked, however, that this comparator was not designed to meet the requirements of such work, but rather for the intercomparison of bars of the same length. Its use for the earlier determinations of the length of the iced bar was suggested by considerations of economy, and it was hoped that with careful manipulation it would answer the purpose. The most important result, however, of this earlier work was a demonstration of the necessity of a different sort of comparator.

The defects of the office comparator for our purpose may be ascribed chiefly to the mode of mounting the microscopes. Being attached as explained above by means of brackets clamped to the I-beam, their focal planes fall at a distance of about $0{\cdot}5^{m}$ below the beam. Any change of curvature or temperature of that beam will therefore cause relative displacements of the microscopes. Resting as the beam does with the friction due to its weight on the piers, it is in general in a state of longitudinal stress, which is frequently relieved by vibrations communicated to the piers by vehicles passing in the adjacent street. The temperature of the comparing room changes very slowly from day to day when not occupied long by the observer, or when lighted for short intervals. But the comparisons in question required occupying the room for some hours per day, and the heat from the observers and electric lights caused notable changes in the temperature of the beam, although it was wrapped with cotton batting and encased by a wooden box. These two sources of disturbance caused large irregular displacements of the microscopes and hence large ranges in the resulting values for the length of B_{17}.

(2.) *Programme of observations.*—The observations were made in such a manner as to eliminate the effects of regular displacements of the microscopes. Thus, calling the 5^{m} bar B_{17}, the prototype metre No. 21 M_{21}, and D the distance between the terminal microscopes of the comparator, the following scheme was adopted in the earliest determinations of July, 1891:

(1) A measure of D with M_{21}.
(*a*) (2) A measure of D with B_{17}.
(3) A measure of D with M_{21} in reverse direction.

In the subsequent determinations of February and March, 1892, the above scheme was supplemented by additional measures with B_{17}. Thus the programme was:

(1) A measure of D with B_{17}.
(2) A measure of D with M_{21}.
(3) A measure of D with B_{17}.
(*b*) (4) A measure of D with B_{17}.
(5) A measure of D with M_{21} in reverse direction.
(6) A measure of D with B_{17}.

In some instances, also, in these later determinations but one measure of D was made in the place of (3) and (4).

In all determinations of length the observations of the two ends of B_{17} and M_{21} were made simultaneously by two different observers. The observers also in every case exchanged positions immediately and read on the opposite ends of the bars, in order to eliminate personal equation, in so far as elimination can be accomplished by such interchange. In measuring the distance between the terminal microscopes of the comparator with M_{21} care was taken to make the dependence on the low-power intermediate microscopes small by confining the measures with them to a few microns in each case. The small inequalities in heights of the focal planes of the microscopes were measured by observing the inclination of a suitably delicate level tube attached to M_{21} when the latter was brought to focus under the microscopes. Repeated observations by this method gave accurate corrections for such inequalities.

(3.) *Computation of length of* B_{17}.—The method of deriving the length of B_{17} from the observations will depend in general on the hypothesis we adopt with respect to the motions of the microscopes. The number of hypotheses available is restricted, also, by the number of observations. Ignoring for the moment the displacement of the intermediate microscopes, and considering the second programme of the preceding section, we may suppose, for example, first, that the distance D between the zeros of the terminal microscopes remains invariable from the time of (1) to the time of (6); second, that D varies uniformly with the time from (1) to (6); third, that D varies uniformly at a rate z_1, say, from (1) to (3), and uniformly at a rate z_2 from (4) to (6); fourth, that D varies at a rate $z + 2wt$ from (1) to (6), z and w being constants and t the time beginning with (1). All these hypotheses were tried in computing the results of the observations of February and March, 1892, made according to the more elaborate programme given above. The residuals brought out by these processes were, however, far greater than could be attributed to errors of the observational class, and they showed clearly that the suppositions were rarely in accord with the facts. The reason for this lack of accord is that the processes do not take adequate account of the irregular motions of the microscopes, and especially of the intermediate microscopes.

It does not seem worth while, therefore, to set down all the formulas which result from the above suppositions and which were tried as stated. It may suffice to give those dependent on the fourth supposition. We observe, first, that the measures (1) and (6) and (2) and (5) were made at equal intervals, respectively, from the observations (3) and (4). The latter two were made so near together that they may be considered simultaneous. Secondly, the intervals (1) to (3) and (4) to (6) were about twice the intervals (2) to (3) and (4) to (5). Now let q_n be the micrometric quantity measured in any of the cases (1) to (6), and

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let v_n be the corresponding correction to q_n. Then, assuming the mean of the times of (3) and (4) as the epoch for D, we have from the several measures—

$$\begin{aligned}
(1)\quad D &= B_{17} - 2z + 4w + q_1 + v_1,\\
(2)\quad &= 5M_{21} - z + w + q_2 + v_2,\\
(3)\quad &= B_{17} + q_3 + v_3,\\
(4)\quad &= B_{17} + q_4 + v_4,\\
(5)\quad &= 5M_{21} + z + w + q_5 + v_5,\\
(6)\quad &= B_{17} + 2z + 4w + q_6 + v_6.
\end{aligned}$$

Putting for brevity

$$\begin{aligned}
x &= 5M_{21} - B_{17},\\
y &= D - 5M_{21},
\end{aligned}$$

the above relations give the following observation equations:

$$\begin{aligned}
x + y + 2z - 4w - q_1 &= v_1,\\
y + z - w - q_2 &= v_2,\\
x + y - q_3 &= v_3,\\
x + y - q_4 &= v_4,\\
y - z - w - q_5 &= v_5,\\
x + y - 2z - 4w - q_6 &= v_6.
\end{aligned} \qquad (1)$$

Calling the weights of the measures of D with B_{17} each unity, and the weights of the measures with M_{21} p, each, the normal equations from (1) give the following values:

$$\begin{aligned}
x &= \tfrac{1}{8}(q_1 + 3q_3 + 3q_4 + q_6) - \tfrac{1}{2}(q_2 + q_5),\\
y &= \tfrac{1}{2}(q_2 + q_5) - \tfrac{1}{8}(q_1 - q_3 - q_4 + q_6),\\
z &= \frac{2q_1 + pq_2 - pq_5 - 2q_6}{8 + 2p},\\
w &= \tfrac{1}{8}(-q_1 + q_3 + q_4 - q_6).
\end{aligned} \qquad (2)$$

It will be observed that the objective quantity x is independent of the weight p. This weight p, in fact, affects only the rates z, z_1, z_2 of the second, third, and fourth assumptions stated above. It may be remarked also that the value of x is the same for the first three assumptions, and differs from the value given by the fourth by the quantity w.

In the case of three measures only of D, as in the determinations of July, 1891, we are virtually restricted to three unknown quantities, and hence to the assumption that D varies at a constant rate. If in this case q_1, q_2, q_3 denote the micrometric quantities of the three values of D we have—

$$\begin{aligned}
x &= q_2 - \tfrac{1}{2}(q_1 + q_3),\\
y &= \tfrac{1}{2}(q_1 + q_3),\\
z &= \tfrac{1}{2}(q_1 - q_3).
\end{aligned} \qquad (3)$$

Since the errors of observation in all determinations of B_{17} on the office comparator are completely masked by the large displacements of the microscopes, and since these determinations are virtually super-

seded by much better ones made as explained in section 13, it does not seem worth while to give anything but the resulting values of x from these earlier determinations.

(4) *Length of B_{17} from observations of July, 1891.*—It being deemed desirable to know the approximate length of B_{17} before using it in the measures of Holton Base, a few determinations were made just prior to shipping the apparatus to Holton, Ind., in July, 1891. The observations were made by Assistants O. H. Tittmann and R. S. Woodward, and the record was kept by Mr. John F. Hayford. The programme (*a*) of section 2 was followed. The computations were made by Mr. Hayford and checked by Mr. Tittmann. Table I following gives the resulting values of $x = 5M_{21} - B_{17}$. The first column gives the date, the second the orientation of M_{21}, the third the values of x for the different orientations of B_{17}. The value of M_{21} at 0° C. is given, on p. 83 of "Rapport sur la construction, les comparaisons et les autres opérations ayant servi à déterminer les équations des nouveaux Prototypes métriques," as

$$M_{21} = 1^m + 2{\cdot}5^\mu \text{ at } 0° \text{ C}.$$

TABLE I.—*Values of $x = 5M_{21} - B_{17}$ at 0° C.**

Date.	End of M_{21} north.	Values of x.	
		A end of B_{17} north.	B end of B_{17} north.
1891.			
July 6	A	$+24{\cdot}0^\mu$	
6		17·8	
7		26·5	
7		17·2	
8	A	22·6	
9	B		$+15{\cdot}8^\mu$
10			15·0
10			36·4
11			22·6
11	B		27·0

*The average temperature shown by thermometers alongside of the I-beam during these comparisons was 21° C.

Since it appears highly probable, as shown below, that the observed length of B_{17} differs with differing aspects of its terminal graduations, we shall derive mean values of x in every case for each of the two orientations of B_{17}. Thus the mean values from the above table are

$$+ 21{\cdot}6^\mu \pm 2{\cdot}0^\mu \text{ for } A \text{ end of } B_{17} \text{ north},$$
$$+ 23{\cdot}4^\mu \pm 2{\cdot}0^\mu \text{ for } A \text{ end of } B_{17} \text{ south}.$$

The probable error assigned to each result here is derived in the usual manner from the ten observations of the two derived quantities. The probable error of a single value of x in the system is $\pm 4{\cdot}5^\mu$, which is a little less than the millionth part of the length of B_{17}.

Length of Iced Bar B_{17}.

To get the corresponding lengths of B_{17}, the above mean values must be increased by $1{\cdot}0^{\mu}$, the correction for inequalities in heights of the focal planes of the microscopes in this case, and the above value of M_{21} applied. Thus we have the following results for length of B_{17} in melting ice.

$$B_{17} = 5^{m} - 10{\cdot}1^{\mu} \pm 2{\cdot}0^{\mu} \text{ for } A \text{ end of } B_{17} \text{ north},$$
$$B_{17} = 5^{m} - 11{\cdot}9^{\mu} \pm 2{\cdot}0^{\mu} \text{ for } A \text{ end of } B_{17} \text{ south}.$$

(5) *Length of B_{17} from observations of February and March, 1892.*—The observations in the more extended series of determinations of February and March, 1892, were made by John S. Siebert and R. S. Woodward. Mr. Siebert also made the record, and in order to facilitate his work all of the micrometer head readings (after the bisections had been made) were observed and called out by Woodward. Programme (*b*) of section 2 above was followed except in a few instances when but one measure was made with B_{17}, instead of (3) and (4) of the programme. Table II following gives the results of these determinations, the arrangement being the same as in Table I. The computations of the quantity x were made by Siebert, in accordance with the simpler assumptions explained in section 3, while the corresponding values resulting from the fourth of those assumptions were derived by Woodward. The two sets of values were checked by means of their difference w. This quantity never exceeded $2{\cdot}8^{\mu}$ and was generally less than $0{\cdot}5^{\mu}$. Its use does not give residuals materially less than the simpler hypothesis.

TABLE II.—*Values of $x = 5M_{21} - B_{17}$ at 0° C.**

Date.	End of M_{21} north.	Value of x. A end of B_{17} north.	Date.	End of M_{21} north.	Value of x. A end of B_{17} south.
1892.			1892.		
Feb. 4	A	$+30{\cdot}5^{\mu}$	Feb. 19	A	$+26{\cdot}4^{\mu}$
5	B	24·4	20	B	13·9
5	B	23·3	23	A	27·0
6	A	33·9	24	B	24·9
8	B	37·6	25	A	23·3
8	B	27·2	25	A	29·8
10	A	33·4	26	B	21·9
10	A	25·4	26	B	35·0
11	B	22·1	26	B	29·1
13	A	26·5	27	A	31·3
13	A	24·4	27	A	25·4
15	B	30·4	Mar. 7	A	33·6
15	B	31·4	7	A	29·1
16	B	22·4	8	B	28·6
16	B	18·6	8	B	28·1
16	B	23·5	9	A	36·7
17	A	13·2	10	B	35·4
17	A	25·2	11	A	34·8
18	A	31·4	12	B	24·0
18	A	30·0			

* During the time these determinations were made, the air temperature in the comparing room varied from about 5° C. up to about 10° C., with an average of about 6° C.

Computing the mean values from Table II in the same manner as the means from Table I were computed, we have,

$$x = + 26{\cdot}7^{\mu} \pm 0{\cdot}8^{\mu} \text{ for } A \text{ end of } B_{17} \text{ north,}$$
$$x = + 28{\cdot}3^{\mu} \pm 0{\cdot}8^{\mu} \text{ for } A \text{ end of } B_{17} \text{ south.}$$

The probable error of a single determination of x from this series is $\pm 3{\cdot}6^{\mu}$, which indicates an appreciable advantage of the programme (b) over (a).

If we group the results in Table II with respect to the orientation of the metre, we find the following mean values of x:

$+ 27{\cdot}4^{\mu} \pm 1{\cdot}2^{\mu}$ for A end of M_{21} north,
$+ 26{\cdot}1^{\mu} \pm 1{\cdot}2^{\mu}$ for A end of M_{21} south. } A end of B_{17} north,

$+ 29{\cdot}7^{\mu} \pm 1{\cdot}0^{\mu}$ for A end of M_{2} north,
$+ 26{\cdot}8^{\mu} \pm 1{\cdot}4^{\mu}$ for A end of M_{21} south. } A end of B_{17} south.

The values of x in Table II require a correction of $+0{\cdot}4^{\mu}$ for inequalities in heights of the focal planes of the microscopes. Applying this correction and the value of $5M_{21}$ at 0° C. there result for the length of B_{17} in melting ice—

$$B_{17} = 5^{m} - 14{\cdot}6^{\mu} \pm 0{\cdot}8^{\mu} \text{ for } A \text{ end of } B_{17} \text{ north,}$$
$$B_{17} = 5^{m} - 16{\cdot}2^{\mu} \pm 0{\cdot}8^{\mu} \text{ for } A \text{ end of } B_{17} \text{ south.}$$

(6) *Sources of error.*—It will be observed that the individual values of x in Tables I and II exhibit wide ranges; amounting to 21^{μ} in the former and to 24^{μ} in the latter, or, at the maximum to 1/200 000 part of the length of B_{17}. Considering errors of observation alone, it appears from the first of equations (2), section 3, that they should give to a single determination of x in Table II a probable error of $\pm \frac{1}{4} \varepsilon \sqrt{45}$, where ε is the probable error of a single microscope reading. The value of ε does not exceed $0{\cdot}5^{\mu}$, so that the probable error of a single value of x due to errors of observation is less than $\pm 0{\cdot}8^{\mu}$. Since the total probable error of a single value of x in Table II is, as given above, $\pm 3{\cdot}6^{\mu}$, the reason for directing attention to other sources of error than those of observation is evident.

This view of the inquiry was reached long before the series of determinations condensed in Table II was completed. During the progress of this work, in fact, many experiments were made with the hope of locating the larger sources of error and devising means for their elimination or correction. Thus, (a), delicate levels were attached to the brackets of the terminal microscopes of the comparator. They indicated that the microscopes were subject to both sudden and progressive displacements, which were frequently large in amount. (b). The question whether these displacements could be due to the varying positions of the observers and the microscope car was examined, with the result that no effects due to the weight of either were produced. (c). The hypothesis was entertained that B_{17} might be subject to longitudinal stress, and hence to changes of length, from friction on the sup-

ports in its Y-trough. This hypothesis was tested by subjecting the trough to tension and compression alternately in quick succession, and observing the length of B_{17} in terms of the distance between the terminal microscopes which were sufficiently stable for the brief interval required. Although the Y-trough was put under much greater stress than that to which it is ever subject in the uses it is designed to meet, no change was discovered in the length of B_{17}. Finally, it may be said, that a careful study of the variations in the intervals between consecutive microscopes, as measured with M_{21}, together with the results of the experiments just outlined, led to the conclusion that the irregular motions of the microscopes would suffice to account for the wide range amongst the derived results.

The important question then arose whether the results in Tables I and II were affected by detrimental constant errors. The application of the levels to the microscope brackets, as explained above, indicated that such might be the case. It became desirable, therefore, to determine the length of B_{17} in some other way. The method which suggested itself by reason chiefly of its economy, and the results attained in its application are explained in the following sections.

DETERMINATION OF LENGTH OF B_{17} BY AID OF B_{18} ON OFFICE COMPARATOR.

(7) *Description of method.*—The time required to measure the distance between the terminal microscopes of the comparator with the metre, according to the direct method, was 12 to 20 minutes. It appeared probable, therefore, that the errors due to motions of the microscopes could be much reduced if not eliminated by diminishing the interval during which dependence must hang on their stability. To accomplish this end, a steel bar of the same dimensions as B_{17} was prepared by the Instrument Division of the Survey and subdivided into metre spaces. It was cut down to its mid depth at intervals of one metre and platinum iridium plugs were inserted to receive the fiducial lines. The plugs were carefully polished and the lines were successfully ruled by Mr. E. G. Fischer. This bar is designated B_{18}. It is marked by a letter A at one end and by a letter B at the other.

The method followed in determining the length of B_{17} by aid of B_{18} consisted essentially in comparing each of the sub-spaces of B_{18} with the metre M_{21}, deriving thus the total length of B_{18}, and then comparing B_{17} with B_{18}.

B_{18} was mounted in the steel Y-trough where it could be accurately aligned and where its flexure could be measured. Each graduation plug having in addition to a single transverse line two longitudinal lines separated by an interval of $0{\cdot}2^{mm}$, B_{18} could be aligned in the vertical plane by means of the five microscopes and the lateral adjusting screws of the trough. For alignment in the horizontal plane, the

long striding level was used. Its feet were placed directly over the graduation plugs, these being temporarily protected by slips of smooth paper. It was thus possible to measure the inequalities in height of the plug surfaces to the nearest tenth of a millimetre when the bar was surrounded by ice. The vertical adjusting screws directly below the plugs were first brought to the proper positions. After these were secured, the intermediate vertical screws were brought up to contact with the bar without disturbing its shape as shown by the striding level; all these adjustments being made after the trough had assumed its shape due to a full load of ice. It is to be remarked, also, that B_{18} remained in the Y-trough in the same position throughout the comparisons. Its alignment as shown by the microscopes did not change at its graduation marks by so much as $0{\cdot}2^{mm}$.

For comparisons with B_{18}, B_{17} was mounted in a wooden box provided with adjusting screws in all respects similar to those of the Y-trough.

The following programme was adopted in this determination of the length of B_{17}:

(1) Six measures of the sub-spaces and hence length of B_{18} in terms of M_{21}.

(2) Nine measures of B_{17} in terms of B_{18}.

(3) Six measures of B_{18} as in (1).

(4) Nine measures of B_{17} as in (2).

(5) Six measures of B_{18} as in (1).

In comparing the subspaces of B_{18} with the metre M_{21} the following programme was observed:

(1) Microscope readings on subspace of B_{18}.

(2) Microscope readings on M_{21}.

(3) Repetition of (1).

(4) Repetition of (2).

(5) Repetition of (1).

Similarly, for the comparison of B_{17} and B_{18} the plan followed was:

(1) Microscope readings on B_{17}.

(2) Microscope readings on B_{18}.

(3) Microscope readings on B_{17}.

B_{17} and B_{18} being interchanged alternately.

The observations at the two ends of the bars or sub-spaces were made simultaneously by two observers who always exchanged places and repeated the readings. The observers were John S. Siebert and R. S. Woodward, Mr. Siebert keeping the record as explained in section 5.

(8) *Computation of results.*—In deriving the lengths of the sub-spaces of B_{18} it is assumed that the distance D_i, say, between the zeros of any pair of microscopes used varied uniformly with the time. Hence, if we designate by q_1, q_2, . . . the micrometric quantities observed in accordance with the second programme of section 7; by v_1, v_2, . . . the corresponding corrections to these observed quantities; by z_i, the rate of

Length of Iced Bar B_{17}.

change of D_i; by s_i, the length of a sub-space; and count time from the epoch of the middle observation, we shall have—

$$\begin{aligned} D_i &= s_i - 2z_i + q_1 + v_1, \\ &= M_{21} - z_i + q_2 + v_2, \\ &= s_i + q_3 + v_3, \\ &= M_{21} + z_i + q_4 + v_4, \\ &= s_i + 2z_i + q_5 + v_5. \end{aligned}$$

If, for brevity, we put—

$$\begin{aligned} x_i &= M_{21} - s_i, \\ y_i &= D_i - M_{21}, \end{aligned}$$

the above relations give the following observation equations:

$$\begin{aligned} x_i + y_i + 2z_i - q_1 &= v_1, \\ y_i + z_i - q_2 &= v_2, \\ x_i + y_i - q_3 &= v_3, \\ y_i - z_i - q_4 &= v_4, \\ x_i + y_i - 2z_i - q_5 &= v_5. \end{aligned} \qquad (1)$$

The normal equations corresponding to these give—

$$\begin{aligned} x_i &= \tfrac{1}{3}(q_1 + q_3 + q_5) - \tfrac{1}{2}(q_2 + q_4), \\ y_i &= + \tfrac{1}{2}(q_2 + q_4), \\ z_i &= \tfrac{1}{10}(2q_1 + q_2 - q_4 - 2q_5). \end{aligned} \qquad (2)$$

It may be observed that the values of x_i and y_i in (2) are precisely the same as those which would result if D_i were assumed to remain invariable, or the same as if z_i were omitted from (1). The introduction of z_i serves simply to give the change in D_i and diminish the residuals v.

The computations were made in duplicate according to the above formulas by Messrs. Siebert and Woodward, the residuals as well as the objective quantity x_i being worked out in every case.

In deriving the length of B_{17} in terms of B_{18}, the computation was in all respects similar to that just outlined, and it was made by the same persons.

(9) *Summary of results.*—Table III following gives the resulting values of x_1, x_2, . . ., or x_i, for the several sub-spaces of B_{18}. x_1 refers to the first sub-space of B_{18} counting from the A end towards the B end; x_2 to the next space in the same order, etc. The first column of the table gives the date of the determination; the second the orientation of the metre M_{21}; the next five give the values of x_i; the eighth gives the sum of the x_i, or the excess of $5M_{21}$ over the whole length of B_{18}; and the last column gives the mean values of Σx_i for the three groups of determinations. It will be remembered that the length of any sub-space s_i is given by the equation

$$s_i = M_{21} - x_i.$$

TABLE III.—*Values of x_i and Σx_i for B_{18} at 0° C.*

Date.	End of M_{21} north.	x_1.	x_2.	x_3.	x_4.	x_5.	Σx_i.	Mean values of Σx_i.
1892.		μ	μ	μ	μ	μ	μ	
Apr. 25	A	+13·4	+0·2	+12·8	+3·2	+29·6	+59·2	
26	B	+14·0	+2·2	+10·3	+4·7	+29·3	+60·5	
26	B	+15·1	−1·3	+9·8	+3·1	+30·9	+57·6	
27	A	+9·8	+3·1	+6·2	+5·7	+30·4	+55·2	
27	A	+13·3	+2·2	+9·3	+4·6	+29·6	+59·0	μ
28	A	+10·7	+4·7	+8·9	+2·9	+30·3	+57·5	+58·2
May 2	B	+11·6	+1·2	+2·6	+0·0	+32·3	+47·7	
2	B	+15·2	+2·3	+8·1	+1·4	+30·8	+57·8	
3	A	+14·0	+2·5	+5·5	+3·2	+25·0	+50·2	
3	A	+11·6	+4·3	+5·2	+4·4	+29·8	+55·3	
4	B	+13·2	+1·3	+3·5	+5·0	+31·8	+54·8	
4	B	+14·9	+3·6	+5·6	+4·6	+29·3	+58·0	+54·0
7	A	+14·6	+1·2	+3·8	−1·0	+30·0	+48·6	
7	A	+12·2	−0·1	+4·7	+2·8	+30·3	+49·9	
9	B	+12·2	+1·5	+7·0	−0·2	+31·4	+51·9	
9	B	+12·4	−6·5	+10·9	+2·4	+33·8	+53·0	
10	A	+12·9	−6·6	+11·9	+1·1	+34·8	+54·1	
10	B	+12·9	−5·5	+9·5	+0·8	+27·9	+45·6	+50·5

Table IV gives the results of the comparisons of B_{17} and B_{18}. The first column gives the date of the comparison, the second the orientation of B_{17}, the third the individual value of $B_{17} - B_{18}$ from a set of observations, the fourth the mean value of the set, and the fifth the group mean.

Each result in the third column is derived from three readings on B_{17} and three on B_{18}, the order of readings being B_{17}, B_{18}, B_{17}, and B_{18}. B_{17}, B_{18}, with an interchange of the bars from set to set. The orientation of B_{18}, as explained above, remained unchanged throughout the work. During the intervals between sets of observations the bars were re-iced and the curvature of B_{17} observed with the long striding level.

TABLE IV.—*Values of $B_{17} - B_{18}$ at 0° C.*

Date.	End of B_{17} north.	$B_{17} - B_{18}$.	Mean of set.	Group mean.
1892.		μ		
Apr. 29	A	+29·4		
		28·0	μ	
		30·2	+29·2	
30	A	28·4		
		29·8		
		30·2	+29·5	
30	A	33·8		
		30·0		μ
		32·5	+32·1	+30·3
May 5	B	+26·0		
		28·6		
		26·5	+27·0	
5	B	32·6		
		33·5		
		31·3	32·5	
5	B	30·8		
		32·4		
		27·4	30·2	+29·9

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(10) *Sources of error.*—The principal sources of error to which the results in Tables III and IV are subject are: (a) those of observation, (b) those due to instability of the microscopes, and (c) those due to flexure of B_{18}. The results involve the aggregate effect of these three sources, but it is practicable to estimate their separate effects. This we now proceed to do.

The quantities q in equations (1) and (2) of section 8 are of the form

$$q = \tfrac{1}{2}(R_n + R'_n - R_s - R'_s),$$

where R_n, R'_n, and R_s, R'_s denote the microscope readings at the north and south ends respectively of M_{21} or of a sub-space of B_{18}, as the case may be. Hence if ε is the probable error of a single microscope reading, the probable error of q due to this source alone is also ε; and it follows from the first of equations (2), section 8, that the probable error of a single determination of x_i is $\pm\ \varepsilon\sqrt{\frac{5}{6}}$. The value of ε, as shown by direct observation, is less than $0{\cdot}5^{\mu}$. Hence the probable error of a single value of x_i due to errors of observation alone is less than $0{\cdot}5^{\mu}$.

The assumption adopted in computing the values of the sub-spaces on B_{18} is that the interval between the zeros of the pair of microscopes used in determining any space changed uniformly with the time required to make the observations. If this is not a true assumption, or if the microscopes were subject to large irregular displacements, we should expect to find the probable errors of the quantities q and x_i derived from the residuals v of equations (1), section 8, larger than those given under (a) above. Such is, in fact, the case.

It is essential to observe here that errors due to flexure of B_{18}, or changes in length of the sub-spaces, if they were at all formidable, did not appear in the residuals v of the equations used. Such changes can only occur slowly, from day to day, or during longer periods, since they are primarily due to the change in shape of the Y-trough, which is affected only by marked changes of the surrounding air temperature. The time required to make a set of observations in a comparison of M_{21} with a sub-space was only 6 to 8 minutes. It appears practically certain, therefore, that the residuals brought out in the computation involve only the two sources (a) and (b).

In the 90 determinations of the 5 sub-spaces there were 450 residuals and 270 unknowns, as shown by the formulas (1) and (2) used in the computation. The sum of the squares of these residuals is 351·4, the unit being a micron. Hence if ε_1 denote the probable error of a single observed value of q,

$$\varepsilon_1 = \pm\, 0{\cdot}6745\sqrt{\frac{351{\cdot}4}{450-270}} = \pm\, 1{\cdot}3^{\mu};$$

whence

$$\text{probable error of one value of } x_i = \varepsilon_1\sqrt{\frac{5}{6}} = \pm\, 1{\cdot}2^{\mu},$$

$$\text{probable error of } \Sigma x_1 = \varepsilon_1\sqrt{\frac{25}{6}} = \pm\, 2{\cdot}7^{\mu}.$$

Comparing these errors with those due to observation alone, it appears that the probable error of a value of q due to instability of microscopes is

$$\sqrt{\varepsilon_1^2 - \epsilon^2} = \pm 1{\cdot}2^{\mu},$$

and that the corresponding value for x_i is only slightly less.

An examination of the individual residuals of the different sets of observations indicates that the assumption of a constant rate of change of a microscope interval was frequently not realized. In the entire number of residuals there were twenty whose values rose to or exceeded 2^{μ}, while six exceeded 3^{μ}, the maximum being $3{\cdot}7^{\mu}$. These large residuals, it is believed, are due to sudden displacements of the microscopes. No other hypothesis appears competent to explain them, unless it be that of sudden changes in temperature of the bars under comparison.

The above probable error of Σx_i, or the probable error of one determination of the length of B_{18} is $\pm 2{\cdot}7^{\mu}$. The corresponding value for a single determination of B_{17} by the direct process, as given in section 6, is $\pm 3{\cdot}6^{\mu}$. Hence it would appear that the process of getting the entire length of B_{18} was subject to considerably less error than the direct determination of B_{17}.

The group means in the last column of Table III indicate a progressive change in the length of B_{18} during the series of comparisons, which extended from April 25 to May 10, 1892. The cause of this change, if real, is important, and much study was given to it.

It may be observed, before considering any other cause, that the two sources of error already examined are not altogether incompetent to account for the range which these group means show. That range is about 8^{μ}. The probable error of one group mean is, if we adopt the probable error of Σx_i given above as a basis, $\frac{5}{6}\,\varepsilon_1 = \pm 1{\cdot}0^{\mu}$. Thus a range of 8^{μ} would be accounted for by errors of opposite signs no greater than four times this probable error, but the probability of such an explanation is too small to be satisfactory. Hence we proceed to examine the question of the effect of flexure of B_{18} on its observed length.

Since the surfaces of the intermediate graduation plugs of B_{18} are not in the neutral surface of the bar, any change in its curvature will tend to displace the fiducial lines on those intermediate plugs. This source of difficulty was anticipated, and the measures of the flexure with the long striding level, as already explained, were made for the purpose of supplying any needed correction. It is obvious that no considerable variation in the whole measured length of B_{18} could occur except through large changes in its curvature—such, for example, as a change from marked concavity upwards to marked convexity upwards. No such change occurred, however, the bar remaining, as will

Length of Iced Bar B_{17}.

presently appear, concave upwards throughout the work. Again, since there could be no displacement of the terminal lines of B_{18} from this cause, the sum of the sub-spaces, as measured at any time, should be sensibly free from the effects of displacements of the intermediate lines, unless the bar was subject to marked changes of curvature during the 30 or 40 minutes required to compare the sub-spaces with M_{21}. There is no indication that B_{18} underwent any changes of curvature other than those due to the slowly fluctuating temperatures of the air in the comparing room, which were sufficient, it would appear, to affect the web of the Y-trough and hence B_{18} by small amounts. Moreover, the observed variations inlength of the sub-spaces, if due to displacements of the intermediate graduation lines arising from slow changes in curvature of B_{18}, ought to appear as a linear function of the observed changes of curvature.

Such, in brief, were the considerations which led to the use of B_{18} for the purpose in question, notwithstanding the obvious objection to its form, if invariability in length of the sub-spaces is required. Let us now examine the facts brought out by the observations.

From an inspection of the results in Table III it is seen that the variations of the quantities x_i are greatest for the intermediate sub-spaces. Taking mean values of x_i for the three groups of observations, we have

Date.	x_1.	x_2.	x_3.	x_4.	x_5.
1892.	μ	μ	μ	μ	μ
Apr. 25–28	+12·7	+1·8	+9·6	+4·0	−30·0
May 2– 4	+13·4	+2·5	+5·1	+3·1	−29·8
7–10	+12·6	−2·7	+8·0	+1·0	+31·4

Thus it appears that while the observed values of the terminal sub-spaces are very accordant in these means, the corresponding values of the intermediate sub-spaces are very discordant. The most striking cases are those of x_2 and x_3, which varied by $5{\cdot}2^{\mu}$ and $4{\cdot}5^{\mu}$, respectively, in their mean values; and one might be led to the inference that the intermediate sub-spaces were subject to marked changes in length. In line with this inference it is instructive to compare the above mean values with the ranges among the corresponding individual values of x_i for the several groups. These ranges are for—

Date.	x_1.	x_2.	x_3.	x_4.	x_5.
1892.	μ	μ	μ	μ	μ
Apr. 25–28	5·3	6·0	6·6	2·8	1·6
May 2– 4	3·6	3·1	5·5	5·0	7·3
7–10	2·4	7·8	8·1	3·8	6·9

From these it would not seem safe to infer that the terminal sub-spaces were subject to less error in their determination than the other spaces, although the mean values of those terminal sub-spaces are by far the most accordant. Moreover, an examination of the above group means of x_i does not indicate such displacements of the intermediate graduation marks as would result from slow changes in curvature of B_{18}; for, since the length of any sub-space s_i is

$$s_i = M_{2i} - x_i,$$

it would appear from those means that, while s_1 and s_5 remained invariable, s_2 and s_4 both increased, the one by $4{\cdot}5^{\mu}$ and the other by $3{\cdot}0^{\mu}$, and that the middle space s_3 increased by $4{\cdot}5^{\mu}$ from the first to the second group and decreased by $2{\cdot}9^{\mu}$ from the second to the third group. It seems very improbable that the intermediate graduations could have been so displaced as to leave s_1 and s_5 invariable and at the same time cause the specified changes in s_2, s_3, s_4.

The observed changes in curvature of B_{18} are given in Table V below. These were measured with the long striding level which reached from any plug to the adjacent plug on either side. The value of one division of the bubble used on this level was $15''$. The level was always reversed in observing the relative heights of adjacent plugs: and as the time required to make such a measure was less than two minutes, the observed quantities are subject to no important errors. The quantities given in the table are the ordinates of the intermediate graduation plugs measured at right angles to the straight line joining the terminal plugs of B_{18}. They are designated by y_i, the suffix corresponding to the number of the plug counting from the A end toward the B end. The minus sign indicates that the plug is below the line of reference.*

* If $h_1, h_2 \ldots h_5$ denote the differences in height of the plugs for the successive spaces, and s the slope per metre of the line joining the terminal plugs, the quantities y_i are given by the following equations:

$$y_1 = h_1 - s,$$
$$y_2 = h_1 + h_2 - 2s,$$
$$y_3 = h_1 + h_2 + h_3 - 3s,$$
$$y_4 = h_1 + h_2 + h_3 + h_4 - 4s,$$
$$0 = h_1 + h_2 + h_3 + h_4 + h_5 - 5s.$$

The quantities h_1, h_2, h_3, h_4, h_5 are observed directly with the striding level.

Length of Iced Bar B_{17}.

TABLE V.—*Observed ordinates of curvature of B_{13}.*

Date.	y_2.	y_3.	y_4.	y_5.	Σy_i.
1892.	*mm.*	*mm.*	*mm.*	*mm.*	*mm.*
Apr. 25	—0·25	—0·23	—0·14	—0·37	—0·99
26	·12	·09	·00	·18	·38
27	·05	·17	·03	·24	·50
27	·15	·24	·12	·33	·85
28	·12	·12	·00	·27	·51
29	·24	·19	—·04	·29	·76
30	·13	·04	+·12	·07	·12
May 2	·27	·28	—·02	·45	1·02
3	·18	·24	·06	·42	0·91
4	·30	·42	·27	·40	1·38
5	·45	·50	·35	·31	1·61
7	·25	·35	·26	·39	1·25
9	·22	·35	·25	·27	1·09
10	·46	·54	·37	·37	1·74
10	·48	·50	·37	·35	1·69

The results in this table show that the bar was concave upwards throughout the work. The mean values corresponding to the three groups of comparisons of the sub-spaces are

	y_2.	y_3.	y_4.	y_5.
	mm.	*mm.*	*mm.*	*mm.*
Apr. 25–30	—0·14	—0·17	—0·06	—0·28
May 2– 4	·25	·31	·12	·42
May 7–10	·35	·43	·31	·34

These, with the exception of y_5, show an increase by small amounts of the concavity with the time. This is precisely what we should expect from the slow seasonal rise of the air temperature in the comparing room. They do not appear to bear any relation, however, to the observed changes in the values of the sub-spaces.

What, then, could have been the cause of the large variations in the values of the group means of the intermediate sub-spaces? I am unable to answer this question except with the suggestion that the shifting of the iced bar and metre from one position to another under the beam to which the microscopes were attached produced at times systematic changes in the intervals between those microscopes. If such changes occurred, one would expect them to be greater for the intermediate microscopes than for the terminal ones, for neither the Y-trough nor the box in which the metre was iced projected beyond the terminal microscopes, and currents of air due to the nearness or remoteness of the ice

loads would obviously be most effective in disturbing the intermediate microscopes.

Finally, recurring to the comparisons of B_{17} and B_{18}, the results of which are exhibited in Table IV, no indication is seen of a change in the entire length of B_{18}. In fact, the results in Table IV do not show any striking anomalies. The range amongst the means of the different sets of comparisons is but $5{\cdot}5^{\mu}$, and the group means agree quite as well as one could expect.

(11) *Adopted length of B_{17} derived by aid of B_{18}.*—In the absence of a satisfactory explanation of the variation of the observed values of B_{18} in terms of M_{21}, we can do no better than to take the mean of the three group means in the last column of Table III for the relation of these two bars. This relation is

$$B_{18} = 5\,M_{21} - 54{\cdot}2^{\mu} \pm 1{\cdot}6^{\mu},$$

the probable error being derived from the discrepancies between the three individual values and their mean. To this value there is to be added a correction of $-0{\cdot}1^{\mu}$, which is the average amount by which the distance between the terminal marks of B_{18} fell short of the sum of the sub-spaces. It is found by projecting the sub-spaces on the line joining the terminal marks, the data for such projection being given in Table V. Applying this correction and introducing the value of M_{21} at 0° C., namely,

$$M_{21} = 1^{m} + 2{\cdot}5^{\mu},$$

there results

$$B_{18} = 5^{m} - 41{\cdot}8^{\mu} \pm 1{\cdot}6^{\mu} \text{ at } 0^{\circ}\text{ C.}$$

In deriving values for the relation of B_{17} to B_{18} we may take the means of the observed values for the two orientations of B_{17} and compute the probable errors of these means from all the discrepancies between them and the individual measures on which they depend. Thus we have from Table IV

$$B_{17} = B_{18} + 30{\cdot}3^{\mu} \pm 0{\cdot}9^{\mu}, \text{ A end of } B_{17} \text{ north.}$$
$$B_{17} = B_{18} + 29{\cdot}9 \pm 0{\cdot}9, \text{ A end of } B_{17} \text{ south.}$$

Introducing in these last equations the value of B_{18}, given above, there result for the length of B_{17} in melting ice

$$B_{17} = 5^{m} - 11{\cdot}5^{\mu} \pm 1{\cdot}8^{\mu}, \text{ A end of } B_{17} \text{ north.}$$
$$B_{17} = 5 - 11{\cdot}9 \pm 1{\cdot}8, \text{ A end of } B_{17} \text{ south.}$$

(12) *Values of sub-spaces of B_{18}.*—Since the subspaces of B_{18} afford multiples of one metre up to five metres, it may not be out of place to give the values of the several sub-spaces resulting from these comparisons.

Length of Iced Bar B_{17}.

Taking means of the three group means given in section 10 for each value of x_i, we find

$$\begin{aligned} x_1 &= +12{\cdot}9^{\mu} \pm 0{\cdot}7^{\mu}, \\ x_2 &= +\ 0{\cdot}5 \pm 0{\cdot}7\ , \\ x_3 &= +\ 7{\cdot}6 \pm 0{\cdot}7\ , \\ x_4 &= +\ 2{\cdot}7 \pm 0{\cdot}7\ , \\ x_5 &= +30{\cdot}4 \pm 0{\cdot}7\ . \end{aligned}$$

The probable errors assigned to these values are derived from the residuals resulting from a comparison of each adopted mean with its three group means. Thus the sum of the squares of all the residuals is 33·0. The number of residuals is 15, and the number of unknowns is 5. Hence the probable error of one group mean is $\pm 1{\cdot}2^{\mu}$, and the probable error of each adopted value of x_i is $\pm 1{\cdot}2^{\mu} / \sqrt{3} = \pm 0{\cdot}7^{\mu}$.

Since any sub-space s_i is expressed by the relation

$$s_i = M_{21} - x_i,$$

wherein

$$M_{21} = 1^{m}_{\cdot} + 2{\cdot}5^{\mu},$$

there results for the sub-spaces on B_{18} when it is packed in melting ice

$$\begin{aligned} s_1 &= 1^{m} - 10{\cdot}4^{\mu} \pm 0{\cdot}7^{\mu}, \\ s_2 &= 1 \ + \ 2{\cdot}0 \pm 0{\cdot}7\ , \\ s_3 &= 1 \ - \ 5{\cdot}1 \pm 0{\cdot}7\ , \\ s_4 &= 1 \ - \ 0{\cdot}2 \pm 0{\cdot}7\ , \\ s_5 &= 1 \ - 27{\cdot}9 \pm 0{\cdot}7\ . \end{aligned}$$

These spaces, it may be again remarked, are the successive spaces on B_{18}, counting from its A end toward its B end. It must be observed, also, that these values appertain to B_{18} when supported and aligned in the Y-trough, as already explained at length.

DETERMINATION OF B_{17} ON NEW COMPARATOR.

(13) *Description of apparatus.*—On completing the determinations of B_{17} by aid of the office comparator, and studying the results after the manner detailed in the preceding sections, it became evident that any value which might be adopted for B_{17} would not have a precision comparable with that attained in the use of the iced bar apparatus on the 100^{m} comparator and standard kilometre of Holton Base. It was plain, also, that the principal source of the difficulty which thus presented itself lay in the instability of the microscopes of the office comparator. The question which logically followed, and which it appeared essential to answer, was whether this instability gave rise to any systematic error in the length of B_{17}. The long series of direct determinations of February to March, 1892, did not seem to justify an affirmative answer to this question; but the indirect determinations by aid of B_{17} did not

warrant a negative answer. There appeared to be no way out of this dilemma except to build a comparator designed especially for the purpose of getting the length of B_{17}. Accordingly plans for a new comparator were drawn up and approved by the Superintendent of the Survey, and the work of construction was completed during June and early July, 1892. It was designed with a view to direct determinations by measuring a distance of 5^m with a Prototype in ice and then transferring this distance to B_{17} in the manner already described.

This comparator was constructed in a vacant lot adjacent to the Survey Office. Briefly described, it consists of six brick piers resting on a continuous foundation of 6 cubic metres of well-rammed concrete. The foundation is 1^m by 1^m in cross section and rises to the level of the ground surface. The foundation and piers are both set in Portland cement and their combined weight is about 12 tons (11 000 kilogrammes). The exposed parts of the foundation and the ground near about are covered to a depth of a decimetre with sawdust. The terminal piers were arranged to receive the Survey microscopes 5 and 6, already described in section 1. Connection between these microscopes and the piers was secured by means of heavy cast-iron plates, held fast by long bolts built into the piers, insuring thus ample rigidity. The intermediate piers had built into them heavy stone posts, on which the post caps of the Repsold microscopes could be mounted as when used in fieldwork. These connections were also very rigid; and the great weight of the piers and foundation, which were virtually one mass, gave a very satisfactory degree of stability. The piers were ranged in an east and west line, nearly. To protect them from the sun and storms a shed $8{\cdot}6^m$ long by 3^m wide was built over them. The north side of the shed was left open until after the comparisons of July and August, 1892, were completed, a temporary cover of canvass being applied at night-time, or when the comparator was not in use. It was thus possible to use daylight for illuminating the microscopes, and a free circulation of air was permitted within the shed. To protect the microscopes from currents of air and from the ice packs of the bars, their tubes and their metallic connections with the piers were heavily wrapped with cotton batting.

The axes of the microscopes were made vertical by means of levels attached to their tubes. Their axes were put into one vertical plane and their focal planes made nearly coincident by bringing them all to focus and bisection on a fine thread stretched under them. The remaining small differences in elevation of the focal planes were measured from time to time by observing the readings of a level (having $12{\cdot}5''$ per division) attached to Prototype No. 21 when it was brought to focus under the microscopes.

The metre and 5^m bars were moved under the microscopes on portable tracks supported on posts isolated from the piers. The track for the metre was a stiff truss of cypress wood. One rail of this truss had a

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cross section shaped like a truncated A, while the other was flat. The car carrying the metre box was made to slide on these rails. By means of suitable lateral and longitudinal stops this truss could be held in such a position that only small lateral motions of the metre box on its car were required. These small lateral motions and small longitudinal motions as well, of the box on the car, were accomplished by means of a system of slow-motion screws. The metre box was also provided with three leveling screws. These provisions made it possible to bring the metre to focus, and to position within a few microns, with ease and rapidity.

The 5^m bar was mounted in its Y-trough and moved on the same cars and track used with it in the fieldwork. It was handled, in fact, in the determinations about to be given, in precisely the same manner as in measuring a line in the field.

Both bars were moved under the microscopes along the south side of the piers. When the metre was not under observation it was removed, together with its car and track, to suitable brackets on the south side of the comparator shed. When B_{17} was not under observation it was run out of the east door of the shed on the portable track and left under an awning alongside the supply of ice and the ice-crusher.

The platforms on which the observers stood were isolated from the piers and from the track supports, so that the movements of the observers produced no disturbance of the piers or of the bars. The piers were about 25^m from the nearest street. Only heavily loaded vehicles, which passed quite infrequently on this street, produced any perceptible vibration in the piers. On no occasion was it considered essential to stop observing on account of such minute vibrations.

The piers and the adjacent parts of the shed on the north side were whitewashed, and this aided in producing an abundance of white light for illuminating the microscopes. In this regard the conditions for ease and comfort in observing were much superior to those afforded by the electric lights of the office comparing room.

(14) *Methods of observation.*—The study of the earlier direct determinations of July, 1891, and February and March, 1892, indicated the desirability of increasing the number of measures with the prototype metre relatively to those with B_{17} in any set of observations. Accordingly the following programme was adopted and adhered to throughout the determinations on the new comparator, the symbol D representing as hitherto the distance between the zeros of the terminal microscopes 5 and 6:

(1) A measure of D with the prototype M_{21}.
(2) A measure of D with B_{17}.
(3) A measure of D with M_{21}.
(4) A measure of D with B_{17}.
(5) A measure of D with M_{21}.

The measures with the metre started alternately from the west and east ends of the comparator. The micrometric readings on either bar were made simultaneously by the two observers, who exchanged positions in every case to eliminate personal equation. Small distances, amounting to a few microns only, were measured with the intermediate Repsold microscopes, whose magnifying powers are but twenty-seven diameters, while the terminal microscopes 5 and 6 have a power of fifty diameters.

The measures with B_{17} were made in the manner described at length in section 10 of Chapter I. That is, the rear (west) end of the bar was first brought to bisection by means of the car lever, while the bisection at the front end was made by turning the micrometer screw at that (east) end. The observers then exchanged positions, and the same operation was repeated in the reverse order, the bar being brought to bisection by the car lever at the east end and the micrometer threads moved to bisection at the west end. Two sets of such observations were generally made.

Both bars were fully iced from half an hour to an hour before observing on them. A slightly greater quantity of ice was used on both bars than in the previous determinations made during the winter in the comparing room, where the air temperature averaged about 6° C. The average air temperature during July and August, 1892, when the present determinations were made, was about 35° C. However, when protected by the felt covers of the metre box and Y-trough, the ice did not appear to melt much faster than it did under the far lower winter temperature. Throughout this series of comparisons the ice about B_{17} was shaken up and replenished just before bringing it under the microscopes for observations, and the prototype was similarly treated at intervals of twenty to thirty minutes. From very careful personal attention to the packing of these bars, I am confident that they were subject to no sensible variations in length due to variations in temperature.

The observations were made by Messrs. John S. Siebert and R. S. Woodward, and the record was kept by Mr. Orville G. Brown. The computations were made and checked in all parts by the same parties as the work of observation progressed.

(15) *Method of computation.*—The considerations on which the method followed in deriving the length of B_{17} depends are similar to those set forth in section 3 above. But since the data were in some respects different, it seems proper to state the present case briefly by itself.

If the positions of the microscopes of the comparator remained invariable, or if the measured quantities were affected by errors of observation only, the distance D between the zeros of the terminal microscopes would be given by each of the following equations, corresponding to the several measures of the programme in section 14.

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$$\begin{aligned}
(1)\quad D &= 5M_{21} + Q_1 + v_1.\\
(2)\quad &= B_{17} + Q_2 + v_2.\\
(3)\quad &= 5M_{21} + Q_3 + v_3.\\
(4)\quad &= B_{17} + Q_4 + v_4.\\
(5)\quad &= 5M_{21} + Q_5 + v_5.
\end{aligned}$$

In these, $Q_1\ Q_2 \ldots$ are the micrometric quantities resulting from the several measures, and $v_1\ v_2 \ldots$ are the corresponding corrections. But the hypothesis of constancy of position of the microscopes, or of invariability of D, is inadequate. It appears, however, from the observations, that the assumption of a constant rate of variation of D is quite satisfactory, and this is the one adopted.

The time intervals between the several measures (1) (2) . . . (5) are nearly equal, being about 7·5 minutes each on the average. Hence, if we call z the rate of change of D and put as hitherto

$$\begin{aligned}
x &= 5M_{21} - B_{17},\\
y &= D - 5M_{21},
\end{aligned}$$

and take the time of the measure (3) as the epoch of D, the following observation equations result:

$$\begin{aligned}
y - 2z - Q_1 &= v_1,\\
x + y - z - Q_2 &= v_2,\\
y - Q_3 &= v_3,\\
x + y + z - Q_4 &= v_4,\\
y + 2z - Q_5 &= v_5.
\end{aligned} \qquad (1).$$

If we assign a weight unity to each of the measures with B_{17}, or to Q_2 and Q_4, and a weight p to each of the measures with M_{21}, or to Q_1, Q_3, and Q_5, the normal equations are

$$\begin{aligned}
2x + (3p+2)y &= p(Q_1 + Q_3 + Q_5) + Q_2 + Q_4,\\
2x + 2y &= Q_2 + Q_4,\\
(8p+2)z &= 2p(Q_5 - Q_1) + Q_4 - Q_2.
\end{aligned} \qquad (2)$$

These give

$$\begin{aligned}
x &= \tfrac{1}{2}(Q_2 + Q_4) - \tfrac{1}{3}(Q_1 + Q_3 + Q_5),\\
y &= \tfrac{1}{3}(Q_1 + Q_3 + Q_5),\\
z &= \frac{2p(Q_5 - Q_1) + Q_4 + Q_2}{8p+2}.
\end{aligned} \qquad (3)$$

Since the quantities Q_1, Q_3, Q_5 depend on the half algebraic sum of twenty independent microscope readings, while Q_2 and Q_4 depend on the half sum of four to eight such readings, the probable errors of the former would stand to those of the latter as $\sqrt{5}$ to 1, or as $\sqrt{5}$ to 1/2, as the case may be, provided the errors involved are of the accidental sort. If the errors are those of observation only, the ratios just given are the proper ones for determining the relative weights. But it is certain that the most formidable errors are not due to defects of observation, and hence some doubt may be entertained as to the best

value of the weight p. In order to test the matter with the actual data, half of the observations were treated on the supposition that $p = 1$ and half on the supposition that $p = 1/5$. The results indicated that the latter supposition is the more appropriate one, and it was therefore adopted for all the work. A mere inspection of the data, in fact, shows that the weights of Q_1, Q_3, Q_5, should be much less than the weights of Q_2 and Q_4.

(16) *Data and results.*—Table VI following gives the principal data of the series of determinations of B_{17} on the new comparator. The observations extended from July 18 to August 10, 1892, and the table embraces every determination made. The first column gives the date; the second gives the times of beginning and ending of a set of observations; the third designates the orientation of the prototype metre M_{21}. The next three columns give the quantities Q_1, Q_3, Q_5, which are the observed excesses of the distance D between the zeros of the terminal microscopes over $5M_{21}$, while the last two columns give the quantities Q_2, Q_4, which are the excesses of D over B_{17}. It is to be remarked that these Q's change from time to time by reason of changes in the adopted zeros of the microscopes, as well as by reason of changes in the relative positions of the microscopes themselves. Those zeros which were most convenient in computing the Q's were, in general, adopted, but the same zeros were retained throughout any day's work, so that the stability of the terminal microscopes is well shown by Q_2 and Q_4 especially for any date. For greater clearness of perception the sets of observations are spaced off with extra leads in groups by dates.

TABLE VI.—*Data for length of B_{17} in ice.*

Date.	Time of day.	End of metre west.	Q_1	Q_3	Q_5	Q_2	Q_4
1892.			μ	μ		μ	μ
July 18	1:15 to 1:56 p. m.	A	+ 6·4	+ 0·8	+ 3·8	+19·0	+19·0
	2:14 3:02		+ 6·4	+11·8	+12·1	+31·9	+34·9
20	10:51 11:29 a. m.	B	—30·9	—26·9	—24·0	+ 3·1	+ 1·0
	1:08 1:41 p. m.		—17·4	—21·5	—12·5	+10·1	+ 8·9
	1:54 2:31		— 9·4	— 7·5	— 8·4	+ 5·1	+12·1
21	10:52 11:27 a. m.	A	+ 9·8	— 0·2	— 3·0	+28·8	+32·7
	11:40 12:13 p. m.		+ 0·5	+ 1·8	+ 0·9	+26·8	+27·3
	1:16 1:46		+ 0·2	— 1·3	+ 6·1	+25·6	+29·9
	1:58 2:26		+ 6·6	+ 1·7	— 4·6	+27·7	+27·0
22	10:37 11:09 a. m.	B	+14·3	+14·2	+12·8	+42·0	+42·2
	11:21 11:51		+13·8	+19·6	+17·5	+39·6	+40·4
	1:18 2:00 p. m.		+25·5	+26·2	+26·8	+52·0	+51·7
26	1:52 2:26 p. m.	A	— 4·4	—10·6	— 6·4	+12·6	+17·0
	2:30 2:59		— 6·4	— 7·2	— 4·4	+ 0·9	+11·4
27	10:45 11:13 a. m.	B	+19·5	+18·2	+17·3	+46·3	+41·9
	11:20 11:48		+18·2	+19·0	+18·7	+38·5	+45·8
	12:00 12:26 p. m.		+22·1	+20·3	+19·4	+46·5	+46·2
	1:15 1:51		+25·9	+26·4	+23·8	+52·9	+54·9

Length of Iced Bar B_{17}.

TABLE VI.—*Data for length of B_{17} in ice*—Continued.

Date.	Time of day.	End of metre west.	Q_1	Q_3	Q_5	Q_2	Q_4
1892.			μ	μ	μ	μ	μ
July 28	11:00 to 11:32 a. m.	A	+12·1	+21·8	+17·1	+40·9	+43·8
	11:38 12:10 p. m.		+16·3	+21·5	+19·3	+41·8	+41·9
	1:07 1:36		+18·5	+25·8	+21·4	+48·3	+45·4
Aug. 4	11:54 12:33 p. m.	A	+13·2	+16·8	+17·7	+42·9	+45·8
	12:36 1:13		+12·2	+20·0	+21·9	+49·2	+46·4
5	10:25 10:58 a. m.	B	+15·6	+11·2	+11·8	+41·5	+41·4
	11:11 11:43		+13·2	+15·6	+17·4	+46·1	+43·3
	11:53 12:23 p. m.		+16·8	+14·2	+11·6	+44·3	+47·6
	1:18 1:48		+15·0	+15·6	+17·8	+48·2	+48·7
6	10:28 11:12 a. m.	A	−16·5	−18·8	−15·6	+16·6	+16·6
	11:22 11:59		−13·6	−12·2	−14·7	+15·8	+15·9
	12:10 12:37 p. m.		−15·7	−14·8	−10·6	+17·0	+20·8
	1:32 1:58		−17·6	−12·6	− 7·2	+22·9	+23·4
8	10:40 11:06 a. m.	A	− 7·3	− 8·3	−10·9	+21·3	+22·7
	11:11 11:38		− 2·1	− 4·0	− 2·9	+24·3	+24·0
	11:59 12:26 p. m.		− 3·1	0·0	+ 1·0	+25·7	+31·2
	1:14 1:41		+ 5·2	+ 0·6	+ 2·2	+32·1	+33·7
9	10:27 11:00 a. m.	B	+42·7	+44·6	+46·3	+73·5	+73·5
	11:05 11:35		+46·9	+46·7	+48·4	+76·0	+80·5
	11:45 12:17		+47·8	+50·1	+49·9	+79·5	+80·4
	1:07 1:36 p. m.		+52·3	+50·7	+52·8	+85·1	+89·0
	1:41 2:04		+60·1	+53·0	+59·6	+92·0	+91·6
	2:08 2:31		+58·7	+58·3	+59·2	+95·3	+92·7
10	10:41 11:06 a. m.	A	+ 8·0	+ 3·0	+ 5·6	+31·0	+35·1
	11:12 11:37		+ 3·0	+ 7·7	+ 5·1	+31·1	+33·7
	11:43 12:07 p. m.		+ 7·5	+ 4·7	+ 9·9	+33·4	+33·2

Table VII following gives the results of the computation, according to equations (1) and (3) of section 15, based on the data of Table VI. The first column gives the date of observation. The second column gives the quantity $x = 5M_{21} - B_{17}$, resulting from a set of observations; the third gives $y = D - 5M_{21}$, the D here being the most probable value of the interval between the terminal microscopes at the middle time of the observations in any set; the fourth gives z, the rate of change of D, or the most probable change in D in 7·5 minutes, about. The next three columns give the residuals v_1, v_3, v_5, or most probable corrections to Q_1, Q_3, Q_5, respectively; while the two following columns give the residuals v_2, v_4, for Q_2, Q_4, respectively. The last column gives the sum of the weighted squares of the residuals, this sum being designated by $[pvv]$. The value adopted for p is 1/5. It may be remarked that the sums $v_2 + v_3 + v_5$, and $v_2 + v_4$ should each be equal to zero within a tenth of a unit. The groups of values for the several dates are spaced off by extra leads as in Table VI. The times of day when the several sets of observations were made are given in Table VI.

TABLE VII.—*Results of computation.*

Date.	x	y	z	v_1	v_3	v_5	v_2	v_4	$[pvv]$
1892.	μ	μ	μ	μ	μ	μ	μ	μ	
July 18	+ [15·3]	+ 3·7	—0·29	—2·1	+2·9	—0·7	—0·3	+0·3	2·84
	[23·3]	+10·1	—1·47	+0·8	—1·7	+0·9	0·0	0·0	0·87
20	+ 29·3	—27·3	+0·18	+3·2	—0·4	+2·9	—1·3	+1·2	6·89
	26·6	—17·1	+0·21	—0·1	+4·4	—4·2	—0·8	+0·8	8·68
	17·0	— 8·4	+2·17	—3·3	—0·9	—4·3	+1·3	—1·3	9·42
21	+ 28·6	+ 2·2	—0·34	—6·9	+2·4	+4·5	+2·3	—2·2	24·88
	25·9	+ 1·1	+0·18	+0·2	—0·7	+0·6	0·0	—0·1	0·19
	26·1	+ 1·7	+1·85	—2·2	+3·0	—0·7	+0·4	—0·3	3·12
	26·2	+ 1·2	—1·44	—2·5	—0·5	+2·9	+1·1	—1·0	5·39
22	+ 28·3	+13·8	—0·11	—0·3	—0·4	+0·8	+0·2	—0·2	0·26
	23·0	+17·0	+0·63	+1·9	—2·6	+0·8	—0·2	+0·2	2·28
	25·6	+26·2	+0·06	+0·6	0·0	—0·5	—0·3	+0·2	0·26
26	+ [21·9]	— 7·1	+1·00	—4·7	+3·5	+1·3	+1·2	—1·2	10·09
	[12·2]	— 6·0	+3·14	—5·9	+1·2	+4·7	+2·2	—2·1	20·92
27	+ 25·8	+18·3	—1·47	+1·7	+0·1	—1·9	—0·7	+0·7	2·28
	23·6	+18·6	+2·08	—3·8	—0·4	+4·1	+1·6	—1·5	11·09
	25·8	+20·6	—0·38	—0·7	+0·3	+0·4	+0·3	—0·2	0·28
	28·5	+25·4	+0·32	—1·1	—1·0	+2·2	+0·7	—0·7	2·39
28	+ 22·8	+19·0	+0·36	+2·0	—2·5	+0·4	—0·4	+0·3	2·33
	25·4	+17·0	+1·36	+2·2	—4·8	+2·6	+0·1	0·0	6·94
	24·9	+21·9	—0·48	+4·4	—3·9	—0·5	—1·0	+0·9	8·77
Aug. 4	+ 28·5	+15·9	+1·31	+0·2	—0·9	+0·8	+0·2	—0·1	0·30
	29·8	+18·0	+0·30	+5·2	—2·0	—3·3	—1·7	+1·7	14·17
5	+ 28·5	+12·9	—0·45	—1·8	+1·7	+0·2	+0·3	—0·4	1·48
	29·3	+15·4	—0·31	+2·8	—0·2	—2·6	—1·1	+1·1	5·35
	31·8	+14·2	+0·36	—3·3	0·0	+3·3	+1·3	—1·2	7·69
	32·3	+16·1	+0·35	+0·4	+0·5	—1·0	—0·2	+0·1	0·33
6	+ 33·6	—17·0	+0·10	—0·7	+1·8	—1·2	—0·1	+0·1	1·05
	29·3	—13·5	—0·09	+0·3	—1·3	+1·0	+0·1	—0·2	0·63
	32·6	—13·7	+1·62	—1·2	+1·1	+0·1	+0·3	—0·3	0·51
	35·7	—12·5	+1·29	+2·5	+0·1	+2·7	—1·0	+1·1	4·92
8	+ 30·8	— 8·8	—0·01	—1·5	—0·5	+2·1	+0·7	—0·7	2·36
	27·2	— 3·0	—0·17	—0·6	+1·0	—0·4	+0·1	0·0	0·31
	29·1	— 0·7	+1·98	—1·6	—0·7	+2·3	+0·7	—0·8	2·80
	30·2	+ 2·7	+0·08	—2·7	+2·1	+0·7	+0·7	—0·7	3·42
9	+ 29·0	+44·5	+0·40	+1·0	—0·1	—1·0	—0·4	+0·4	0·72
	30·9	+47·3	+1·42	—2·4	+0·6	+1·7	+0·8	—0·9	3·25
	30·7	+49·3	+0·48	+0·5	—0·8	+0·4	0·0	+0·1	0·22
	35·1	+51·9	+1·14	—2·7	+1·2	+1·4	+0·8	—0·9	3·59
	34·2	+57·6	—0·17	—2·2	+4·4	—2·3	0·0	0·0	5·90
	35·3	+58·7	—0·67	+1·3	+0·5	—1·8	—0·6	+0·6	1·76
10	+ 27·5	+ 5·5	+0·87	—4·1	+2·5	+1·6	+1·1	—1·2	7·77
	27·1	+ 5·3	+0·96	+0·4	—2·4	+2·1	+0·3	—0·3	2·25
	25·9	+ 7·4	+0·21	—0·5	+2·7	—2·1	—0·3	+0·3	2·57

(17) ***Corrections to the computed values of x.***—The computed values of x in Table VII are subject to two small corrections. The first of

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these depends on the inequalities in heights of the focal planes of the microscopes. The microscopes were adjusted to near equality in heights of their focal planes by the method explained in section 13, but owing to some unexplained cause, microscope 6, which was attached to the east pier of the comparator, was given a position about 2^{mm} too high. This maladjustment was discovered on July 19, but since it could be measured accurately it was not corrected until August 3. Measurements of the inequalities in heights of the focal planes were made on July 19, 20, 26, and August 3, by means of a level attached to the prototype metre, as already explained. The microscopes were arranged in the order 5, 1, 2, 3, 4, 6; 5 being on the west pier, 6 on the east pier, and the others on the intermediate piers. The following are the mean values of the differences in height of the focal planes resulting from the measures, seven in all, for the determinations extending from July 18 to July 28. The plus sign preceding a result shows that the focal plane of the east microscope of any pair is the higher.

	mm.
Difference for 5 and 1	−0·34,
1 and 2	+0·52,
2 and 3	−0·30,
3 and 4	+0·32,
4 and 6	+1·85,
Sum	+2·05.

These results show that the sum of the projections of the metric spaces on the horizontal focal plane of microscope 5 is 2·0$^{\mu}$ less than the sum of those spaces. Likewise, since the plane of microscope 6 is 2·05^{mm} higher than the plane of 5, the line joining the foci of 5 and 6 is 0·4$^{\mu}$ longer than the projection of this line on those planes. Hence the effective correction to the quantities Q_1, Q_3, Q_5 is $-2{\cdot}0^{\mu} + 0.4^{\mu} = -1{\cdot}6^{\mu}$, or the correction to x is $+1{\cdot}6^{\mu}$. This is to be applied to every value of x up to July 28, inclusive.

On August 3, microscope 6 was re-adjusted, and it and the others were then left undisturbed throughout the remainder of the comparisons. Measures of the relative heights of the focal planes were made on August 5 and 10. The resulting mean values from the three measures are as follows:

	mm.
Difference for 5 and 1	−0·26,
1 and 2	+0·48,
2 and 3	−0·31,
3 and 4	+0·40,
4 and 6	−0·12,
Sum	+0·19.

The corresponding correction to x is $+0{\cdot}3^{\mu}$, and this is to be applied to all values of x from August 4 to 10, both inclusive.

The second correction depends on the curvature of B_{17} in a vertical plane. This curvature is measured by means of the long striding level as explained in Chapter I, section 9. The correction is so small that it has been neglected in all previous work, though its amount was less in that work than in the work now considered. The theory of the computation of this correction from the level readings is given in Supplement A. It may suffice here to give the values of the computed corrections. They are as follows:

	μ.
July 18, 1892,	0·01,
21,	·09,
27,	·35,
Aug. 4,	·05,
6,	·07,
8,	·09,
9,	·31,
10,	·13.

These are so small that they might be ignored without material error, but since it is believed that they represent determinable quantities, we shall here apply them. It may be remarked that their variability is undoubtedly due to the effect of the external air temperature on the web of the Y-trough so long as the vertical adjusting screws under the bar are left undisturbed. The observations show, in fact, that if the bar is made straight for any given external temperature, it becomes convex upwards for an increase of that temperature. The adjusting screws were left undisturbed during the series of determinations extending from July 18 to July 28, and it will be seen by reference to Table VIII following that the maximum correction of July 27 corresponds to the maximum air temperature of the series. A like remark applies to the series extending from August 4 to 9, the maximum correction corresponding to the maximum air temperature of August 9. (See Table IX).

Considering the small number of determinations of corrections in the series of July, we may adopt a single mean value for application to x, viz, 0·2$^{\mu}$. For the series extending from August 4 to 9 we shall adopt 0·1$^{\mu}$ for all dates except August 9, and the observed value 0·3$^{\mu}$ for the latter date. The value adopted for August 10 is also the observed value, the bar having been turned end for end and re-adjusted on that day.

Since the curvature of B_{17} diminishes the interval between its terminal lines, these corrections must be applied to the quantities x with the negative sign.

(18) *Adopted individual and mean values of x.*—Tables VIII and IX following show the adopted values of x and various data pertaining thereto for the two orientations of B_{17} in the Y-trough.

In making up these tables the results of July 18 and 26 have been rejected for the following reasons: On these dates the microscopes were taken apart, cleaned, and wrapped with cotton batting just

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before observing with them. In addition, the leveling and clamp screws of the intermediate microscopes 1, 2, 3, 4 were tightened. It is thought that the anomalous results of these two days are due to movements of the microscopes before reaching their most stable positions after replacing their objectives and rewrapping them in cotton. Subsequently, when any such adjustment was made, the apparatus was allowed to stand at least one day before using it. Similarity of conditions, then, if there were no other reason, would seem to require that these results be omitted.

Table VIII gives the results obtained with the A-end of B_{17} west, and Table IX those obtained with the A-end of B_{17} east. The first column of either table gives the date; the second the maximum air temperature of the day as observed under the comparator shed; the third specifies the orientation of the metre; the fourth gives the computed value of x from Table VII; the fifth the correction for inequality in heights of focal planes of the microscopes, or slopes of metre; the sixth the correction for curvature of B_{17}; the seventh the adopted value of x, and the eighth the residual, found by subtracting any observed x from the mean value.

TABLE VIII.—*Adopted values of* $x = 5M_{21} - B_{17}$ (*A-end of* B_{17} *west*).

Date.	Max. air temperature of day.	End of metre west.	Computed value of x.	Correction for—		Adopted x.	Mean x for date.	v.
				Slopes of metre.	Curvature of B_{17}.			
1892.	°		μ	μ	μ	μ	μ	μ
July 20			+29·3	+1·6	—0 2	+30·7		—3·8
			26·6	1·6	·2	28·0		—1·1
	32·0 C.	B	17.0	1·6	·2	18·4	+25·7	+8·5
21			28·6	+1·6	—0·2	30·0		—3·1
			25·9	1·6	·2	27·3		—0·4
			26·1	1·6	·2	27·5		—0·6
	31·0	A	26·2	1·6		27·6	28·1	—0·7
22			28·3	+1·6	—0·2	29·7		—2·8
			23·0	1·6	·2	24·4		+2·5
	33·5	B	25·6	1·6	·2	27·0	27·0	—0·1
27			25·8	+1·6	—0·2	27·2		—0·3
			23·6	1·6	·2	25·0		+1·9
			25·8	1·6	·2	27·2		—0·3
	38·5	B	28·5	1·6	·2	29·9	27·3	—3·0
28			22·8	+1·6	—0·2	24·2		+2·7
			25·4	1·6	·2	26·8		+0·1
	36·0	A	24·9	1·6	·2	26·3	25·8	+0·6
Aug. 10			27·5	+0·3	—0·1	27·7		—0·8
			27·1	·3	·1	27·3		—0·4
	36·0	A	25·9	·3	·1	26·1	27·0	+0·8

Mean value of x, 26·91μ $\pm$ 0·4μ.

TABLE IX.—*Adopted values of* $x = 5M_{21} - B_{17}$ (*A-end of* B_{17} *east*).

Date.	Max. air temperature of day.	End of metre west.	Computed value of x.	Correction for—		Adopted x.	Mean x for date.	v.
				Slopes of metre.	Curvature of B_{17}.			
1892.	°		μ	μ	μ	μ	μ	μ
Aug. 4			+28·5	+0·3	—0·1	+28·7		+2·6
	34·5 C.	A	29·8	·3	·1	30·0	+29·4	+1·3
5			28·5	+0·3	—0·1	28·7		+2·6
			29·3	·3	·1	29·5		+1·8
			31·8	·3	·1	32·0		—0·7
	30·0	B	32·3	·3	·1	32·5	30·7	—1·2
6			33·6	+0·3	—0·1	33·8		—2·5
			29·3	·3	·1	29·5		+1·8
			32·6	·3	·1	32·8		—1·5
	32·5	A	35·7	·3	·1	35·9	33·0	—4·6
8			30·8	+0·3	—0·1	31·0		+0·3
			27·2	·3	·1	27·4		+3·9
			29·1	·3	·1	29·3		+2·0
	34·5	A	30·2	·3	·1	30·4	29·5	+0·9
9			29·0	+0·3	—0·3	29·0		+2·3
			30·9	·3	·3	30·9		+0·4
			30·7	·3	·3	30·7		+0·6
			35·1	·3	·3	35·1		—3·8
			34·2	·3	·3	34·2		—2·9
	37·5	B	35·3	·3	·3	35·3	32·5	—4·0

Mean value of x, $31{\cdot}34^{\mu} \pm 0{\cdot}4^{\mu}$.

The individual values of x in Table VIII or IX are, so far as known, of equal weight. The values of x in the two tables differ systematically, however, for reasons given at length in section 20 below. Hence we shall, for the sake of distinction, call the mean of the adopted values of x in Table VIII, x_w, and the mean of those in Table IX, x_e, the suffixes defining the orientation of B_{17}. These means are

$$x_w = +\ 26{\cdot}9^{\mu} \pm 0{\cdot}4^{\mu} \text{ for A-end of } B_{17} \text{ west,}$$
$$x_e = +\ 31{\cdot}3\ \pm 0{\cdot}4 \text{ for A-end of } B_{17} \text{ east.}$$

Since $$x = 5M_{21} - B_{17}$$
and $$5M_{21} = 5^{m}\ \ +\ 12{\cdot}5^{\mu},$$
these equations give

$$B_{17} = 5^{m} - 14{\cdot}4^{\mu} \pm 0{\cdot}4^{\mu} \text{ for A-end west,}$$
$$B_{17} = 5\ \ - 18{\cdot}8\ \pm 0{\cdot}4 \text{ for A-end east.}$$

The probable errors assigned to these mean values are derived from the residuals in the last columns of the tables. Thus, the sum of the squares of the residuals in Table VIII is 135·5, and the corresponding sum from Table IX is 118·1. The number of residuals is forty and the number of unknowns two. Hence the probable error of an individual value of x in either table is $\pm 1{\cdot}7^{\mu}$; and since the weights of x_w and x_e are each 20, their probable errors are each equal to $\pm 0{\cdot}4^{\mu}$.

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(19) *Sources of accidental error.*—The principal accidental errors affecting the quantities x, y, z of Table VII, and likewise the adopted values of x in Tables VIII and IX, are, it is believed, errors of observation and errors arising from irregular movements of the six microscopes of the comparator. An idea of the relative magnitudes of these two species of error can be derived in the following manner: Referring to equations (1), (2), and (3) of section 15, it is seen that the resultant of the errors involved in the quantities Q, on which the values in Table VII depend, appears in the sums of the weighted squares of the residuals given in the last column of that table. These sums vary very greatly for different sets of observations, but a trustworthy value for the resultant error may be derived from the aggregate of the sums given in the last column of the table. Omitting the values of July 18 and 26 for the reasons assigned in section 18, the aggregate of the sums is

$$[pvv] = 168{\cdot}8.$$

The total number of residuals in the 40 sets of determinations is 200, and the total number of derived values of x, y, z is 120. Thus the probable error of an observed Q of weight unity (or of any Q_2 or Q_4) is

$$\pm\, 0{\cdot}6745\left(\frac{168{\cdot}8}{200-120}\right)^{\frac{1}{2}} = \pm\, 1{\cdot}0^{\mu}.$$

This applies to a single measure of the distance between the terminal microscopes with the 5^{m} bar B_{17}. Since the weight of any Q_1, Q_3, Q_5, or of a single measure of that same distance with the metre M_{21} is 1/5, the probable error of one such measure is $\pm\, 2{\cdot}2^{\mu}$.

Since the Q's which enter the formulas of section 15 are independent, it follows that the probable error of a single value of x derived by this process is

$$\pm 1{\cdot}0^{\mu}\sqrt{\frac{39}{18}} = \pm\, 1{\cdot}5^{\mu}.$$

This value is only $0{\cdot}2^{\mu}$ less than the valuederived by a different process in the preceding section. The latter process, it will be observed, takes account of any changes in the quantities x which may occur from set to set of observations or from day to day. The close agreement of the two probable errors is, therefore, an indication that such changes were small, and also that the assumption on which the method of computation depends is essentially correct.

An examination of the residuals in Table VII shows a satisfactory distribution. Thus, of the whole number of residuals used, namely, 200, 99 are plus, 93 are minus, and 8 are zero each. The maximum residual in the class whose weight is unity is $2{\cdot}3^{\mu}$. The maximum residual in the class whose weight is 1/5 is $6{\cdot}9^{\mu}$, which is about three times the corresponding probable error, or $\pm\, 2{\cdot}2^{\mu}$, as given above.

It will be noticed that the $[pvv]$ for the earlier sets of determinations in July are considerably larger on the average than for the later sets in August. The only reasons I can assign for this are, first, that the

piers, which were completed only a short time before beginning to use them in July, grew more and more stable with the drying and hardening of the cement; and, second, that the smaller time interval required to make a set of observations after the observers had become well accustomed to the work diminished the chance of error from movements of the microscopes. It does not appear, however, as shown in the preceding section, that the resulting values of x were appreciably less precise in the earlier than in the later comparisons.

Now, as to the relation of the errors of observation to the aggregate errors, we may assume as hitherto that the probable error of a single microscope reading does not exceed $0{\cdot}5^{\mu}$. The probable error from this source, then, in an observed Q_2 or Q_4 is $\pm 0{\cdot}5^{\mu}$, if we suppose but four readings to be made on B_{17}. The corresponding value for an observed Q_1, Q_3, or Q_5 is $\pm 0.5^{\mu}\sqrt{5} = \pm 1{\cdot}1^{\mu}$. These are exactly half as great as the values derived from the resultant errors. Hence we conclude that the probable error in a single determination of x, due to instability of microscopes, etc., or to all extra-observational sources, is

$$\pm 1{\cdot}5^{\mu}\sqrt{1-\frac{1}{4}} = \pm 1{\cdot}3^{\mu}.$$

This shows a very satisfactory degree of stability of the microscopes, since it is but little more than 1/4 000 000th part of the length of B_{17} or of the distance between the terminal microscopes.

Finally, from whatever point of view we regard the adopted values of x in Tables VIII and IX, their accordance leaves little to be desired. It appears plain that for either orientation of B_{17} the same quantity, essentially, was observed on every date. This is well shown by the constancy of the mean values of x for the several dates. In Table VIII the maximum divergence of any date mean from the adopted mean of all values of x is but $1{\cdot}2^{\mu}$, and the corresponding maximum divergence in Table IX is but $1{\cdot}9^{\mu}$. The average of these is no greater than the probable error of a single value of x as derived in this section, namely, $\pm 1{\cdot}5^{\mu}$; and it is less than the probable error of a single value of x derived in the preceding section, namely, $\pm 1{\cdot}7^{\mu}$.

Taking this last probable error as a basis it appears that the mean of four determinations of B_{17} in terms of M_{21} on the new comparator would have a probable error less than 1^{μ} or less than 1/5 000 000th of B_{17}. We must conclude, therefore, that the temperatures of M_{21} and B_{17} were fixed within very narrow limits by the melting ice with which they were surrounded.

(20) *Systematic error.*—The precision of the results attained on the new comparator brought to light an unexpected source of systematic error. This source of error appears to be due to the fact that the transverse graduation marks at the ends of B_{17} differ much in width, and to the fact that the personal equations of the observers differ widely in amount. As already stated the width of the graduation mark at the A end of B_{17} is $16{\cdot}3^{\mu}$, while the width of the mark at B end is $36{\cdot}2^{\mu}$. It had been known from the experience of the observers, Siebert and

Woodward, on Holton Base that their personal equations differed; but it was not known that their equations varied with differing widths of graduation marks and micrometer wire intervals until the fact was developed in the course of the determinations detailed above. It was then easily seen by direct observation that such difference might introduce an appreciable constant error.

Fortunately in nearly all the work done with the apparatus the observers have been the same, namely, Siebert and Woodward; the only exception being the few determinations of length of B_{17} made by Tittmann and Woodward in July, 1891.

Before proceeding to examine this question minutely it may be remarked that all determinations of B_{17} indicate a systematic difference in its length depending on its orientation in the Y-trough. To show this clearly we will collect the results given in the preceding sections. For this purpose it is desirable to introduce a more general designation for the orientation of B_{17} than that used hitherto. We shall adopt the convention that the A end of B_{17} is left or right according as it is to the left or right of the observer as he stands alongside of the bar while reading a microscope. Thus A end left corresponds to A end west for the work on the new comparator and to A end north for the work on the office comparator.

Collecting the results of the direct determinations for length of B_{17} from sections 4, 5, and 18 we have—

July,	1891,	$B_{17} = 5^m$	$-10{\cdot}1^\mu \pm 2{\cdot}0^\mu$	for A end left,
		$B_{17} = 5$	$-11{\cdot}9 \pm 2{\cdot}0$	for A end right;
Feb.–Mar.,	1892,	$B_{17} = 5$	$-14{\cdot}6 \pm 0{\cdot}8$	for A end left,
		$B_{17} = 5$	$-16{\cdot}2 \pm 0{\cdot}8$	for A end right;
July–Aug.,	1892,	$B_{17} = 5$	$-14{\cdot}4 \pm 0{\cdot}4$	for A end left,
		$B_{17} = 5$	$-18{\cdot}8 \pm 0{\cdot}4$	for A end right.

The differences shown here are all of the same sign. Their amounts with their probable errors are

July,	1891,	$1{\cdot}8^\mu \pm 2{\cdot}8^\mu$,
Feb.–Mar.,	1892,	$1{\cdot}6 \pm 1{\cdot}1$,
July–Aug.,	1892,	$4{\cdot}4 \pm 0{\cdot}6$.

A difference with the same sign but of much smaller amount is shown by the comparisons of B_{17} and B_{18}, as may be seen by reference to Table IV, section 9. Its amount is $0{\cdot}4^\mu \pm .1{\cdot}3^\mu$.

Although these quantities differ widely amongst themselves their probable errors show that they are not inconsistent with one another, while their persistence in sign renders it highly probable that they are due to a common cause.

The only cause which appears adequate to explain this discrepancy is that mentioned at the beginning of this section, and although the evidence of this cause is not conclusive there would seem to be little doubt that it is the principal cause. We proceed to examine the question at length and to present all the available evidence.

It is essential to the inquiry, in the first place, to know precisely how personal equation or error may affect the observed data. Since nearly all the observations were made by Siebert and Woodward it will be convenient to use a notation referring especially to their equations. Let—

S_a, S_b = the corrections to the microscope readings of Siebert on the A end and B end of B_{17}, respectively,

W_a, W_b = the corresponding corrections to the readings of Woodward.

R_1, L_2 = the observed readings of Siebert on the right and left hand ends of the bar, respectively,

R_2, L_1 = the corresponding readings of Woodward;

Then, supposing the microscope readings to increase from right to left,* we have for the distance D between the zeros of the terminal microscopes

(A end of B_{17} left.)

$$D = B_{17} + L_1 + W_a - R_1 - S_b,$$
$$= B_{17} + L_2 + S_a - R_2 - W_b.$$

The sum and difference of these give

$$D = B_{17} + \tfrac{1}{2}(L_2 + L_1) - \tfrac{1}{2}(R_2 + R_1) + \tfrac{1}{2}(S_a + W_a) - \tfrac{1}{2}(S_b + W_b), \quad (1)$$
$$(S_a - W_a) + (S_b - W_b) = -(L_2 - L_1) + (R_2 - R_1). \quad (2)$$

When B_{17} is reversed in the Y-trough we may assume, in order to be entirely general, that owing to change in the aspects of the marks and to the difference in wire intervals of the two microscopes the corrections $S_a, W_a, \ldots$ change in value. Designating the new values of the corrections and of the new readings by the same letters with an accent, we have—

(A end of B_{17} right.)

$$D = B_{17} + \tfrac{1}{2}(L'_2 + L'_1) - \tfrac{1}{2}(R'_2 + R'_1) - \tfrac{1}{2}(S'_a + W'_a) + \tfrac{1}{2}(S'_b + W'_b), \quad (3)$$
$$(S'_a - W'_a) + (S'_b - W'_b) = -(L'_2 - L'_1) + (R'_2 - R'_1). \quad (4)$$

For brevity, put

$$\begin{aligned} u &= \tfrac{1}{2}(S_a + W_a) - \tfrac{1}{2}(S_b + W_b), \\ u' &= \tfrac{1}{2}(S'_a + W'_a) - \tfrac{1}{2}(S'_b + W'_b), \\ v &= (S_a - W_a) + (S_b - W_b), \\ v' &= (S'_a - W'_a) + (S'_b - W'_b). \end{aligned} \quad (5)$$

If now we denote the mean values for length of B_{17} corresponding to its two orientations by V_1 and V_2, the above equations (1) and (3) show that

$$\begin{aligned} B_{17} + u &= V_1 \text{ for A-end of } B_{17} \text{ left}, \\ B_{17} - u' &= V_2 \text{ for A-end of } B_{17} \text{ right}. \end{aligned} \quad (6)$$

If the personal equations remain the same for both positions of the bar, $u = u'$, and the half sum of (6) gives B_{17} free from this source of error. On the other hand, if those equations differ, the error of B_{17} derived from the mean of (6) is

$$\tfrac{1}{2}(u - u') = \tfrac{1}{4}(S_a - S'_a + W_a - W'_a - S_b + S'_b - W_b + W'_b). \quad (7)$$

* The microscopes were so used on the new comparator and in all the work with the iced bar on Holton Base. The increase was in the opposite direction with the microscopes as mounted on the office comparator. It will be noticed that the corrections S_a, W_a change sign with such change of direction.

Length of Iced Bar B_{17}.

It appears from the equations (1) to (4) that the observations are insufficient to determine the corrections S_a, W_a, . . . without additional relations between them. It is clear, however, that if the widths and aspects of the graduations were the same at both ends of the bar, and if the wire intervals of the micrometers were equal, no error from this source would appear in the resulting lengths of the bar, and it is a matter of regret that these theoretical considerations were not duly appreciated when the bar was constructed. It is clear also that the orientation of B_{17} ought to have been changed from day to day, for the errors in question would thus have been more certain of elimination in the final mean.

We may nevertheless gain some idea of the magnitude of the corrections desired from equations (2) and (4), which give the quantities v and v' defined by the last two of (5). Each set of observations on B_{17} gives a value for v or v', as the case may be, and there were 4 to 12 such sets for each day's work. The mean values of the resulting v and v' are collected in Table X following, which is arranged in two parts corresponding to the two orientations of B_{17}. The first column gives the date, the second the number of individual values of v, and the third the mean value of v for the date. The second half of the table gives the corresponding values pertaining to v'.

TABLE X.—*Observed values of v and v', July and August, 1892.*

A-end of B_{17} left.			A-end of B_{17} right.		
Date.	Number of values observed.	Mean value of v.	Date.	Number of values observed.	Mean value of v'.
1892.		μ	1892.		μ
July 20	6	+5·9	Aug. 4	4	−0·9
21	8	+4·8	5	8	+0·1
22	6	+6·0	6	8	+2·6
27	8	+4·1	8	8	+1·0
28	6	+4·2	9	12	−1·5
Aug. 10	6	+4·2			

If we assign to the several mean values in this table weights proportional to the numbers of observed values on which those means depend, the following weighted means result:

$$v = +\ 4{\cdot}8^{\mu} \pm 0{\cdot}2^{\mu}, \qquad v' = +\ 0{\cdot}2^{\mu} \pm 0{\cdot}6^{\mu}.$$

It thus appears that the quantities v and v' are no less definite than the corresponding values for length of B_{17} already derived. But the fact that v and v' differ so widely makes it necessary to suppose that the personal equations changed with the change in aspect of the bar. How such a change can arise I am unable to explain, but some additional observational evidence indicates that it took place. This evidence was obtained by measuring the differences $(S_a - W_a)$, $(S_b - W_b)$,

$(S'_a - W'_a)$, and $(S'_b - W'_b)$ directly. For this purpose the observers made readings alternately at either end of the bar while it was held still in its Y-trough. Several such series of readings were made during August 30 and 31, 1892. They were not very satisfactory for the reasons that the bar was not iced and that it could not be kept very stable. The following mean values were derived:

$$\left.\begin{aligned} S_a - W_a &= +0{\cdot}78^\mu \\ S_b - W_b &= +0{\cdot}78 \end{aligned}\right\}, \text{ A-end of } B_{17} \text{ left,}$$

$$\left.\begin{aligned} S'_a - W'_a &= +1{\cdot}80 \\ S'_b - W'_b &= -1{\cdot}09 \end{aligned}\right\}, \text{ A-end of } B_{17} \text{ right.}$$

The sums of these pairs of values give in accordance with the last two of equations (5):

$$v = +1{\cdot}6^\mu, \qquad v' = +0{\cdot}7^\mu.$$

These values are much less definite than the mean values given above and differ widely from them, but the discordance is so much greater than can be attributed to other sources of error, that it seems essential to entertain, the hypotheses first, that the personal equations of the observers varied from time to time; or second, that the work on the new comparator was subject to some constant error which changed sign with the change in orientation of the bar; or third, that both these possible causes were operative. These views are, in fact, suggested by the variations of the values of v' in the second half of Table X.

It becomes desirable, therefore, to glean such information on this point as is afforded by the observations on B_{17} during February to May, 1892, on the office comparator, and likewise by the observations in the use of the apparatus on Holton Base in 1891. The values of v and v' resulting from the direct determinations of B_{17} on the office comparator are given below in Table XI, which is arranged in the same manner as Table X.

TABLE XI.—*Observed values of v and v', February to March, 1892.*

A-end of B_{17} left.			A-end of B_{17} right.		
Date.	Number of values observed.	Mean value of v.	Date.	Number of values observed.	Mean value of v'.
1892.		μ	1892.		μ
Feb. 4	3	+0·8	Feb. 19	4	+2·9
5	8	2·8	20	4	2·8
6	4	2·4	23	4	3·8
8	8	4·4	24	4	5·1
10	8	4·0	25	8	1·6
11	4	1·0	26	12	1·9
13	8	2·0	27	8	2·0
15	8	3·4	Mar. 7	8	1·2
16	12	1·9	8	8	2·0
17	8	3·0	9	4	2·0
18	8	4·1	10	4	1·9
			11	4	2·0
			12	4	3·3

Length of Iced Bar B_{17}.

Giving the values in this table weights proportional to the number of measures for each, there result the following weighted means:

$$v = + 2{\cdot}9^{\mu} \pm 0{\cdot}2^{\mu}, \qquad v' = + 2{\cdot}3^{\mu} \pm 0{\cdot}2^{\mu}.$$

These values, it is seen, not only agree well with each other but are determined with a precision quite equal to that of the corresponding mean values from Table X. It may be remarked that the precisions should not differ much in the two cases since the readings from which v and v' are obtained are made in less than one minute, and hence stability of the microscopes is depended on for this short period only. There are, however, nearly twice as many determinations represented in Table XI as in Table X, and this fact accounts mainly for the smaller probable errors of v and v' derived from Table XI. The range amongst the daily means is nearly the same in the two tables and amounts at most to $4{\cdot}1^{\mu}$.

The observations on B_{17} at the times it was compared with B_{18} give the following values:

Apr. 29, 1892, $v = + 2{\cdot}8^{\mu}$ (mean of 10 measures,)
30, 1892, $v = + 2{\cdot}2$ (mean of 17 measures),
May 5, 1892, $v' = + 1{\cdot}9$ (mean of 27 measures).

These values are likewise fairly accordant with each other and with the values just given above.

Finally, the values of v'* derived from the observations on the 100^{m} comparator of Holton Base in 1891 are (see Chapter III, section 6):

Aug. 4 to Aug. 7, 1891, $v' = + 3{\cdot}8^{\mu}$,
Sept. 24 to Aug. 6, 1891, $v' = + 3{\cdot}3$.

With respect to the latter results, it is to be remarked that they were obtained with the lower power (27 diameters) Repsold microscopes, and may not be strictly comparable with the results obtained with microscopes 5 and 6, which have a power of 50 diameters. They do not differ materially, however, in the mean from the average values of v or v' obtained on the office comparator.

Collecting all these mean values for v and v' and taking their half sums and half differences, we have the following condensed statement of all the direct information bearing on the constancy of the personal equations of the observers:

Date.	$\frac{1}{2}(v+v')$.	$\frac{1}{2}(v-v')$.
Feb. to Mar., 1892	$+2{\cdot}6^{\mu}$	$+0{\cdot}3^{\mu}$
Apr. to May, 1892	$+2{\cdot}2$	$+0{\cdot}2$
July to Aug., 1892	$+2{\cdot}5$	$+2{\cdot}3$

The agreement of the quantities $\frac{1}{2}(v + v')$ in this statement is no less remarkable than the disagreement of the last of the quantities $\frac{1}{2}(v - v')$ from the other two. It appears that the cause of the divergence of

* B_{17} was used with the A-end right in all the work on Holton Base. Hence the quantity v' only appears in this work.

$\frac{1}{2}(v - v')$ in July and August, 1892, whatever it may have been, was such as to increase v and decrease v' by about the same amount. A change of this kind might have occurred in several ways through changes in the eight quantities S_a, W_a, . . . as may be seen by reference to the last two of equations (5); but the data do not indicate that the change is more likely to have occurred in one of these ways rather than in another.

In view of all the evidence presented in this section, it appears certain that the observers assigned different lengths to B_{17}, depending on its orientation in the Y-trough. It appears equally certain that this difference is due to personal equation of the observers or to change in aspect of the graduation marks, or to both these causes. The two causes are entangled with one another and can not be separated without additional data. We conclude, however, that the effects arising from these causes are well eliminated in the mean of lengths of the bar in its two orientations, or that the error of this mean length, expressed by equation (7), is small.

The conclusion just stated does not require the assumption of constancy and equality of the quantities u and u' of equations (5), but it seems best to adopt this assumption in deriving the final lengths of B_{17} for A-end left and A-end right. The justification of this view is to be found, I think, in an examination of the observed values of $\frac{1}{2}(v + v')$, $\frac{1}{2}(v - v')$, and $u + u'$, given above. Thus the first of these quantities varies by $0{\cdot}4^{\mu}$ only for three different sets of determinations extending over half a year's time, the second varies by $2{\cdot}1^{\mu}$, and the third by $4{\cdot}0^{\mu}$. With respect to this last, however, or to the quantity $u + u'$, we must remark that it might be expected to vary in observed value much more than the others, since it comes from data dependent on the stability of all parts of the apparatus, while the other two depend only on the stability of B_{17} and the terminal microscopes. But the constancy of $u + u'$ in the determinations of July to August, 1892, renders it highly probable that the value then derived is a real quantity, which was masked by errors of the less precise determinations on the office comparator.

ADOPTED LENGTHS FOR B_{17}.

(*21*) *Summary of results.*—Collecting the results for length of B_{17} given in sections 4, 5, 11, and 18, we have the following summary in Table XII:

TABLE XII.—*Summary of results for length of B_{17}.*

Date.	B_{17}	
	A-end left.	A-end right.
July, 1891	$5^{m} - 11{\cdot}0^{\mu} \pm 2{\cdot}0^{\mu}$	$5^{m} - 11{\cdot}9^{\mu} \pm 2{\cdot}0^{\mu}$
Feb. to Mar., 1892	$5 - 14{\cdot}6 \pm 0{\cdot}8$	$5 - 16{\cdot}2 \pm 0{\cdot}8$
Apr. to May, 1892	$5 - 11{\cdot}5 \pm 1{\cdot}8$	$5 - 11{\cdot}9 \pm 1{\cdot}8$
July to Aug., 1892	$5 - 14{\cdot}4 \pm 0{\cdot}4$	$5 - 18{\cdot}8 \pm 0{\cdot}4$

Length of Iced Bar B_{17}.

The probable errors in this table differ widely, but so far as is known they are the best available indexes of the precision of the several results.

In accordance with the conclusions and assumption stated in the preceding section we shall derive three values for length of B_{17}, namely: First, a length for A-end of B_{17} left; second, a length for A-end of B_{17} right; and, third, a mean length, which it is assumed will be free from constant error.

It will be recalled that the results of July, 1891, were obtained by Tittmann and Woodward, while all the rest were obtained by Siebert and Woodward, who used the bar also in the position A-end right in all the work of Holton base. We might, therefore, use the results of 1891 in getting the mean length of B_{17}, but they should be excluded in getting the other lengths. In any case, however, it can make but little difference whether we include or exclude the results of 1891, since the weights we are constrained to adopt amount practically to the exclusion of those results. The same remark applies also to the results of the indirect determinations of April and May, 1892, since their weights must be small in comparison with the weights of the results of the direct determinations.

(22) *Derivation of lengths of B_{17} in terms of M_{21}.*—It is assumed, as already stated, that the length of B_{17} observed by Siebert and Woodward is greater or less than its mean length according as the A-end is left or right by a constant which we may denote by u. In conformity with this assumption, if B_{17} denote the mean length of the bar, or the length free from systematic error, $B_{17}+u$ will be the observed length for A-end left and $B_{17}-u$ the observed length for A-end right.

The observed lengths given in Table XII afford a series of observation equations from which B_{17} and u may be determined. In writing these equations the question whether the theoretical weights of the observed quantities should be modified in any way presents itself. Using the probable errors of the observed lengths of 1892 as a basis, the weights of these lengths are proportional to the numbers 1, 0·2, and 4. These seem to present great disparity, but in view of all the considerations set forth in the preceding sections, I am not disposed to change them. Other numbers presenting less disparity might be used, but the resulting values for B_{17} and u would not be much changed thereby. Adopting these theoretical weights, then, and writing for brevity, $\beta = B_{17} - 5^m$, the observation equations given by the results in Table XII become

		Weight.
$\beta + u + 14{\cdot}6^\mu$	$= +0{\cdot}27^\mu,$	1,
$\beta - u + 16{\cdot}2$	$= -1{\cdot}83,$	1,
$\beta + u + 11{\cdot}5$	$= -2{\cdot}83,$	0·2,
$\beta - u + 11{\cdot}9$	$= -6{\cdot}13,$	0·2,
$\beta + u + 14{\cdot}4$	$= +0{\cdot}07,$	4,
$\beta - u + 18{\cdot}8$	$= +0{\cdot}77,$	4.

The normal equations from these are

$$10{\cdot}4\,\beta = -168{\cdot}28^{\mu},$$
$$10{\cdot}4\,u = +\ 19{\cdot}28,$$

whence

$$\beta = -16{\cdot}18^{\mu} \pm 0{\cdot}40^{\mu},$$
$$u = +\ 1{\cdot}85\ \pm 0{\cdot}40.$$

The sum of the weighted squares of the residuals is 14·93. Hence the probable error of an observed quantity of unit weight is $\pm 1{\cdot}3^{\mu}$, and the probable error of either β or u is $\pm 0{\cdot}40^{\mu}$. Likewise, the probable error of $\beta + u$ or $\beta - u$ is $\pm 0{\cdot}8^{\mu}$, or exactly twice that of β or u.

Hence the values we adopt for the length of B_{17} in terms of Prototype Metre M_{21} are the following:

$B_{17} = 5^{m} - 14{\cdot}3^{\mu} \pm 0{\cdot}8^{\mu}$ for A end left,
$B_{17} = 5\ \ -18{\cdot}0\ \ \pm 0{\cdot}8$ for A end right,
$B_{17} = 5\ \ -16{\cdot}2\ \ \pm 0{\cdot}4$ for mean.

It is to be remarked that the first two values here given apply especially to the work done with the bar by the observers Siebert and Woodward. Other observers having different personal equations might assign different lengths for the left and right orientations of the bar. The last value, on the other hand, is supposed to be free from systematic error dependent on personal equation and on the orientation of the bar. It is the length which should be applied in deriving the mean length of a line measured the same number of times for each orientation of the bar.

With respect to the adopted mean length of B_{17}, it may be further remarked that it differs but $0{\cdot}9^{\mu}$ from the mean of the direct determinations of February and March, 1892, and but $0{\cdot}4^{\mu}$ from the mean of the direct determinations of July and August, 1892. These two series of determinations were the most extensive and accordant of all the determinations made, though differing much in precision. It is seen, however, that if this difference in precision were ignored, the simple mean of the two values for either orientation or for the mean of both would not differ widely from the adopted values. Thus, the mean with equal weights for A-end left is $5^{m} - 14{\cdot}5^{\mu}$; that for A-end right is $5^{m} - 17{\cdot}5^{\mu}$; and for the mean of these two $5^{m} - 16{\cdot}0^{\mu}$. Finally, it may be remarked that the results of July, 1891, and those of April and May, 1892, which are practically rejected by reason of their small weights, do not differ at most from the adopted values by amounts much in excess of the millionth part of the bar's length; the discrepances being, in fact, $3{\cdot}3^{\mu}$ for A-end left, $6{\cdot}1^{\mu}$ for A-end right, and $4{\cdot}8^{\mu}$ in the mean. Moreover, these discrepances are quite within the range indicated by the probable errors of the inferior results. It appears proper to conclude, therefore, that the values for B_{17} adopted above, are trustworthy to the extent shown by their probable errors.

Length of Iced Bar B_{17}.

(23) *Lengths of B_{17} in terms of the International metre.*—The values for length of B_{17} derived in the preceding section are expressed in terms of Prototype Metre No. 21. It remains, then, to combine the probable error of the length of this metre with the probable errors of those derived lengths in order to get the probable errors of the lengths of B_{17} when expressed in terms of the International Metre. In the Rapport sur la construction, les comparaisons et les autres Opérations ayant servi à déterminer les Équations des Nouveaux Prototype Métrique, p. 82, the probable errors of the Prototypes are said to vary from $\pm 0{\cdot}1^{\mu}$ to $\pm 0{\cdot}2^{\mu}$ between such limits of temperature as are ordinarily met in metrology. Since in the application of M_{21} to the determination of B_{17} only one temperature, namely, that of melting ice, has been used, we shall adopt the extreme value $\pm 0{\cdot}2^{\mu}$ for the probable error of the length of M_{21}. The probable error of $5M_{21}$ is then $\pm 1{\cdot}0^{\mu}$, and the probable error of the length of B_{17} for A-end left or right becomes $\pm 1{\cdot}3^{\mu}$, and for the mean length $\pm 1{\cdot}1^{\mu}$. Hence we have for the length of B_{17} in terms of the International Metre, the following values, the first two of which apply only to the work done with the bar by Siebert and Woodward:

$$B_{17} = 5^{m} - 14{\cdot}3^{\mu} \pm 1{\cdot}3^{\mu} \text{ for A-end left.}$$
$$B_{17} = 5 \quad - 18{\cdot}0 \quad \pm 1{\cdot}3 \quad \text{for A-end right.}$$
$$B_{17} = 5 \quad - 16{\cdot}2 \quad \pm 1{\cdot}1 \quad \text{for mean.}$$

(24) *Concluding remarks.*—In designing and using the iced-bar apparatus many questions not referred to in the preceding pages were raised and considered. Some of these have important bearings on the practical success of the apparatus, and most of them will naturally arise in the mind of the critical reader. It is proper, therefore, to devote some attention to them here.

The most obvious and important of these questions is whether the bar assumes the temperature of melting ice when fully packed in it. No absolutely crucial mode of answering this question is known to me, but there appears to be no reason to suppose that it takes a materially different temperature. Repeated observations on mercurial thermometers placed in the ice alongside of the bar show that they read zero C. within the unavoidable errors of a few hundredths of a degree. That the bar assumes a fixed length within very narrow limits is, it would seem, demonstrated by the small range amongst the determinations of its length on the new comparator, and the equally small range amongst the measures made with the apparatus on the 100^{m} comparator and the standard kilometre of the Holton Base. Considering these ranges, and the fact that the bar's expansion is about 5^{μ} per degree C., there is but very small margin for variation in the bar's length. An independent and perhaps more conclusive answer to this question is afforded by the results of the two most important groups of determinations of the length of the bar, namely, the group of February to March, 1892, and that of July and August, 1892. During the period covered by the first group

the air temperature about the Y-trough varied from 5° to 10° C., while for the latter group the air temperature varied from 25° to 40° C. If the bar is affected to any appreciable extent by the external air temperature it ought to have been longer at the time of the second group of comparisons than at the time of the first, since the average air temperature for the second group was higher by about 30° C. than for the first group. The mean lengths for those groups differ, however, by only 1·2", the indicated length of the bar being so much shorter at the higher than at the lower temperature. My conclusion, then, is that the bar assumes a sensibly constant temperature when properly iced in the Y-trough.

The time required by the bar to reach a stable length is less than ten minutes. The rate of temperature change is so great in the early stages of freezing that 90 per cent or more of the contraction of the bar occurs within a minute after it is well surrounded by ice. The corresponding time required by the prototype metre to reach a stable length appears to be less than five minutes, which is less than the time required to properly pack it in ice. In the use of the apparatus, however, the bar and metre have been, in general, fully packed in ice from half an hour to an hour before observing on them.

A question which received attention in designing the apparatus is whether the friction of the bar on its supports in the Y-trough may not affect its length appreciably. Since the cross section of the bar is 2·56 square centimetres, and the modulus of elasticity of steel about 2×10^6 kilogrammes per square centimetre, it would require a longitudinal stress of 1 kilogramme to change the bar's length 1 micron if the bar were free to respond to such stress. From this calculation and from a consideration of the actual circumstances under which the bar is used, it was concluded that this friction would produce no appreciable effect. Some direct experiments to test this point were made, however, during the course of the comparisons with the Prototype Metre of February and March, 1892. In these experiments the Y-trough was subjected alternately in quick succession to tension and compression while readings were made on the bar. Several such trials were made without disclosing any change in the bar's length. It is, indeed, difficult to see how any frictional stress could manifest itself, especially when the apparatus is trundled along on its cars as in measuring a line or as used on the new comparator, since the jarring incident to such motion ought to relieve the bar from all but the smallest stress.

A query may also arise as to the effect on the bar's temperature of the exposed graduated surfaces. It might be inferred that the bar is perceptibly hotter by reason of these exposed parts, small though they are, than it would be if entirely covered with ice. But a study of the physical relations here presented does not justify such an inference. The reason is that the conductivity of the steel is so much higher than its emissivity in air that the exposed parts of the bar cannot differ in temperature by more than a few thousandths of a degree from the

unexposed parts.* Whatever this excess may be, it can affect only short portions of the bar, and hence produce only vanishing changes of length. The same conclusion, it would appear, applies with greater certainty to the Prototype Metre. A simple and direct experiment bearing on this question has also been frequently tried. It consisted in microscope readings on the ends of the bar just before and just after pressing the bare finger for a period of ten to thirty seconds against one of the graduation plugs. If the heat thus conducted from the hand to the bar is not speedily dissipated, the bar should have been measurably lengthened thereby. No such change, however, was observed.

An obstacle which sometimes proves troublesome in the use of the apparatus, is the condensation of moisture on the graduated surfaces of the bar. The most effective means found for removing this is a dry camel's-hair brush. Such brushes were used in the long series of comparisons of the bar and Prototype Metre in July and August, 1892, and no material delay from moisture occurred.

Finally, experience with the apparatus leads me to recommend a bar having a Y-shaped rather than a rectangular cross section. Such a bar would possess the requisite rigidity with small mass per unit length. It could be subdivided into metre spaces by marks on its neutral surface, and its entire length determined by measuring those spaces separately as well as by the method dependent on the use of six microscopes. Its fiducial lines should be of equal width within a micron or two, so that personal equation would be certain of elimination by exchange of positions of the observers for either orientation of the bar. Provision might also be made for protecting the graduated surfaces from direct contact with the ice. For this purpose hard rubber tubes might be fitted around those surfaces, and if desirable they could be made long enough to extend up through the ice pack. Such tubes would prevent abrasion of the polished surfaces by grit in the ice, and would also protect them to some extent from currents of air, which, if laden with moisture, produce troublesome condensation. The length in melting ice of a bar possessing this degree of refinement could be readily determined by the method followed on the new comparator. It would appear, in fact, from our experience in the use of that comparator that the length of such a bar in terms of a prototype metre would be known with a probable error not exceeding 1 micron from the mean of four measures, which might be made in one day, or in two days at most.

*This conclusion is based on an investigation made by the author in 1888, namely, On the diffusion of heat in homogeneous rectangular masses, with a special reference to bars used as standards of length. Annals of Mathematics, vol. 4, pp. 101–127. It appears from this investigation, which is founded on Fourier's methods, that a bar of small cross section (less than $4^{cm} \times 4^{cm}$, say), cooling or heating in air from an initial uniform temperature, maintains a sensibly uniform temperature throughout its mass during the entire process of cooling or heating. That is, the difference between the temperatures within and at the surface of the mass is very small when the cross section is small.

CHAPTER III.

MEASUREMENTS MADE WITH ICED BAR APPARATUS AT HOLTON BASE IN 1891.

(1) *Plan of operations with apparatus in 1891.*—The plan submitted by me to the Superintendent of the Survey for the use of this apparatus on the Holton Base of the transcontinental triangulation in Indiana. contained the following recommendations which were approved and carried out during the summer of 1891: (*a*) To construct a 100-metre comparator near the Holton Base; to standardize this comparator by repeated measurements with the iced bar, and to use this comparator in turn to standardize and study the behavior of 100^{m} tapes or those of less length, or any other form of base apparatus; (*b*) to use the iced bar in addition to make several measures of a kilometre at least of the base line, so that the efficiency of the different forms of apparatus used in measuring the whole base could be tested on the actual ground over which they were applied.

Before giving the results of the measures made with the iced bar it is proper to give a brief description of the long comparator and of the kilometre whereon the apparatus was used.

(2) *The 100-metre comparator.*—The 100-metre comparator of the Holton Base was a line 100^{m} long fitted for measurement with the iced bar apparatus. Twenty-one beech-wood microscope posts, 1·8^{m} long and 15^{cm} × 15^{cm} in cross section, were set firmly in the ground, 5^{m} apart, on a level plat near the north end of the base. Alongside of the posts a stationary railway track was laid, the support posts of which were half way between the microscope posts. The ends of the line were marked by brass, spherical-headed bolts cemented into the upper ends of stone posts, which latter were well set in beds of concrete. The comparator was covered by a shed 110^{m} long by 3^{m} wide. Its length extended nearly east and west. It was covered at the ends and on the south side as well as overhead, but the north side was left open in order to permit free access of daylight and air. (See illustration No. 31.)

This comparator was built by Assistant A. T. Mosman after plans drawn up by Mr. Siebert. It answered its purpose very satisfactorily. An efficient auxiliary applied by Assistant Mosman was a sawdust covering to the ground along the comparator. This covering absorbed the dust and moisture, and prevented the transmission of disturbances through the ground to the microscope posts. The stability of these posts may be inferred from the measures of the comparator interval given below.

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(3) *The standard kilometre.*—A nearly level portion, one kilometre in length, of the Holton Base was selected by Assistant Mosman for measurement with the iced bar aparatus. The base line, whose entire length is 5·5 kilometres, runs in a nearly north and south direction across the Crawfish Flats of Southern Indiana. The portion selected for the iced bar measures passes for 600^{m} of its length through a dense forest growth, leaving about 200^{m} at either end in open fields. The whole kilometre is on low ground and the part within the forest is, in a wet season, subject to partial inundation. The soil along the kilometre is a stiff clay, which is very firm when dry, but which assumes a jelly-like mobility and elasticity when saturated with water.

The way through the forest was cleared, and the end stones of the kilometre were set under the direction of Assistant Mosman during May and June, 1891. During the latter half of the following August and early September the microscope and track posts were set along the line. Owing to frequent and heavy rains this was a tedious operation. Many of the posts were set in the water which filled the post holes as fast as they were dug. It is impossible, therefore, to present any statistics as to the speed with which this work can be done under usually favorable circumstances. It may be remarked, however, that it is a work which requires but little skilled labor. In addition, it should be said that the microscope posts were set with considerable precision. Accurate spacing of the posts to 5^{m} apart was secured by means of 100^{m} and shorter steel tapes, while the posts were aligned by means of a theodolite. The probable error in position of a post face with respect to the kilometre line does not, I think, exceed $\pm$ 3^{mm}, while the probable error of the reference line fixed on the posts, as explained in section 9, Chapter I, does not exceed $\pm$ 1^{mm}.

The bolts marking the termini of the kilometre were cemented in the end stones by Assistant Mosman early in August, after their proper relative positions had been determined by Assistant O. H. Tittmann with the Survey secondary apparatus. Intermediate stones dividing the kilometre into four nearly equal sections were set on September 7, 1891. Each of them consisted of a half cubic metre of concrete set in the ground so that its upper surface was about even with the ground surface. On the top of each stone was cemented one of the Repsold cut-off plates, which are provided with spherical headed bolts for use with the cut-off cylinder previously described.

When these intermediate stones were set, the ground along the line was so wet that it was a matter of difficulty to keep the water out of the excavations while the concrete was being rammed into place. These stones did not become dry and hard until deep trenches were dug about them on September 18, 1891. For this reason it is probable that these stones were much less stable during the first two measures

of the kilometre (September 10, 15) than during the last two measures, (September 26, 30).

(4) *Results of measures of comparator interval.*—The distance between the terminal spheres of the 100m comparator was measured nineteen times. The first nine measures were made between July 30 and August 7, 1891, and the remainder between September 24 and October 6. A measure as here designated means a determination of the comparator interval without interchange of the observers in reading on the ends of the bar. Such interchange of observers took place in all cases except for the first five measures, during which direct observations for personal equation were made on the bar. These observations proceeded on the assumption that the relative equation of the observers was the same at both ends of the bar. But, as shown in section 20 of Chapter II, this assumption is not correct. It becomes essential, therefore, to recur to the investigation given in the section just referred to in order to understand how the personal equation affects the measures in question.

In all measures of the comparator interval the A end of B_{17} was right, and the observers were S (Siebert) and W (Woodward). When a measure proceeded from left to right (or west to east) S always observed at the right or A end of the bar and W at the other end, if they did not exchange positions. When they did exchange positions, S always observed first on the A end and W at the B end of the bar. When a measure proceeded from right to left, S always observed first at the B end of the bar and W at the A end.

In conformity, then, with the notation of section 20, Chapter II, let L_1, L_2, R_1, R_2, denote the left and right hand readings respectively of a pair of microscopes for any bar length; and let S_a, S_b, W_a, W_b denote the corrections to these readings for personal equations of the observers at the A end and B end of the bar, respectively. Then, supposing the measure to proceed from left to right, the distance between the pair of microscopes considered is

$$B_{17} + L_1 - R_1 - (S_a - W_b), \qquad (a)$$

for the first position of the observers, and

$$B_{17} + L_2 - R_2 + (S_b - W_a), \qquad (b)$$

for the second position.

Now the derived length of B_{17} for A end right corresponds to the mean of these two expressions (a) and (b); but when, as in the case of the first five measures of the comparator interval, the expression (a) only was observed, the use of the derived value of B_{17} gives a distance too large by $(S_a - W_b)$ for each bar length. Similarly, the distance in the case of the use of expression (b) only is too small by $(S_b - W_a)$ per bar length. It is necessary, therefore to treat the first group of measures of the comparator interval in such a way as to take

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due account of the inequality of the numbers of measures in the two cases denoted by the expressions (a) and (b).

Let D_1 denote the most probable value of the apparent distance between the terminal spheres when the expression (a) is used, and D_2 the corresponding value when the expression (b) is used. Then the resulting observation equations will be of the forms

$$\begin{aligned} D_1 - 20\,B_{17} - Q' &= v', \\ D_2 - 20\,B_{17} - Q'' &= v'', \end{aligned} \tag{1}$$

wherein Q' and Q'' are the quantities measured with the microscopes and cut-off cylinders, and v' and v'' are their most probable corrections. If, on the other hand, D denotes the most probable value of the actual interval between the terminal spheres, we shall have

$$D = \tfrac{1}{2}\,(D_1 + D_2). \tag{2}$$

In the case of the second group of measures, the observers exchanged positions in every case, so that we might use the values resulting from the means of the expressions (a) and (b); but inasmuch as it is desirable to get an idea of the constancy of the personal equation effect, we shall treat the second group in the same manner as the first; that is, by means of equations (1) and (2).

The greater part of the measures were made by moving the bar from west to east (or left to right) along the comparator. Some of the later measures were made in the opposite direction, and they disclose, apparently, a source of systematic though minute error, depending on the direction of measure. The same sort of error is indicated also by the measures of the standard kilometre given in detail below. We shall exhibit the data, then, in a way to disclose these effects as well as those arising from personal equation.

Table I following gives all of the data observed for the length of the comparator except the special observations for personal equation alluded to above and referred to again below. The first column gives the date of the observations. The second column gives the direction in which the measure proceeded; W to E, for example, signifying that the measure started from the west end of the comparator and proceeded towards the east end, or proceeded from left to right. The third column gives the observation equations in the forms (1) above.

TABLE I.—*Data for length of 100^{m} comparator.*

Date.	Direction of measure.	Observation equation.		
1891.			*mm.*	*mm.*
July 30	W to E	$D_1 - 20\,B_{17} - 39{\cdot}532 =$	$+$	0·055
30	W to E	$D_1 - 20 -$ ·574 =	$+$	·013
30	W to E	$D_1 - 20 -$ ·660 =	$-$	·073
31	W to E	$D_1 - 20 -$ ·548 =	$+$	·039
Aug. 3	W to E	$D_1 - 20 -$ ·643 =	$-$	·056
4	W to E	$D_1 - 20 -$ ·624 =	$-$	·037
4	W to E	$D_2 - 20 -$ ·476 =	$-$	·026
7	W to E	$D_1 - 20 -$ ·528 =	$+$	·059
7	W to E	$D_2 - 20 -$ ·424 =	$+$	·026
Sept. 24	W to E	$D_1 - 20\,B_{17} - 39{\cdot}272 =$	$+$	0·134
24	W to E	$D_2 - 20 -$ ·191 =	$+$	·149
Oct. 2	W to E	$D_1 - 20 -$ ·394 =	$+$	·012
2	W to E	$D_2 - 20 -$ ·320 =	$+$	·020
2	E to W	$D_1 - 20 -$ ·432 =	$-$	·026
2	E to W	$D_2 - 20 -$ ·369 =	$-$	·029
6	W to E	$D_1 - 20 -$ ·422 =	$-$	·016
6	W to E	$D_2 - 20 -$ ·370 =	$-$	·030
6	E to W	$D_1 - 20 -$ ·512 =	$-$	·106
6	E to W	$D_2 - 20 -$ ·449 =	$-$	·109

(5) *Derivation of lengths of Comparator.*—An inspection of the results in the above table indicates that the length of the comparator interval changed during the period of about two months which elapsed between the determinations of the two groups. That such a change might occur seems not improbable, since the stones carrying the terminal spheres rose to a level with the ground surface, and since a marked change in the amount of moisture in the ground took place during the period. The results of the second group of determinations indicate that such a change was taking place at the time they were made. Additional evidence that such changes are likely to occur is afforded by the measures of the sections of the standard kilometre, as explained below. It will be assumed, therefore, that the values of D for the two groups should differ, and the observed quantities will be treated accordingly.

Attributing equal weights to the observation equations, we have from the first group

$$mm.$$
$$D_1 = 20B_{17} + 39{\cdot}587$$
$$D_2 = 20B_{17} + 39{\cdot}450$$

The sum of the squares of the residuals, using the millimetre as unit, is 0·0193. The number of equations is 9 and the number of unknowns 2. Hence the probable error of a single equation is $\pm$ 0·036^{mm}, and the apparent lengths D_1 and D_2 have probable errors of $\pm$ 0·014^{mm} and $\pm$ 0·026^{mm}, respectively.

But the objective quantity is $D = \frac{1}{2}(D_1 + D_2)$. Using the above numerical values and observing that the probable error of D is

$$\pm\, 0{\cdot}026^{mm}\sqrt{\frac{9}{56}},$$

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we have from the first group of observations—

$$D = 20B_{17} + 39{\cdot}518^{mm} \pm 0{\cdot}010^{mm}.$$

From the second group of measures we find

$$\begin{aligned} &mm. \\ D_1 &= 20B_{17} + 39{\cdot}406 \\ D_2 &= 20B_{17} + 39{\cdot}340 \end{aligned}$$

The sum of the squares of the residuals is 0·066 and the probable error of a single equation is $\pm 0{\cdot}061^{mm}$. Hence we find for the length of the comparator from the second group of measures

$$D = 20B_{17} + 39{\cdot}373^{mm} \pm 0{\cdot}019^{mm}.$$

(6) *Discussion of results of comparator measures.*—The two values of D just derived differ by $0{\cdot}145^{mm}$, a quantity 14·5 times the probable error of the first value of D and 5·4 times the probable error of the second value of D. It seems probable, therefore, that, whatever may be the cause of this difference, it is a real quantity, although it is but 1 689 000th part of the comparator interval.

I am unable to explain the greater magnitudes of the residuals in the second group of comparisons over those in the first group. It may be observed, however, that the hypothesis of a uniform increase of the interval D_1 or D_2, from September 24 to October 6, satisfies the observed values very closely and renders them quite as accordant as the first group of values. So far as skill in handling the apparatus could affect the measures, those of the latter group should be the more precise, since in the interval between the two groups the observers had the benefit of a large amount of experience with the apparatus on the standard kilometre. Judging from our experience on that kilometre during the second set of measures of it, when the conditions were about equally favorable to those presented on the comparator, it would seem impossible for the normal errors of measurement to produce a range as great as that shown in the second group of comparator measures.

The length of the comparator interval was used in turn to standardize the 100^m steel tapes, as explained in Chapter IV, and also to standardize the bars of the secondary apparatus. To get working lengths of the comparator for these purposes, it will be assumed that the length changed uniformly with the time which elapsed between the two groups of measures. Fortunately the greater part of the determinations of the tape lengths was made at about the times of the groups of measures of the comparator interval.

If we take the difference between D_1 and D_2 for the two groups of comparison, we find

$$\begin{aligned} &\mu \\ &+ 137 \text{ for the first group,} \\ &+ 66 \text{ for the second group.} \end{aligned}$$

These quantities represent twenty times the function $(S_a - W_b) + (S_b - W_a)$ of the personal equations of the observers, as shown by the expressions (a) and (b) of section 4 above. This function is also the same as v' of section 20, Chapter II. Hence we have

$$v' = + 6{\cdot}8^{\mu} \text{ for the first group,}$$
$$v' = + 3{\cdot}3 \text{ for the second group.}$$

The first of these values is less trustworthy than the second, since the latter is independent of all sources of error save that of the character of personal equation, while the former involves other sources of error. If we use the observations of August 4 and 7 only in the first group, the resulting value of v' is $+3{\cdot}8^{\mu}$, which does not differ materially from the value coming from the second group.

Had we assumed the values of $(S_a - W_b)$ and $(S_b - W_a)$ equal, as was done in a preliminary account of this work published in the American Journal of Science for January, 1893, the resulting value of D for the second group would have been identical with that derived above, while that for the first group would have differed from the value adopted by only 4^{μ}.

The direct observations for personal equation referred to in section 4, and published in the number of the American Journal of Science just cited, were made on July 30, 31, August 3, 4, 1891. They give $2{\cdot}4^{\mu}$ for the value of $(S_a - W_b)$ or $(S_b - W_a)$, supposing these to be equal, and $4{\cdot}8^{\mu}$ for the quantity corresponding to v'.

It appears, then, that the relative personal equation of the observers would have been an appreciable source of error in this work had not care been taken to eliminate its effect.

Finally, attention may be called to the indication of systematic error, due to the direction of measurement, furnished by the measures of October 2 and 6, 1891. Referring to Table I, it is seen that on

		mm.
Oct. 2,	$D = 20B_{17} +$	39·357 for direction W. to E.
	$D = 20B_{17} +$	39·400 for direction E. to W.
Oct. 6,	$D = 20B_{17} +$	39·396 for direction W. to E.
	$D = 20B_{17} +$	39·480 for direction E. to W.

It thus appears that the measure from E. to W. exceeded that from W. to E. on October 2 by $0{\cdot}043^{mm}$ and on October 6 by $0{\cdot}084^{mm}$, and in the mean by $0{\cdot}064^{mm}$. This amounts to $1{\cdot}6^{\mu}$ per bar length relatively to the mean of a forward and backward measure.

Taken by itself this evidence would not appear to justify the assumption of the existence of such systematic error; but since a similar discrepancy is shown by the measures of the kilometre (as explained in sections 9 and 10 below), it is important to note that the circumstances attending the comparator measures of October 2 and 6 were closely comparable with those attending the last set of measures of the kilo-

metre. The extent of the error in the latter measures is somewhat less than that just indicated, but there seems to be little doubt of its reality. A probable explanation of the source of this error in the kilometre measures and in the comparator measures of October 2 and 6 is given in section 10 below, but some doubt is entertained as to its application to all the comparator measures. If the explanation referred to is correct and applicable to the comparator measures, the computed values of D may need a correction of about $0{\cdot}03^{mm}$; but since we can not establish the validity or magnitude of such correction, the only way to allow for it is to increase the assigned probable errors of the values of D. This will be done in the following section.

(7) *Adopted lengths for the 100m comparator.*—Since we have assumed that the length of the 100^m comparator changed during the period which elapsed between the two groups of determinations, it is proper to assign an epoch to each derived length given in section 5. If we attribute weights to the several dates of observation proportional to the number of measures made on each, these epochs are August 3 and October 2, 1891. The derived lengths and their epochs are then

$$D = 20B_{17} + 39{\cdot}518\,mm. \pm 0{\cdot}010\,mm. \text{ for August 3, 1891.}$$
$$D = 20B_{17} + 39{\cdot}373\,mm. \pm 0{\cdot}019\,mm. \text{ for October 2, 1891.}$$

The probable errors attached to these values do not include the probable error of B_{17}. The latter error for A-end right, as given in section 23, Chapter II, is $\pm 1{\cdot}3^{\mu}$. To allow for the possible source of error mentioned at the close of the preceding section, it is considered sufficient to increase the average of the probable errors in the above equations, or $\pm 0.015^{mm}$, by one-third its amount. This will make the probable error dependent on errors of measurement $\pm 0{\cdot}020^{mm}$ for each value of D. Combining this with $\pm 1{\cdot}3^{\mu} \times 20$, we have $\pm 0{\cdot}033^{mm}$ as the probable error of either value of D when expressed in terms of the International Metre. This is equivalent to the 1 3 000 000th part of the length of the comparator.

The values we adopt, then, for the length of the comparator in terms of the International Metre, are

$$D = 20\,B_{17} + 39{\cdot}518\,mm. \pm 0{\cdot}033\,mm. \text{ for August 3, 1891,}$$
$$D = 20\,B_{17} + 39{\cdot}373\,mm. \pm 0{\cdot}033\,mm. \text{ for October 2, 1891.}$$
$$B_{17} = 5^m - 18{\cdot}0^{\mu}.$$

(8) *Results of measures of standard kilometre.*—The method followed in making the measures of the kilometre of Holton Base has been briefly explained in section 10 of Chapter I. The specimen record given in section 11 of the same chapter serves in all essential respects to show how the records of the kilometre measures were kept. In the latter work, however, grades were determined by means of the sector attached

to the Y-trough, and the defect of alignment, instead of being made zero for each bar length, was measured with a millimetre scale.

In all measures of the kilometre sections the observers exchanged positions to eliminate personal equation, so that we have here to consider results of the form $D = \frac{1}{2}(D_1 + D_2)$ only, instead of the separate quantities D_1 and D_2, as in the case of the first group of the comparator measures.

Throughout these kilometre measures the A end of B_{17} was right as in the comparator measures.

The kilometre lies in a north and south direction, nearly, and the direction of measurement will be designated as N. to S. (north to south) or S. to N., as the case may be. The direction N. to S. corresponds to the direction left to right of our adopted convention and to W. to E. (west to east) in the designation of the direction of the measures of the 100^m comparator.

As explained in section 3, the kilometre was divided into four nearly equal sections by intermediate marking stones set a few days before the first measure was begun. Subsequently it was thought desirable to have a section 100^m long at the north end of the kilometre measured with the iced bar, for the purpose of studying the behavior of the bars of the secondary apparatus. Accordingly, a section stone was set on September 10, 1891, 100^m south of the north end of the kilometre.

The first set of measures of the kilometre began on September 9, 1891. Starting at the north end of the kilometre on the afternoon of this date, it was thought best to measure the first section for practice only, since the men were nearly all unaccustomed to the work, and since the portable tracks and some minor parts of the apparatus were then used for the first time. Owing to some delays but 150^m were measured before nightfall, when the four microscopes were left in place on their posts to mark the terminus of the day's measure. The work was continued to the end of the section on the following date. The result of this trial measure is not included in the adopted results given below, though it is given in a footnote to Table II following. Although obtained under the unfavorable conditions stated, it does not differ materially from the results for the same section obtained a few days later.

In all the kilometre measures the observers on the bar were J. S. Siebert and R. S. Woodward. Siebert in all cases adjusted the microscope at the front end of the bar, while Woodward held the trough and bar by means of the lever, as explained fully in section 10 of Chapter I. The grade-sector readings and the alignment measures were observed by John F. Hayford. The record was kept by Robert Penington, except for the last day's work, September 30, when Mr. Hayford performed this task in addition to his work of observation.

The computations entailed by the work were made by Siebert, Hayford, and Woodward in the field, and carefully revised at the office by Siebert and Woodward.

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Table II following gives the results of all the measures of the kilometre sections except that of the trial measure alluded to above. Since the several corrections resulting from these measures are large relatively to those of the measures of the 100m comparator, it is considered desirable to give them in full detail. The first column gives the date of the measure. The second column designates the section measured as well as its length, 0m–100m, for example, denoting the section between the north terminal sphere of the kilometre and the next sphere south. The third column gives the direction in which the measure proceeded. The following four columns give in succession the micrometer, grade, alignment, and cut-off corrections. The last column gives the sum of these corrections, or the excess of the length of the section over a round number of bar lengths, which number is 20, 30, or 50, according as the section is 100m, 150m, or 250m long.

TABLE II.—*Data for lengths of sections of standard kilometre.**

Date.	Section.	Direction of measure.	Micrometre correction.	Grade correction.	Alignment correction.	Cut-off correction.	Sum of corrections.
1891.	*m m*		*mm*	*mm*	*mm*	*mm*	*mm.*
Sept. 15	0–100	N. to S.	—0·478	—0·338	—0·245	— 4·814	— 5·875
15		S. to N.	—0·247	—0·328	—0·225	— 5·009	— 5·809
26		N. to S.	—0·084	—0·324	—0·145	— 4·914	— 5·467
30		S. to N.	—0·145	—0·315	—0·137	— 5·003	— 5·600
15	100–250	N. to S.	— 0·125	—0·270	—0·292	+12·909	+12·222
15		S. to N.	+ 0·205	—0·264	—0·334	+12·722	+12·329
26		N. to S.	+0·607	—0·280	—0·222	+12·336	+12·441
30		S. to N.	+0·876	—0·258	—0·352	+12·400	+12·666
10	250–500	N. to S.	+0·086	—0·289	—0·197	— 5·030	— 5·430
15		S. to N.	—0·752	—0·302	—0·439	— 3·060	— 4·553
26		N. to S.	—0·670	—0·290	—0·214	— 4·321	— 5·495
29		S. to N.	—0·022	—0·336	—0·303	— 4·940	— 5·601
11	500–750	N. to S.	—0·354	—0·962	—0·472	+20·864	+19·076
14		S. to N.	+1·831	— 1·048	—0·535	+19·118	+19·366
28		N. to S.	—0·100	—0·950	—0·376	+21·095	+19·669
29		S. to N.	+0·079	—0·982	—0·350	+21·019	+19·766
11	750–1000	N. to S.	—0·812	—5·852	—0·577	—13·323	—20·564
14		S. to N.	+0·265	—5·860	—0·461	—13·223	—19·279
28		N. to S.	—0·960	—5·812	—0·495	—14·016	—21·283
29		S. to N.	—0·364	—5·889	—0·391	—14·549	—21·193

* The length of the section 0m to 250m derived from the practice measure of September 9 and 10, 1891, was 50 B_{11} + 6·45 mm.

(9.) *Derivation of the lengths of the kilometre sections.*—As explained in section 3 and more fully in section 10 below, the conditions which obtained during the time of the first two measures of the kilometre were very unfavorable as compared with those met during the time of the last two measures. In addition, it is probable that the section stones underwent some small though appreciable displacements dur-

ing the interval which elapsed between the two sets of measures. For these reasons and others which will appear below, it is deemed best to consider the two sets of measures separately.

Since the division of the first quarter section of the kilometre into two parts by the 100m stone did not materially affect the precision of the sum of those parts relatively to the precisions of the other quarter sections, we shall in the first place derive the lengths of the several quarter sections, and consider the measures of the 100m section subsequently by themselves.

As remarked in the discussion of the measures of the 100m comparator in section 6 above, the measures of the kilometre sections indicate a systematic difference depending on the direction of measurement. It is desirable, therefore, to exhibit the results in such a way as to clearly bring out this peculiarity. Accordingly the following statement is derived from Table II, the sums of the quantities therein being rounded to the nearest hundreth of a millimetre:

TABLE III.—*Statement of results of measures of standard kilometre.*

[First set, September 10 to 15, 1891.]

No. of measures.	Direction of measure.	Section 0m to 250m.	Section 250m to 500m.	Section 500m to 750m.	Section 750m to 1,000m.	0m–1,000m.
		mm.	*mm.*	*mm.*	*mm.*	*mm.*
1	N. to S.	+6·35	—5·43	+19·08	—20·56	—0·56
2	S. to N.	+6·52	—4·55	+19·37	—19·28	+2·06
1—2		—0·17	—0·88	— 0·29	— 1·28	—2·62

[Second set, September 26 to 30, 1891.]

3	N. to S.	+6·97	—5·50	+19·67	—21·28	—0·14
4	S. to N.	+7·07	—5·60	+19·76	—21·19	+0·04
3—4		—0·10	+0·10	— 0·09	— 0·09	—0·18

It is seen that in seven out of the eight independent cases presented in this statement, the measure made from north to south was less than that made in the reverse direction; and it appears worth while to give an idea of the extent of this discrepancy before adducing what appears to be its cause. To this end, let s_1, s_2, s_3, s_4 denote the most probable values of the excesses of the measured lengths of the several sections over fifty bar lengths, on the assumption that the mean of a forward and backward measure of a section is free from systematic error; and let x denote the most probable value of the systematic error on the suppositions that it is proportional to the distance measured and that it changes sign with the direction of measurment. Then a pair of

measures of a section will give a pair of observation equations of the form

$$s + x - n_1 = v_1,$$
$$s + x - n_2 = v_2;$$

and each set of measures of the sections of the kilometre will give eight equations of this type containing five unknowns, namely, s_1, s_2, s_3, s_4, and x. On the other hand, we may entertain the simpler assumption that no systematic error is eliminated in the mean of a forward and backward measure; in which case the quantity x will disappear from the above observation equations, but the values of the quantities s will be the same as before. The validity of the quantity x will therefore depend, so far as the purely observational evidence goes, on the effect its use has in diminishing the residuals which result from the simpler supposition of no systematic error. In order to compare the two systems of residuals we shall give those resulting from the suppression of x alongside and to the right of those resulting from the use of x in the observation equations below.

Referring, then, to Table III, the first set of measures of the sections gives the following system of equations:

	mm.	*mm.*	*mm.*
$s_1 - x -$	6·35	$= -$ 0·24,	$+$ 0·09,
$s_1 + x -$	6·52	$= +$ 0·25,	$-$ 0·08,
$s_2 - x +$	5·43	$= +$ 0·11,	$+$ 0·44,
$s_2 + x +$	4·55	$= -$ 0·11,	$-$ 0·44,
$s_3 - x -$	19·08	$= -$ 0·19,	$+$ 0·14,
$s_3 + x -$	19·37	$= +$ 0·18,	$-$ 0·15,
$s_4 - x +$	20·56	$= +$ 0·31,	$+$ 0·64,
$s_4 + x +$	19·28	$= -$ 0·31,	$-$ 0·64.

These give

mm.	*mm.*
$s_1 = +$ 6·44,	$s_2 = -$ 4·99,
$s_3 = +$ 19·22,	$s_4 = -$ 19·92,

mm.

$$x = + 0{\cdot}33,$$

together with the residuals in the second members of the observation equations.

The sum of the squares of the residuals in the above system wherein x is used is 0·4050. Hence the probable error of one equation is $\pm$ 0·25^{mm}, that of either value of s is $\pm$ 0·18^{mm}, and that of x, $\pm$ 0·09^{mm}.

The sum of the squares of the residuals resulting from the suppression of x is 1·263. The probable error of a single equation is $\pm$ 0·38^{mm}, and that of either value of s, $\pm$ 0·27^{mm}.

The corresponding data of the second set of measures give the following system of equations and values:

		mm.	*mm.*	*mm.*
$s_1 - x -$	6·97	$=$	+ 0·03,	+ 0·05,
$s_1 + x -$	7·07	$=$	− 0·03,	− 0·05,
$s_2 - x +$	5·50	$=$	− 0·07,	− 0·05,
$s_2 + x +$	5·60	$=$	+ 0·07,	+ 0·05,
$s_3 - x -$	19·67	$=$	+ 0·03,	+ 0·05,
$s_3 + x -$	19·76	$=$	− 0·02,	− 0·04,
$s_4 - x +$	21·28	$=$	+ 0·02,	+ 0·04,
$s_4 + x +$	21·19	$=$	− 0·03,	− 0·05,

	mm.		*mm.*
$s_1 =$	+ 7·02,	$s_2 =$	− 5·55,
$s_3 =$	+ 19·72,	$s_4 =$	− 21·24,

$$x = + 0{\cdot}02 \text{ mm.}$$

The sum of the squares of the residuals in the system involving x is 0·0142. Hence the probable error of one equation is $\pm$ 0·046^{mm}, that of either s is $\pm$ 0·033^{mm}, and that of x is $\pm$ 0·016^{mm}. Suppressing x, the sum of the squares of the residuals is 0·0182, the probable error of one equation is $\pm$ 0·045^{mm} and the probable error of either s is $\pm$ 0·032^{mm}.

(10) *Discussion of results of kilometre measures.*—Judging from the numerical evidence as presented in the preceding section it would appear that there was an active cause of systematic error during the time of the first pair of measures of the kilometre, and that this cause, though present, produced very small effects during the time of the second pair of measures. As already stated in section 3, the ground along the kilometre at the time of the first two measures was very wet and in some places inundated, and in view of the differences in magnitude of the discrepancies (1 — 2) shown in Table III, for the several sections, it is important to consider a number of circumstances which were present during the time of the first pair of measures but absent during the time of the second pair.

Those portions of the kilometre whereon the stability of the microscopes was least, fell in the sections 250^m to 500^m and 750^m to 1,000^m. Along about 150^m of the first of these the ground was more or less under water and very elastic, so that when a length of the portable track was carried forward considerable vibrations were transmitted through the soil to the microscope posts. Along the other section the ground was in general much drier, and hence the posts were in general much stabler. But near the middle of the section was a low stretch of about 50^m where the ground was wet, and where the posts were set to a less depth than elsewhere in order to avoid steep gradients; so that the microscopes had here much less than the average stability. On recurring to Table III it is seen that the differences (1 — 2) for these sections are much larger than those for the other sections, and it

is difficult to avoid the inference that the lack of average stability of the microscope posts on these sections gave rise in some way to the error in question. A way which naturally suggests itself is the following:

Consider a measure of the kilometre proceeding from north to south. It was the habit of the microscope porter to stand on the south side of the microscope posts when clamping the microscopes to them. He thus turned the clamp screw which fell on the east side of any post with his right hand. If in this operation he habitually pushed the posts towards the north, they would not speedily recover their normal positions when set in wet or viscous soil. Any jarring about them, however, would tend to hasten the recovery of their normal positions. Such jarring was always caused by the movements of the operatives with the heavy car tracks and by the rolling of the cars along the tracks. Under these conditions each microscope would undergo more or less displacement towards the south *after* it was used at the front end of the bar and *before* it was used at the rear end. This would make the measured value of the distance too small. On the other hand, when the measure proceeded in the opposite direction, the porter stood on the same side of the posts as before stated in fastening the microscopes, and hence their displacements would still be towards the south. The measured value of the distance would thus be, in this case, too great.

This hypothesis explains all the discrepancies between the forward and backward measures of the kilometre as well as those of the comparator measures of October 2 and 6, to which attention is called in section 6 above; for in the measures of the comparator on the two latter dates the porter who moved and clamped the microscopes was the one who handled them in all the kilometre measures, and it is practically certain that whatever habit he followed in the latter work was adhered to in the former.*

In the light of this hypothesis the larger values of the discrepancies developed in the measures of the kilometre sections on wet and plastic ground, and the practical disappearance of such discrepancies in the second set of measures made when the ground was dry and the microscopes very stable, are easily understood.

In view of all the evidence, the explanation just given is regarded as the most plausible one assignable for the systematic error in question. But there are some other circumstances affecting the first two measures of the south half of the kilometre which are worthy of mention. In the first measure of the section 500^m to 750^m one stop was made and the measure transferred for a half hour to a cut-off plate, while some posts which had been slightly misplaced were put in proper position. In the first measure of the section 750^m to $1{,}000^m$ a stop was

* During the earlier measures of the comparator the microscopes were handled by another porter whose habit, unfortunately, was not observed.

made by reason of a shower of rain. This delay brought us to the end of the section after nightfall, and a still further delay was caused at that end by a maladjustment of the terminal microscope post with reference to the cut off sphere. Lamps for illuminating the microscopes were at hand, so that no difficulty on this score resulted, but during the delay in the last instance dependence had to be placed on the stability of the microscopes. Although great care was taken in all these cases to avoid the introduction of error, the opportunities for its occurrence were, of course, greater than when no delay took place. During the reverse or second measure of this section the rear microscope was set forward or backward five times by means of its slide rest in order to bring the forward microscope within reach of the end of the bar. This method of shifting the microscopes was also resorted to three times during the second measure of the section 500^m to 750^m.* These displacements were measured with a millimetre scale, and it is possible, though I deem it very improbable, that some formidable error was introduced thereby. Finally, a fear was entertained that some displacement of the cut-off sphere at the 750^m stone took place during the interval of two days which elapsed between the first two measures of the south half of the kilometre. The basis of this fear lay in the fact that by accident or design on the part of some one unconnected with the work a heavy block of wood used as a cover to the stone was allowed to fall on the cap of the cut-off sphere. The cap was thus driven on the sphere so tightly that considerable force was requisite to remove it; and it is possible, though not very probable, that this double disturbance produced some displacement.

With regard to the circumstances attending the second pair of measures of the kilometre, it should be said that they were in nearly every way more favorable than those attending the first pair. The ground along the line had become dry and firm; the intermediate section stones had had time to settle and to become hard; all of the difficulties in the use of the apparatus had become familiar to the observers, and the measurements proceeded without a single unexpected stop or delay. During the course of these measures several tests were made of the stability of the microscope posts as affected by the nearness of the operatives to them. It was found that no appreciable motion of the microscopes occurred unless the operatives approached much closer to the posts than was permitted in the actual work. In a trial made on one of the least stable portions of the line, the presence of five men alongside the Y-trough produced no displacements of the microscopes at the ends of the bar so long as the man nearest to a post

*In no other cases was this questionable method of shifting the microscopes resorted to. It should be said, in fact, that the experience gained during the first two measures of the kilometre enabled us to avoid not only this difficulty, but other difficulties which, while they may not have vitiated the work perceptibly, caused more or less delay and vexation.

Iced Bar Measures at Holton Base.

was distant two decimetres. It is believed, then, that the microscope posts were very stable during these measures, although it appears not improbable that they underwent slight displacements due to the pressure applied in clamping the microscopes to them.

It remains to consider briefly the question whether we can safely assume that the terminal and section marks of the kilometre maintained constant relative positions. Assuming that the systematic error is well eliminated in the mean of a forward and backward measure, an idea of the stability of the end and section marks may be gained from Table III by computing the differences between the mean lengths of the several sections resulting from the first pair and second pair of measures. Thus we have:

Mean of first pair *minus* mean of second pair of measures for—

	m.		*m.*	*mm.*
Section	0	to	250.	= − 0·58
	250		500.	= + 0·56,
	500		750,	= − 0·50,
	750		1000,	= + 1·32.

These figures indicate considerable movements of the marking stones; and it seems not improbable that movements of such magnitude did actually occur, since the stones rose to the surface of the ground and the moisture in the ground varied from the extreme of saturation to the extreme of dryness during the interval of fifteen days which elapsed between the two sets of measures. It will be seen also, from these figures, that a movement towards the south of $0{\cdot}5^{mm}$ of the first section stone would account for the discrepancies presented by the measures of the first two sections. This stone, set originally in ground saturated with water, is perhaps more likely to have moved than the other two. Similarly, it is inferred that the stone at 750^{m} is more likely to have moved than those adjacent to it, although its movement alone would leave appreciable outstanding discrepancies.

Additional evidence of relative motions of the marking stones is afforded by the measures of the 100^{m} section of the kilometre at its north end. Thus we have from Table II the following values for the measured excesses of that section over twenty bar lengths:

	mm.
Sept. 15, 1891, N. to S., excess	= − 5·88,
Sept. 15, 1891, S. to N., excess	= − 5·81,
Mean excess	= − 5·84.
Sept. 26, 1891, N. to S., excess	= − 5·47,
Sept. 30, 1891, S. to N., excess	= − 5·60,
Mean excess	= − 5·53.

Mean of first pair of measures *minus* mean of second pair $= -0{\cdot}28^{mm}$ This result, so far as it goes, agrees with the first of those given above in indicating a movement of the terminal stone towards the north.

From the evidence just submitted, together with that furnished by the measures of the 100^{m} comparator, and from an intimate knowledge of the conditions attending the work, it is impossible to doubt that we were dealing to a quite appreciable extent with actual movements of the marking stones. For this reason it would seem essential to the highest precision in measures with the iced-bar apparatus to place the marking stones well under ground and to give them plenty of time for settlement before using them.

(11) *Probable errors of the kilometre measures.*—From the data and discussion given in the preceding sections, it appears that the errors affecting the kilometre measures as expressed in terms of B_{17} are of three kinds, namely: First, error of operation of the accidental class; second, error of operation of the systematic class; and third, error arising from motions of the marking stones, which latter may be assumed to be of the accidental sort.

In the case of the first two measures of the kilometre it would seem that the systematic error surpassed by far the resultant of the other two. In the case of the last two measures the systematic error, though probably still the controlling one, was very small and to some extent concealed by the resultant of the other two. It would seem quite safe then to deduce the probable error due to the accidental errors of operation from the second set of measures on the supposition that the two other sources of error were inoperative, since the value thus reached will be too large. No such limiting value, however, can be assigned for either of the probable errors due to the other sources, since the uneliminated portion of the systematic error is entangled with the error due to stone movements.

Recurring to the second system of equations and values given in section 9, it is seen that the probable error of a single measure of a section (of 250^{m} length) is $\pm$ 0·045^{mm}; and this is the upper limiting value for the probable error of operation when the apparatus is used on stable ground. From this it follows that the corresponding probable error for one measure of a kilometre is $\pm$ 0·09^{mm}.

It is of interest to compare this with the value deducible from the measures of the 100^{m} comparator, where the conditions were essentially the same as those which obtained during the second set of measures of the kilometre. Referring to section 5, we have $\pm$ 0·036^{mm} and $\pm$ 0·061^{mm} as the probable errors of a single measure of 100^{m} without exchange of observers to eliminate personal equation. Taking the average of these two values, dividing it by $\sqrt{2}$ to allow for exchange of observers, and multiplying the quotient by $\sqrt{10}$, we have for the probable error of one measure of a kilometre, as inferred from the comparator work, $\pm$ 0·11^{mm}, which agrees well with the value just given above.

It is clear from these figures that, with stable microscopes and with stable section marks, the error of measurement may be rendered quite insignificant in comparison with that of the length of the measuring bar.

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(12) *Adopted lengths and probable errors of the kilometre sections and kilometre.*—In deriving final values for the lengths of the sections and for the whole kilometre, we should deal only with the means of the forward and backward measures, since in these means the systematic error is partly, if not largely, eliminated. The lengths in question should appertain to an epoch falling nearer to the epoch of the second set than to that of the first set of measures of the kilometre, since it was used from September 8 to October 8, 1891, as a standard in testing the behavior of the 100m steel tapes and the secondary apparatus. Whatever lengths we derive will be affected by the changes which took place in the relative positions of the marking stones. It is believed that these changes were considerable in comparison with the actual errors of measurement affecting the mean results in either set of measures. In view of these considerations, it is deemed best to assign double weight to the mean results of the second set of measures, and compute probable errors for the weighted mean values of the sections accordingly.

In conformity with this plan, then, the mean values given in Table III, or in section 9, furnish the following system of observation equations and values, s_1, s_2, s_3, s_4, being, as in section 9, the most probable values of the excesses of the sections over $50B_{17}$:

	mm.		*mm.*	Weights.
s_1	− 6·44	=	+ 0·39,	1.
s_1	− 7·02	=	− ·19,	2,
s_2	+ 4·99	=	− ·37,	1,
s_2	+ 5·55	=	+ ·19,	2,
s_3	− 19·22	=	+ ·33,	1,
s_3	− 19·72	=	− ·17,	2,
s_4	+ 19·92	=	− ·88,	1,
s_4	+ 21·24	=	+ ·44,	2.

mm.	*mm.*
$s_1 = +$ 6·83,	$s_2 = -$ 5·37,
$s_3 - +$ 19·55,	$s_4 = -$ 20·80.

The sum of the weighted squares of the residuals in this system of equations is 1·76. Hence the probable error of one equation of weight 1 is $\pm 0{\cdot}45^{mm}$ and that of either s is $\pm 0{\cdot}26^{mm}$. Thus it appears that when the movements of the marking stones are included with the errors of operation, the probable error of our adopted values for the quarter sections of the kilometre amounts to the millionth part of their length.

If it were proper to infer the probable error of the whole kilometre or $(s_1 + s_2 + s_3 + s_4)$, from the above calculation, we should find $\pm 0{\cdot}45^{mm} \times \frac{2}{3}\sqrt{3} = \pm 0{\cdot}52^{mm}$. But this is manifestly too large if the measured values of the whole length of the kilometre are free to any extent from the effects of the motions of the intermediate marking stones. The mean values of the excesses of the kilometre over $200B_{17}$

for the two pairs of measures are as seen from Table III, section 9, $+0{\cdot}75^{mm}$ and $-0{\cdot}05^{mm}$. Giving the second of these double weight, their weighted mean is $+0{\cdot}22^{mm} \pm 0{\cdot}26^{mm}$. The probable error is here only half that given above, and thus confirms the supposition that the values for the whole kilometre are not to any considerable extent affected by the movements of the intermediate marking stones.

For the whole length of the kilometre we shall adopt the value just derived. This length is

$$200B_{17} + 0{\cdot}22^{mm} \pm 0{\cdot}26^{mm}.$$

The values of the quarter sections of the kilometres are not needed, but the halves of the line were used in testing the secondary apparatus. Deriving the lengths of these halves and their probable errors by the method just applied in getting the entire line, we find the

$$\text{North half} = 100B_{17} + 1{\cdot}46^{mm} \pm 0{\cdot}19^{mm}.$$
$$\text{South half} = 100B_{17} - 1{\cdot}25 \quad \pm 0{\cdot}19.$$

It remains now to combine the probable error of B_{17} with the probable errors attached to the above values in order to get their total probable errors when expressed in metres. The probable error of B_{17} for A end right, as given in Chapter II, section 23, is $\pm 1{\cdot}3^{\mu}$, and hence the probable errors of $100B_{17}$ and $200B_{17}$ are $\pm 0{\cdot}13^{mm}$ and $\pm 0{\cdot}26^{mm}$, respectively. Finally, therefore, we have for the lengths of the kilometre and its halves, when expressed in terms of the International Metre,

$$\text{Whole kilometre} = 200B_{17} + 0{\cdot}22^{mm} \pm 0{\cdot}37^{mm},$$
$$\text{North half} = 100B_{17} + 1{\cdot}46 \quad \pm 0{\cdot}23,$$
$$\text{South half} = 100B_{17} - 1{\cdot}25 \quad \pm 0{\cdot}23,$$
$$B_{17} = 5^{m} - 18{\cdot}0^{\mu}.$$

(13) *Adopted lengths and probable error of 100^{m} section.*—The 100^{m} section at the north end of the kilometre was used in some experiments with the secondary bars, and it is important to adopt some lengths for it. Since the dates of use of this section did not correspond with the dates of its measurement with the iced bar, it seems best to adopt a mean length from each set of measures. Then for any date of use of the section a value for its length may be interpolated, assuming that the relative positions of the marking stones changed proportionally with the time. Thus we have from Table II, section 8,

Mean length 100^{m} section—

$$\text{September 15, 1891, } = 20B_{17} - 5{\cdot}84^{mm} \pm 0{\cdot}04^{mm},$$
$$\text{September 28, 1891, } = 20B_{17} - 5{\cdot}53 \quad \pm 0{\cdot}04.$$

Introducing the probable error of B_{17} we have finally, for the lengths of this section when expressed in metres

$$\text{September 15, 1891, } 100^{m} \text{ section} = 20B_{17} - 5{\cdot}84^{mm} \pm 0{\cdot}05^{mm},$$
$$\text{September 15, 1891, } 100^{m} \text{ section} = 20B_{17} - 5{\cdot}53 \quad \pm 0{\cdot}05,$$
$$B_{17} = 5^{m} - 18{\cdot}0^{\mu}.$$

CHAPTER IV.

THE METALLIC TAPE BASE APPARATUS.

A.—DESCRIPTION OF APPARATUS AND ITS USE.

(1) *Historical note.*—The investigation of the practicability of using long metallic tapes for measuring lines of precision was taken up by the Coast and Geodetic Survey in the autumn of 1890. This work was assigned to the writer, who devised the apparatus and methods described herein.

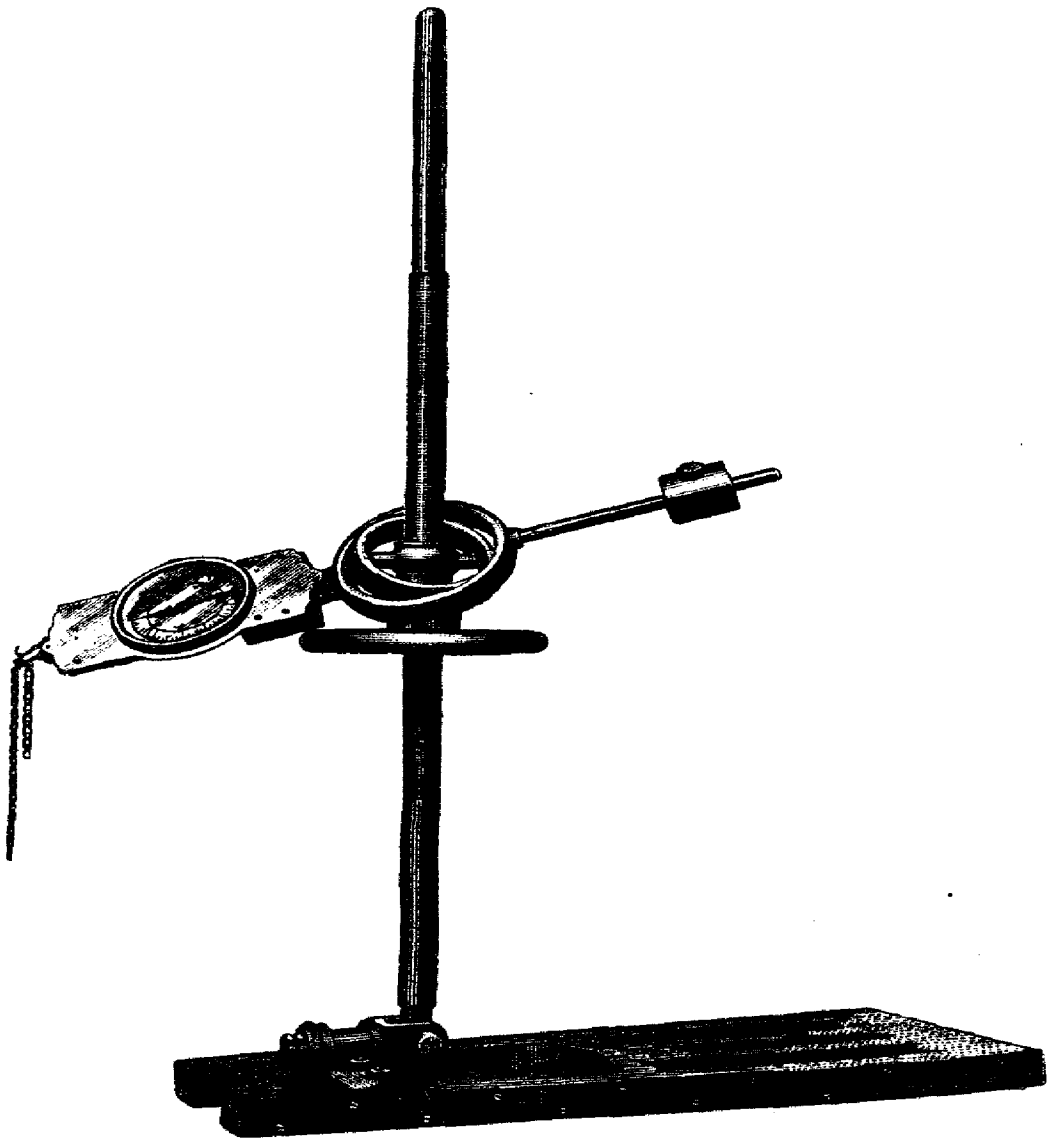
During the winter of 1890 and 1891 a great number of experiments were made with a view to determine the best method of stretching and aligning a tape, and especially with a view to discovering the most effective way of getting the tape's temperature. For this purpose a 100^{m} comparator was set up in the Botanical Garden at Washington, D. C. This comparator consisted simply of a series of equidistant stakes, 10^{m} apart, bearing support nails for the tape, and a post at either end, on which were attached scales for defining the tape's position at any time. In addition, a large number of laboratory experiments were made with various thermometers to determine their sensitivities under differing conditions. This preliminary work served to fix attention on most of the points essential to the use of tapes in precise work.

During the spring of 1891 the apparatus as herein described was constructed by D. Ballauf of Washington, D. C., and by the Instrument Division of the Survey, and in July following, the apparatus was taken to Holton Base, where the long tapes were standardized on the 100^{m} comparator described in section 2 of Chapter III. The tapes were in turn used to measure Holton Base during the same summer. The five sections of this base (of 5 500^{m}) were measured from six to thirty times each under various circumstances as to temperature, time of day, etc., and the behavior of the tapes was well shown by the measures of the kilometre section which was standardized by means of the iced bar apparatus as explained in Chapter III. In the following year, namely, October, 1892, another base (of 3 900^{m}) at St. Albans, W. Va., was measured.

Mercurial thermometers were used in all these measures to determine the temperature of the tape. An attempt was made in 1891 to use a bronze tape in connection with the steel ones for temperature indications, but the bronze tape was found to have too low a rate of expansion for successful competition with mercurial thermometers. The

feasibility of stretching two tapes simultaneously was demonstrated, however, and if a suitable metal for use with steel can be found it may prove advantageous to discard the mercurial thermometers.

In all this work, save that pertaining to St. Albans base, very efficient service was rendered by Mr. J. S. Siebert, who made most of the working drawings for the construction of the apparatus and who partici-



pated equally in the large amount of observation and computation entailed.

(2) *The 100^{m} tapes.*—These tapes are of steel. They are 101·01^{m} long, and are 6·34^{mm} by 0·47^{mm} in cross section. They weigh 22·3 grammes per metre of length. They are subdivided into 20^{m} spaces by graduations ruled on the surface of the tape itself. Being 101·01^{m} long, the end graduations fall about 0·5^{m} from the tape ends, which terminate in loops formed by annealing and riveting the tape back on itself. The

Description of tape apparatus and its use.

surface of the tape, where it is not polished to receive the graduations, is of a dull black color. When not in use the tapes are rolled up on reels, and they may be thus transported with ease and safety.

(3) *The tape-stretchers.*—The nature of the tape-stretchers used to give tension and alignment to the tape may be best understood by a glance at the cut on page 414. They consist of a lever hinged by a universal joint to a platform on which the operator stands. This lever is made of a piece of steel tubing terminating in a hickory handle. Along the upper two-thirds of the tube is cut a screw thread on which a wheel nut plays freely. This nut gives a vertical motion to a gimbal-jointed support to which a spring balance is attached. The balance is connected with the tape by means of a short piece of sash chain. It is evident from the figure that the stretcher has all the motions necessary to enable the operator to align the tape and to give it the proper tension.

The balance, which is simply a commercial article adapted to the purpose, reads to ounces, and may be easily held to the nearest ounce by the operator.

An important item for the safety of the tape used is a breaking link, or a link which will part under a tension of about 14 kilogrammes or 30 pounds. A link of this sort is provided at each end of the tape, so that it can not be overstrained by accident.

It will be noticed that the balance carries two hooks. The extra hook, which is attached to the balance frame, was intended to carry the additional tape in case it proved advantageous to use tapes of different metals simultaneously. This plan of stretching two tapes worked very satisfactorily, the tension of the steel tape being given by one balance and that of the bronze by the other; but, for the reason stated above, this plan was followed only in making a few of the tape comparisons mentioned below. The extra hook, it may be stated, is provided with a longitudinal screw motion, so that by means of this and the links of the connecting chains the tapes may be always brought to the proper tension, whatever their temperature.

When a single tape is used, the rear end is attached to the extra hook of the rear end balance, while the desired tension is given by the balance at the front end of the tape.

An objection to this form of stretcher lies in its weight, which is about 45 pounds. Contemplated improvements will lighten it by several pounds and bring the balance to a position concentric with the lever shaft. Experience has shown, however, that an able-bodied operator can handle the stretcher successfully when measuring at the rate of 2 kilometres per hour. During the measurement of St. Albans base, in 1892, 8 kilometres were measured on one occasion between 7:30 p. m. and midnight without excessive fatigue to the operators.

(4) *The thermometers.*—The thermometers used are of the Centigrade type and are graduated on their stems to half degrees. They were

made by Green, of New York. They were originally mounted on brass scales. The preliminary experiments showed that the mass of these scales caused the thermometers to lag very considerably with respect to the tape temperature. Hence the scales were removed, and light wire loops were attached to the upper ends of the stems, so that the thermometers could be suspended by a cord and thus isolated from adjacent masses or whirled in the air when necessary. Experiments also showed these thermometers to be slightly more sensitive when the bulbs were covered with sheaths of blackened aluminum foil, though this increase, if real, is probably unimportant. During the measurement of St. Albans base, in 1892, the aluminum sheaths were replaced by steel sheaths, made by coiling and annealing very thin steel tape into lengths just long enough to slip over the bulbs and clasp the thermometer stems. These are undoubtedly superior to the aluminum sheaths in simulating the surface of the tape, but they are of somewhat greater mass than the aluminum sheaths and cause more lag of the thermometers with respect to the tape. In any case, it seems desirable to blacken the bulbs of thermometers used with steel tapes presenting a blackened surface.

A great number of experiments, made in the Botanical Garden during the winter of 1890–'91, showed that with the thermometers arranged as just described (i. e., with aluminum sheaths) they followed the tape temperature without serious lagging when observations were made under a clear sky at night.

Subsequent experience in determining the tape lengths on the 100^m comparator of Holton Base showed a very satisfactory accordance of the tape and thermometer temperatures, whether observations were made during the day or night. This comparator was covered, however, except on its north side, by a shed which screened the tape from the direct rays of the sun.

Sometimes three of the thermometers were used with the tape, and at other times two. When three were used, they were placed one at the middle of the tape's length and one at one-sixth that length from either end. When two were used they were generally placed at one-fourth the tape's length from its ends, respectively. For the most precise work, as in comparisons for determining the tape lengths, the three thermometers were each read twice, once just before and once just after the observations on the tape. The mean of the six thermometer readings thus made gave the tape's temperature with a probable error of about $\pm 0{\cdot}2^\circ$ C., equivalent to $\frac{1}{500000}$ part of the tape length, when observations were made at night or under the comparator shed.

(5) *Method of supporting and marking position of tape.*—When in use for measuring a line the tape is supported at equal intervals of 10^m or 20^m throughout its length. The supports found most convenient and amply sufficient are steel-wire nails driven into stakes set at the proper intervals along the line. These nails are ranged into a straight line for

any tape length by means of a small theodolite whose telescope axis may be brought to the right height at one end of that tape length.

The ends of the tape are supported by the tape stretchers, by means of which the tape is aligned over the marking stakes. The latter are stakes of scantling cut off or driven down to the proper height and capped with a small table, on which a plate of zinc is held in place by light nails. These tables and plates are rectangular, 5^{cm} by 20^{cm}, say, and the longer sides are placed parallel to the line. On the plates the position of the forward-end graduation of the tape is marked as the measure progresses by means of a sharp brad-awl held against the edge of a small try-square, which is aligned against the longer edge of the marking plate. The rear-end graduation of the tape is in turn brought into coincidence with the successive marks on the zinc plates.*

(6) *Operation of measurement.*—The marking and support stakes for the measurement of a line are most advantageously ranged out and adjusted beforehand. This is especially essential for work of the highest precision carried on at night. Assuming this to have been done, and that night work is contemplated, the operation of measurement, if carried on rapidly, will require twelve men, to wit, two observers of the tape, one at the front and one at the rear end; two observers of thermometers, who also help to carry the tape forward; one recorder; two operators of the stretchers; five men to handle lamps and to carry the tape forward.

The co-operation of the observers and other operatives is secured by means of a code of signals made with whistles held by the observers. Thus, when the observer at the rear end of the tape is ready to have the tension applied, he gives one blast of his whistle. The stretcher at the front end immediately brings his balance to the proper reading, when the observer at that end announces the fact with one blast of his whistle. With the tape under tension the rear-end observer adjusts the rear-end graduation of the tape to coincidence with the defining mark on the zinc plate at that end. When this is accomplished he gives two blasts from his whistle. On hearing these, the recorder, who stands at the middle of the tape length, immediately lifts the tape a few centimetres from the support nail near him and lets it fall back. The effect of this is to make the tape straight and relieve it from friction on the support nails. When the vibration of the tape ceases the rear observer gives one blast, which signifies that the position of the front end of the tape may be marked on the zinc plate. The mark is made by the front-end observer, who, on completing the operation, gives one blast from his whistle. Then the thermometer readings, which are observed while the mark is made, are called out to and

*The use of such plates was introduced in 1885 by Mr. O. B. Wheeler, assistant engineer, Missouri River Commission.

repeated by the recorder. Finally, the rear observer sounds two blasts from his whistle and the tape and tape stretchers are carried forward to a new position.

The speed attained in this process has been about 2 kilometres per hour, though a single kilometre has been measured in twenty minutes.

By reason of the expansion and contraction of the tape, it is frequently desirable to set the tape back or forward by a round number of centimetres. This may be done on the zinc plates by means of a suitable pocket scale. Such displacements are recorded as "set-backs" and "set-ups." Their amount is engraved on the plates, which may be appealed to if any doubt arise concerning the record. When "set-backs" or "set-ups" are made, and when more than one measure is recorded on the plates, it is advantageous to number, orient, and date them for filing as part of the records.

B.—DETERMINATION OF EQUATIONS OF TAPES.

(7) *Apparatus for observing lengths of tapes.*—It is obviously necessary to the successful use of long tapes in measuring lines of precision to provide suitable means for observing the tape lengths at different temperatures. Such data for any tape give its length at some assumed temperature and its rate of expansion, and thus enable us to write an equation from which the length at any observed temperature can be computed. In order to meet this requirement in a way which would leave least room for doubt concerning the actual behavior of the tapes, the 100^{m} comparator described in section 2, Chapter III, was devised. The features essential to a study of the tapes embodied by the comparator were, 1st, that its length could be determined by means of the iced bar apparatus with a precision superior to that attainable with the tapes; 2d, that the tape could be stretched and aligned on the comparator in precisely the same way as when used in the field; and 3d, that the tape length at any time could be quickly and accurately referred to that of the comparator.

As explained in section 2, Chapter III, the ends of the comparator were marked by spherical-headed bolts, or cut-off spheres. The end marks or graduations of a tape when under comparison were observed by means of micrometer microscopes mounted over the cut-off spheres as when used in measuring the length of the comparator with the iced bar apparatus. The positions of the tape ends could thus be accurately referred to the cut-off spheres through the medium of the cut-off cylinders described in section 8, Chapter I.

The tape stretchers were mounted on the stationary car track of the comparator, and by means of their vertical adjusting screws the graduated surfaces of the tape could be easily brought to focus under the microscopes.

The tapes were supported at intervals of 10^{m} on wire rods attached to the alternate microscope posts of the comparator; and the method of handling the tape was in every way similar to that followed in field

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measurements with the tape, except that its graduation marks were observed with micrometer microscopes.

The thermometers were hung from light wooden supports attached to the microscope posts of the middle of the comparator and at 15^{m} from either end. The stems of the thermometers were upright and the bulbs were kept at about the same height as the tape.

(8) *Method of observation.*—The programme adhered to in all determinations of the tape lengths made on the 100^{m} comparator is the following:

The tape being mounted on its support nails and attached to the tape stretchers, was first put under tension, aligned, and brought to focus under the microscopes at the ends of the comparator. When this operation was completed at the east end of the comparator, the observer of that end gave one blast from his whistle. The observer at the west end answered this signal by one blast from his whistle as soon as the adjustments at his end were completed. The observers then read the thermometers nearest to them, while the recorder who stands near the middle of the comparator read the thermometer at that point. On announcing their thermometer readings to the recorder and hearing his repetition of them, the observers returned to their respective ends of the comparator and re-examined their adjustments of the tape and microscopes with respect to one another; and the tension, which was always applied at the west end of the comparator, was brought to its normal value. On completing his adjustment the observer at the east end gave one blast from his whistle. This was responded to by one blast from the whistle of the observer at the west end as soon as his adjustments were satisfactory. The observer at the east then gave two blasts from his whistle, whereupon the recorder lifted the tape a few centimetres from and let it fall back on the middle support nail. This served to relieve the tape from friction on its support nails; and as soon as the tape ceased vibrating the observer at the east end gave one blast from his whistle. This was responded to by one blast from the whistle of the other observer, and the microscopes were immediately brought to bisection on the graduation marks of the tapes. This completed, the observers exchanged signals, and proceeded at once to re-read the thermometers. Finally, after completing the record of the thermometer readings, the observers referred their respective microscopes to the cut-off spheres of the comparator.

The time required to make such a set of observations, exclusive of the cut-off measures, was two to three minutes, while the cut-off measures required about two minutes additional.

In observing on the tape it was generally found most convenient to move the whole microscope by means of the longitudinal screw of its slide rest.

The reading of the balance index was noted immediately after

observing on the tape, so that a correction could be applied for any departure from the normal tension.

Tables I and II following give the records of a complete set of observations for temperature and length of 100^{m} steel tape No. 88 on the comparator.

As will be seen by a glance at Table I, the thermometers were read to tenths of a degree. In order to make such readings with ease, hand-magnifying glasses were used.

The standard tension adopted for the 100^{m} steel tapes was 25 pounds, 9 ounces (11·596 kilogrammes). This is the tension given by the balances used when they are horizontal, when they are free from index error, and when they read 25 pounds even. The departure from this even reading was observed to the nearest ounce; or what amounts to the same, the actual reading of the balance at the time of observing with the microscope on the tapes was noted.

Since it was generally easiest to move the whole microscope by means of the longitudinal screw of its slide rest in reading on the tape. the micrometer screws were usually set to the same reading at the two ends of the comparator in order to simplify the computation which requires the difference of the microscope readings.

The records of the cut-off measures are in all respects similar to those obtained in measurements with the iced bar, and fully set forth in section 11, Chapter I, to which reference may be made for an explanation of the meaning attached to the symbols in Table II.

TABLE I.—*Record of observed temperature and length of steel tape No. 88, October 4, 1891.*

Time of day.	Thermometer readings.			Balance reading.	Microscope readings.	
	No. 5598.	No. 5599.	No. 5601.		West end, *L*.	East end, *R*.
a. m.						
10·10	11·7° C.	11·7° C.	11·5° C.	25 pounds.	20·00	20·00
10·11	11·7	11·9	11·6	2 ounces.		

TABLE II.—*Record of cut-off measures corresponding to observations in Table I.*

West end—Repsold cylinder.						East end—U. S. C. and G. Survey cylinder.					
Time of day.	End of scale, east.	Graduation observed.	Microscope reading.	Level readings.		Time of day.	End of scale, east.	Graduation observed.	Microscope readings.	Level readings.	
				l.	*r*.					*l*.	*r*.
a. m.						*a. m.*					
10·12	A.	22 I.	24·00	9·0	16·0	10·12	A.	1 I.	19·61	5·0	9·5
	B.	21 I.	16·69	15·0	10·0		B.	1 I.	22·72	8·0	6·5

(9) ***Length of tape in terms of comparator interval.***—Let T denote the

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length of the tape under comparison at the time of any set of observations, and let D denote the comparator interval, or distance between the terminal spheres of the comparator, at the same time. Also, as in section 11, Chapter I, let

L, R = readings of the microscopes on the tape at the left and right hand ends of the comparator, respectively,

L_c, R_c = corresponding mean readings of the same microscopes on the cut-off scales,

S_l, S_r = divisions of cut-off scales observed at the respective ends of the comparator,

I_l, I_r = corresponding inclinations of cut-off cylinders,

H_l, H_r = corresponding heights of cut-off scales.

Finally, let $\Delta\tau$ be the excess in ounces of the applied tension over the normal tension of the tape.

Then

$$T = D \mp S_l \pm S_r - (L - R) + (L_c - R_c) - H_l I_l + H_r I_r - 0{\cdot}05^{mm} \times \Delta\tau. \quad (1)$$

In this formula the upper signs apply to S_l and S_r when the graduation numbers appear inverted (I), and the lower ones when those numbers appear erect (E). The inclinations I_l and I_r are positive when the left-hand end of the bubble is the higher. The tension excess $\Delta\tau$ is positive when the balance reading is greater than 25 pounds.

The corrections for inclination of the cut-off cylinders $H_l I_l$, $H_r I_r$ were generally very small, and hence negligible. The factors by which the difference of the sums of the level bubble readings must be multiplied to get these corrections are, for the Repsold cylinder 1·6″ and for the U. S. Coast and Geodetic Survey cylinder 6·1″. These apply, of course, only to the special values of the heights H of the cylinders as used on the comparator.

The value of the coefficient of $\Delta\tau$ used in the actual computations of length was $0{\cdot}047^{mm}$ as determined by experiments on the comparator in 1891. Additional determinations made in 1892 indicate that the value $0{\cdot}05^{mm}$ per ounce when the normal tension is 25 pounds is more precise.

The computation of the length T by formula (1) in the case of the observations in Table I and II is then as follows:

		mm.
$\mp S_l$	=	− 21·500
$\pm S_r$	=	+ 1·000
$-(L - R)$	=	0·000
$+(L_c - R_c)$	=	− 0·082
$- H_l I_l$	=	+ 0·003
$+ H_r I_r$	=	− 0·018
$-0{\cdot}047^{mm} \times \Delta\tau$	=	− 0·092
	Sum =	− 20·689

Hence we have

$$T = D - 20{\cdot}689^{mm}.$$

The value actually used was $D - 20{\cdot}67^{mm}$, which comes from neglecting the corrections for inclination of cut-off cylinders and rounding all computed corrections to the nearest hundredth of a millimetre.

(10) *Thermometer corrections.*—The thermometers used in all determinations of tape lengths on the 100m comparator were Nos. 5598, 5599, and 5601 by Green, of New York. Their corrections were determined by Mr. O. H. Tittmann, assistant in charge of the Office of Standard Weights and Measures of the Survey.

Table III following gives the corrections essential to reduce the observed readings of these thermometers to the hydrogen scale as determined by Assistant Tittmann on August 19, 1890.

TABLE III.—*Thermometer corrections August 19, 1890.*

Temperature.	Correction to—		
	5598.	5599.	5601.
2·6° C.	—0·20° C.	—0·19° C.	—0·21° C.
3·6	·24	·18	·16
10·2	·19	·19	·16
15	·18	·18	·18
19·6	·18	·21	·24
25	·17	·17	·17
30	·14	·22	·17
34·9	·19	·14	·29

The readings of these thermometers when packed in melted ice on August 19, 1890, were + 0·25°, + 0·08° and 0·00°, respectively. Their freezing point readings were not again determined until January 21, 1892, when they read + 0·15°, + 0·17°, and 0·00° respectively.

To take account of these changes in the freezing point readings it is assumed that they varied directly as the time during the interval which elapsed between the two determinations of August, 1890, and January, 1892. This assumption gives for the zero point readings at the epoch of the tape comparisons, September, 1891, say, +0·17°, +0·15°, and 0·00°, respectively; and these are the values adopted.

The changes which these readings entail in the corrections in Table III are found by adding +0·08° to the corrections for No. 5598, by adding —0·07° to the corrections for No. 5599, while the corrections for No. 5601 remain unchanged.

Since the thermometers always had the same readings within a degree, it has been considered sufficient to apply the mean of their corrections to the mean of their readings in computing results from comparions on the 100m comparator. Accordingly, the mean corrections in Table IV following have been applied in deducing the temperatures of the tapes from the observations on the comparator.

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TABLE IV.—*Means of corrections to thermometers September, 1891.*

Temperature	2·6°	3·6°	10·2°	15·0°	19·6°	25·0°	30·0°	34·9°
Correction	—0·20	—0·19	—0·18	—0·18	—0·20	—0·17	—0·17	—0·20

The means of the six thermometer readings were carried to hundredths in the computation, and the above corrections were then applied in getting the adopted tape temperatures, which are likewise carried to hundredths of a degree. This is to some extent a needless refinement, but its application cost little labor.

(11) *Adopted lengths of the comparator.*—As explained in section 5, Chapter III, it is assumed that the comparator interval or length D changed during the period of two months which elapsed between the two groups of determinations of that interval. Fortunately, the largest groups of tape comparisons were nearly coincident in time with those of the comparator measures. For the small number of tape comparisons made between August 30 and September 5, a length of the comparator interval is derived on the assumption that it changed directly as the time between the two epochs for which lengths are given in section 7, Chapter III. Hence the following values are adopted for the comparator interval D in terms of the iced bar B_{17}:

$$\begin{aligned} &\text{Aug. 1 to 8, 1891,} && D = 20B_{17} + 39{\cdot}52 \text{ mm.},\\ &\text{Aug. 30 to Sept. 5, 1891,} && D = 20B_{17} + 39{\cdot}44 \text{ mm.},\\ &\text{Oct. 3 to 9, 1891,} && D = 20B_{17} + 39{\cdot}37 \text{ mm.}\end{aligned}$$

$$B_{17} = 5^{m} - 18{\cdot}0^{\mu}.$$

(12) *Correction for index error of balances.*—The spring balances used to give tension to the tapes were tested from time to time by the method explained in section 4, Chapter V. They were also tested frequently with known weights to see that they gave the tensions indicated.*

The standard tension adopted was 25 pounds 9 ounces, which is the tension given by the balances when horizontal, when they are free from index error, and when they read 25 pounds even. By reason of some accidents to the balances their index errors changed to some extent during the season. They were always accurately observed, however, and the corrections applied are without sensible error.

The correction for index error is of the same form as that for excess of tension explained in section 9 above. The adopted elongation of the tape per ounce was $0{\cdot}047^{mm}$; and the corrections actually applied to the lengths of the tapes as computed by equation (1), section 9, are the following:

Aug. 1 to Aug. 8, 1891,	0·00,
Aug. 30 to Sept. 5, 1891,	—0·16,
Oct. 3 to Oct. 9, 1891,	+0·16.

*See Chapter V, section 4, for details of observed index corrections.

(13) *Observed data for lengths and rates of expansion of steel tapes Nos. 85 and 88.*—Tables V and VI following give the results of all observations made on the 100^{m} comparator to determine the equations of the steel 100^{m} tapes Nos. 85 and 88. The first column gives the date of observation, and the second the time when the observations were made. The third column gives the temperature of the tape corrected to the hydrogen scale as explained in section 10 above. The fourth column gives the length of the tape in terms of the comparator interval D as computed from equation (1), section 9. The fifth column gives the length of the tape in terms of the iced bar B_{17}, in accordance with the relations given in section 11. The corrections to standard tension of 25 pounds 9 ounces given in section 12 are also applied in deriving the values in this column. The last column gives the residuals which result from the equations of the tapes deduced in the following section.

TABLE V.—*Results of observations for length and rate of expansion of 100^{m} steel tape No. 85.*

Date.	Time of day.	Observed temperature of tape.	Observed length of tape.		Residual (computed minus observed value).
			$D-$	$20\ B_{17}+$	
1891.			*mm.*	*mm.*	*mm.*
Aug. 6	11:29 a. m.	29·02° C.	4·01	35·51	+0·12
	1:02 p. m.	30·56	1·83	37·09	— ·34
	2:06	30·07	3·37	36·15	+ ·63
	3:05	29·88	2·67	36·85	— ·28
	4:02	28·92	4·38	35·14	+ ·38
	5:00	28·01	4·94	34·58	— ·06
	9:02	19·58	14·86	24·66	+ ·63
	9:12	18·44	15·72	23·80	+ ·25
7	7:02 a. m.	20·96	12·68	26·84	— ·03
	8:02	24·53	8·68	30·84	— ·13
	9:02	26·73	6·55	32·97	+ ·15
	10:02	27·96	4·95	34·57	— ·11
	11:01	29·26	3·57	35·95	— ·06
	11:48	30·11	1·98	37·54	— ·70
	2:04 p. m.	30·21	2·24	37·28	— ·35
	7:16	31·35	1·05	38·47	— ·29
	7:32	21·51	12·48	27·04	+ ·37
	8:38	20·15	14·21	25·31	+ ·60
8	7:28 a. m.	24·66	9·09	30·43	+ ·43
	8:34	27·06	5·63	33·89	— ·41
	9:35	29·14	3·63	35·89	— ·22
	10:32	30·34	2·62	36·90	— ·17
	11:31	31·39	1·13	38·39	— ·17
	12:38 p. m.	32·12	0·29	39·23	— ·21
	1:33	31·54	0·80	38·72	— ·33
	2:34	30·32	2·75	36·77	+ ·28
	7:41	21·48	12·42	27·10	+ ·27
	8:34	20·93	12·96	26·56	+ ·21
	9:34	20·38	13·32	26·20	— ·03

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TABLE V.—*Results of observations for length, etc.*—Continued.

Date.	Time of day.	Observed temperature of tape.	Observed length of tape.		Residual (computed minus observed value).
			$D-$	$20\ B_{17}+$	
1891.			*mm.*	*mm.*	*mm.*
Sept. 5	2:40 p. m.	14·88	19 08	20·20	— ·05
	3:24	14·70	19·51	19·77	+ ·18
	5:04	14·26	19·91	19·37	+ ·10
	7:35	13·88	20·02	19·26	— ·21
	9:08	13·98	20·16	19·12	+ ·04
Oct. 3	10:40 a. m.	26·73	6·28	33·25	— ·13
	11:48	27·24	5·99	33·54	+ ·14
	1:41 p. m.	27·06	6·15	33·38	+ ·10
4	4:54 p. m.	13·85	20·12	19·41	— ·39
	6:50	12·67	21·60	17·93	— ·20
	8:08	12·32	22·35	17·18	+ ·17
	8:58	11·70	22·68	16·85	— ·18
	10:20	11·38	23·35	16·18	+ ·14
5	5:00 a. m.	7·20	27·86	11·66	+ ·07
	6:04	6·66	28·34	11·19	— ·04
	6:54	7·97	26·89	12·64	— ·05
	8:04	10·24	24·82	14·71	+ ·36
	9:00	11·80	22·99	16·54	+ ·24
	12:38 p. m.	15·13	19·23	20·30	+ ·12
	7:03	7·07	27·95	11·58	+ ·02
	8:08	6·72	28·51	11·02	+ ·20
	8:58	6·38	28·82	10·71	+ ·13
	10:16	5·04	30·01	9·52	— ·14
6	4:32 a. m.	3·51	31·95	7·58	+ ·12
	5:28	3·47	31·78	7·75	— ·09
	6:28	4·76	30·52	9·01	+ ·06
	7:56	9·59	25·30	14·23	+ ·13
	8:54	11·95	22·75	16·78	+ ·16
8	7:54 a. m.	8·62	26·03	13·50	— ·20
	8:48	10·89	23·51	16·02	— ·24
	9:52	12·55	21·57	17·96	— ·36
	10:46	13·60	20·97	18·56	+ ·19
	11:52	14·63	19·55	19·98	— ·11
	12:58 p. m.	15·02	19·29	20·24	+ ·06
	2:06	15·63	18·76	20·77	+ ·20
	3:00	14·67	19·77	19·76	+ ·16
	4:06	13·87	20·76	18·77	+ ·27
	5:52	9·45	25·26	14·27	— ·07
	7:12	7·38	27·70	11·83	+ ·11
	11:16	7·25	27·49	12·04	— ·24
	11:58	7·70	27·05	12·48	— ·19
9	2:04 a. m.	5·31	29·73	9·80	— ·13
	3:04	5·49	29·48	10·05	— ·18
	4:08	5·49	29·56	9·97	— ·10
	5:06	4·19	30·77	8·76	— ·29
	6:10	4·47	30·58	8·95	— ·20
	6:56	6·68	28·15	11·38	— ·21
	8:00	9·33	25·31	14·22	— ·15

TABLE VI.—*Results of observations for length and rate of expansion of 100m steel tape No. 88.*

Date.	Time of day.	Observed temperature of tape.	Observed length of tape.		Residual (computed minus observed value).
			$D -$	$20\,B_{17} +$	
1891.			*mm.*	*mm.*	*mm.*
Aug. 1	10:14 a. m.	27·46° C.	3·66	35·86	+0·42
	1:52 p. m.	29·28	0·86	38·66	— ·40
	2:50	24·96	6·58	32·94	+ ·61
	3:32	24·61	6·04	33·48	— ·31
	4:10	22·86	8·17	31·35	— ·09
	4:56	23·34	7·11	32·41	— ·63
	5:56	23·97	7·63	31·89	+ ·58
	7:25	20·46	11·35	28·17	+ ·46
3	7:52 a. m.	14·99	16·79	22·73	— ·06
	8:29	14·46	17·69	21·83	+ ·26
	9:59	14·30	16·95	22·57	— ·65
	11:05	14·70	17·56	21·96	+ ·39
	1:32 p. m.	16·45	15·35	24·17	+ ·09
	2:08	16·95	14·73	24·79	+ ·02
4	8:22 a. m.	20·84	10·40	29·12	— ·07
	9:02	21·66	9·70	29·82	+ ·13
	10:02	23·24	7·93	31·59	+ ·08
	10:56	24·14	6·61	32·91	— ·26
	12:02 p. m.	24·01	6·65	32·87	— ·37
	1:02	25·01	5·92	33·60	+ ·00
	4:46	24·77	6·49	33·03	+ ·31
	9:04	17·72	14·36	25·16	+ ·49
5	7:15 a. m.	21·71	9·57	29·95	+ 0·5
	8:32	24·29	7·02	32·50	+ ·32
	9:32	25·28	6·05	33·47	+ ·43
	10:32	25·71	5·69	33·83	+ ·54
	11:32	28·01	2·44	37·08	+ ·21
	1:32 p. m.	28·01	2·53	36·99	+ ·12
	2:32	28·27	2·49	37·03	+ ·13
	3:32	27·89	2·92	36·60	+ ·15
	4:30	26·31	4·78	34·74	+ ·28
	5:32	26·27	4·64	34·88	+ ·10
	8:56	17·67	14·46	25·06	+ ·54
30	6:46 p. m.	16·41	15·35	23·93	+ ·29
	8:14	14·26	17·79	21·49	+ ·39
	9:30	13·15	18·63	20·65	+ ·01
Sept. 5	12:08 p. m.	14·55	16·70	22·58	— ·39
	1:16	15·10	15·92	23·36	— ·57
	1:42	15·78	15·19	24·09	— ·56
	2:24	15·28	15·79	23·49	— ·50
	3:45	14·28	17·32	21·96	— ·06
	4:54	14·26	17·26	22·02	— ·15
Oct. 3	10:55 a. m.	27·08	3·22	36·31	— ·46
	11:39	26·99	3·09	36·44	— ·67
	1:49 p. m.	26·93	3·09	36·44	— ·74
4	4:44 p. m.	13·83	17·59	21·94	— ·54
	7:02	12·60	19·37	20·16	— ·10
	8:01	12·30	19·55	19·98	— ·25
	9:10	11·53	20·61	18·92	— ·03
	10:10	11·50	20·67	18·86	— ·10

Determination of equations of tapes.

TABLE VI.—*Results of observations for length, etc.*—Continued.

Date.	Time of day.	Observed temperature of tape.	Observed length of tape.		Residual (computed minus observed value).
			$D -$	$20\ B_{17} +$	
1891.			*mm.*	*mm.*	*mm.*
Oct. 5	5:12 a. m.	7·03	25·38	14·15	— ·18
	5:56	6·46	25·87	13·66	— ·30
	7:00	8·25	24·10	15·37	— ·06
	7:56	9·97	22·63	16·90	+ ·29
	9:04	11·70	20·68	18·85	+ ·23
	12:30 p. m.	15·35	16·48	23·05	+ ·01
	7:12	7·02	25·82	13·71	+ ·25
	8:02	6·42	26·26	13·27	+ ·05
	9:08	6·54	26·11	13·42	+ ·03
	10:04	5·12	27·50	12·03	— ·13
6	4:42 a. m.	4·39	28·33	11·20	— ·10
	5:22	3·46	29·40	10·13	— ·04
	6:34	4·66	28·24	11·29	+ ·11
	7:48	8·82	23·41	16·12	— ·18
	9:02	12·70	19·52	20·01	+ ·16
8	7:50 a. m.	8·15	23·96	15·57	— ·36
	8:54	11·20	20·96	18·57	— ·04
	9:48	12·14	20·09	19·44	+ ·16
	10:52	13·85	18·29	21·24	+ ·19
	11:46	14·05	18·43	21·10	+ ·54
	1:04 p. m.	15·25	16.72	22·81	+ ·14
	2:00	16·12	15·98	23·55	+ ·35
	3:06	14·62	17·32	22·21	+ ·06
	4:02	14·22	17·80	21·73	+ ·10
	5:58	9·23	23·45	16·08	+ ·20
	7:06	7·57	25·09	14·44	+ ·13
	11:06	7·35	24·86	14·67	— ·34
9	12:08 a. m.	7·52	24·82	14·71	— ·19
	1:56	5·39	26·96	12·57	— ·38
	3:10	5·29	27·49	12·04	+ ·04
	4:04	5·67	26·97	12·56	— ·06
	5:12	4·21	28·67	10·86	+ ·04
	6:04	4·11	28·69	10·84	— ·04
	7:02	7·07	25·28	14·25	— ·22
	7:56	9·10	22·82	16·71	— ·47

(14) *Derivation of equations of tapes.*—Let t denote the observed temperature of the tape at any time, and x the excess of the tape's length over $20\ B_{17}$ at the same time. Let x_o denote the corresponding excess in length at any assumed temperature t_o. Then if we call the rate of expansion of the tape y, and denote the most probable corrections to x and t by Δx and Δt, respectively, each pair of observed values in Tables V and VI will give an equation of condition of the form

$$x_o + y\,(t + \Delta t - t_o) - x - \Delta x = 0. \qquad (2)$$

If now we denote the weights of x and t by p and q, respectively, the method of least squares requires that, subject to the conditions (2),

$$[p\,(\Delta x)^2] + [q\,(\Delta t)^2] = \text{a minimum}. \qquad (3)$$

The solution of the problem presented by equations (2) and (3) is

not difficult, especially when the ratio of the weights p and q is constant. The method usually followed in treating this problem is based on the assumption that $q = \infty$, or that the temperature observations are not subject to error. In the present application, it seems more fitting to make the other extreme assumption, viz, that $p = \infty$, since the temperature errors surpass by far the resultant effect of all other errors. Both these extreme assumptions were made in the computations as actually carried out, and it may be said that the resulting values of x and y do not differ appreciably, so that for practical purposes one method is as good as another. Hence we shall adopt the usual method, which converts (2) into an observation equation of the form

$$x_o + y\,(t - t_o) - x = \Delta x, \text{ with weight } p. \tag{4}$$

Since t_o is arbitrary, we may conveniently give it the value

$$t_o = \frac{[pt]}{[p]}, \tag{5}$$

since this facilitates the solution of the normal equations and the computation of the probable error of the tape length at any temperature. Thus, with this value of t_o, we have

$$x_o = \frac{[px]}{[p]}, \quad y = \frac{[p\,(t - t_o)\,(x - x_o)]}{[p\,(t - t_o)^2]}. \tag{6}$$

The observations in Tables V and VI would give equations of equal weight, but instead of treating these equations separately they have been grouped into means with respect to temperature limits in order to learn what extent the errors of observation are compensating. The resulting values of x_o and y are, however, the same as would come from considering the several equations separately.

Table VII following gives the values in detail which result from the processes of grouping just mentioned as applied to the data for Tape No. 85. The first column gives the limits of the temperatures included in any group. The second column gives the number of observed temperatures or lengths in the group, or the weight p of the mean temperature or mean length for the group. The third column gives the mean temperature t and the fourth the mean length excess x for the group. The last column gives the residual which results from the substitution of the computed values x_o, y and the group means t and x in equation (4).

Table VIII gives the corresponding data for Tape No. 88.

Determination of equations of tapes.

TABLE VII.—*Summary of data for equation of tape No. 85.*

Temperature range.	Weight p.	Mean value of t.	Mean value of x.	Residual Δx. (Computed *minus* observed value.)
° °		°	*mm.*	*mm.*
30 to 35	11	30·72	37·64	—0·150
25 30	11	27·92	34·42	+ ·004
20 25	8	21·82	27·54	+ ·207
15 20	6	16·44	21·66	+ ·197
10 15	18	12 94	18·01	+ ·016
5 10	18	7·18	11·78	— ·059
0 5	5	4·08	8·41	— ·083

TABLE VIII.—*Summary of data for equation of tape No. 88.*

Temperature range.	Weight p.	Mean value of t.	Mean value of x.	Residual Δx. (Computed *minus* observed value.)
° °		°	*mm.*	*mm.*
25 to 30	15	26·89	35·66	—0·004
20 25	13	23·07	31·47	+ ·017
15 20	12	16·09	23·85	+ ·019
10 15	22	13·14	20·68	— ·030
5 10	18	7·22	14·18	+ ·009
0 5	5	4·19	10·86	+ ·022

From the data in Tables VII and VIII we find by means of equations (5) and (6)

$$\left.\begin{aligned} t_o &= 16{\cdot}89^\circ \text{ C.},\\ x_o &= +22{\cdot}35^{mm}, \text{ with weight } 77,\\ y &= 1{\cdot}0947^{mm}, \text{ with weight } 6436, \end{aligned}\right\} \text{Tape No. 85.}$$

$$\left.\begin{aligned} t_o &= 15{\cdot}72^\circ \text{ C.},\\ x_o &= +23{\cdot}47^{mm}, \text{ with weight } 85,\\ y &= 1{\cdot}0914^{mm}, \text{ with weight } 4687, \end{aligned}\right\} \text{Tape No. 88.}$$

These values substituted in equation (4) give, in connection with the mean values of t and x, the residuals shown in the last column of Tables VII and VIII. Likewise, these values of t_o, x_o, and y, when substituted in equation (4), give in connection with the individual values of t and x the residuals of the individual determinations of length shown in the last column of Tables V and VI.

The salient features of the latter residuals are shown in the following statement:

	Tape 85.	Tape 88.
Number positive residuals =	36	45
Number negative residuals =	41	40
Average residual =	0·20^{mm}	0·25^{mm}
Maximum residual =	0·70^{mm}	0·74^{mm}
Sum of squares of residuals =	4·82	8·62

From the sums of the squares of the residuals just given, it follows that the probable error of a single determination of length of these tapes on the 100^{m} comparator is

mm.
± 0·17 for tape No. 85,
± 0·22 for tape No. 88.

Applying the weights of x_0 and y, given above, there result

	Tape 85.	Tape 88.
	mm.	*mm.*
Probable error x_0 =	±0·020	±0·024
y =	±0·0021	±0·0032

If, now, we denote the lengths of tapes No. 85 and No. 88 by T_{85} and T_{88}, respectively, the preceding data enable us to write the following equations:

$$T_{85} = 20B_{17} + 22{\cdot}35^{mm} \pm 0{\cdot}020^{mm} + (1{\cdot}0947^{mm} \pm 0{\cdot}0021^{mm})\,(t - 16{\cdot}89^{\circ}\ \text{C.}) \quad (7)$$

$$T_{88} = 20B_{17} + 23{\cdot}47^{mm} \pm 0{\cdot}024^{mm} + (1{\cdot}0914^{mm} \pm 0{\cdot}0032^{mm})\,(t - 15{\cdot}72^{\circ}\ \text{C.}) \quad (8)$$

(15) *Discussion of results.*—An examination of the residuals in Tables V and VI shows that, on the whole, the observed lengths are well represented by equation (4). Thus, the whole number of the residuals in the two tables is 162, and of these, as just shown above, 81 are positive and 81 negative, while the probable error of a single determination of length is ± 0·17^{mm} for tape 85 and ± 0·22^{mm} for tape 88. Taking the average of these two probable errors, or ± 0·20^{mm}, as representative of the whole group of residuals, we may say that the probable error of a single determination of the lengths of these tapes was 1/500 000th part of their length. The maximum residual is 0·74^{mm}, which is less than four times its probable error. Taking the values ± 0·17^{mm} and ± 0·22^{mm} for the probable errors in the two cases as a basis for computation, and then

combining the two groups of residuals into a single group, the following comparison of the actual with the theoretical distribution of the residuals results:

Between limits.		Actual number of residuals.	Theoretical number.
mm.	*mm.*		
0·0 and	0·2	85	84
0·2	0·4	51	52
0·4	0·6	17	21
0·6	0·8	9	5

The agreement thus shown between the actual and theoretical distribution of the residuals is, in general, quite close. As is usual in such comparisons, the departure from theory is greatest in the case of the large errors.

On the whole, therefore, it appears that the errors to which these determinations of tape lengths were subject are of the compensating class. A confirmation of this view is afforded by the residuals of the group mean values shown in Tables VII and VIII. If the errors to which the individual determinations are subject were not of the compensating class we should expect large residuals to result from those group means. The residuals are largest in the case of the results for tape 85; but they reach only $0{\cdot}2^{mm}$ at the maximum, and this for group means having relatively small weight. The degree of compensation was manifestly best for the comparisons of tape 88, since the maximum group residual shown in Table VIII is only $0{\cdot}03^{mm}$, while the individual residuals for the comparisons of tape 88 were, on the average, somewhat greater than those of tape 85. The average of the residuals in Tables VII and VIII, regardless of weights, is $0{\cdot}06^{mm}$. This is equivalent to an error of 0·06° C. in temperature of the tape and to 3/5 000 000ths of a tape length.

Notwithstanding the compensating character of the errors in question when taken as a whole, a careful examination of the residuals in Tables V and VI indicates that the determinations were subject more or less to some systematic errors. These seem to be chiefly of two kinds, namely: First, errors due to lag of the tape temperature relatively to that of the thermometers; and second, that due probably to peculiarities of radiation which produced at times somewhat persistent differences in temperature between the tape and thermometers. The lag effect is best shown, perhaps, in the observations of October 8 and 9, 1891, which were continuous for twenty-four hours except for an interval of four hours on October 8, when the apparatus was used in measuring a section of Holton Base. The amount of this lagging may have been as much as 0·2° C., though it is entangled with other

errors to such an extent that one cannot draw a definite conclusion from the residuals of the tables. Examples of what I have provisionally called the "radiation effect" are afforded by the observations of September 5, October 3, and October 4, in the comparisons of tape 88. For these dates the residuals are all relatively large and of one sign. On the same dates, however, no such persistence in sign is shown by the residuals of tape 85 which was alongside and within two centimetres of tape 88 on the comparator. It seems impossible that there could have been such a difference in temperature of the tapes thus situated as would appear from the residuals just cited; but similar persistent differences between the tape and thermometer temperatures appeared at times in the measures of Holton Base, on cloudy days, especially, and there seems to be little doubt of their reality. (See section 8, Chapter V.) The only reason I am able to assign for the difference in behavior of the two tapes when lying alongside of one another on the comparator on the dates referred to, is that the lower surface of tape No. 88 had become somewhat polished, presenting a bright, rather than a dull black surface, from much use in preparing the standard kilometre and other sections of Holton Base. Otherwise the tapes were in all respects alike, having been cut from the same coil.

This radiation effect, or whatever it may be, was observed to be present in a somewhat greater degree in some experiments on the behavior of the tapes made in the open air and in sunshine on a portion of St. Albans Base in 1892. Since these experiments will be referred to in section 26, Chapter V, I may dismiss the question here with the remark that it is the only anomaly presented by the tapes in my experience, and that it is probably the only serious obstacle in the way of making the long tape a standard whose equation can be determined with a precision little short of that attained with laboratory standards.

(*16*) *Probable errors of tape lengths.*—The probable errors of the mean lengths of the tapes given in equations (7) and (8), section 14, are due to the errors of comparison only. The total probable errors of those mean lengths, when expressed in metres, depend, first, on the errors of comparison just mentioned; second, on the error in length of the comparator as measured with the iced bar B_{17}; third, on the error in length of B_{17} when expressed in terms of the International Metre, and fourth, on the thermometer errors with respect to the hydrogen scale.

For the probable error due to the first source of error we may adopt the mean of those given in equations (7) and (8) above, namely, $\pm 0{\cdot}022^{mm}$. This amounts to 1/4 550 000th part of a tape length.

The resultant probable error due to the second and third sources of error is given in section 7, Chapter III. It is $\pm$ 0·033^{mm}, or 1/3 000 000th part of a tape or comparator length.

An idea of the uncertainty remaining in the mean of the corrected readings of the three thermometers used may be formed by an inspection of the data for the thermometer corrections given in section 10

Determination of equations of tapes.

above. It is estimated that the probable error due to this source can not exceed ± 0·02° C. This is equivalent to ± 0·020^{mm} or 1/5 000 000th part of a tape length.

Taking the square root of the sum of the squares of these independent probable errors, there results ± 0·044^{mm} for the total probable error of the length of either tape when expressed in terms of the International Metre, and when its temperature is the mean of those observed during the comparisons. This probable error is equivalent to 1/2 270 000th part of the tape length.

The probable errors of the rates of expansion of the tapes given in equations (7) and (8) are ± 0·0021^{mm} and ± 0·0032^{mm}, and for the sake of uniformity we may adopt the mean of these, or ± 0·0026^{mm}, as a working value for either tape.

With these values of the probable error of a tape length at its mean temperature and the probable error of the rate of expansion, it follows that the probable error of a tape length at any temperature t is given by the expression

$$\pm \left\{ (0{\cdot}044^{mm})^2 + (0{\cdot}0026^{mm})^2 (t - t_o)^2 \right\}^{\frac{1}{2}}, \qquad (9)$$

wherein t_o = 16·89° C. for tape 85,
= 15·72 for tape 88.

The values given by expression (9) for such extreme temperatures as 0° and 30° C. are ± 0·062^{mm} and ± 0·055^{mm}, respectively, for tape No 85, and ± 0·060^{mm} and ± 0·058^{mm}, respectively, for tape No. 88. The square root of the average of the squares of the errors given by (9) between 0° and 30°, supposing all temperatures within these limits of equal frequency, is ± 0·050^{mm}. Hence it is concluded that for such temperatures as are likely to be had in the use of these tapes, the probable errors of their absolute lengths will not on the average exceed ± 0·05^{mm}, or 1/2 000 000th part of their length.

(*17*) *Practical form of equations of tapes.*—For the purposes of computation it is most convenient to combine the products of the mean temperatures by the rates of expansion in equations (7) and (8), of section 14, with the mean excesses over $20 B_{17}$. Thus, making this combination, omitting the probable errors, and adding the numerical value of B_{17} as given in Chapter II, section 23, we have

$$T_{85} = 20\ B_{17} + \underset{mm.}{3{\cdot}86} + \underset{mm.}{1{\cdot}0947t},$$
$$T_{88} = 20\ B_{17} + 6{\cdot}31 + 1{\cdot}0914t,$$
$$B_{17} = 5^m - 18{\cdot}0^s,$$
Standard tension = 25 pounds 9 ounces.

In these, t is expressed in degrees of the centigrade scale, and B_{17} is

the length of the steel 5m bar No. 17 when in melting ice. Introducing the value of B_{17} there result, finally, for the equations of these tapes

$$T_{85} = \underset{m.}{100} + \underset{mm.}{3{\cdot}50} + \underset{mm.}{1{\cdot}0947t},$$

$$T_{88} = 100 + 5{\cdot}95 + 1{\cdot}0914t,$$

Standard tension = 25 pounds 9 ounces.

CHAPTER V.

MEASUREMENT OF HOLTON AND ST. ALBANS BASES WITH THE TAPE APPARATUS.

A.—HOLTON BASE MEASUREMENT.

(1) *Location of base and preparation for measurement.*—Holton Base is a line in the Transcontinental Chain of Triangulation along or near the thirty-ninth parallel of latitude. It is situated near the village of Holton, Ripley County, Ind. It is about 5 500^{m} long, and extends in a nearly north and south direction across what is known as the Crawfish Flats. As will be inferred from this designation, the ground along the line is generally level, although a few rather abrupt changes in altitude are met in crossing the drainage courses.

This base was located in 1890 by Assistant Mosman, who made a preliminary measure of its length with a 50^{m} tape the same year. Early in the following summer Assistant Mosman set the terminal and section stones of the base and provided all materials requisite to the measures made with the 100^{m} tapes.

During August, 1891, the marking and support stakes for the measures with the long tapes, were set under the supervision of the writer. The support stakes were set at equidistant intervals of 10^{m} throughout the length of the line, except on the standard kilometre section, where the microscope posts for the iced-bar apparatus rendered such stakes unnecessary. The tapes were supported on steel wire nails driven to the depth of 2·5^{cm} into the stakes. Since these nails projected horizontally from the stakes for a distance of about 12^{cm}, no great precision was essential in placing the support stakes along the line. The marking stakes, which fell 100^{m} apart, were aligned with considerable precision, and the support nails were placed at closely equal intervals of 10^{m} by the method explained in section 5, Chapter IV.

(2) *Measurements of the sections of the base.*—Since the tape apparatus used was new and untried, except in the experiments of the Botanical Garden and in the comparisons on the 100^{m} comparator, it was essential to a considerable extent to develop the methods of operation suitable to field measurement as the work progressed. At the same time that this work was under way, the work of tape comparisons, and the preparation for and measurement of the standard kilometre with the iced-bar apparatus, were also in progress; while the employés of the party were often needed for other work conducted by Assistants Mosman and Tittmann. Hence no attempt was made to measure the whole base on any one date; but the various sections were measured from time to time as opportunities occurred. The earliest measures were

made on the section 5 000^{m} to 5 500^{m} during the last days of August, 1891, and the work continued at intervals until October 8, 1891. The several sections were measured from six to thirty times each, the largest number of measures being made on the kilometre section, whose precise length was designed to give a thorough test of the effectiveness of tape apparatus.

Most of the measures of the sections were made at night, but a considerable number also were made during cloudy or partly cloudy days. Some of the latter were made under circumstances of wind, rain, and sunshine, which would now be considered quite unfavorable; but since it was desired to learn as much as practicable in the allotted time concerning the behavior of the tapes, a wide variety of circumstances was purposely encountered.

In general, the measures were made by laying each tape length once as the work proceeded along a section. In a few cases, however, the experiment was tried of laying each tape length from two to five times and reading the thermometers as many times; and in order to make these measures independent, the rear end of the tape at starting was placed successively in coincidence with marks a centimetre apart. This experiment served to give an idea of the error in laying or placing a tape length.

In all these measures but two thermometers were used, and these were read but once for each tape length. When a sufficient number of operatives were available the thermometers were carried and read by observers who did naught else but assist in carrying the tape. At other times the thermometers were carried and read by the observers who fixed the ends of the tape. In the former case, the thermometers were held at arm's length by light strings so that the bulbs were near to and at about the same height from the ground as the tape; while in the latter case the thermometers were suspended from a nail or brad-awl attached to a support stake so as to fall in about the same relative position with respect to the tape as before. In both cases, the thermometers were placed at 20^{m} from the tape ends. The same thermometers used in the determinations of length of the tapes on the 100^{m} comparator were used also in the field measures, and the thermometer corrections given in section 10, Chapter IV, were applied in computing the tape temperatures for these measures.

In all measures of the sections of Holton Base, Mr. J. S. Siebert was the observer at the rear end of the tape and R. S. Woodward at the front end. During the earlier measures the temperatures were observed for the most part by Messrs. O. H. Tittmann and J. F. Hayford. The records were kept by Messrs. Hayford and Robert Penington.

(3) *Determination of grade corrections.*—The differences in height of the marking stakes or tables, essential to the computation of grade corrections, were measured in duplicate by Messrs. J. S. Siebert, Th. Gjertsen and J. F. Hayford.

Tape measures of Holton Base.

The method of computing grade corrections from such data is very simple and easy. Thus, let h be the difference in altitude of two consecutive marking tables, or what is essentially the same thing, the difference in altitude of the two ends of the tape when placed in measurement over these tables. Let s be the length of the tape or right line distance between its terminal graduations when in place over these tables. Then if we call c the grade correction, or the quantity which must be added to s to give its horizontal projection,

$$c + s = \sqrt{s^2 - h^2},$$

whence

$$c = -\frac{h^2}{2s} - \frac{h^4}{8s^3} - \ldots.$$

Since s is in general large in comparison with h, this series for c converges very rapidly. For a 100^m tape

$$\frac{h^4}{8s^3} < 0{\cdot}01^{mm} \text{ for } h < 2{\cdot}9^m.$$

Values of h as great as 3^m are rarely met. Hence for such length of tape, in nearly all cases,

$$c \text{ in millimetres } = 5\,(h \text{ in metres})^2.$$

The grade corrections for the sections of Holton Base were computed in the field by J. S. Siebert and R. S. Woodward, and revised subsequently, in 1892, by M. V. Safford and Woodward. Table I following gives the grade corrections for the several sections of the base. The zero end of the line is here taken as coincident with its southern extremity, or South Base. The section 3900^m to 4900^m is the standard kilometre measured with the iced bar apparatus.

TABLE I.—*Grade corrections for sections of Holton Base.*

Section. 0^m–1200^m.	Section 1200^m–2100^m.	Section 2100^m–3000^m.	Section 3000^m–3900^m.	Section 3900^m–4900^m.	Section 4900^m–5000^a.	Section 5000^m–5500^m.
mm.	*mm.*	*mm.*	*mm.*	*mm.*	*mm.*	*mm.*
—25·02	—1·64	—5·92	—49·83	—5·39	—0·84	—8·72

(4) *Correction for index error of balances.*—If the spring balance used to give tension to the tape is adjusted to give correct tensions when in a vertical position, it will indicate (by its face readings) less than the actual tension when in a horizontal position. Such balances are liable also from wear and other causes to have an appreciable index correction.

To determine the actual tension in any case with such balances, let r be the index correction. It is the reading of the index when the balance is vertical, hook end down, and without load on the hook; it is *minus* when the index reads greater than zero and *plus* when it reads less. Let R be the reading of the index when the balance is suspended

by its hook, hook end up. Let W be the total weight of the balance found by weighing it on another balance. Then if T_1 denote the observed or face reading of the balance when horizontal and T denote the corresponding actual tension—

$$T = T_1 + r + \tfrac{1}{2}(W - R - r)$$
$$= T_1 + \tfrac{1}{2}(W - R + r).$$

The standard tension adopted in all the base-line measures, as well as in all the determinations of the tape lengths on the 100^m comparator, is 25 pounds 9 ounces, this being the value of T when the balances are horizontal, when they are free from index error, and when they read 25 pounds even. We are concerned here, then, only with the excess of the actual over the observed or nominal tension in any case; but in order to show the data for these excesses, which apply to the tape comparisons as well as to the measures of the sections of the base, we give the observed values of W, R, and r and the resulting values of $\frac{1}{2}(W - R + r)$ for both balances in full in Tables II and III following. The last column of these tables gives, also, the corresponding corrections to the standard length of the tape. These corrections are found by multiplying the excess of $\frac{1}{2}(W - R + r)$ over 9 ounces by 0·047^{mm}, the observed elongation of the tape per ounce excess in tension when the actual tension is near to the standard value, 25 pounds 9 ounces. Later determinations of this constant indicate that it should be increased to about 0·050^{mm} per ounce, but the value given above was used in the actual computation. (See section 5, Supplement B.)

It should be observed that the value + 0·16^{mm} was used in the computations for the correction to the tape length when under tension of balance No. 52 for the dates August 6 to September 18, 1891, and the mean value + 0·44^{mm} for the dates September 25 to October 3. In addition, it may be remarked that the variations in the values of W and R were due partly to the occasional removal of the extra hooks from the balances, while the large change in r in the case of No. 52 was the result of an accident to that balance. It was used but little after this accident, although the computed correction due to it in this case could have given rise to no appreciable error.

TABLE II.—*Data for tensions by spring balance No. 52.*

Date.	W	R	r	$\frac{1}{2}(W-R+r)$	Resulting correction to tape length.
1891.	*Ounces.*	*Ounces.*	*Ounces.*	*Ounces.*	*Millimetres.*
Aug. 6	50·0	32·0	0·0	+ 9·0	+0·00
30	49·5	28·0	+3·5	12·5	+ ·16
Sept. 3	44·5	23·5	+3·2	12·1	+ ·15
18	45·0	23·5	+3·2	12·4	+ ·16
25	44·2	17·0	+9·8	18·5	+ ·45
Oct. 3	49·0	22·2	+9·5	18·1	+ ·43

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TABLE III.—*Data for tensions by spring balance No. 53.*

Date.	W	R	r	$\frac{1}{2}(W-R+r)$	Resulting correction to tape length.
1891.	*Ounces.*	*Ounces.*	*Ounces.*	*Ounces.*	*Millimetres.*
Aug. 6	50·0	35·5	—3·5	5·5	—0·16
30	49·5	35·2	—3·0	5·6	— ·16
Sept. 25	49·2	35·0	—3·0	5·6	— ·16
Oct. 3	44·5	30·2	—3·2	5·6	— ·16
8	44·8	30·2	—3·5	5·6	— ·16

(5) *Specimen of record and computation.*—A form of record suitable for tape measures will readily suggest itself to the reader; but for the sake of completeness, and in order to present data for the computation of the measured value of a section, a specimen of the records actually made in the Holton Base work is given below in Table IV:

TABLE IV.—*Specimen of Record. Holton Base, September 8, 1891.*

[Measure of standard kilometre.]

No. of tape length.	Time of day.	Thermometer readings—			Remarks.
		No. 5601.	No. 5599.	Mean t.	
		° C.	° C.	° C.	
1	6·37	13·4	13·0	13·20	Steel tape No. 88. Tension 25 pounds by balance No. 52.
2	·42	13·6	13·3	13·45	
3	·46	13·3	13·5	13·40	Index correction +3·2 ounces.
4	·50	13·0	13·2	13·10	
5	·53	12·8	13·2	13·00	Measure begins at south end of kilometre.
6	·56	12·4	12·3	12·35	
7	·59	12·4	12·6	12·50	Cloudiness 0·0.
8	7·05	12·6	12·7	12·65	Set back 100·0mm at rear end of No. 3.
9	·09	11·8	12·3	12·05	Set back 80·2mm at rear end of No. 8.
10	·12	12·1	12·5	12·30	North end of 10th tape length was 10·6mm north of terminal mark.

The computation of the length of the standard kilometre from the data given in the above table may be made as follows:

The actual or working length of the tape T, say, may be most conveniently expressed in the form

$$T = A + a + \alpha t,$$

wherein A is the nominal length of the tape, a round number of metres or lengths of some standard; a is a small excess over that round number; α is the rate of expansion of the tape, and t is the tape's temperature in degrees centigrade. Then the sum of N such lengths, t being supposed to vary from one length to another, will be

$$\Sigma T = NA + Na + \alpha \Sigma t.$$

To this sum must be applied the sum of the set-ups, the sum of the

end corrections, and the grade correction for the whole section of N tape lengths.

Thus, to illustrate by means of the data in Table IV above, we have, first, from section 17 of Chapter IV, for the length of tape No. 88,

$$T_{88} = 20B_{17} + 6{\cdot}31^{mm} + 1{\cdot}0914^{mm}\, t$$

for the standard tension of 25 pounds 9 ounces. The tension in this case was 3·2 ounces in excess of the standard value, and hence the tape was, as explained in section 4 above, $0{\cdot}16^{mm}$ longer than the standard length just given. We have, therefore,

$$\begin{aligned} A &= 20B_{17}, \\ a &= +\,6{\cdot}31^{mm} + 0{\cdot}16^{mm} = +\,6{\cdot}47^{mm}, \\ \alpha &= 1{\cdot}0914^{mm}, \\ N &= 10. \end{aligned}$$

Likewise, from Table IV, the sum of the observed temperatures is 128·00°, and the sum of the corrections to this for errors of the thermometers used is $-\,2{\cdot}12°$, as found from the table of thermometer corrections given in section 10, Chapter IV; so that there results

$$\Sigma t = 125{\cdot}88°.$$

The several terms of the computation are, then, the following:

NA	$= 200B_{17}$,	
Na	$=$	$+\ 64{\cdot}70^{mm}$,
$\alpha \Sigma t$	$=$	$+137{\cdot}39$,
Sum of set-ups	$=$	$-180{\cdot}20$,
Sum of end corrections	$=$	$-\ 10{\cdot}60$,
Grade correction	$=$	$-\ 5{\cdot}39$,
Resulting length	$= 200B_{17}$,	$+\ 5{\cdot}90$.

In this particular case a correction of $-\,0{\cdot}4^{mm}$ is to be added to the resulting length in order to refer it to the kilometre as measured with the iced-bar apparatus, the reference marks for the tape measures having been to that extent eccentric with respect to the terminal cut-off spheres.

The computations of the lengths of the sections of Holton Base were made in the field in 1891 by Messrs. Siebert, Hayford, and Woodward. They were revised in 1892 by Messrs. Safford and Woodward.

(6) *Results of the measures of the sections of Holton Base.*—Table V following gives all of the results obtained from the various measures of the several sections of Holton Base. The first column of the table gives the date of the measure; the second, the time of day when the measure was made; the third, the number of the tape used; the fourth, the direction in which the measure proceeded; the fifth, the mean temperature of the tape for the measure; the sixth, the temperature range for the measure, or the difference between the highest and lowest observed temperatures; the seventh indicates by the letters

Tape measures of Holton Base.

F, R, V whether the temperature was falling, rising, or variable during the course of the measure; the eighth gives the degree of cloudiness, estimated on a scale of tenths; and the ninth column gives the length of the section, computed in the manner explained above, in terms of the iced bar B_{17}, the excess over a round number of tape or bar lengths having been measured with a millimetre scale. Results for length obtained from measures made before sundown are inclosed in square brackets.

TABLE V.—*Results of measures of Holton base.*

[Section south base to 1,200m stone, 240 B_{17}.]

Date.	Time of day.	No. of tape used.	Direction of measure.	Mean temperature.	Temperature range.	Temperature falling, F; rising, R; variable, V.	Cloudiness	Length of section.
1891.				°	°			*mm.*
Sept. 1	5:55– 6:39 p. m.	88	S. to N.	17·77	4·1	F	0·0	+ 41·3
4	8:04– 8:44 a. m.	88	N. S.	13·90	0·9	F	1·0	[32·1]
4	8:52– 9:46	88	S. N.	15·44	4·9	R	1·0	[18·1]
16	7:03– 7:56 p. m.	88	N. S.	19·20	1·4	F	0·0	31·4
16	8:12– 8:59	88	S. N.	18·25	1·2	F	0·0	38·5
16	9:20–10:07	88	N. S.	18·03	1·8	F	0·0	28·2
16	10:14–11:03	88	S. N.	18·42	2·8	R, F	0·0	32·0
[Section 1,200m stone to 2,100m stone, 180 B_{17}.]								
Sept. 2	5:59– 6:25 p. m.	88	S. to N.	19·10	2·6	F	0·2	— 23·0
3	4:46– 5:09	88	N. S.	18·05	1·4	F	1·0	27·5
4	9:52–10:30 a. m.	88	S. N.	16·68	2·6	F	1·0	[40·5]
17	6:53– 7:25 p. m.	88	N. S.	19·31	1·2	V	0·0	27·7
17	7:44– 8:15	88	S. N.	18·20	1·0	V	0·0	27·4
17	8:24– 8:58	88	N. S.	17·82	0·9	V	0·0	29·0
17	9:07– 9:34	88	S. N.	17·15	1·6	F	0·0	26·9
17	9:40–10:10	88	N. S.	16·74	2·4	R	0·0	26·0
[Section 2,100m stone to 3,000m stone, 180 B_{17}.]								
Sept. 3	2:01– 2:30 p. m.	88	S. to N.	23·92	3·4	F	0·8	+[52·8]
3	4:21– 4:42	88	N. S.	19·69	0·8	F	0·9	[62.0]
4	10:3[illegible]–11:09 a. m.	88	S. N.	16·16	1·1	V	1·0	[58·0]
18	7:40– 8:33 p. m.	88	N. S.	17·82	1·8	V	0·0	70·9
18	8:47– 9:23	88	S. N.	17·16	1·9	F	0·0	71·4
19	7:50– 8:17	88	N. S.	18·33	1·4	F	0·0	73·3
19	8:30– 9:06	88	S. N.	17·07	0·9	V	0·0	68·1
[Section 3,000m stone to 3,900m stone, 180 B_{17}.]								
Sept. 3	2:41– 3:33 p. m.	88	S. to N.	23·45	1·7	V	0·7	+ [8·0]
3	3:35– 4:12	88	N. S.	21·98	2·4	F	0·9	[15·7]
18	6:38– 7:37	88	S. N.	18·62	1·7	V	0·0	22·3
18	9:32–10:16	88	S. N.	16·51	3·0	F	0·0	18·4
19	6:48– 7:32	88	N. S.	18·56	1·3	V	0·0	20·8
19	9:14– 9:55	88	S. N.	16·25	2·2	F	0·0	19·3

TABLE V.—*Results of measures of Holton Base*—Continued.

Date.	Time of day.	No. of tape used.	Direction of measure.	Mean temperature.	Temperature range.	Temperature falling, F; rising, R; variable, V.	Cloudiness.	Length of section.
			[Section 3,900m stone to 4,900m stone, or standard kilometre, 200 B_{17}.]					
1891.				°	°			*mm.*
Sept. 8	5:25– 5:54 p. m.	88	S. to N.	14·82	3·0	F, R	0·1	+ 3·3
8	5:58– 6:33	88	N. S.	13·52	1·8	V	0·0	+ 1·5
8	6:37– 7:12	88	S. N.	12·58	1·4	F	0·0	+ 5·5
23	6:27– 7:03	88	N. S.	20·72	1·4	V	0·0	— 2·4
23	7:16– 7:49	88	S. N.	19·90	1·6	V	0·0	— 3·6
23	7:56– 8:23	88	N. S.	19·43	1·6	F	0·0	— 4·3
23	8:37– 9:32	88	S. N.	18·65	1·8	V	0·0	— 1·1
30	6:33– 7:11	88	N. S.	13·94	2·4	R	0·0	+ 0·1
30	7:17– 7:55	88	S. N.	13·67	2·7	F	0·0	— 0·6
Oct. 1	6:50– 7:25	85	N. S.	18·52	0·5	F	0·0	— 2·3
1	7:32– 8:07	85	S. N.	18·07	2·8	V	0·0	+ 1·5
2	6:48– 7:18	85	N. S.	19·88	1·0	V	0·0	+ 1·4
2	7:25– 7:55	85	S. N.	19·45	1·6	R, F	0·0	+ 3·1
3	2:56– 3:18	85	N. S.	24·03	2·4	F, R	0·9	— [5·1]
3	3:26– 3:48	85	S. N.	23·62	2·4	F, R	0·9	— [4·1]
3	3:53– 4:13	85	N. S.	28·16	1·5	F	0·9	— [6·9]
3	4:17– 4:38	85	S. N.	22·66	1·1	V	0·9	— [1·3]
				14·09	3·2			— [8·5]
				14·22	2·5			—[11·1]
7	10:02–11:31 a. m.	85	N. S.	14·20	2·4	F, R	0·8	— [9·6]
				14·20	2·6			— [8·1]
				14·26	3·5			—[11·9]
				6·53	1·3			— 1·1
				6·54	1·6			+ 0·1
8	7:44– 8:45 p. m.	88	N. S.	6·55	1·6	R, F	0·2	+ 1·7
				6·51	1·9			+ 2·1
				5·75	1·1			— 2·0
				5·84	1·2			— 0·9
8	8:53– 9:58	88	S. N.	5·87	1·2	R, F	0·6	— 1·4
				5·89	1·3			— 0·0
			[Section 4,900m stone to 5,000m stone, 20 B_{17}.]					
Aug. 27	4:24 p. m.	88	N. to S.	20·32			0·7	—[52·4]
28	7:37	88	N. S.	11·44			0·0	53·0
				14·08				[55·9]
				14·48				[56·9]
Oct. 7	9:30– 9:55 a. m.	85	N. S.	14·48	0·6	V	0·8	[55·6]
				13·83				[55·9]
				14·03				[55·1]
				5·75				55·7
				5·75				55·3
8	10:05–10:08 p. m.	88	N. S.	5·70	0·1	V	0·6	55·7
				5·65				55·4
				5·65				55·4
			[Section 4,900m stone to 5,000m stone, 100 B_{17}.]					
Aug. 27	4:24– 5:10 p. m.	88	S. to N.	20·26	0·2	V	0·7	+[803·9]
28	7:37– 8:13	88	S. N.	11·05	0·8	F, R	0·0	808·9
29	5:58– 6:10	88	N. S.	16·12	3·0	F, R	0·2	808·3
31	4:35– 5:07	88	S. N.	21·67	1·2	F	0·6	[800·4]
31	5:52– 6:03	88	N. S.	17·92	1·1	F	0·3	807·6
31	6:07– 6:17	88	S. N.	16·95	0·8	V	0·2	808·0
31	6:22– 6:33	88	N. S.	15·86	0·4	F	0·2	808·3

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(7) *Interpretation of results.*—It is essential to a proper interpretation of the results in the above table to state certain additional information relative to the conditions under which the measures were made.

The day measures of September 3, 1891, were made during the afternoon of a day whose earlier part had been hot and sultry. The field notes of the measures of this day state that, although the sky was generally cloudy, the tape and thermometers were occasionally exposed to the direct rays of the sun. The measures of September 4 were made while a strong breeze, rising to a gale at intervals, was blowing. This breeze, accompanied by more or less rain, sufficed at times to blow the tape when under tension off its supports. For these reasons the results of September 4 can not be regarded as at all trustworthy in comparison with results obtained under ordinarily favorable conditions.

The day measures of the standard kilometre of October 7 were made at a time of general cloudiness, but the tape and thermometers were exposed at intervals to direct sunshine. When the sun appeared on this date the measures were suspended until the clouds again cut off his rays, but in a number of cases the later measures of the set of five made on this occasion were not completed before the sun's heat was manifest in the thermometer readings.

The night measures were made, in general, at times when dew was deposited to a greater or less extent in open fields. The only marked exception to this rule applies to the measures of the standard kilometre of September 23, 1891. The night of this day was an unusually warm and dry one, and the wooded portion of the kilometre, which is about 600^{m} long, had a notable store of radiant heat.

The measures of the short section between the 4900^{m} and 5000^{m} stone of August 27 and 28, 1891, present a wide divergence from the later measures of the same section. These earlier measures were the first made with the apparatus in the field, and it was in laying this particular tape length that the operatives, with the exception of Siebert and Woodward, acquired their first experience. On these dates, also, a temporary and rather unstable marking table was used at the 4900^{m} stone, pending the preparation of a rigid post which served subsequently as a microscope post at the terminus of the standard kilometre, and as a stable support for the marking table used in the tape measures of September and October. For these reasons the measures of August 27 and 28 must be regarded as untrustworthy in comparison with the later measures of this short section.

All of the measures save those of August 27 and 28 and September 4, alluded to above, were made under favorable conditions, it is believed, so far as the manipulation of the apparatus was concerned.

With these explanations in mind attention may now be called to a marked peculiarity exhibited by all the results of the day measures

except that of August 27 of the short section 4900^{m} to 5000^{m}. A glance at Table V shows that those results are all small relatively to the results of the night measures save in the single instance noted. Disregarding those measures of August 27 and September 4, concerning which there are grave doubts as already explained, the discrepancy in question is most marked in the results of August 27 and 31 (Section 5000^{m} to North Base), September 3 (Section 2100^{m} to 3900^{m}), and October 7 (Section 3900^{m} to 4900^{m}). It appears, also, from the measures of August 31 (Section 5000^{m} to North Base), September 3 (Section 1200^{m} to 2100^{m}), and October 3 (Section 3900^{m} to 4900^{m}), that this discrepancy disappears, or tends to do, so with the going down of the sun. The unexpected fact thus brought to light is that under the conditions presented by the dates cited, the thermometers gave a temperature too low for the tape by amounts ranging up to about a degree Centigrade at the maximum. How this fact is to be explained I am unable to say at present, but of its existence there can be no doubt. It is probably due to the differing capacities of the tape and thermometers for absorption and radiation under different conditions.

In view of this peculiarity of the day measures it becomes essential to inquire carefully into the behavior of the tapes in the night measures during which the conditions as to radiation differed much less from those of the 100^{m} comparator whereon the tape equations were determined. The data for this inquiry are furnished by the measures with the tapes of the standard kilometre, whose length was determined with the iced bar, as explained in Chapter III, and we proceed to examine these measures in detail in the following section.

(8) *Conclusions from tape measures of standard kilometre.*—In order to give a clear idea of the performance of the tapes in the numerous measures of the standard kilometre, a summary of those measures is drawn up in Table VI following. The first column of this table gives the date of a set or group of measures; the second gives the limiting times of the date within which the set of measures was made; the third gives the number of measures in the set; the fourth gives the mean of the results for length of the kilometre, and the last column gives the error of the mean length for any date, the true length of the kilometre being assumed to be $200B_{17} + 0{\cdot}2^{mm}$ as found from the iced-bar measures and given in Chapter III, section 12. The results from the day measures are inclosed in square brackets:

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TABLE VI.—*Summary of results of tape measures of kilometre.*

Date.	Time of day.	No. of measures.	Mean length for date.	Actual error of mean.
1891.	*p. m.*		*mm.*	*mm.*
Sept. 8	5:25– 7:12	3	$200B_{17}$ + 3·4	+ 3·2
23	6:27– 9:32	4	— 2·8	— 3·0
30	6:33– 7:55	2	— 0·2	— 0·4
Oct. 1	6:50– 8:07	2	— 0·4	— 0·6
2	6:48– 7:55	2	+ 2·2	+ 2·0
3	2:56– 4:38	4	—[4·4]	—[4·6]
	a. m.			
7	10:02–11:31	5	—[9·8]	—[10·0]
	p. m.			
8	7:44– 9:58	8	— 0·2	— 0·4

It is clear from the results in this table that the errors of the night measures are small in comparison with those of the day measures. The mean of the night measures, giving weights to the tabular means proportional to the number of results on which they depend, is $200B_{17}$ + 0·0^{mm}, which differs but 0·2^{mm} from the value derived from the iced bar measures. The maximum error of any group mean from the night measures is 3·2^{mm}, or 1/300 000th part of the kilometre; and the average error of such group means (disregarding weights) is only 1·6^{mm}, or 1/600 000th part of the kilometre. On the other hand, the day measures are largely in error and in one direction only, the maximum error of one group mean being 1/100 000th part of the kilometre.

It seems safe to conclude, therefore, from these measures of the standard kilometre, that the temperatures of the tapes when used at night in the open air were substantially as well defined by the mercurial thermometers as when used under the shed of the 100^{m} comparator. Moreover, this conclusion is substantiated so far as it can be by the evidence afforded by the measures of the other sections of the base, which do not present a single well-defined exception to the rule that the day measures are relatively short.

In accordance with this conclusion, we shall use the night measures only in deriving the adopted mean lengths of the several sections of the base. It may be observed, however, that another method, which does not require the rejection of the day measures, is available for the determination of the mean lengths of those sections. This method consists in deriving the working lengths of the tapes for the day measures from the day measures of the standard kilometre whose length was measured precisely with the iced-bar apparatus. A glance at the data shown in Table V makes it apparent that this method would bring the day measures into close agreement with the night measures of any section. The computation according to this method has been made, and the resulting mean lengths of the sections will be given below.

It will be remarked, also, that the numerous night measures of the standard kilometre afford sufficient data for a good determination of the equations of the tapes, especially for tape No. 88, since the temperature range for it in the kilometre measures is about 15° C. But, as shown above, the equations of the tapes derived from observations on the 100^{m} comparator are in so close accord with those data that there is no need of appealing to them for supplementary information concerning the equations of the tapes for the night work.

Finally, attention may be called to the substantial agreement with each other and with the true value of the means of the night measures of the standard kilometre from the two tapes. These means are $200B_{17} + 0{\cdot}9^{mm}$ for tape No. 85 and $200B_{17} - 0{\cdot}2^{mm}$ for tape No. 88. The actual errors of these measures are thus, with respect to the iced bar value ($200B_{17} + 0{\cdot}2^{mm}$), only $+ 0{\cdot}7^{mm}$ and $- 0{\cdot}4^{mm}$, which are in each case less than the millionth part of the length of the line.

(9) *Mean values of lengths of sections of base.*—As stated in the preceding section, the lengths we shall adopt for the sections of the base will be made to depend on the night measures only, for the reason that these were made under circumstances closely comparable with those which obtained on the 100^{m} comparator in determining the equations of the tapes. It will be of interest, however, to see to what extent the mean lengths of the sections and the length of the whole base would be affected if the day measures were included and given equal weight along with the night measures. Likewise, it will be instructive to learn the effect of the inclusion of the results of the day measures when computed by means of working lengths of the tapes derived from the day measures of the standard kilometre. Accordingly, the values which result from each of these three suppositions are given in Table VII following. The data for this table are derived from Table V. The first column designates the section of the base. The second column gives the mean length of the section derived from the night measures alone, these being considered of equal weight. The third column gives the mean length of the section derived by considering all day and night measures of the same weight. The fourth column gives the mean by equal weights of all measures of a section after the day measures have been increased by $0{\cdot}74^{mm}$ per tape length. The results of August 27 and 28 for the short section 4 900^{m} to 5 000^{m} and those of September 4 for the sections South Base to 3 000^{m} are, however, excluded in the first and third of these computations for the reasons given in section 8. The correction $+ 0{\cdot}74^{mm}$ per tape length is a mean of the corrections furnished by the kilometre measures of October 3 and 7. A more precise value might be derived, it is believed, by making such correction depend on the time of day, since the evidence indicates that the error in question was due to solar radiation; but the data are insufficient to justify such a degree of refinement.

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TABLE VII.—*Mean values of lengths of sections of Holton Base.*

Section of base.	Mean of night measures.	Mean, including uncorrected day measures.	Mean, including corrected day measures.
m.	*mm.*	*mm.*	*mm.*
South Base- 1200	$240B_{17}$ + 34·3	$240B_{17}$ + 31·7	$240B_{17}$ + 34·3
1200–2100	180 — 27·0	180 — 28·5	180 — 27·0
2100–3000	180 + 70·9	180 + 65·2	180 + 68·6
3000–3900	180 + 20·2	180 + 17·4	180 + 19·6
3900–4900	200 + 0·0	200 — 2·2	200 + 0·4
4900–5000	20 — 55·5	20 — 55·2	20 — 55·3
5000–North Base	100 + 808·2	100 + 806·5	100 + 807·5
South Base to North Base.	$1100B_{17}$ + 851·9	$1100B_{17}$ + 834·9	$1100B_{17}$ + 848·1

It will be observed that the means in the third column of this table are in every case smaller than the corresponding values in the second column, so that there can be no doubt of the wide divergence of the day measures from the night measures obtained with the apparatus as used on the Holton Base. The total length of the base found by including the day measures is 17·0^{mm} shorter than that found by excluding them. This amounts to 1/325 000th part of the length of the base. This total length, however, is affected to a marked extent by the measures of September 4, made during rain and wind as already explained. These adverse circumstances afford sufficient reason for the rejection of those measures; but it is proper to explain that the general effect of the wind on this date was to increase the distance between the terminal marks of the tape, and hence to make the resulting computed lengths of the sections measured too small. This arose from the fact that the wind blew briskly from a northerly direction, or nearly in the direction of the base, and hence it tended to straighten and thus increase the working length of the tape, just as a strong breeze tends to straighten a string attached to a fixed point at one end or to render it parallel to the direction of the current, which is in general horizontal. The maximum increase which such a cause could produce in the distance between the terminal marks of the tape as used is 1·55^{mm}. This increase is much greater, of course, than any which is likely, but it seems probable that a quite appreciable fraction of that amount per tape length entered as an actual error into the measures of the date in question.

As to the results in the last column in Table VII, it may be said that their agreement with those of the second column is fairly satisfactory. It appears, however, that the day measures are not sufficiently increased.

(10) *Errors of measurement.*—The errors which give rise to the discrepancies between different results for the length of a line measured with a long tape may be conveniently divided into the two following

classes: (1) Errors of manipulation, or errors incident to the mere operation of placing or laying a tape length. They include errors due to lack of coincidence of the terminal marks of the tape with the defining marks on the zinc or other marking plates, errors due to incorrect tension of the tape, errors due to friction of the tape on its supports, and errors due to defective alignment of the tape. (2) Temperature errors. These include all errors tending to give incorrect temperatures to the tape, except such constant errors of the thermometers as may affect all measures alike.

Relatively, the second class of errors is by far the most formidable, since they tend at times toward the constant type of errors, but it is important also to give such information as is available with respect to the minor source. Accordingly we shall endeavor to separate the two classes of error.

It may be remarked that the errors of manipulation enumerated above are all of the compensating sort, except that due to defective alignment of the tape. This error must, however, be very small, since flipping the tape when under tension tends to make it perfectly straight on its supports. Even if the middle of a 100^{m} tape lay one decimetre to one side of the straight line joining its ends (an error of alignment which may be regarded as impossible) the resulting length would be in error only $0{\cdot}2^{mm}$; so that errors of this kind, it is believed, are absolutely negligible.

It appears from an inspection of the results in Table V, especially those for the length of the standard kilometre, that any set of measures of a section made within a short interval of time was affected by more or less persistent temperature errors. This is, as already explained, very marked in the case of the day measures as compared with the night measures; but the latter exhibit also some traces of the same sort of error, which arises, it is thought, from the relatively constant conditions of radiation pertaining to any short line during a limited period of time. The full range of the resultant error of measurement, therefore, can be expected to appear only in numerous measures of a section made under widely varying conditions. Measures satisfying this requirement were made on the standard kilometre. The various measures of the other sections of the base, when taken as a whole, satisfy the requirement also, and we thus have two independent sets of data for determining the aggregate probable error of measurement of a section.

In computing this error we shall consider the night measures only, since they alone, as shown by the measures of the standard kilometre, conform to the adopted equations of the tapes, and they alone are used in deriving the adopted length of the base.

The standard kilometre was so accurately known from the iced bar measures of it that the discrepancies shown by the tape measures may be treated as actual errors, and, other things equal, as affording the

Tape measures of Holton Base.

best data for estimating the probable error of a single measure of a kilometre. The residuals found by subtracting the iced bar value of this kilometre ($200\ B_{17} + 0{\cdot}2^{mm}$) from the several night measures, taken individually, range from $-\ 4{\cdot}5^{mm}$ to $+\ 5{\cdot}3^{mm}$. The number of these residuals is 21 and the sum of their squares is 117·2; so that the probable error of a single night measure of the standard kilometre was $\pm\ 1{\cdot}6^{mm}$.

On the other hand, if we consider all the night measures of the several sections, including the standard kilometre (but excluding the short section of 100^{m} for the reason that one tape length does not afford an opportunity for the development of the errors in question), there are in all 46 measures of 6 sections. These sections vary in length from 500^{m} to $1{,}200^{m}$, and hence it is essential to have recourse to weights in order to derive the probable error of one measure of a kilometre. Assuming that the resultant error desired is of the compensating class, the weights of the different sections will vary inversely as their lengths; and if we call the weight of one measure of a kilometre 1, the weight of one measure of a section n tape lengths long ($n100^{m}$) will be $10/n$. In order to exhibit the data furnished by the several sections for the probable error desired, the details are brought together in Table VIII below. The first column of the table gives the designation of the section; the second gives the number of measures of the section; the third gives the range among those measures; the fourth gives the weight of the section; and the fifth gives the sum of the weighted squares of the residuals found by comparing the individual measures of a section with their mean.

TABLE VIII.—*Data for error of measurement.*

Section.		No. measures of section.	Range of results.	Weight p.	$[pvv]$
m.	*m.*		*mm.*		
0 to	1200	5	13·1	10/12	97·6
1200	2100	7	6·0	10/9	23·99
2100	3000	4	5·2	10/9	1·40
3000	3900	4	3·9	10/9	·80
3900	4900	21	9·8	1	6·58
5000	5500	5	1·3	2	1·82

The total sum of the quantities $[pvv]$ in the above table is 265·55. Hence the probable error of one measure of a kilometre as shown by them is

$$\pm\ 0{\cdot}6745\ \sqrt{\frac{265{\cdot}55}{40-6}} = \pm\ 1{\cdot}74^{mm}.$$

This, it will be seen, is slightly larger than the value derived from the kilometre measures alone.

Now, by means of this value and the data afforded by the determinations of the tape lengths on the 100^{m} comparator, it is possible to derive an estimate of the magnitude of the error due to manipulation. By reference to Chapter IV, section 14, it is seen that the error of one determination of a tape length on the comparator was $\pm$ 0·20^{mm}, and this is due almost wholly to error in the assigned temperature from the mean of six thermometer readings. Assuming that the temperature errors were no greater in field work than on the comparator, the probable error of one tape length from the mean of two in place of six thermometer readings should be $\pm$ 0·20$^{mm}\sqrt{3}$; and hence the probable error in the length of a kilometre, or the sum of 10 tape lengths, should be $\pm$ 0·20$^{mm}\sqrt{30} = \pm$ 1·1^{mm}. Since the probable error of measurement due to all sources of error was found above to be $\pm$ 1·7^{mm}, at most, we infer that the probable error due to errors of manipulation in one measure of a kilometre is

$$\pm \sqrt{(1\cdot7)^2 - (1\cdot1)^2} = \pm 1\cdot3^{mm}.$$

It appears, then, that the probable errors due to the two classes of error (error of manipulation and error of temperature) were about equal in the case of the night measures of Holton Base. Some doubt may be entertained as to the correctness of the above estimate of $\pm$ 1·1^{mm} for the probable error due to temperature errors. Subsequent experience, on St. Albans Base, in 1892, under somewhat different conditions, as explained in sections 20–27 below, does not confirm this estimate, but seems to show conclusively that the error of operation is quite negligible in comparison with that of the temperature.

(11) *Probable errors of mean lengths of sections.* (*a*) *Probable errors due to errors of measurement.*—The mean lengths of the sections we shall adopt are those given in the second column of Table VII above. The probable errors of these mean values may be derived in two ways, namely: First, by considering the several measures of a section by themselves; second, by considering the measures of all the sections as a connected group of observations with weights dependent on the lengths of the sections for the individual measures and weights dependent on the numbers of measures of the sections for their mean lengths. The data for these two probable errors for the several sections and the resulting values of the errors are given in Table IX below. The first column of the table designates the section; the second gives the number of measures of the section m; the third gives the sum of the squares of the discrepancies between the individual measures of a section and their mean, which sum is denoted by $[vv]$; the fourth column gives the weight of the section p, which is the reciprocal of the number of tape lengths in the section, unit weight being here attributed to one measure of a single tape length, or 100^{m}; the fifth column gives the sum of the

weighted squares of the residuals, or $[pvv]$, and the last two columns give the probable errors of the mean values of the sections computed from the formulas

$$(1) = \pm 0{\cdot}6745 \sqrt{\frac{[vv]}{m\,(m-1)}}, \text{ and } (2) = \pm 0{\cdot}6745 \sqrt{\frac{[pvv]}{mp\,(51-7)}}.$$

TABLE IX.—*Data for and probable errors of mean lengths of sections.*

Section.	No. measures m.	$[vv]$	Weight p.	$[pvv]$	Probable error.	
					(1)	(2)
m. *m.*					*mm.*	*mm.*
0–1200	5	117·55	1/12	9·55	±1·64	±0·81
1200–2100	7	21·59	1/9	2·40	± ·48	± ·59
2100–3000	4	13·85	1/9	1·54	± ·73	± ·78
3000–3900	4	8·82	1/9	0·98	± ·58	± ·78
3900–4900	21	116·58	1/10	11·66	± ·36	± ·36
4900–5000	5	0·14	1	0·14	± ·06	± ·23
5000–5500	5	0·91	1/5	0·18	± ·15	± ·52

It will be observed that the probable errors in the last two columns of the above table agree fairly well, and thus justify the assumption that the errors of measurement with the tape apparatus are of the compensating sort, or that the aggregate error of measurement increases as the square root of the distance measured. We conclude, also, that the probable errors (2) are, on the whole, the more trustworthy, since they are not unduly affected by the excessively large or the excessively small errors which appear in the measures of some of the sections.

The probable error of laying a single tape length, as shown by the process (2), is $\pm 0{\cdot}524^{mm}$. Hence the probable error of one measure of a kilometre is, by this process,

$$\pm 0{\cdot}524^{mm} \sqrt{10} = \pm 1{\cdot}66^{mm};$$

and this may be regarded as the most trustworthy value afforded by the measures of Holton Base.

The probable error of measurement in the whole length of the base is, if we regard the measures of the different sections as wholly independent, the square root of the sum of the squares of the probable errors (1) in Table IX. This value is $\pm 1{\cdot}99^{mm}$. On the other hand, by the process (2) the probable error of the whole length of the base is

$$\pm 0{\cdot}524^{mm}\left(\frac{12}{5}+\frac{9}{7}+\frac{9}{4}+\frac{9}{4}+\frac{10}{21}+\frac{1}{5}+1\right) = \pm 1{\cdot}65^{mm}.$$

These are equivalent to the 1/2 770 000th and the 1/3 330 000th part, respectively, of the whole base.

(*b*) *Probable errors due to errors of tape lengths.*—The probable error of the absolute length of either of the 100^{m} tapes used in the measures of Holton Base may be taken as not exceeding $\pm 0{\cdot}05^{mm}$, as shown by the discussion in section 16, Chapter IV; and we shall adopt this limiting value in computing the probable errors of the absolute length of the sections. Multiplying this error, $\pm 0{\cdot}05^{mm}$, by the numbers of tape lengths per section, there result for the probable errors of the several sections from this source

	m. *m.*	*mm.*
Section	0–1200	$\pm$ 0·60,
	1200–2100	$\pm$ ·45,
	2100–3000	$\pm$ ·45,
	3000–3900	$\pm$ ·45,
	3900–4900	$\pm$ ·50,
	4900–5000	$\pm$ ·05,
	5000–5500	$\pm$ ·25.

It is worthy of remark that these errors are in every case but one smaller than the errors of measurement shown in Table IX above. The exceptional case is that of the standard kilometre, whose adopted mean length depends on twenty-one measures. The probable error of the whole base due to this source is $\pm 55 \times 0{\cdot}05^{mm} = \pm 2{\cdot}75^{mm}$, or 1/2 000 000th part of the base.

(12) *Adopted lengths and errors of sections.*—Collecting the mean lengths of the sections given in the second column of Table VII, and combining the probable errors in the last column of Table IX with those given under (*b*) of the preceding section, there result for the lengths we adopt for the several sections

	m. *m.*		*mm.*	*mm.*
Section	0–1200 = 240B_{17}	+	34·3 $\pm$	1·01,
	1200–2100 = 180	—	27·0 $\pm$	·75,
	2100–3000 = 180	+	70·9 $\pm$	·90,
	3000–3900 = 180	+	20·2 $\pm$	·90,
	3900–4900 = 200	+	0·0 $\pm$	·62,
	4900–5000 = 20	—	55·5 $\pm$	·24,
	5000–5500 = 100	+	808·2 $\pm$	·58,

$$B_{17} = 5^{m} - 18{\cdot}0^{\mu}.$$

(13) *Adopted length and probable error of Holton Base as measured with the tapes.*—The length we adopt for Holton Base as measured (not reduced to sea level) is the sum of the lengths of its parts given in the preceding section; that is,

$$1100B_{17} + 851{\cdot}9^{mm},$$

wherein

$$B_{17} = 5^{m} - 18{\cdot}0.^{\mu}$$

The probable error of this value, so far as it depends on errors of meas-

urement or those developed in the repeated measures of the sections, is given under (*a*) of section 12 as $\pm 1{\cdot}99^{mm}$ or $1{\cdot}65^{mm}$, according as we adopt the process (1) or the process (2) of that section. The probable error in the whole length due to error in the adopted tape lengths is, as stated in (*b*) of section 12, $\pm 55 \times 0{\cdot}05^{mm} = \pm 2{\cdot}75^{mm}$.

It remains to consider the possible effects of errors of grades and alignment. These errors, though quite negligible in the length of a single section, may yet rise to an appreciable quantity in the length of the whole base.

As to errors in grade corrections, it will be observed that they are of the compensating class, since errors in determining the differences in altitude of consecutive marking tables are as likely to be of one sign as another. The grade correction c for any tape length, as explained in section 3, is given by

$$c = -\frac{h^2}{2s},$$

where h is the difference in altitude of the two ends of the tape and s its length. The effect of an error Δh in h is expressed by

$$\Delta c = -\frac{h}{2s} 2\Delta h = -2c\frac{\Delta l}{h}$$

For a constant value of Δh, therefore, the value Δc will vary as h. The greatest value of h for the tape lengths on Holton Base was $1{\cdot}72^{m}$, giving rise to a value of $14{\cdot}79^{mm}$ for c. Since the values of h were measured by two different observers at different times, and since all discrepant values were redetermined, it seems impossible that an error as great as 2^{cm} could have entered any adopted h. In the case of the above maximum value of h, the error in c due to an error $\Delta h = 2^{cm}$ would be

$$\Delta c = (14{\cdot}79^{mm}) \times 4/172 = 0{\cdot}34^{mm}.$$

The average value of h for the base was less than $0{\cdot}4^{m}$, so that the average correction for grade was thus less than $0{\cdot}8^{mm}$ per tape length. Hence, since the errors under consideration are of the compensating sort, it is believed that their aggregate effect on the whole line was inappreciable.

Errors of alignment in tape measures may manifest themselves in two ways. In the first place, the tape as stretched from length to length may not be parallel to the base. A lack of parallelism with the base to the extent of a decimetre in the length of a tape 100^{m} long would give rise to an error of $0{\cdot}05^{mm}$. Similarly, any departure of the tape from a vertical plane through its ends would give rise to an error of the same sort and sign. In the second place, any imperfections in the alignment of the support nails to a uniform slope for any tape

length would also produce a cumulative error. Both of these alignment errors make the measured length of the base too great. Both errors are, however, very small. An error as great as $0{\cdot}2^{mm}$ from crookedness of the tape with respect to a vertical plane through its ends is, it is thought, of very rare occurrence. Errors due to defects in setting the support nails to a uniform grade must be smaller per tape length than the former and of still rarer occurrence. The method of setting the support nails is so easy and accurate that an error in position as great as 1^{cm} can arise only through accidents or blunders.

Notwithstanding the indications that the errors in grades and alignment are negligible, we shall attribute $\pm 1^{mm}$ as a probable error to each of these sources, since it is possible that some blunders occurred in this first application of the method and apparatus.

With respect to the two errors of measurement, $\pm 1{\cdot}99^{mm}$ and $\pm 1{\cdot}65^{mm}$, given above, we shall adopt the larger, in order that the precision may be underestimated rather than overestimated.

Collecting the several independent probable errors, which are to be combined to produce the resultant probable error of the base, we have the following statement:

Probable error due to errors of measurement	$\pm 1{\cdot}99^{mm}$,
Probable error due to errors in grade	$\pm 1{\cdot}00$,
Probable error due to errors of alignment	$\pm 1{\cdot}00$,
Probable error due to errors of tape length	$\pm 2{\cdot}75$.

Taking the square root of the sum of the squares of these, there results for the total probable error of the base as measured with the tapes

$$\pm 3{\cdot}68^{mm}.$$

This is equivalent to the 1/1 500 000th part of the base.

For the measured length, then, of Holton Base, when expressed in terms of the International Metre, we may write

$$1100B_{17} + 851{\cdot}9^{mm} + 3{\cdot}68^{mm} = 5500{\cdot}832^{m} \pm 3{\cdot}68^{mm}.$$

(14) *Data for reduction to sea level.*—The heights of the marking tables used in the tape measures were all determined with reference to the tops of the marking stones at the ends of the base. The stone at north base was found to be $2{\cdot}61^{m}$ above the stone at south base. The average heights of the tape lengths for the several sections above the stone at south base are given in the following statement. The values are derived by taking the average of the half sums of the heights of the ends of the several tape lengths for the sections. The slopes of the sections as measured with the tapes are so gentle that these average values are amply accurate for computing the reductions to sea level.

Tape measures of St. Albans Base.

Average height of tape above south base:

Section south base to 1200^{m} stone 4·45^{m},
1200^{m} stone to 2100^{m} stone 6·46,
2100^{m} stone to 3000^{m} stone 5·49,
3000^{m} stone to 3900^{m} stone 3·48,
3900^{m} stone to 4900^{m} stone 4·63,
4900^{m} stone to north base 3·99.

B.—ST. ALBANS BASE MEASUREMENT.

(15) *Location of base and preparation for measurement.*—St. Albans base is a line in the transcontinental chain of triangulation along or near the thirty-ninth parallel of latitude. It is situated in the valley of the Great Kanawha river, near the village of St. Albans, Kanawha County, West Virginia. It is about 3 900^{m} long, and extends in a nearly east and west direction along a closely level portion of the valley. It is on the north side of and is parallel to the Chesapeake and Ohio railway, the west end being 17·8^{m} and the east end 18·4^{m} from the nearest rail of the main track in 1892.

This base was located in the summer of 1891 by Sub-Assistant Walter B. Fairfield. The terminal marking stones were provided and set under the direction of Assistant Mosman during the autumn of the same year. A profile of the line and a determination of its height with respect to an adjacent triangulation station of known height were also made under Assistant Mosman's direction by Mr. J. S. Siebert.

This base was measured with the tape apparatus in the early part of October, 1892, by a party in charge of the writer, aided by Assistant E. E. Haskell, Recorders M. V. Safford and Orville G. Brown. The line was cleared of brush, brambles, grass, and other obstacles, and the marking and support stakes for the tapes were set during the days October 1–9, 1892. These stakes were set 10^{m} apart, and the marking stakes were all carefully aligned with a theodolite. Special pains were also taken in this work to make the spacing of the support nails uniform and to bring them accurately into coincidence with the line of slope for any tape length. Some special provision had to be made for supporting the tapes at an elevation of 3^{m} to 5^{m} for two tape lengths where the line crossed a ravine. But the supports for these two tape lengths were no less stable than at other parts of the line, and no difficulty was met in handling the tapes successfully at such altitudes.

(16) *Measurements of the base.*—A measure of the base was made during the afternoon of October 10 with the 100^{m} steel tape No. 85 in

order to afford practice in manipulating the apparatus to the operatives, all of whom, except the writer, had had no experience with the apparatus. In this practice measure no record was kept of the set-ups, and some tape lengths were omitted, so that no results can be derived from it. The work of actual measurement was begun on October 11 and continued until October 14. Two measures of the base, one in each direction, were made at night with each of the steel tapes Nos. 85 and 88; and a daytime measure, in bright sunshine, was made with tape No. 88.

In all the measures, Mr. E. E. Haskell was the observer at the rear end of the tape and Mr. R. S. Woodward the observer at the front end. The thermometers from which the tape temperature was inferred were also read by these observers. An additional thermometer with a bright bulb, as explained more fully below, was read at the middle of the tape by Mr. M. V. Safford, who also attended to the operation of flipping the tape and assisted in carrying it forward. The records were kept by Mr. Orville G. Brown. The whole number of operatives employed in the work was ten.

The base was divided into four sections. These were all nearly one kilometre in length, except the easternmost one, which was about 869^{m}. The fractional part of a tape length, namely, 69^{m}, in this latter case, was measured on the tape when under the usual tension; 60^{m} of this fraction (or, rather, three-fifths of a tape length) was obtained directly from the 20^{m} sub-spaces on the tapes, while the remaining portion of 9^{m}, about, was measured by means of an auxiliary 15^{m} Chesterman steel tape divided to millimetres. The particular portion of the latter tape used was tested on the Mural Standard of the Survey on returning from the field, and found to accord sufficiently well with its nominal value. The whole fraction of a tape length (69^{m} about) was on each occasion fixed on the tape by means of a light transverse line ruled with a brad awl. The distance between the 60^{m} mark on the tape and the temporary line was measured in each case with the auxiliary tape immediately after ruling this line, but as an additional check the same distance was remeasured in the daytime, so that it is practically certain that no appreciable error enters this fractional part of a tape length.

The several positions of the tapes were marked on zinc plates which were left in position until the work was completed, when they were numbered, oriented, and filed as part of the records. The "set-ups" and "set-backs" made were measured and recorded at the time, but they were all carefully remeasured by daylight, so that no appreciable errors from this source can enter the measures.

The measures were referred directly to the terminal marks in the monuments at the ends of the base and to the marking plates at the intermediate ends of the sections. These plates were nailed to very stable stakes, and there was no indication of any appreciable movements of them during the short period of four days covered by the

measures. Any such movements, however, which might take place from day to day would not affect the interval between the terminal stones, since the base was measured continuously from end to end whenever any measurement was made.

(17) *Determination of grade corrections.*—The differences in altitude of the consecutive marking tables were measured independently by Messrs. Haskell and Safford. Whenever any discrepancies appeared, the work was re-examined by them, and it is believed that the adopted differences are trustworthy within half a centimetre in every case, while the two largest of those differences were determined with a much greater precision.

The maximum difference in height of the tables for any tape length was 3·571^{m}, which gives a grade correction of 63·76^{mm}. The average difference in height per tape length for the several sections counting from the west end were 0·37^{m}, 0·89^{m}, 0·29^{m}, and 0·18^{m}.

The grade corrections were computed in duplicate by Messrs. Safford and Woodward, and the following are the resulting values for the several sections:

	mm.
Section West Base to Stake 10 —	8·52,
Stake 10 to Stake 20 —	93·58,
Stake 20 to Stake 30 —	7·20,
Stake 30 to East Base —	3·40.

(18) *Determinations of tape temperatures.*—Mercurial thermometers of the same sort as those used on Holton Base were used to give the tape temperatures in the measurement of St. Albans Base. But in the hope that the thermometers could be made to follow the tape temperatures with greater certainty the aluminum sheaths used in the Holton work were replaced by steel sheaths. These were made of very thin steel tape which was coiled into cylinders just long and large enough to slip over the thermometer bulbs and clasp their stems. By annealing, the surface of these coils was given a dull black color closely resembling that of the measuring tapes. A disadvantage, however, of these steel sheaths over the aluminum ones lies in their greater mass, which is about 0·50 grammes, and is about three times the mass of the aluminum sheaths. This increase in mass caused a notable lagging of the thermometers with respect to the tapes, whereas such lagging was very small when the lighter sheaths were used. The effects of this lagging, however, appear to have been well eliminated by the process of measurement adopted.

With the hope, also, that some light might be shed on the obscure question of the persistent difference between the tape and the thermometric temperatures during the day measures of Holton Base, a thermometer with no sheath attached to it (or one with bright instead

of black bulb) was read in all the measures of St. Albans Base. The salient result revealed by this experiment is that during the daytime the thermometers with black bulbs (or sheaths) read uniformly higher and at night uniformly lower than the thermometer with the bright bulb. This fact confirms what might be, of course, expected, namely, that the tape and thermometers may have persistently differing temperatures unless they have the same capacity for absorption and radiation. Another result of this experiment which may prove of value in further studies with the tapes is that the period when thermometers with bright and black bulbs give the same readings occurs shortly before sundown, and appears to coincide with the period of steady atmosphere.* But aside from these results, interesting though they be in a general way, nothing came from the use of the thermometer with the bright bulb. The temperature of the tape was inferred altogether from the readings of the two thermometers supplied with the steel sheaths. That the latter gave temperatures closely coincident with those of the tapes, except for the lag effect, which was in general well eliminated, appears to be demonstrated by the results of the several measures of the four sections of the base.

The thermometers used in all work on St. Albans Base were Nos. 5598, 5620, and 5621 by Green. Table X following gives the corrections to be applied to the readings of these thermometers to bring them into accordance with the standard hydrogen scale. The corrections to 5598 are those given in section 10, Chapter IV, the freezing point reading adopted being that of January 21, 1892. The freezing point readings of 5620 and 5621 were determined by Mr. Louis A. Fischer, of the office of Standard Weights and Measures, in September, 1892.

TABLE X.—*Corrections to thermometer readings.*

Temperature.	Correction to—		
	5598.	5620.	5621.
°	°	°	°
0	—0·10	—0·20	—0·25
5	— ·12	— ·15	— ·20
10	— ·09	— ·21	— ·26
15	— ·08	— ·25	— ·32
20	— ·08	— ·27	— ·34
25	— ·07	— ·25	— ·32
30	— ·04	— ·25	— ·25
35	— ·09	— ·23	— ·28

(19) *Tension correction and working lengths of tapes.*—Spring balance No. 53 was used in all the work on St. Albans Base to give tension to

* These two periods are probably due to the same cause, to wit, equality in the quantities of radiant heat received and rejected by the earth at that time of day.

Tape measures of St. Albans Base.

the tapes. Its index correction was observed on October 10, 12, 14, 18, and 21. The mean of the resulting values, which vary from — 2·0 ounces to — 3·0 ounces, is — 2·55 ounces; or in the formula

$$T = T_1 + \tfrac{1}{2}(W - R + r),$$

given in section 4, $r = -2{\cdot}55$ ounces.

The following values were also observed:

$$W = 49{\cdot}5 \text{ ounces},$$
$$R = 35{\cdot}0 \text{ ounces}.$$

The nominal value of the tension used was

$$T_1 = 25 \text{ pounds}.$$

Hence the actual tension applied was

$$T = 25 \text{ pounds } 6 \text{ ounces}.$$

Since the standard tension is 25 pounds 9 ounces, the tension correction is — 3·0 ounces, and the correction to the standard lengths is $-3 \times 0{\cdot}047^{mm} = -0{\cdot}14^{mm}$. The lengths of the tapes as used are then (see Chapter IV, section 17)

$$T_{85} = 20\,B_{17} + 3{\cdot}72\,mm. + 1{\cdot}0947t\,mm.,$$
$$T_{88} = 20\,B_{17} + 6{\cdot}17\,mm. + 1{\cdot}0914t\,mm.$$

(20) *Results of the measures of St. Albans Base.*—Table XI following gives the results of the measures of St. Albans Base. The first column gives the date; the second, the time of day when the measure was made; the third, the direction in which the measure proceeded; the fourth, the mean temperature of the tape; the fifth, the temperature range, or the difference between the highest and lowest observed temperature of the thermometers; the sixth indicates by the letters R and F whether the temperature was rising or falling during the measure of a section, and the order of the letters indicates the order in which the changes occurred; the seventh gives the length of the sections in terms of the iced bar B_{17}, the excess over a round number of tape and bar lengths having been measured with a millimetre scale, and with a 15^m auxiliary steel tape in case of the fourth section; and the last column gives the number of the tape used. The values for the lengths given in the seventh column are not corrected for grades. Since the whole base only was ultimately used, the total grade correction will be applied to the sum of the lengths of the sections.

TABLE XI.—*Results of measures of St. Albans Base.*

WEST BASE TO STAKE 10.

Date.	Time of day.	Direction of measure.	Mean temperature.	Temperature range.	Temperature rising, R, falling, F.	Value for distance.	No. of tape.
1892.	*p. m.*		°C.	°C.		*mm.*	
Oct. 11	7:20– 8:02	W. to E.	9·04	3·40	F	200 B_{17} + 258·2	88
12	9:03– 9:44	E. W.	7·62	2·70	R	244·5	88
13	7:06– 7:40	W. E.	13·30	3·00	F	252·6	85
13	10:49–11:21	E. W.	9·56	1·10	R	251·5	85
14	2:53– 3:16	W. E.	32·11	1·40	R, F	248·1	88

STAKE 10 TO STAKE 20.

Oct. 11	8:02– 8:44	W. to E.	5·92	2·80	F, R	200 B_{17} + 534·8	88
12	9:44–10:25	E. W.	6·39	2·25	F, R	535·5	88
13	7:40– 8:12	W. E.	10·38	2·55	F, R	533·0	85
13	11:21–11:53	E. W.	9·05	0·80	F, R, F	536·0	85
14	3:16– 3:49	W. E.	30·47	0·80	F, R	536·0	88

STAKE 20 TO STAKE 30.

Oct. 11	8:44– 9:26	W. to E.	6·23	1·50	R, F, R	200 B_{17} + 527·0	88
12	7:40– 8:22	E. W.	7·84	1·60	R, F	524·0	88
13	8:12– 8:44	W. E.	9·92	1·10	F, R, F	527·8	85
13	9:45–10:17	E. W.	8·75	0·70	R, F, R	528·9	85
14	3:49– 4:02	W. E.	30·08	1·50	R, F, R	534·0	88

STAKE 30 TO EAST BASE.

Oct. 11	9:36–10:08	W. to E.	4·80	0·75	F, R	172 B_{17} + 9 325·5	88
12	8:22– 9:03	E. W.	7·92	2·65	F, R	9 3 3·0	88
13	8:44– 9:16	W. E.	9·24	0·50	F, R	9 321·4	85
13	10:17–10:49	E. W.	8·57	1·00	F, R, F	9 328·6	85
14	4:02– 4:26	W. E.	29·80	0·95	R, F, R	9 333·0	88

(21) *Interpretation of results.*—An inspection of the results in the above table shows the ranges amongst the individual measures of the several sections to be 13·7^{mm}, 3·0^{mm}, 10·0^{mm}, and 11·6^{mm}, respectively; and the question which immediately suggests itself is, why should the range for the section stake 10 to stake 20 be so much less than for the other sections? The hypothesis we offer in answer to this question affords a satisfactory explanation of the ranges just mentioned, especially those of the night measures, and clears up some of the peculiarities noted in the measures of Holton Base. It is based on the following considerations:

Tape measures of St. Albans Base.

In case there is any relative lagging of the tape and thermometric temperatures, its effect will be more or less eliminated in the mean of two measures of a section made the one under a rising and the other under a falling temperature. A similar elimination of the lag effect will occur in a single measure of a section if the temperature is rising for one-half and falling for the other half of the measure; and, in general, the lag effect will tend to eliminate itself if the temperature fluctuates to any considerable extent about its mean trend. The conditions which obtained during the measures of St. Albans Base, so far as they pertain to this lag effect are, in general, well indicated by the data in the fifth and sixth columns of the above table; but it should be stated that the section stake 10 to stake 20 was specially favorable by reason of its topographic conformation for eliminating the lag effect. The middle portion of this section was on low marshy ground compared with the ground at either of its ends, so that there was always a marked fall of temperature on approaching the low ground and a marked rise on leaving it, regardless of the direction of measurement.

This answer, it may be conceded, explains the small range among the results for the section stake 10 to stake 20; but if it is the correct answer, we should expect the means of the forward and backward measures of the other sections to show a similar degree of accordance, since, as shown in the sixth column of the table, the temperature gradient was in general reversed with a reversal of the direction of measurement. The two night measures with each tape are in this respect strictly comparable, since they were made under conditions in all respects similar; i. e. the sky was cloudless or nearly so, there was little or no wind, the preceding days had been very much alike, and the measures were made at times when dew was condensing along most parts of the line. That these means do so accord is shown by the results in Table XII following, which gives the means of the night measures of the sections made with the two tapes. For brevity the sections are numbered 1, 2, 3, 4, respectively:

TABLE XII.—*Means of forward and backward measures of the sections.*

No. tape.	Section 1.	Section 2.	Section 3.	Section 4.	Whole Base.
	mm.	*mm.*	*mm.*	*mm.*	*mm.*
88	+251·4	+535·2	+525·5	+9 324·2	10 636·3
85	252·0	534·5	528·4	9 325·0	10 639·9

The accordance of these results leaves little to be desired; but for the reasons already adduced and for others given below, it is difficult to see how it can be otherwise than real.

Although the fact that means of forward and backward measures of

a section indicated a marked elimination of some source of error was noticed in certain of the measures of Holton Base, the explanation of that fact, presented above, did not occur to me until after the night measures of St. Albans Base had been made. The full import of the explanation, indeed, was not appreciated until after the day measure of October 14 had been made; otherwise a reverse measure of the line would have been made on the latter day. The particular measures of Holton Base just referred to are those of sections 2 100^{m} to 3 000^{m} and 3 000^{m} to 3 900^{m}. On referring to Table V of section 7 it will be seen that each of these sections was measured in both directions on September 18 and again on September 19, 1891, under conditions favorable for the elimination of the lag effect. The excesses over $180B_{17}$ of the means of the forward and backward measures of the section 2 100^{m} to 3 000^{m} for these two days are + 71·2^{mm} and + 70·7^{mm}; and for the section 3 000^{m} to 3 900^{m} the corresponding excesses are + 20·4^{mm} and + 20·0^{mm}. These exhibit a degree of accordance equal to that shown in Table XII. A similar degree of accordance is shown in the forward and backward measures of the standard kilometre of Holton Base of September 30, October 1, and October 8, 1891, on which dates the temperature gradients were favorable to the elimination of the lag effect. Attention has also been called in section 11 to the fact that the measures of the standard kilometre show a less probable error than the measures of the other sections of Holton Base. The reason for this is found, I believe, in the topographic conformation of that kilometre. It passed for 600^{m} of its length through a dense forest, leaving 200^{m} at either end in open fields. In nearly all the night measures the thermometer readings show a rise of temperature on passing from the open fields to the forest, and vice versa; which conditions, as explained above, were favorable for the elimination of the lag effect in a single measure of the line.

To recur to the measures of St. Albans Base, it is seen from the results in Tables XI and XII that the largest discrepancies are found in every case where, according to the hypothesis considered, they would be expected to appear. Thus, in Table XI the range is greatest in the measures of the section West Base to stake 10, which presented the steepest temperature gradients; and in Table XII the range is greatest in the case of the third section, which, as appears from the data in the sixth column of Table XI, presented the least favorable temperature gradients.

It should be noted, however, that the difference between the night measures of the section West Base to stake 10 with tape No. 88 is much greater than that between the night measures with tape No. 85. The temperature gradients, in fact, indicate that the former difference ought to exceed the latter, but it is not clear that the excess should be as great as it is. This apparent anomaly has led to a comparison of the separate tape lengths of the night measures of this first section

Tape measures of St. Albans Base.

with the two tapes. It shows that the measures with tape No. 88 were for every tape length apparently longer in the one case and shorter in the other by about 0·7^{mm}; in other words, if we are dealing with a lag effect, the thermometers gave a temperature too high to the tape under a falling gradient and too low under a rising gradient by about 0°·7 C., and this error was persistent throughout both measures of the section. Attention has been called in Chapter IV, section 15, to a similar discrepancy in the behavior of these two tapes when placed alongside of one another on the 100^{m} comparator of Holton Base. It is most probably due, as suggested in the section just referred to, to some difference in the emissivities of the two tapes, but a study of all the data has not up to the present time led me to any clearer view of the question.

It remains to consider the results of the day measure of the base of October 14, 1892. This was made on a cloudless day between 2:53 and 4:26 p. m., and the mean temperatures for the several sections varied from 32°·11 C. to 29°·80 C. Including this measure, the mean temperatures of the tape ranged for the different day and night measures of each section through nearly 25° C.; so that the accordance of the day measures with the mean of the night measures affords a pretty severe test of the equations of the tapes and of the thermometeric temperatures attributed to them. A glance at the data in the sixth and seventh columns of Table XI shows that the accordance of the day measure is best, as it ought to be in conformity with the considerations adduced above, in the case of the first and second sections, and worst, as would be expected, in the case of the last two sections.*

The differences between the day measures of the several sections and the means of the night measures of the same are

	mm
For the first section	− 3·6
second section	+ 1·2
third section	+ 7·0
fourth section	+ 8·4

The average of these regardless of sign is 5·0^{mm}, or 1/200 000th of a section; while the average of the first two of them, which must be largely free from the lag effect, is only 2·4^{mm}, or 1/400 000th part of a section. From any point of view these discrepancies, when we bear in mind the wide range of temperature involved, do not seem large; but when we consider them in connection with the temperature gradients which obtained during the day measure, it appears clear that, except

* It is now plainly regrettable that a reverse measure of the base was not made immediately after this one, since it appears almost certain that the means of such day measures would have shown a very close agreement with the means of the night measures.

for the lag effect, the thermometers as fitted with the steel shields gave temperatures in very close agreement with those of the tapes during both the day and the night measures of the base.

Briefly summarized, then, our interpretation of the results of the measures of St. Albans Base is that the most formidable errors to which they were subject arose from lag of the thermometers as fitted with the steel shields; that these errors were well eliminated in the means of the forward and backward night measures with either tape for all sections, and well eliminated in all the individual measures of the second section by reason of its topographic conformation; and finally, that the result of the day measure is too great by reason of uncompensated lag effect which lies chiefly in the lengths of the two easternmost sections of the base.

(22) *Values for measured lengths of base and probable error of measurement.*—The value we shall adopt for the entire length of the base is the mean of the four night measures, since all our experience indicates that the temperature uncertainty is in general least in night measures. and since it appears certain that a considerable lag effect was eliminated in the means of the forward and backward measures of this base. Accordingly, the value we adopt is

$$772B_{17} + 10638{\cdot}1^{mm},$$

or the mean of the results in the last column of Table XII.

If, on the other hand, we include the day measure and give it and each of the night measures the same weight, the result is

$$772B_{17} + 10640.7^{mm},$$

which differs but $2{\cdot}6^{mm}$ from the adopted value.

The results of the several measures of the entire base, in the order in which they were made, are

	mm.
Oct. 11, 1892,	$772B_{17} + 10645{\cdot}5$,
12, 1892,	27·0,
13, 1892,	34·8,
13, 1892,	45·0,
14, 1892,	51·1.

The range among these is $24{\cdot}1^{mm}$, or 1/160 000th part of the base. The range of the first four, or the night measures, is $18{\cdot}5^{mm}$, or 1/209 000th part. The greatest divergence from the adopted mean is that of the last, or day measure. The amount is $13{\cdot}0^{mm}$, or 1/298 000th part.

The mean lengths from the forward and backward measures with the two tapes are

	mm.
Tape No. 85,	$772B_{17} + 10636{\cdot}3$,
No. 88,	$772B_{17} + 10639{\cdot}9$.

The difference of these is $3{\cdot}6^{mm}$, or 1/1 070 000th part of the base.

Tape measures of St. Albans Base.

In computing the probable error of measurement from the data afforded by Tables XI and XII, we may proceed according to several hypotheses. Using 1 kilometre as the unit of distance to which such probable error refers, we may assume (a) that all of the night and day measures of the several sections of the base are of equal weight, and that no elimination of systematic error results in the mean of a forward and backward measure; (b) that the night measures should be considered by themselves and treated as if they involved no errors due to lagging of the thermometers; (c) that the means of the forward and backward measures given in Table XII are more or less free from errors due to lagging, and that such means are alone competent to give an idea of the attainable precision.

The errors which result from each of these hypotheses will now be given. Let ε_a, ε_b, ε_c = the probable error of one measure of a kilometre according to the suppositions (a), (b), (c), respectively.

m = the total number of independently observed quantities, or total number of measures of the sections,
= 20 for supposition (a),
= 16 for supposition (b),
= 8 for supposition (c),

$[vv]$ = sum of squares of the discrepancies between the individual measures of the sections and their means, respectively.

Then from Tables XI and XII we find*·

$[vv] = 250{\cdot}57$ for supposition (a),
$= 142{\cdot}94$ for supposition (b),
$= 4{\cdot}80$ for supposition (c).

According to the method of least squares, when we have m measures of equal weight of μ unknown quantities, the probable error of a single measure is expressed by

$$\pm 0{\cdot}6745\sqrt{\frac{[vv]}{m-\mu}}.$$

In the present case the number of unknown quantities, or μ, is 4, the number of sections of the base. Hence for the several suppositions we have $m - \mu = 16, 12, 4$, respectively. These data then give

	mm.
$\varepsilon_a =$	$\pm 2{\cdot}67$
$\varepsilon_b =$	$\pm 2{\cdot}33$
$\varepsilon_c =$	$\pm 0{\cdot}74$

* The sections are treated as of equal length, the modification arising from the fact that the section from stake 30 to East Base is 130^m short of a kilometre being unimportant.

It may be observed that if the mean values in Table No. V, from which ε_c is derived, are not to a marked extent free from systematic error, as is assumed, we should have by the theory of errors

$$\varepsilon_c = \tfrac{1}{2}\varepsilon_b \sqrt{2} = 0{\cdot}71\varepsilon_b.$$

But the actual relation is $\varepsilon_c = 0{\cdot}32\varepsilon_b$, or the ratio of the two errors is less than half as great as it should be if the hypothesis (*c*) were false.

It may be observed, also, that the work of determining the tape lengths on the 100^m comparator of Holton Base affords an indirect means of estimating the extent of the temperature error involved in the mean of a forward and backward measure of a kilometre; and hence some additional light may be gained concerning the magnitude of the error of manipulation, which latter, as revealed in the tape measures of Holton Base, was briefly discussed in section 10 above. As shown in Chapter IV, section 15, the probable error of a single determination of a tape length on the 100^m comparator was $\pm 0{\cdot}20^{mm}$. This, as explained in the chapter referred to, is due almost wholly to temperature errors in the mean of six thermometer readings. It involves the lag effect to a slight extent, and hence ought to indicate a larger temperature component in the probable error of a mean measure of a section of St. Albans Base than that shown from the actual observations, if in these latter the lag effect was well eliminated. Assuming the above error of $\pm 0{\cdot}20^{mm}$ per tape length to apply to the field work if six thermometer readings were made, the value for the actual case, wherein but two such readings were observed, is $\pm 0{\cdot}20^{mm}\sqrt{3}$. With this as the probable error per tape length, the probable error of the sum of ten such lengths is $\pm 0{\cdot}20^{mm}\sqrt{30}$, and hence that for the mean of two such measures is

$$\pm 0{\cdot}20^{mm}\sqrt{15} = \pm 0{\cdot}77^{mm}.$$

This is slightly greater than the value of ε_c given above. It shows, so far as it goes, a satisfactory elimination of the lag effect in the means of the forward and backward measures of the sections of St. Albans Base, and indicates that the error of manipulation is really much less than was inferred in section 10 from the measures of Holton Base alone.

Lastly, it should be remarked that the value $\varepsilon_b = \pm 2{\cdot}33^{mm}$, derived by using the night measures of St. Albans Base, is considerably larger than the corresponding value deduced from the night measures of Holton Base, namely, $\pm 1{\cdot}74^{mm}$, given in section 10 above. The main reason for this lies, undoubtedly, in the fact that the steel sheaths used on the bulbs of the thermometers in 1892 caused much greater lagging than the aluminum sheaths used in 1891. The exceptionally favorable conditions of the standard kilometre of Holton Base for eliminating the lag effect have been alluded to in the preceding section, and this circumstance also tended to make the probable error of a single measure of a kilometre in the Holton work small.

Tape measures of St. Albans Base.

Now as to probable error due to errors of measurement in the whole length of St. Albans base, three values dependent on the suppositions (a), (b), (c), may be derived; and a fourth value may be computed from the difference between the mean lengths for the whole base from the night measures with the two tapes. The latter process, it will be noted, should give the smallest probable error, since it does not involve the possible errors due to movements of the intermediate section stakes, though such movements, it is believed, were very small. The first three of these probable errors are

$$\begin{aligned} &mm. \\ \varepsilon_a\sqrt{4/5} &= \pm 2{\cdot}39 \text{ according to supposition (a),} \\ \varepsilon_b\sqrt{4/4} &= \pm 2{\cdot}33 \text{ according to supposition (b),} \\ \varepsilon_c\sqrt{4/2} &= \pm 1{\cdot}04 \text{ according to supposition (c),} \end{aligned}$$

For the fourth probable error we find $\pm 0{\cdot}9^{mm}$.

Expressed as fractions of the length of the base, these probable errors vary from 1/1 620 000th to 1/4 300 000th.

The errors just discussed, or those developed in repeated measures of a line, appear to be the only errors of measurement which need be considered in connection with St. Albans Base. In comparison with these, the errors in grade corrections, in alignment, in transfers to permanent marks, and in the measurement of fractions of tape lengths, are trifling. As already explained, special pains were taken in the alignment of the support and marking stakes and in the determination of grade corrections. We may allow, however, for possible contributions of error from the latter sources by adopting $\pm 2{\cdot}0^{mm}$ as the probable error of measurement. This, it will be observed, is but little less than that given by supposition (b), which takes no account of the elimination of the lag effect, and would appear to materially underestimate the precision actually attained.

(23) ***Probable error of base due to errors of tape lengths.***—From the evidence adduced in Chapter IV, section 16, it appears that the probable errors of the tape lengths as determined on the 100^m comparator of Holton Base can not exceed $\pm 0{\cdot}05^{mm}$ within such ranges of temperature as are likely to be met in ordinary work. For such temperatures as obtained during the night measures of St. Albans Base, the probable error per tape length, if computed from the formula given in the section of Chapter IV just referred to, would be somewhat less than $\pm 0{\cdot}05^{mm}$. One circumstance, however, leads me to adopt a larger value. It is this: If we use the mean lengths of the sections of St. Albans Base given in Table XII to compute a correction to the difference in lengths of the tapes at 0° C., the mean value of such correction is found to be $\pm 0{\cdot}09^{mm} \pm 0.095^{mm}$, indicating that the difference in length of the tapes at 0° C. should be $2{\cdot}54^{mm}$ instead of $2{\cdot}45^{mm}$, as

given by the equations of the tapes derived in Chapter IV. Subsequently to the measurement of St. Albans Base the tapes were compared directly on a 100^{m} section of the base. An account of this work is given in section 26 below. It indicates a correction to the difference of the tapes at 0° C. of about the same amount as that just mentioned and with somewhat greater certainty. The amount of this discrepancy, it will be observed, is of about the same magnitude as its probable error derived from the expression (9), section 16, Chapter IV, namely:

$$\sqrt{(0{\cdot}055^{mm})^2 + (0{\cdot}058^{mm})^2} = \pm\, 0{\cdot}08^{mm},$$

so that no great weight can be attached to it when considered by itself. But inasmuch as the lengths of the tapes were determined by observing on them with micrometer microscopes having a power of 27 diameters, while in applying them to measure a base a hand magnifier or the unaided eye is used, it is possible that some constant error may have thus entered the measures of St. Albans Base. For this reason the extreme value of $\pm\, 0{\cdot}06^{mm}$ will be taken for the probable error of the tape lengths in those measures. This, as shown in section 16, Chapter IV, is the probable error of the tape lengths given by their equations for such temperatures as 0° and 30° C.

Since there were 38·7 tape lengths in the base, the probable error of $\pm\, 0{\cdot}06^{mm}$ per tape length gives $\pm\, 2{\cdot}32^{mm}$ as the probable error of the whole base due to this source.

(*24*) *Corrections to measured length of base.*—To the adopted mean length of the base given in section 22 above, three corrections are to be applied, namely: (1), correction for grades; (2), correction for error in thermometer corrections used in the computation of the lengths of the sections of the base; (3), reduction to sea level.

(1) Since the entire length only of the base is needed, the sum of the grade corrections may be applied to the adopted measured length given in section 22. This sum, the terms of which are given in section 16, is $-\,112{\cdot}7^{mm}$.

(2) The thermometer corrections which should have been applied in computing the length of the base are those given in Table X, section 17. The means of the corrections to Nos. 5598 and 5620, which alone were used to give the tape temperatures, vary from $-\,0°{\cdot}135$ to $-\,0°{\cdot}175$, giving thus a practically constant value of $-\,0°{\cdot}155$, which we adopt as the correction to be applied to the means of those thermometer readings for the entire work. The corrections actually applied in computing the lengths of the sections of the base were, by reason of an error in making up a table of corrections, all numerically larger than those by the following amounts for the several sections of the base counting from the west end:

First section, 0°·028,
Second section, 0 ·022,
Third section, 0 ·021,
Fourth section, 0 ·021.

Tape measures of St. Albans Base.

Since the thermometer corrections are all negative, and since the rate of expansion of the tapes is $1{\cdot}09^{mm}$, the computed length of the base is too short by

$$1{\cdot}09^{mm}\ \{10\ (0{\cdot}028 + 0{\cdot}022 + 0{\cdot}021) + 8{\cdot}7\ (0{\cdot}021)\} = 0{\cdot}97^{mm}.$$

The correction, then, to the measured length of the base from this source of error is

$$+\ 0.97^{mm}.$$

(3) Complete data for reduction to sea level are not at present available. Table XIII following, however, gives the heights of the several marking tables with respect to the top of the marking stone at West Base, and also the mean heights of the several tape lengths referred to the same datum and counting from West Base toward the east. Every tape length is to be understood as 100^{m} long, except the thirty-ninth or last, which is but 70^{m} long. The *plus* sign signifies that the tables or mid tape lengths were higher, and the *minus* sign that they were lower than the datum.

TABLE XIII.—*Heights of marking tables and mean heights of tape lengths.*

Height of table at—	Feet.	Mean height of tape length. Feet.	Height of table at—	Feet.	Mean height of tape length. Feet.
West Base	+ 1·191		Stake 21	— 6·575	— 7·112
Stake 1	+ 1·848	+ 1·519	22	— 5·184	— 5·880
2	+ 0·655	+ 1·252	23	— 5·495	— 5·340
3	— 0·079	+ 0·288	24	— 2·966	— 4·230
4	+ 2·035	+ 0·978	25	— 2·719	— 2·842
5	+ 2·894	+ 2·464	26	— 2·461	— 2·590
6	+ 4·154	+ 3·524	27	— 4·139	— 3·300
7	+ 3·938	+ 4·046	28	— 4·299	— 4·219
8	+ 2·723	+ 3·330	29	— 4·615	— 4·457
9	+ 1·269	+ 1·996	30	— 2·925	— 3·770
10	+ 3·592	+ 2·430	31	— 2·596	— 2·760
11	+ 5·800	+ 4·696	32	— 2·239	— 2·418
12	+ 3·670	+ 4·735	33	— 2·249	— 2·244
13	— 8·044	— 2·187	34	— 2·175	— 2·212
14	— 13·360	— 10·702	35	— 2·160	— 2·168
15	— 13·368	— 13·364	36	— 2·252	— 2·206
16	— 13·773	— 13·570	37	— 0·675	— 1·464
17	— 8·856	— 11·314	38	+ 0·995	+ 0·160
18	— 9·492	— 9·174	East Base	+ 2·101	+ 1·548
19	— 8·588	— 9·040	Stone, E. B.	+ 1·533	
20	— 7·648	— 8·118			

The sum of the mean heights of the tape lengths in this table is —103·7 feet. Hence the average height of the tape above the stone at West Base was —2·66 feet.

For the several sections of the base, counting them as hitherto from the west end, the average heights of the tape were +2·18 feet, —6·80

feet, — 4·37 feet, and — 1·53 feet, respectively, above the stone at West Base.

(25) *Adopted length and probable error of St. Albans Base.*—The value for the computed length of the base adopted in section 22 is

$$772B_{17} + 10638{\cdot}1^{mm}.$$

where B_{17} is the length of the iced-bar No. 17, as used on the 100^m comparator of Holton Base. This length is, as given in Chapter II, section 23, in terms of the International Metre,

$$B_{17} = 5^m - 18{\cdot}0^\mu.$$

Thus

$$772B_{17} = 3860^m - 13{\cdot}896^{mm}.$$

Hence, introducing this value, and applying the corrections for grade and error of applied temperatures giving in the preceding section, there results for the measured length of St. Albans Base 3870·512^m.

The adopted values of the independent errors whose resultant gives the probable error of this length are given in sections 22 and 23. They are—

Probable error due to errors of measurement, $\pm 2{\cdot}00^m$,
" " " " " of tape lengths, $\pm 2{\cdot}32$.

Taking the square root of the sum of the squares of these, we have for the total probable error of the base as measured, $\pm 3{\cdot}06^{mm}$. This amounts to 1/1 260 000th part of the base.

We write, then, for the measured length of St. Albans Base*

$$3870{\cdot}512^m \pm 3{\cdot}1^{mm}.$$

(26) *Results of special experiments with tapes on St. Albans Base.*—After the measurements of St. Albans Base were completed, certain experiments with the tapes were made with a view, first, to redetermine their relative lengths; and second, to learn more of their behavior under varying temperature conditions. Incidentally, also, the questions of friction of the tape on its supports and its elongation for small increments to the normal tension, were reexamined.

For these purposes, a tape length of the base presenting average conditions was selected and used as a comparator. In place of the marking stakes for this tape length, well seasoned wooden posts were set firmly in the ground. On one of these posts a zinc plate having a fine reference line ruled on it was fastened. On the other post a heavy brass plate carrying an ivory scale 5^{cm} long divided to half millimetres was secured. These two terminal marks and the intermediate support nails were put in the same horizontal plane. The tapes were stretched and aligned on this comparator in precisely the same way as in field

* The approximate elevation of the base above sea level is 180^m. This gives a correction of — 109^{mm} to reduce the measured length to sea level.

Experiments with tapes on St. Albans Base.

work, except that the stretcher at the rear end was fastened once for all in a rigid framework, and the position of the tape controlled by the slow-motion screw of the balance hook to which the tape was attached. Variations in length of the tapes were thus observed at the front end by means of the ivory scale, which could be easily read to tenths of millimetres. The thermometers provided with steel shields were placed 10^m from either end of the tape and read by the observers at the ends of the tape, while a thermometer with bright bulb was read at the middle of the tape length by the observer at that point, who also flipped the tape as in field work.

Observations on the tapes were made on this comparator on October 18, 19, 20, 21, and 24, 1892. On the first four dates the observations were made according to the following programme: One of the tapes being put under tension and brought to near coincidence with the reference mark at the rear end, the thermometers were first read; the observer at the rear end then brought the rear end graduation of the tape accurately to coincidence with the reference mark at that end and signaled this fact by one blast from his whistle. Thereupon the observer at the middle of the tape flipped it. As soon as the vibration of the tape ceased the observer at the rear end signaled the observer at the front end to read the scale. This being done the thermometers were again read. Then the other tape was put under tension and a similar set of observations made on it and the thermometers. The time required to make such a series of observations with the two tapes was on the average about five minutes.

On October 24 the tapes were observed on separately, one during the forenoon and one during the afternoon, and the thermometers were read but once, and that immediately after the observations on the tape.

The experiments were carried on regardless of the weather except that they were discontinued when it was rainy. They began, as a rule, about 9 a. m., and continued until about 9 p. m. Very variable conditions were thus encountered. The early mornings of the dates of observation were generally cool and foggy. As the fog was dissipated the temperature rose rapidly and then fell less rapidly toward sundown. The total temperature range was from 5° to 35° C., and the daily ranges varied from 8° to 23° for the different dates.

The results of the experiments were computed in the field by Messrs. Haskell and Woodward, and subsequently revised at the office by Mr. Safford, who also plotted all the temperature, tape-length, and cloudiness curves supplied by the data. A study of the numerical data and of their graphical representation reveals the following special characteristics of the experiments:

(1) That the temperatures of the tapes were influenced to a considerable extent by the local conditions, which forced them to undergo certain

typical changes of length dependent on the type of day. The thermometers with steel sheaths, while following the changes of tape temperature fairly well, indicate some systematic differences dependent on the local conditions and on lagging. The discrepancy between the tape and thermometric temperatures rose in extreme instances of rapidly varying temperature, as with alternating cloud and sunshine, and with alternating currents of hot and cool air, to 3° C.

(2) That the systematic character of the discrepancies between tape and thermometric temperatures disappeared largely after sundown.

In order to derive the difference in length of the two tapes the quantity $D - T_0$, where D is the distance between the terminal marks of the comparator and T_0 is the length of either tape at 0° C., was computed for each set of observations by means of the known rates of expansion of the tapes. It was assumed for any date when the two tapes were simultaneously under comparison that D remained constant, though the difference in lengths of the two tapes is practically free from any change which may have occurred from day to day in that interval. Table XIV following gives a summary of the results of the tape comparison computed in the way just described. The first column gives the date, the second the number of comparisons on that date, the third and fourth columns give the mean values of the resulting differences $D - T_0$ for the respective tapes, and the last column gives the difference in length of the two tapes at 0° C., found by subtracting the result in the third column from the corresponding one in the fourth:

TABLE XIV.—*Observed difference of tape lengths at 0° C.*

Date.	No. of comparisons.	Mean value of $D - T_0$ for—		$T_{88} - T_{85}$ at 0° C.
		Tape 85.	Tape 88.	
1892.		*mm.*	*mm.*	*mm.*
Oct. 18	14	−0·68	−3·37	+2·69
19	13	+0·10	−2·74	2·84
20	21	−0·78	−3·41	2·63
21	24	−0·70	−3·05	2·35

In deriving a final mean value of $T_{88} - T_{85}$ from the results in the last column of the above table it is thought best to assign weights proportional to the number of comparisons, except in the case of the result of October 20. On this date the observations were not extended into the night, and the result from them is for this reason of less weight than the other results. Hence we assign a weight 2 to the result of October 21 and a weight 1 to each of the other results. Their weighted mean is then

$$+ 2{\cdot}57^{mm} \pm 0{\cdot}07^{mm}.$$

This is larger by $0{\cdot}12^{mm}$ than the difference in length of the tapes

furnished by their equations given in section 14, Chapter IV. This latter difference is

$$2{\cdot}45^{mm} \pm 0{\cdot}08^{mm},$$

as shown in section 23 above. The discrepancy $0{\cdot}12^{mm}$ is thus only once and a half times its probable error, so that we can not infer with any considerable probability that any change in the relative lengths of the tapes has occurred since their absolute lengths were determined on the Holton comparator. It is possible, however, as remarked in section 23, that this discrepancy may arise from a lack of defining power of the hand-magnifying glasses used in the tape comparisons at St. Albans. Such power would not, it is believed, preclude the occurrence of an actual error equal to this discrepancy. It may be remarked, however, that the maximum possible error of St. Albans Base due to this discrepancy, even if real, is only slightly greater than one two-millionth part of its length.

Finally, it seems worth while to give the results of certain observations made on October 24, 1892, for the purpose of getting an idea of the error involved in the operation of laying and stretching the tape. It happened on this date that very favorable conditions for determining this error presented themselves. That is, the temperature was very steady, the sun being obscured by clouds; there was equilibrium of radiation as shown by the near equality in readings of the thermometers with black and bright bulbs, and there was little if any wind. Under these circumstances three sets of observations were made with the tape and thermometers in the same way that observations are made in measuring a line, except that the tape was not removed from the support nails of the comparator, and that the position of its front end was read on the millimetre scale instead of being marked on a zinc plate. The tension of the tape was in each case brought up from zero to the standard amount, and all other operations performed in precisely the same manner as in the work of measurement. Table XV below gives the results of these observations. The first column gives the limiting times of day of the groups of observations; the second gives the temperature observed by the thermometers; the third gives the reading of the tape on the scale at its front end, the tape being set in each case to coincidence with the mark at its rear end; the fourth gives the values of $D - T_0$, or the difference between the comparator interval and the length of the tape at 0° C.; and the last column gives the discrepancies v between the individual results for any group and their mean.

TABLE XV.—*Results of observations of error in laying a tape length.*

Time.	Temperature.	Scale reading.	$D - T_0$.	v (C—O).
1892.	°	*mm.*	*mm.*	*mm.*
Oct. 24, 4:29	11·51	12·5	+0·10	+0·05
	11·36	12·3	·14	+ ·01
	11·36	12·2	·24	— ·09
	11·26	12·1	·23	— ·08
	11·21	12·1	·17	— ·02
	11·11	12·1	·06	+ ·09
	11·16	12·1	·12	+ ·03
4:35	11·11	12·0	·16	— ·01
4:50	10·27	11·0	+0·24	+0·02
	10·32	11·0	·30	— ·04
	10·32	11·0	·30	— ·04
	10·37	11·1	·25	+ ·01
	10·32	11·1	·20	+ ·06
	10·32	11·0	·30	— ·04
	10·32	11·0	·30	— ·04
4:57	10·42	11·2	·21	+ ·05
5:04	10·22	11·1	+0·09	+0·01
	10·22	11·1	·09	+ ·01
	10·17	11·0	·13	— ·03
	10·22	11·0	·19	— ·09
	10·22	11·1	·09	+ ·01
5:10	10·17	11·1	·03	+ ·07

The residuals given in the last column of this table arise from errors in defining the positions of the tape at its ends, from friction of the tape on its supports, and from variations in temperature of the tape not registered by the thermometers. Presumably, the latter source contributed little to the resultant error during the short periods of time covered by the groups of obervations; but whether allowance be made for the temperature error or not, it appears that the operation of placing and stretching the tape was subject to very small errors. Thus, the average value of the residuals in the table is $\pm 0{\cdot}04^{mm}$, the maximum $0{\cdot}09^{mm}$, and the corresponding probable error of a single value of $D - T_0$ is $\pm 0{\cdot}03^{mm}$. The probable error of laying a single tape length is undoubtedly somewhat greater than this amount under such conditions as are met on the average in field work; but it seems essential to conclude from the above observations that the more formidable errors which affect such tape measurements as have been described in the preceding pages arise from defective means of getting the tape's temperature rather than from imperfections of manipulation.*

* The results of the experiments on the elongation of the tapes for increments to the normal tension are given in Supplement B, section 5.

SUPPLEMENT A.

METHOD OF COMPUTING CORRECTION FOR FLEXURE OF 5m BAR B_{17}.

(1) *Nature of flexure considered.*—As explained in Chapter I, section 3, the normal length of the 5m bar B_{17} is the distance between its end transverse graduations when the upper surfaces of the alignment plugs are in one plane and when the longitudinal marks on those plugs are in the same straight line. Whenever the bar is flexed, the distance between the graduation lines is less than the normal distance. The flexure here considered is that due to changes in temperature of the vertical web of the Y-trough and to variable load on the same. This flexure is in a vertical plane, and it is measured by means of the long striding level in the manner described in section 9, Chapter I. The level reaches from any plug to the second adjacent plug on either side, consecutive plugs being 495mm apart.

Before trying the apparatus it was feared that this flexure might be considerable during the course of a day presenting wide temperature variations, and hence adequate provision was made for measuring the curvature of the bar and computing a correction therefor. Experience, however, has shown that such fear was ill-founded. It has shown, in fact, that by means of the long striding level the bar may be made so straight in the Y-trough that no appreciable variation in its length will result in the course of any ordinary day. The largest correction for such flexure which has occurred in our experience with the bar is 0·35μ, which may well be regarded as a negligible quantity. But it might happen that so close an adjustment would not be secured always, or it might happen that in the course of a considerable length of time wide variations in temperature of the external air would occur and produce sensible changes through flexure in length of the bar. It seems worth while, therefore, to give the results of my study of this question, although it need not be a very important one with the iced bar apparatus.

Before proceeding to the details of the question, it may be remarked that the flexure here considered is due almost wholly to temperature changes in the web of the Y-trough. Such changes are noticeable from day to day, as a rule, rather than during a given day. In our experience they have partaken of the character of seasonal changes, or of such slow changes as accompany a hot or cold wave. Thus, for example, during the last set of measures of the standard kilometre of Holton Base, in September, 1891, the air temperature was pretty high and the

bar was concave upwards. About ten days later, in early October, the air temperature had fallen about 15° C., and the bar at this time when used on the 100m comparator was markedly convex upwards.

In the construction of the bar, pains were taken to make the upper surfaces of the alignment plugs equally distant from its neutral surface. This was accomplished with precision by means of the striding level referred to above; and we may assume that a surface similar to and parallel with the neutral surface of the bar can be drawn tangent to all of the alignment plugs. The sinuosities of this surface, or, more definitely, of the line in this suface cut out by the vertical alignment plane of the bar, are measured with the striding level; and they accord sensibly with the sinuosities of the corresponding line in the neutral surface.

(2) *Theory of computation.*—If s be the length of any curve whose rectangular co-ordinates are x and y,

$$ds = dx\left(1 + \frac{dy^2}{dx^2}\right).$$

In the case of the 5m bar here considered, dy/dx is always very small, so that we may write

$$ds = dx + \tfrac{1}{2}\left(\frac{dy}{dx}\right)^2 dx - \ldots,$$

whence the correction for curvature of the bar is

$$[s - x] = \tfrac{1}{2}\int\left(\frac{dy}{dx}\right)^2 dx \qquad (1)$$

taken between proper limits.

Consider the sinuous curve defined above. If it could be completely determined, it would be tangent to the plugs 1, 3 . . . 11 whereon the long striding level is applied. Refer this curve to a system of rectangular axes, the origin being at plug No. 1 and the axis of x being the straight line joining the terminal plugs 1 and 11. The level readings at any time give the co-ordinates of the plugs 3, 5 . . . 11 relatively to a system of axes having the same origin and a horizontal line tangent to plug No. 1 as axis of obscissas. Calling the co-ordinates in the latter system x', y', and denoting the sine of the inclination of the two systems of axes by σ, we have, since σ is always small,

$$x = x',$$
$$y = -\sigma x' + y'.$$

The quantity σ will vary from time to time, or from set to set of observations, while the values of y for any plug will remain constant for a given adjustment of the bar and for stable temperature of the Y-trough.

Correction for flexure of iced bar B_{17}.

Let h_1, h_2, . . . h_5 be the measured values at any time of the relative heights of plugs 1 and 3, 3 and 5, . . . 9 and 11, respectively; and let v_1, v_2, . . . v_5 be the most probable corrections to h_1, h_2 . . . respectively. Thus if we call the distance between the feet of the striding level l, and denote the ordinates of the plugs 3, 5, 7, 9, 11 in the first system of co-ordinates by y_1, y_2, . . . y_5, we have

$$\begin{aligned}
y_1 &= h_1 + v_1 - l\sigma,\\
y_2 &= h_1 + v_1 + h_2 + v_2 - 2l\sigma,\\
y_3 &= h_1 + v_1 + h_2 + v_2 + h_3 + v_3 - 3l\sigma,\\
y_4 &= h_1 + v_1 + h_2 + v_2 + h_3 + v_3 + h_4 + v_4 - 4l\sigma.\\
y_5 &= h_1 + v_1 + h_2 + v_2 + h_3 + v_3 + h_4 + v_4 + h_5 + v_5 - 5l\sigma.
\end{aligned}$$

From these it follows that

$$\begin{aligned}
l\sigma + y_1 \qquad\quad - h_1 &= v_1,\\
l\sigma + y_2 - y_1 - h_2 &= v_2,\\
l\sigma + y_3 - y_2 - h_3 &= v_3,\\
l\sigma + y_4 - y_3 - h_4 &= v_4,\\
l\sigma + y_5 - y_4 - h_5 &= v_5.
\end{aligned} \tag{2}$$

Now the function of x, $\varphi(x)$ say, which is to represent the curve whose length is sought must evidently fulfill the following conditions, namely:

$$\begin{aligned}
\varphi(x) &= 0 \text{ for } x = 0,\\
\varphi(x) &= 0 \text{ for } x = 5l.
\end{aligned}$$

The most general form of such a function is

$$y = \varphi(x) = a \sin 2\pi \frac{x}{10l} + b \sin 4\pi \frac{x}{10l} + \ldots, \tag{3}$$

wherein a, b, . . . are constants. Since the number of observation equations (2) is limited to 5 (in this case), the number of constants or terms in (3) must be limited to 4.

Giving to x in (3) the values l, $2l$, . . . $5l$ successively, the values of y_1, y_2, . . . y_5 result; and if we write for brevity

$$\begin{aligned}
a' &= 2a \sin \tfrac{1}{10}\pi,\\
b' &= 2b' \sin \tfrac{2}{10}\pi,\\
&\ldots;
\end{aligned} \tag{4}$$

$$\alpha = \tfrac{1}{10}\pi,\ \beta = \tfrac{2}{10}\pi,\ \gamma = \tfrac{3}{10}\pi,\ \ldots;$$

the above equations (2) become

$$\begin{aligned}
l\sigma + a' \cos\alpha + b' \cos\beta + c' \cos\gamma + \ldots - h_1 &= v_1,\\
l\sigma + \cos 3\alpha + \cos 3\beta + \cos 3\gamma + \ldots - h_2 &= v_2,\\
l\sigma + \cos 5\alpha + \cos 5\beta + \cos 5\gamma + \ldots - h_3 &= v_3,\\
l\sigma + \cos 7\alpha + \cos 7\beta + \cos 7\gamma + \ldots - h_4 &= v_4,\\
l\sigma + \cos 9\alpha + \cos 9\beta + \cos 9\gamma + \ldots - h_5 &= v_5.
\end{aligned} \tag{5}$$

By the method of least squares there result from equations (4) and (5) the following values, wherein n is successively 1, 3, 5 . . . 9:

$$l\sigma = \tfrac{1}{5}[h],$$

$$a = \frac{[h \cos n\,\alpha]}{5 \sin \alpha}, \qquad b = \frac{[h \cos n\,\beta]}{5 \sin \beta}, \tag{6}$$

$$c = \frac{[h \cos n\,\gamma]}{5 \sin \gamma}, \ldots .$$

The constants a, b, c, . . . in (3) are thus found from the observed quantities h.

If now for brevity we put

$$\theta = 2\pi \frac{x}{10l},$$

equation (3) may be written

$$y = a \sin \theta + b \sin 2\theta + c \sin 3\theta + \ldots ,$$

whence,

$$\frac{dy}{dx} = \frac{\pi}{5l}(a \cos \theta + 2b \cos 2\theta + 3c \cos 3\theta + \ldots).$$

This value substituted in equation (1) gives

$$\begin{aligned}
[s - x]_0^{5l} &= \tfrac{1}{2}\int_0^{5l}\left(\frac{dy}{dx}\right)^2 dx \\
&= \frac{\pi}{10l}\int_0^{\pi}(a \cos \theta + 2b \cos 2\theta + \ldots)^2\, d\theta \qquad (7) \\
&= \frac{\pi^2}{20l}(a^2 + 4b^2 + 9c^2 + \ldots).
\end{aligned}$$

The last member of (7) gives the correction for flexure $[s - x]$ by an easy computation after the values of a, b, c, . . . have been found from equations (6). In all cases to which the formulas have been applied the first three terms, or those in a, b, c, have sufficed.

(3) *Example of computation.*—As an example of the application of the preceding formulas the following case, which is the case of greatest flexure observed with the apparatus may suffice. It occurred on July

Correction for flexure of iced bar B_{17}.

27, 1892. The observed quantities h and the computed quantities $h \cos n\alpha$, $h \cos n\beta$, etc., are—

h.	$h \cos n\alpha$.	$h \cos n\beta$.	$h \cos n\gamma$.	$h \cos n\delta$.
mm.	*mm.*	*mm.*	*mm.*	*mm.*
— 0·324	— 0·308	— 0·262	— 0·191	— 0·100
— ·108	— ·064	+ ·033	+ ·103	+ ·087
— ·018	·000	+ ·018	·000	— ·018
+ ·288	— ·169	— ·089	+ ·274	— ·233
+ ·720	— ·685	+ ·582	— ·423	+ ·222
Sum	— 1·226	+ 0·282	— 0·237	— 0·042

Then the values of a, b, c, d, found from (6), and a^2, $4b^2$, etc., for use in (7) are

	mm.		
a	= — 0·793,	a^2	= 0·629,
b	= + ·096,	$4b^2$	= ·037,
c	= — ·058,	$9c^2$	= ·030,
d	= — ·009,	$16d^2$	= ·001,
		Sum	0·697

Hence, since $l = 990^{mm}$,

$$[s - x] = 0{\cdot}35^{ft}.$$

SUPPLEMENT B.

MATHEMATICAL THEORY OF METALLIC TAPES.

(1) *Object of investigation.*—The use of long metallic tapes or wires in measures of precision gives rise to a number of questions which can only be answered by a competent mathematical theory or by a series of elaborate experiments. Thus, in the use of such tapes it is essential to know how to compute the change in length due to a change in tension, the change in length due to a change in the number of supports on which the tape rests, the change in length due to a change in weight per unit length of the tape, etc. It is the object of this brief investigation to give so much of the theory of such tapes or wires as is needed to answer these practical questions.

(2) *Equations of equilibrium of elastic tapes.*—The theory of the equilibrium of elastic strings, tapes, etc., is given in one form or another in various treatises on mechanics. It generally proceeds, however, on the assumption of "perfect flexibility" of the material of the tape, or on some other doubtful assumption as to the internal stresses of that material. Properly speaking, the inquiry belongs to the theory of elasticity, and its correct basis has been clearly set forth by Lamé in sections 39–41 of his Leçons sur la Théorie Mathématique de L'Élasticité des Corps Solides.*

A tape as here considered is an elastic body of small cross section subject to tension applied at its ends and to any bodily forces, but free from surface tractions. The cross section must be so small that the tension per unit area may be considered constant over the entire area of any section. Let ω be the cross section and ρ the density of the tape at any point of its length defined by rectangular coordinates x, y, z. Let T be the total normal tension on the section ω, and denote the rectangular components of the bodily forces by X, Y, Z, respectively. Then the investigation of Lamé just referred to shows that the equations of equilibrium of the element of mass $\rho\omega ds$ are

$$\begin{aligned} \frac{d\left(T\frac{dx}{ds}\right)}{ds} + \rho\omega X &= 0, \\ \frac{d\left(T\frac{dy}{ds}\right)}{ds} + \rho\omega Y &= 0, \\ \frac{d\left(T\frac{dz}{ds}\right)}{ds} + \rho\omega Z &= 0. \end{aligned} \tag{1}$$

* Second edition, Gauthier-Villars, Paris, 1866.

Theory of Metallic Tapes.

These equations contain the general theory. To adapt them to our purposes we observe, first, that the only bodily force acting on a tape when used as a standard of length is gravity, and, secondly, that the curve assumed by such a tape lies wholly in a vertical plane. Thus, if we take this plane to be that of xy, y being considered positive upwards, we may write $X = 0$, and $Y = -g$ in the first two of equations (1), and they become

$$\left.\begin{aligned} \frac{d\left(T\frac{dy}{ds}\right)}{ds} &= \rho\omega g, \\ \frac{d\left(T\frac{dx}{ds}\right)}{ds} &= 0. \end{aligned}\right\} \tag{2}$$

Further, let ρ_0, ω_0, ds_0 be the density, cross section, and length of the element of the tape considered when it is not under tension. ds is thus the stretched and ds_0 the unstretched length of this element. Also, let μ be the reciprocal of the modulus of elasticity of the tape multiplied by the area of its cross section. Then, since the mass of the element remains constant and Hooke's law applies to its elongation,

$$\left.\begin{aligned} \rho\omega ds &= \rho_0\omega_0 ds_0, \\ ds &= (1 + \mu T)ds_0. \end{aligned}\right\} \tag{3}$$

Since the tapes we consider are of uniform cross section when unstretched, we may write for the weight per unit of unstretched length

$$w = \rho_0\omega_0 g. \tag{4}$$

By means of the relations (3) and (4), equations (2) may be given the following forms:

$$\left.\begin{aligned} (1 + \mu T)\frac{d\left(T\frac{dy}{ds}\right)}{ds} &= w, \\ \frac{d\left(T\frac{dx}{ds}\right)}{ds} &= 0, \end{aligned}\right\} \tag{5}$$

$$\left.\begin{aligned} \frac{d\left(T\frac{dy}{ds}\right)}{ds_0} &= w, \\ \frac{d\left(T\frac{dx}{ds}\right)}{ds_0} &= 0. \end{aligned}\right\} \tag{6}$$

(3) *Integration of equations.*—A first integral of the second of equations (5) is

$$T\frac{dx}{ds} = \text{a constant} = \tau, \text{ say},$$

where τ is the horizontal component of the tension, or the tension at the lowest point of the curve assumed by the tape. From this equation

$$T = \tau\frac{ds}{dx},$$

which, substituted in the first of (5), gives, after some obvious changes,

$$\frac{\tau\frac{d}{dx}\left(\frac{dy}{dx}\right)}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}} + \mu\tau^2\frac{d}{dx}\left(\frac{dy}{dx}\right) = w.$$

The first integral of this equation, if we take the origin at the lowest point of the curve so that $dy/dx = 0$ for $x = 0$, is

$$\tau\log\left(\frac{dy}{dx} + \sqrt{1+\left(\frac{dy}{dx}\right)^2}\right) + \mu\tau^2 dy = wx.$$

In order to integrate again, we may expand dy/dx as a function of the small quantity μ by means of Maclaurin's series. Thus, the last equation gives for $\mu = 0$,

$$\frac{dy}{dx} = \tfrac{1}{2}\left(e^{+wx/\tau} - e^{-wx/\tau}\right),$$

$$\frac{d\left(\frac{dy}{dx}\right)}{d\mu} = \tfrac{1}{4}\tau\left(e^{+2wx/\tau} - e^{-2wx/\tau}\right),$$

$$\cdots,$$

in which e is the Napierian base. On account of the minute value of μ for metallic tapes, the higher differential coefficients of dy/dx with respect to μ are not needed. Hence, to terms of the first order in μ, we have

$$\frac{dy}{dx} = \tfrac{1}{2}\left(e^{+wx/\tau} - e^{-wx/\tau}\right) + \tfrac{1}{4}\tau\left(e^{+2wx/\tau} - e^{-2wx/\tau}\right)\mu,$$

whence, since the origin is on the curve,

$$y = \tfrac{1}{2}\frac{\tau}{w}\left(e^{+\frac{1}{2}wx/\tau} - e^{-\frac{1}{2}wx/\tau}\right)^2 + \tfrac{1}{8}\frac{\tau^2}{w}\left(e^{+wx/\tau} - e^{-wx/\tau}\right)^2\mu, \qquad (7)$$

Theory of Metallic Tapes.

For brevity put

$$a = \frac{w}{\tau}. \tag{8}$$

Then equation (7) gives by expansion

$$\begin{aligned} y = {} & \tfrac{1}{2}\, a x^2 + \tfrac{1}{24}\, a^3 x^4 + \tfrac{1}{720}\, a^5 x^6 + \ldots \\ & + (\tfrac{1}{2}\, x^2 + \tfrac{1}{6}\, a^2 x^4 + \tfrac{1}{120}\, a^4 x^6 + \ldots)\, \mu\, w. \end{aligned} \tag{9}$$

Referring now to equations (6), their first integrals are

$$\begin{aligned} T \frac{dy}{ds} &= w\,(s_0 + \beta), \\ T \frac{dx}{ds} &= \tau, \end{aligned} \tag{10}$$

where β and τ are constants, the latter being as explained above the horizontal component of the tension. The sum of the squares of (10) gives

$$T^2 = \tau^2 + w^2\,(s_0 + \beta)^2. \tag{11}$$

Eliminating T and ds from (10) by means of (3) and (11), there result

$$\begin{aligned} dy &= \mu\, w\,(s_0 + \beta)\, ds_0 + \frac{w\,(s_0 + \beta)\, ds_0}{\sqrt{\tau^2 + w^2\,(s_0 + \beta)^2}}, \\ dx &= \mu\, \tau\, ds_0 + \frac{\tau\, ds_0}{\sqrt{\tau^2 + w^2\,(s_0 + \beta)^2}}. \end{aligned}$$

If the origin be taken on the curve, the integrals of these equations are*

$$\begin{aligned} y &= \tfrac{1}{2}\, a\,(1 + \mu\,\tau)\,\{(s_0 + \beta)^2 - \beta^2\} - \tfrac{1}{8}\, a^3\,\{(s_0 + \beta)^4 - \beta^4\} + \ldots, \\ x &= (1 + \mu\,\tau)\, s_0 - \tfrac{1}{6}\, a^2\,\{(s_0 + \beta)^3 - \beta^3\} + \ldots \end{aligned} \tag{12}$$

The differential equations of equilibrium (5) and (6) are thus completely integrated, and the form of the curve assumed by the tape is assigned by either (7), (9), or (12).

It will be essential in what follows to have an expression for the unstretched length of the tape s_0 in terms of x for the case in which the ends of the tape are in the same horizontal line. In this case, if we take the origin at the lowest point of the curve, or at the middle point of its length, $\beta = 0$, and the second of (12) gives

$$x = (1 + \mu\tau)\, s_0 - \tfrac{1}{6}\, a^2\, s_0^3 + \tfrac{3}{40}\, a^4\, s_0^5 - \cdots;$$

* The finite forms of the integrals of these equations are not so well adapted to our purposes as the series given in the text.

whence, by reversion, we have to a sufficient degree of approximation

$$s_0 = (1 - \mu\tau)\, x + \tfrac{1}{6}\, a^2\, x^3 - \cdots. \qquad (13)$$

(4) *Sag of tape.*—The sag of a tape is the vertical distance of its lowest point below either of its adjacent points of support. It is the value of y in equation (7) or (9) when x is the abscissa of the point of support to which the sag is referred. When the supports of a section of a tape are in the same horizontol line, the sag is found from (9) by substituting for x half the span of the section.

It will be of interest to consider a numerical example with reference to the tapes used with the tape base apparatus described in Chapter IV. Thus we have for steel tape No. 85:

Weight per metre of length $= w = 22{\cdot}32$ grammes,
applied tension (25·5 pounds) $= \tau = 11567$ grammes.

Hence the constant

$$a = (w/\tau) = 193 \times 10^{-5}.$$

We also have for this tape

$$\mu = 16 \times 10^{-9} \text{ for the gramme as unit,}$$
$$= 450 \times 10^{-9} \text{ for the ounce as unit.}$$

With these data the computation of the sag for a span of 40^m ($x = 20^m$) by equation (9) runs as follows:

	m.
$\frac{1}{2}\, a\, x^2 =$	0·386000
$\frac{1}{24}\, a^3\, x^4 =$	·000048
$\frac{1}{2}\, \mu\, w\, x^2 =$	·000072
Sag $= y =$	·386120

It appears therefore that for such a tape and such a tension, the sag for spans less than 40^m in length is given with sufficient accuracy by the simple formula $y = \frac{1}{2}\, a\, x^2$.

(5) *Change in length of tape due to change in tension.*—Suppose a tape supported at a number of equidistant points all in the same horizontal line. Let

n = the number of sections into which the tape is divided by the equidistant supports,
l = the span of any such section,
l_0 = the unstretched length of the tape in the same section,
L = the normal length of the entire tape, or right line distance between its terminal marks when under standard tension $= \Sigma\, l$.

Theory of Metallic Tapes.

Making $\beta = 0$ in the second of equations (12) (since the supports are assumed to lie in a horizontal line), and putting

$$x = \tfrac{1}{2} l, \qquad s_0 = \tfrac{1}{2} l_0,$$

we get

$$l = (1 + \mu \tau) l_0 - \tfrac{1}{24} a^2 l_0^3 + \cdots. \tag{14}$$

Since the sections are of equal length we have

$$\Sigma l = L = (1 + \mu \tau) n l_0 - \tfrac{1}{24} n a^2 l_0^3 + \cdots. \tag{15}$$

Differentiating this with respect to τ, the change in length of the tape, ΔL_1 say, due to a change $\Delta\tau$ in τ is found to be

$$\Delta L_1 = n l_0 \mu \Delta\tau + \tfrac{1}{12} a^2 n l_0^3 \frac{\Delta\tau}{\tau}. \tag{16}$$

Obviously one may use l for l_0 in this formula, a roughly approximate value of l_0 being in general quite sufficient for computing the small quantity ΔL_1.

Example.—For tape No. 85 when supported at equidistant intervals of 10^m we have—

$n = 10$,
$l = 10^m$,
$w = 22{\cdot}32$ grammes per metre,
$\tau = 25{\cdot}5$ pounds $= 11567$ grammes,
$a^2 = (w/\tau)^2 = 372 \times 10^{-8}$,
$\mu = 16 \times 10^{-9}$, for gramme as unit,
$= 450 \times 10^{-9}$ for ounce as unit.*

Hence for $\Delta\tau = 1$ ounce,

	mm.
$nl\mu\Delta\tau = 10 \times 10^m \times 450 \times 10^{-9}$	$= 0{\cdot}0450$
$\tfrac{1}{12} a^2 \mu l^3 \frac{\Delta\tau}{\tau} = \tfrac{1}{12} \times 372 \times 10^{-8} \times 10^7 \times \tfrac{1}{408}$	$= 0{\cdot}0076$
ΔL_1	$= 0{\cdot}0526$

It may be observed that the quantity ΔL_1 can be measured directly by increasing and decreasing the tension τ in the vicinity of its standard value. From a small number of observations made in 1891 by this process the value $\Delta L_1 = 0{\cdot}047^{mm}$ per ounce was found for tape 85 with $\tau = 25{\cdot}5$ pounds. A more elaborate series of observations made in

* Since the cross section of the tape is $6{\cdot}34^{mm} \times 0{\cdot}47^{mm}$, or 0·0298 square centimetres, the value of μ corresponds to a modulus of $2{\cdot}1 \times 10^6$ kilogrammes per square centimetre or 30×10^6 pounds per square inch. This value of μ was determined by measuring the variations of length of a piece of the tape due to variations in tension when the tape lay on the flat Mural Standard of the Coast and Geodetic Survey.

1892 with the same tape and tension gave $\Delta L_1 = 0{\cdot}053^{mm}$ per ounce, which agrees well with the value just computed from the weight per unit of length and the modulus of elasticity of the tape. The value $\Delta L_1 = 0{\cdot}05^{mm}$ may be adopted for tapes 85 and 88 when $\tau = 25{\cdot}5$ pounds.

(6) *Change in length of tape due to change in number of supports.*—Suppose the tape divided by its equidistant supports (all in the same horizontal line) first, into n_1, sections of length l_1, and secondly, into n_2 sections of length l_2. Then, assuming the tension the same in both cases, the difference in distance between the terminal marks of the tape, ΔL_2 say, as shown by equation (14) is

$$\Delta L_2 = \Sigma\,(l_2 - l_1) = \tfrac{1}{24} a^2\,(n_1 l_1^3 - n_2 l_2^3), \qquad (17)$$
$$n_2 > n_1.$$

When a tape is supported throughout its length on a horizontal plane, as on a mural standard, n_2 in (17) becomes infinite and l_2 infinitesimal. If we denote the length of the tape in this case by $L_\infty = \Sigma l_2$ and call its length in the case of n sections $L_n = \Sigma l$, equation (17) gives

$$L_\infty - L_n = \tfrac{1}{24}\, a^2\, nl^3. \qquad (18)$$

To find the shortening of a tape due to the omission of one support, we have only to make $n_2 = 2$, $n_1 = 1$, and $l_1 = 2l_2 = 2l$ say in (17). It gives thus

$$\Delta L_2 = \tfrac{1}{4}\, a^2\, l^3, \qquad (19)$$

l being the length of a section when no supports are omitted.

Similarly the omission of m consecutive supports shortens a tape by

$$\tfrac{1}{24}\, m\,(m+1)\,(m+2)\, a^2\, l^3,$$

where l is the length of a section when no supports are omitted.

Numerical examples.—(1) For the 100^m tape No. 85, $n_1 = 5$ and $l_1 = 20^m$ when the supports are 20^m apart; and $n_2 = 10$ and $l_2 = 10^m$ when the supports are 10^m apart. Hence in this case, since $a^2 = 372 \times 10^{-8}$, when the tension is 25·5 pounds, (17) gives

$$\Delta L_2 = \tfrac{1}{24} \times 372 \times 10^{-8} \left(5\,(20)^3 - 10\,(10)^3\right) = 4{\cdot}65^{mm}.$$

(2) For the same tape the omission of one support, supposing them to be 10^m apart, or l to be 10^m, shortens it by $0{\cdot}93^{mm}$ as shown by (19).

(3) Equation (18) shows that tape No. 85 would be $1{\cdot}55^{mm}$ longer when lying on a horizontal surface than it is when supported at equidistant intervals of 10^m, the tension in both cases being 25·5 pounds.

(7) *Change in length of tape due to change in weight per unit length.*—It is of some interest to have an expression for the change in length of a tape due to a change in its weight. Such changes may occur from wear or from a deposit of dew on the tape. An expression is found

by differentiating (15) with respect to w. Thus, calling the change in question ΔL_3, we find, since $a = w/\tau$,

$$\Delta L_3 = -\tfrac{1}{12}\, n\, a^2\, l^3\, \frac{\Delta w}{w}. \tag{20}$$

This formula shows, for example, that when the 100^m tape No. 85 is supported at equidistant intervals of 10^m and is under a tension of 25·5 pounds, or when

$$\begin{aligned} l &= 10^m, \\ n &= 10, \\ a^2 &= 372 \times 10^{-8}, \\ \Delta L_3 &= -3{\cdot}1^{mm} \times \frac{\Delta w}{w}. \end{aligned}$$

Thus, in order to produce a change of the millionth part in this tape's length, or in order to make $\Delta L_3 = 0{\cdot}1^{mm}$, we must have $\Delta w = w/31$. Obviously, no such change as this is likely to occur in the length of a tape.

(8) *Change in length of tape due to slope of its supports.*—In the use of long tapes for measuring lines of precision the supports for any tape length are placed in the same straight line. It is plain that if this line is inclined, the distance between the terminal marks of the tape will be greater than when the line is horizontal. It is plain also that the elongation of a tape due to ordinary slopes can not be very great, but their use in refined work requires that we should know the nature and extent of such elongation.

The question may be investigated in the following manner: Equations (12) give the rectangular coordinates of any section of the tape. The origin for these coordinates is any point on the curve, and is defined by the constant β. For the present purpose let us place the origin at the left-hand support of the section considered, and let x_1 y_1 be the coordinates of the other end or support of the section. Then the span of the section, or its length as used in measurement, is $\sqrt{x_1^2 + y_1^2}$. But the length of the same section when the supports are in the same horizontal line is given by (14) as

$$l = (1 + \mu\, \tau)\, l_0 - \tfrac{1}{24}\, a^3\, l_0^3 + \ldots,$$

where l_0 is the unstretched length of the tape in the section. The elongation we desire is then

$$\sqrt{x_1^2 + y_1^2} - l$$

per section of the tape. To evaluate this quantity we have to find β for the assumed position of the origin, to substitute its value in the

equations (12), and thus get x_1 and y_1 in terms of s_o, or rather l_o as we shall here denote the unstretched length of tape in the section.

Since in the first of (12) $y = y_1$ for $s_o = l_o$, we find by Maclaurin's series

$$\beta = -\frac{1}{2} l_o + \frac{1}{a}\left(\frac{y_1}{l_o}\right)(1 + \mu\tau)^{-1} + \left(\frac{1}{2a} - \frac{1}{16} a^2 l_o^2\right)\left(\frac{y_1}{l_o}\right)^3 (1 + \mu\tau)^{-4} + \ldots,$$

or nearly enough for our purpose

$$\beta = -\frac{1}{2} l_o + \frac{1}{a}\left(\frac{y_1}{l_o}\right) + \frac{1}{2a}\left(\frac{y_1}{l_o}\right)^3.$$

Again, since y_1 is always small with respect to x_1, we may use the approximate expression

$$\sqrt{x y_1^2 + y_1^2} - l = x_1 + \frac{y_1^2}{2x_1} - l.$$

Hence, substituting the values of β, x_1, and y_1 as just explained, there results for the elongation per section of the tape to terms of the fourth order inclusive in the slope $\left(\frac{y_1}{l_o}\right)$,

$$\frac{1}{12} a^2 l_o^3 \left(\frac{y_1}{l_o}\right)^2 + \frac{1}{8} l_o \left(\frac{y_1}{l_o}\right).$$

Let n denote the number of equal sections of the tape, L the total length of the tape, h the difference in altitude of its ends, and ΔL_4 the total elongation due to its slope. Then, since

$$y_1 = h/n \text{ and } l_o = L/n,$$

the above expression gives

$$\Delta L_4 = \frac{1}{12n^2} a^2 L^3 \left(\frac{h}{L}\right)^2 + \frac{1}{8} L \left(\frac{h}{L}\right)^4. \tag{21}$$

For a tape 100^m long the value of h will rarely if ever equal 5^m. For such a value and for the tapes Nos. 85 and 88 when supported at equidistant intervals of 10^m we have, if $a^2 = 372 \times 10^{-8}$,

$$\frac{1}{12n^2} a^2 L^3 \left(\frac{h}{L}\right)^2 = \frac{1}{1200} \times \frac{372}{10^8} \times (100)^3 \left(\frac{5}{100}\right)^2 = 0{\cdot}008^{mm},$$

$$\frac{1}{8} L \left(\frac{h}{L}\right)^4 = \frac{1}{8} \times 100 \times \left(\frac{5}{100}\right)^4 = 0{\cdot}078.$$

Thus in this case $\Delta L_4 = 0{\cdot}086^{mm}$. Hence for such a tape we may

regard this correction as insensible for slopes less than 5 per cent or values of h less than 5^m.*

(9) *Change in length of tape due to inequality in spacing of its supports.*—In setting the support nails for a tape length it is impossible to make the intervals between them exactly equal. Practically, therefore, it is important to know how great the inequality may be without introducing appreciable error in precise measurements. This question is easily answered by means of equation (14). Thus, let $l_1, l_2, \ldots l_n$ be the lengths or spans of the n sections of the tape supposed unequal; and let $l_o + \Delta l_1, l_o + \Delta l_2 \ldots l_o + \Delta l_n$ be the corresponding unstretched lengths of the tape for the same sections. Then, according to (14), observing that $\Sigma \Delta l = 0$,

$$\Sigma l_n = (1 + \mu\tau)\, n l_o - \frac{1}{24}\, n a^2 l_o^3 - \frac{1}{8}\, a^2 l_o \Sigma(\Delta l)^2.$$

If the sections were all of equal length l we should have, to the same order of approximation, for the whole length of the tape

$$nl = (1 + \mu\tau)\, n l_o - \frac{1}{24}\, n a^2 l_o^3.$$

Calling the difference of these two expressions ΔL_5, we have

$$\Delta L_5 = \frac{1}{8}\, a^2 l_o \Sigma(\Delta l)^2. \qquad (22)$$

This gives the shortening of the tape due to the departures from equality of the several sections. To get an idea of its extent, suppose $\Delta l_1 = -\Delta l_2 = \Delta l_3 = -\Delta l_4 \ldots = 0{\cdot}1^m$ in the case of the 100^m tapes when supported at equidistant intervals of 10^m and when $a^2 = 372 \times 10^{-8}$. With these data (22) gives

$$\Delta L_5 = \frac{1}{8} \times 372 \times 10^{-8} \times 10 \times 10 \times 10^{-2} = 0{\cdot}46^{\mu}.$$

It is seen, therefore, that in spacing the supports for such a tape, equality to the nearest decimeter, which is easily attained, is amply sufficient.

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* This investigation ignores the effect of the change in $a^2 = (w/\tau)^2$ due to the slope of the tape. This change is $\Delta a^2 = -2a^2\,(\Delta\tau/\tau) = a^2\,(h/L)^2$, which amounts to only 1/400th part of a^2 in the case cited.

III. THE NEW SECONDARY BASE-APPARATUS OF THE COAST AND GEODETIC SURVEY AS USED IN THE MEASUREMENT OF THE HOLTON BASE, INDIANA.

A report by O. H. TITTMANN, Assistant.

Submitted for publication November 30, 1891, and corrected for adopted value of five-metre standard in February, 1894.

The duty of measuring the Holton Base with the new secondary apparatus, (five-metre contact slide rods 13 and 14) was assigned to me by a letter of instructions from the Superintendent of March 28, 1891.

As soon as circumstances permitted I reported to Assistant A. T. Mosman, in camp at Holton. He had made all the necessary preparation of selecting, clearing, and marking the line and had organized the force of men. As soon therefore as the instruments arrived the measuring operations were begun by me on July 28. The men had not been previously drilled in their duties, but gained the requisite skill and rapidity of manipulation as the measurement progressed.

The base, including two extra measurements of the standard kilometre, was measured twice between July 28 and August 13.

As soon as the measurement had been finished, the lengths of the rods were determined on the Standard 100-metre Comparator, the length of which had been found by Prof. R. S. Woodward, Assistant Coast and Geodetic Survey, and the relation of the thermometer indications to the actual temperatures of the bars was investigated.

This report will therefore first treat of the results of this investigation and its bearing on the length of the base, after which the details of the measurements of the base line and of the apparatus, will be described.

COEFFICIENT OF EXPANSION OF THE MEASURING RODS.

As will be seen by reference to the latter part of this report, this was determined at the office in May, 1891, between 0° and 37° centigrade.

The temperature range was ample for a mean value, but the intermediate points were not sufficiently numerous to deduce the value of the expansion depending on the second power of the temperature.

The results are:

	μ.
For rod 14 the expansion =	58·509 per degree C.
	± ·131
For rod 13 the expansion =	58·861 per degree C.
	± ·127

giving as the mean expansion for the two rods

$$\begin{array}{l} \mu. \\ 58{\cdot}7 \text{ per degree C.} \\ \pm\ {\cdot}09 \end{array}$$

LENGTH OF THE RODS.

The lengths of the rods depend upon the five-metre steel bar No. 17 whose length was determined at this office by Assistant Woodward, by comparing it with National Prototype Metre No. 21. In these comparisons the Prototype as well as No. 17 was kept in melting ice.

The resulting length of No. 17 was found to be at

$$\begin{array}{rl} & \mu. \\ 0^\circ \text{ C.} = 5^m & - 18{\cdot}0 \\ & \pm\ 0{\cdot}8 \end{array}$$

No. 17 is a line measure, while rods 13 and 14 are end measures. Before taking the latter into the field they were compared on three days with No. 17. The latter was kept in melting ice while Nos. 13 and 14 were at, or very near, the temperature of the comparing vault, about 22°·2 C.

The comparisons were made under microscopes, the pointings on the rods being made on the agate knife edge on the one hand, and on the other by pointing on the space between the direct and reflected images of a spider thread stretched across the plane end surface of the rod.

These comparisons gave

$$\begin{array}{lll} & \mu \quad \mu. & \\ \text{Rod 13} = 5^m + 1278 \pm 4{\cdot}0 & & \\ & & \text{at } 22^\circ{\cdot}2 \text{ C.} \\ \text{Rod 14} = 5^m + 1297 \pm 3{\cdot}0 & & \end{array}$$

It was subsequently found in the field that the knife edges and abutting surfaces were not in perfect contact while the bars were used for measurement, owing to the circumstance that the contact surfaces are not at right angles to the longitudinal axes of the bars. The effect of this mechanical defect was measured at Holton with considerable precision by means of a microscope mounted over the ends of the rods when in contact.

The amount of this correction 30$^\mu$ when added to the above values gives for the combined length of the bars at

$$\begin{array}{rl} & \mu. \\ 22^\circ{\cdot}2 \text{ C } 10^m & + 2605 \\ & \pm\ 5 \end{array}$$

A few days after the measurement of the base had been finished, the standard 100-metre comparator was measured with this apparatus in order to deduce the combined length of the rods.

The measurements used for determining the lengths are given in full:

Date.	Hour.	Corr. temp.	At 22°·2 resulting length of 20 bars.
		°	*m.* *mm.*
Aug. 17	11:00 a. m.	24·47	100 + 26·18
17	4:36 p. m.	27·76	25·97
18	8:17 a. m.	23·32	25·97
18	9:24 a. m.	23·74	25·95
18	5:00 p. m.	28·52	26·23
		Mean at 22°·2 =	100 + 26·06

hence

$$\text{at } 22°{\cdot}2 \text{ rod } 13 + 14 = 10^{m} + \underset{\pm 4}{2606}\ \mu.$$

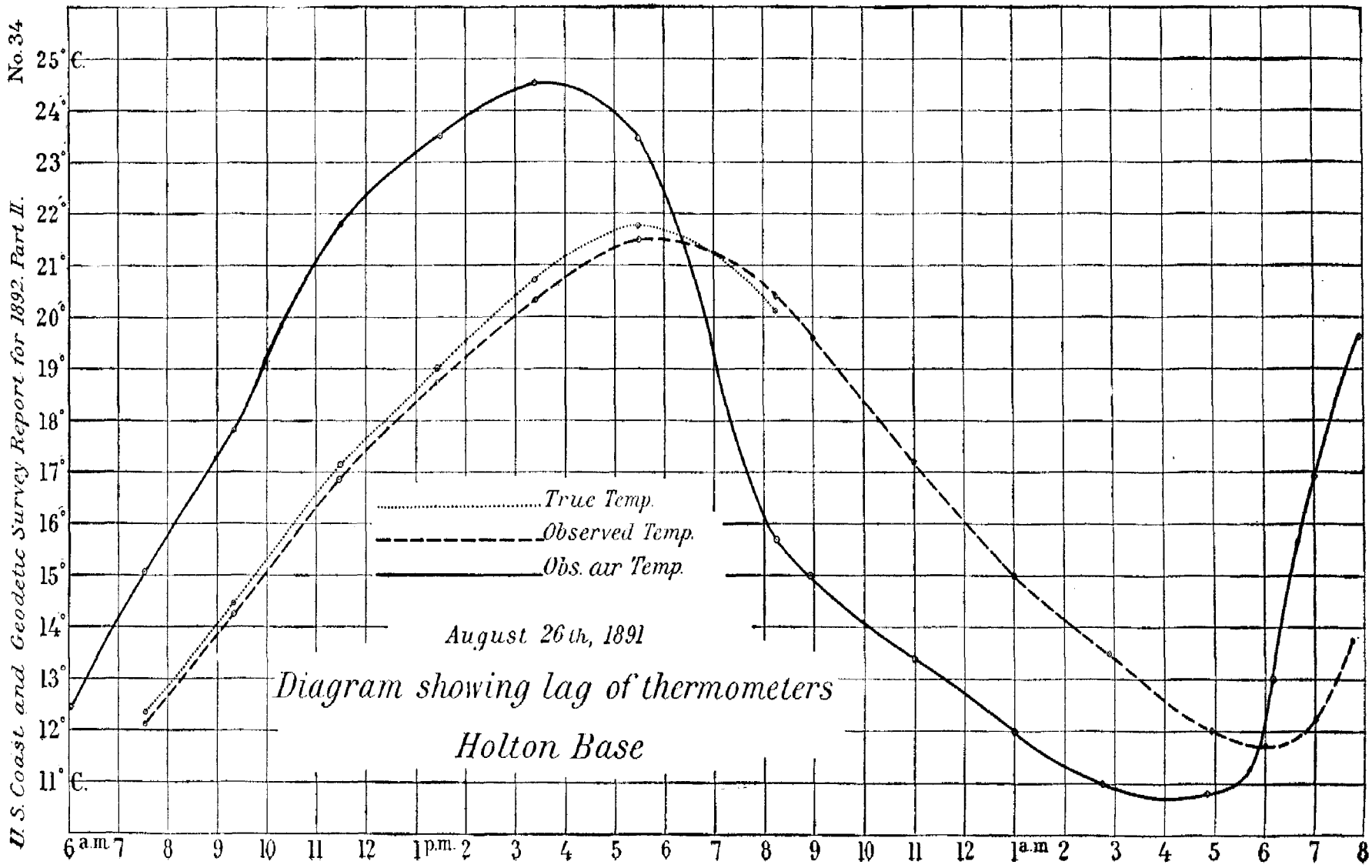
In view of this agreement with the office value, and owing to the relatively constant temperature night and day owing to cloudiness and rain, the results obtained on August 17 and 18 were considered sufficiently accurate, and further observations were discontinued until a change in the weather occurred on August 22; thereafter the daily temperature range of the air was much increased. Observations made on Aug. 22 and the two following days disclosed a difference between the observed and true temperatures of the bars, and in order to determine its amount, a systematic series of comparisons at intervals of about two hours was carried out on August 26.

Abstract of results.

Date.	Hour.	Obs. temp.	Length of 20 bars reduced to 22·2°.	Computed lag of thermometers.
		°	*m.* *mm.*	°
Aug. 26	7:33 a. m.	12·16	100 + 26·25	+0·16
26	9:20	14·19	26·29	+ ·20
26	11:29	16·85	26·45	+ ·33
26	1:28 p. m.	18·73	26·45	+ ·33
26	3:26	20·32	26·57	+ ·43
26	5:29	21·48	26·38	+ ·27
26	8:17	*20·43	25·72	— ·29

*Falling temperature.

The lag, or the correction to the observed temperature, is computed from the known length of the combined bars as determined August 17


25° C.
24°
23°
22°
21°
20°
19°
18°
17°
16°
15°
14°
13°
12°
11° C.
6 a.m. 7 8 9 10 11 12 1 p.m. 2 3 4 5 6 7 8 9 10 11 12 1 a.m. 2 3 4 5 6 7 8
True Temp.
Observed Temp.
Obs. air Temp.
August 26th, 1891
Diagram showing lag of thermometers
Holton Base

and 18, the known coefficient of expansion and the lengths as determined August 26.

It will be seen that during August 26 the thermometers indicated too low a temperature throughout the day while the temperature was rising, amounting on the average to 0°·29, say 0°·30 centigrade, and that as soon as the temperature fell the conditions were reversed, the rods following the temperature of the air faster than the thermometers which again lagged 0°·3 behind.

The result graphically delineated on the accompanying diagram (illustration No. 34) shows that the maximum lag corresponded to (it probably followed) the hour of maximum heat of the day and to the greatest difference between the air and bar temperatures.

The curves also exhibit the range of temperature to which the bars were subjected during the twenty-four hours following the first observation.

Inasmuch as the measurements on which these results depend were made in the shade, that is, in the comparing shed, the question naturally presented itself what the relation between the indicated and true temperatures of the rods might be with the apparatus exposed to the sun.

It was therefore decided to measure a section of the standard kilometre in the bright sunshine under circumstances which should be very similar to those under which the whole base was measured.

An additional stone was therefore planted 100 metres south of the north end of the standard kilometre and the distance was measured by Prof. Woodward during his measures of the kilometre.

On September 21 this distance was accordingly measured with the secondary apparatus 5 times, and on the following day 8 times, using the trestles and measuring exactly as in the base measurement, with the exception that the beginning and end of each 100 metre measure was referred to the section stones by means of microscopes and the cut-off apparatus.

The results of these measurements may be discussed on two suppositions:

1. By giving equal weight to the four measures of the 100^m distance, made by Prof. Woodward, on this supposition, the distance $= 100^m - 6{\cdot}04^{mm}$.

2. By giving double weight to the 3d and 4th measures and this, because the 1st and 2d measures were made on the same day and not long after the 100 metre stone had been set, while the 3d and 4th measures were made on different days and after the ground had dried.

According to this supposition the distance is $100^m - 5{\cdot}99^{mm}$.

Computing on supposition 1 the results of these measurements are as below given in detail and show an astonishingly high degree of precision. They disclose the fact that the lag was rendered practically insensible by the bright sunshine, amounting to 0°·07.

The first table shows the length of the bars deduced from the observed temperatures and the distance as measured with the iced bar by Prof. Woodward, namely, $100^{m}—6{\cdot}04^{mm}$.

The second table shows the distance as computed from the observed temperatures and the adopted length of the bars derived August 17 and 18.

As the object of this measurement was to show the thermal behavior of the apparatus when subjected to conditions similar to those which prevailed during the measurement of the base, and since the whole base was measured during rising temperatures, and nearly all of it during bright sunshine, the single measure made with falling temperatures, September 21, 5 p. m., may be thrown out of the second table. The effect of throwing it out is to increase slightly the indicated lag of the thermometers, but to decrease the probable error of measurement, which then becomes only 1-1470000 for a single measure of the 100-metre distance.

Computing on supposition 2, based on the following iced bar measures, namely—

		m.	*Weight.*
Sept.	15	100 — 6·24	1
	15	6·17	1
	26	5·83	2
	30	5·96	2
Weighted mean		100 — 5·99	

it appears that the error of the result of the measurement with the secondary apparatus amounts to $-0{\cdot}14^{mm}$, in 100^{m}, or $\frac{1}{714286}$ of the distance.

TABLE I.

Date.	Hour.	Obs. temp.	Length of 20 bars at 22°·2.	True minus indicated temp.
	h. *m.*	°	*m.*	°
Sept. 21	9 57 a.m.	23·31	100 + 26·26	+ 0·17
	11 28	26·91	26·21	+ ·13
	2 40 p.m.	32·89	26·19	+ ·11
	3 53	34·29	26·10	+ ·03
	[5 00	34·26	*25·58	[— ·41]
Sept. 22	8 05 a.m.	20·41	26·21	+ ·13
	9 07	22·77	26·16	+ ·09
	10 06	25·04	26·30	+ ·20
	11 00	26·75	26·02	— ·03
	1 32 p.m.	30·83	26·23	+ ·14
	2 33	32·13	26·11	+ ·02
	3 29	32·89	25·97	— ·08
	4 24	33·15	25·99	— ·06
	Sept. 21, mean		26·19	+ ·11
	Sept. 22, mean		26·12	+ ·05
	Weighted mean		100 + 26·14	+ ·07

* Falling temperature.

Measurement of Holton Base with the New Secondary Base Apparatus.

TABLE II.

The distance between stones as measured with the secondary apparatus, rods 13+14 = 10^{m} + 2606^{m} at 22·2° C. corresponding to the dates and times given in Table I, becomes—

m.	*mm.*	*v.*	v^2.
100 —	6·24	0·11	·0121
	6·19	·06	·0036
	6·17	·04	·0016
	6·08	·05	·0025
	[5·56]		
	6·19	·06	·0036
	6·14	·01	·0001
	6·28	·15	·0225
	6·08	·05	·0025
	6·21	·08	·0064
	6·09	·04	·0016
	5·95	·18	·0324
	5·97	·16	·0256
m.			
Mean 100 —	6·13		·1145
	± ·018		

and for a single measure the *p. e.* = 0·068^{mm}, or 1–1470000 of the distance.

As before stated, the distance found by Prof. Woodward is 100^{m} — 6·04^{mm} on supposition 1 and 100^{m} — 5·99^{mm} on supposition 2.

MEASUREMENT OF THE STANDARD KILOMETRE.

The standard kilometre, which is a part of the base line, was measured four times during the progress of the base measurement proper.

During these four measures the ground in the woods, that is over about 600 metres of the distance, was very wet and springy and unfavorable for the stability of the trestles, which were set directly into the soft ground.

After the ground had dried it was measured twice on September 24 and 25. These six measures were made under about the same temperature conditions as those prevailing during the base measures except that the line over one-half of the kilometre passes through woods. This becomes important in view of the results of the measures made in the sunshine, since the shade would tend to create lag in the thermometers, for it has been shown that the average correction to the thermometers for lag, if the bars are shaded, amounts to about 0°·3 when the temperature of the air is rapidly rising.

It is therefore to be expected that the measures of the kilometre will not show as high a degree of accuracy as those made in the sunshine, owing chiefly to the uncorrected thermal effect of the shade.

The results of the 6 measures of the standard kilometre are:

	Temp.	Resulting length.	
	°	*m.*	*mm.*
Aug.	23·86	1000	0·6
	26·75		— 5·2
	24·52		— 9·2
	25·37		— 5·5
Sept.	24·01		— 3·9
	26·18		— 5·2
	Mean,	1000	— 4·9
			± ·76

and the probable error of a single measure = ± 1·86 or 1–538000.

The measures made on the 100-metre comparator and on the 100 metres measured on the standard kilometre show that the lag or the correction to the thermometers becomes minus with falling temperatures, and that its amount approximates closely to 0°·3.

Five determinations made in the early part of the evening give a mean of 0°·44, but the average during a period of say 8 hours at night would probably be less, to judge from the curves on the morning of August 26, as shown on the diagram.

To substantiate the conclusion that with falling temperatures the rods would be shorter than the length indicated by the thermometers, it became desirable to measure the standard kilometre under such conditions. As, however, during the time alloted to the work, falling temperatures, during say 8 consecutive hours, could not be had without measuring at night, and as it was not deemed desirable to complicate the question of precision by night measures, an artifice was resorted to.

The bars were exposed during the morning to bright sunshine until about noon. They were then carried into the woods to the middle of the kilometre and allowed to remain there in the shade for two hours, after which the measurement of the half kilometre was commenced. By the time the measurement extended outside of the woods the temperature was falling rapidly and the sun's rays were feeble, it being near sundown.

In this way one-half of the kilometre was measured in three hours on October 5, and the other half on October 6, giving as the result: Standard kilometre = $1000^{m} + 2{\cdot}0^{mm}$. Since the true length found by Prof. Woodward is $1{,}000^{m} - 3{\cdot}4^{mm}$ the measurement is in error $5{\cdot}4^{mm}$, and if we ascribe the total error to the thermometer indications the lag amounted to — 0°·47.

LENGTH OF THE BASE.

The computed length of the base without applying any correction for lag of the thermometers is $5500{\cdot}808^{m} \pm 3{\cdot}2^{mm}$ where the *p. e.* includes that of the unit employed.

Measurement of Holton Base with the New Secondary Base Apparatus.

The whole base was measured during rising temperatures and much the greater part during bright sunshine.

All the experiments show that under such conditions the bars are never shorter than the indicated temperatures imply, and that therefore the result, if it is in error on account of thermometer lag, is too small.

The experiments on the 100m measured in the bright sunshine, September 21 and 22, show that the lag on supposition 1, explained in pages 7 and 8, amounts to about 1 part in one million, and on supposition 2 to $\frac{1}{741286}$, and it is therefore a fair inference that its effect on the whole base is of that order of magnitude, and that it should be applied to the uncorrected length of the base. Using the value 0·14mm per 100m the measured length should be increased by $55 \times 0{\cdot}14 = 7{\cdot}7^{mm}$.

In order not to claim too high a precision for the length above given it seems safe to assert that its absolute error does not exceed its 1-500000 part.

An abstract of the results of the measurements of the base is appended:

Section.	No. of bars.	Corr. temp.	Resulting length.	$1-2d$.	$e=1/3\,d$.	$e^2=2$.
		°	*m.* *mm.*			
S B to 240	240	26·27	1 200+ 22·8			
		23·83	21·2	1·6	·53	·281
240 420	180	26·80	900— 36·8			
		26·88	32·9	3·9	1·30	1·690
420 600	180	25·62	900+ 62.6			
		27·49	62·3	0·3	+ ·10	·010
600 780	180	22·18	900+ 17·4			
		27·99	20·1	2·7	·90	·810
780 980	200	23·86	1 000— 0·6			
		26·75	— 5·2			
		24·52	— 9·2			
		25·37	— 5·5			
		24·01	— 3·9			
		26·18	— 5·9		·76	·578
980 N B	120	21·57	600+743·7			
		31·23	744·6	0·9	·30	·090

First measure, 5 500+805
Second measure, 810

Length of base, 5 500+808 $+7{\cdot}7^{mm} = 5{,}500.816^{m}$.
$\pm$ 1·9

The probable error of measurement without regard to constant errors is $\pm$ 1·9mm.

The probable error in the length of a 100-metre unit from comparisons with the 100-metre standard is ± 0·04.

The probable error of the 5-metre iced bar at 0° C = ± 0·0008.

The probable error of measuring the 100-metre standard (August measurements) from Prof. R. S. Woodward's determinations = ± 0·018, hence the *p. e.* of the base arising from these values is

$$55\sqrt{(0{\cdot}018)^2+(0{\cdot}016)^2+(0{\cdot}04)^2}=2{\cdot}57^{mm}$$

and the *p. e.* of the base is

$$\sqrt{(2{\cdot}6)^2+(1{\cdot}9)^2}=\pm 3{\cdot}2^{mm}.$$

The reduction to sea level and the *p. e.* of the leveling (mean altitude of base line) remain to be incorporated.

GENERAL CONCLUSIONS IN REGARD TO THE USE OF THIS APPARATUS.

1. The average speed of measurements, including the time requisite for training the operators, may be taken at 1 kilometre per day of eight hours, on level ground, or on a uniform grade. With trained men the speed can be increased to 1·5 kilometres per day without detriment to accuracy.

2. If the temperature of the bars were known, the accuracy of measurement would leave nothing to be desired. The probable error of a single measurement would then be about one part in a million.

3. As usual the principal source of error is in the uncertainty between the indicated and true temperature of the bars.

This relation depends on the range of the diurnal temperature to which the bars have been exposed, and its value must be subject to some uncertainty. It appears, however, that it is possible to assign the limits of error due to lag of the thermometers.

The absolute lag of the thermometers can safely be taken to lie between 0 and ± 0°·5 C. on the average in a day's measurement.

Hence, if an error of one part in 340 000 shall not be exceeded, a single measurement with this apparatus will secure that degree of accuracy if the following rules be observed:

1. If the temperature is rising, apply a correction for lag of + 0°·25 to the mean temperature of the thermometers for the day, and a correction of — 0°·25 if it is falling.

2. If one-half of the day's measures are made during rising and the other half during falling temperatures, the mean may be taken without correction.

3. If the measurement is made during hot weather and in the bright sunshine with rising temperatures, no correction need be made.

It may also be pointed out that where it is possible the bars, if day measurement only is intended, should be protected as much as possible against change of temperature at night.

MEASUREMENT OF HOLTON BASE, RIPLEY COUNTY, IND.

For description of base monuments and locality see Assistant A. T. Mosman's report, pp. 330, 331.

The 5-metre contact slide rods Nos. 13 and 14 were used in this measurement.

This apparatus is of the usual pattern. It was made at the Coast and Geodetic Survey Office and was completed in July, 1891.

The measuring rods are of cylindrical steel, 5 metres long and 9^{mm} in diameter, with the usual contact slides. The slide end of each rod carries a white agate knife edge 3^{mm} wide, the other a circular plane surface about 3^{mm} in diameter. The index and slide piece of each rod have each three lines ruled on them. When the middle lines on the slide and index are in coincidence, the distance between the agate plane at one end and the knife edge at the other is the length of the rod.

Each steel rod is encased in a built up wooden bar and rests directly in a longitudinal groove in its middle. The bars are $7{\cdot}5^{cm}$ wide, 14^{cm} high, and 4·9 metres long, so that a length of about 10^{cm} of each rod is exposed to the air.

Each rod carries two thermometers. They are inserted in opposite sides of the bar and are in contact with the rod and not with the wood. Each one is 1 metre from the end of the bar.

The wooden bar in turn is surrounded with a white canvas cover lined with half an inch of cotton.

THE TRESTLES.

They are of the usual wedge and tripod pattern and are the same as those used in the Yolo base. Throughout all the measurements, except those on the comparator, the trestles were used without foot plates, the legs being merely firmly stuck into the ground.

THE SECTORS.

The sectors are segments of arcs of circles. Each carries two (2) verniers 180° apart and is so graduated that the position in which the bar is horizontal reads nearly 20° on the front vernier. The verniers read to 10″. Before beginning work the contact ends of each bar were put in a horizontal plane by means of a leveling instrument, theodolite, or engineer's transit.

In general the sector errors were determined three times a day.

Each sector was protected against the sun by a hood made of canvas stretched on a wire frame attached to the bar.

During all the measures the two verniers of each sector were read by different persons.

THERMOMETERS.

The thermometers attached to the rods are centigrade divided to half degrees, and were read by means of a magnifying hand lens to tenths.

They were made by H. J. Green, of New York, and were compared at the office in June, 1891, with Tonnelot thermometers 4655 and 4656.

Bar 13 carries thermometers 1 and 2.
14 carries thermometers 3 and 4.

Calling that end of the bar on which the aligning telescope is mounted the front end:

Thermometer 1 is on same side as the sector, rear end.
2 is on opposite side of the sector, front end.
3 is on same side as the sector, rear end.
4 is on opposite side of the sector, front end.

The maker's numbers corresponding to those given are—

No. 1, 5609.
2, 5604.
3, 5606.
4, 5616.

On August 28 Green 5613 was substituted for 5616 (No. 4), the latter having been broken on that date after its zero point had been determined.

The corrections of these thermometers to reduce them to the hydrogen scale were determined in June, 1891, to be as follows:

At	No. 1, 5609.	No. 2, 5604.	No. 3, 5606.	No. 4, 5616.	Mean.	5613
°	°	°	°	°	°	°
0	—0·10	+0·05	—0·15	—0·10	—0·08	—0·08
3	— ·16	— ·24	— ·25	— ·23	— ·22	— ·25
6	— ·11	— ·19	— ·31	— ·19	— ·20	— ·19
11	— ·13	— ·18	— ·33	— ·21	— ·21	— ·18
16	— ·17	— ·17	— ·34	— ·17	— ·21	— ·19
21	— ·26	— ·21	— ·23	— ·21	— ·23	— ·21
25	— ·16	— ·16	— ·36	— ·26	— ·24	— ·26
32	— ·21	— ·21	— ·36	— ·26	— ·26	— ·28
34	— ·22	— ·22	— ·32	— ·22	— ·25	— ·32
37	— ·26	— ·28	— ·28	— ·28	— ·28	— ·28

On August 28 the zero corrections to these thermometers were redetermined. See Vol. II of the record, with the following results:

At	No. 1.	No. 2.	No. 3.	No. 4.	Mean.
°	°	°	°	°	°
0	—0·22	—0·16	—0·27	—0·22	—0·22

Measurement of Holton Base with the New Secondary Base Apparatus.

ALIGNMENT.

The bars were aligned during the measurement by bringing the rear end of the rod over the starting point or into contact with the forward agate of the rear bar and keeping it there while the forward end of the bar was aligned by pointing a small telescope on a signal usually mounted on the next section stone ahead.

A small aligning telescope is mounted on each bar 25 centimetres from the front end. The base of the uprights carrying the telescope has a sliding motion on an arc of a circle concentric with the measuring rod. It carries a level, by means of which it can be clamped in a horizontal position in a vertical plane passing longitudinally through the rod.

In order that the telescope may transit in this plane the adjusting screws of the base must be properly set.

The adjustment was not made with satisfactory accuracy during the first measurement of the distance between south base and the fourth kilometre stone. On reaching this stone a second and more perfect adjustment was made.

Subsequently the deviations from a straight line during this fractional part of the first measure were traced on the ground, and it was found that the error due to the deviation of the measured line from the base line amounted to not more than 1 part in 15 000 000, which can therefore be neglected.

Experiments were made to determine the probable error of alignment of a single bar with the telescope adjusted as in the remainder of the measures and it was found to be 38″.

GROUND TRAVERSED.

The greater part of the measurement was over grassy soil. Several plowed fields were crossed.

The surface of the ground near south base is broken, and between the 3 000 and 3 900 metre stone it is also very irregular. The first four measures of the Standard Kilometre were made when the soil was wet and springy, but during the measures made on September 24 and September 25 it was dry and hard.

A distance of 200 metres at each end of the Standard Kilometre is in the open fields; the remainder is in the woods.

DUTIES PERFORMED BY MEMBERS OF THE PARTY.

Prior to September 21, during the measurements, the bars were aligned by Prof. Gore, who also read the forward thermometer; the rear thermometer was read by Mr. Pennington; one vernier of each sector was read by Mr. Gjertsen, the other by Mr. Hayford, who also

kept the record and called attention to any apparent or real errors in the readings. After September 20 the alignments were made by Mr. Hayford.

Throughout the work the contacts were made by myself, using a small lens magnifying 3 diameters.

SECTION MARKS.

The base and its sections, excepting the Standard Kilometre, were marked by lines traced at right angles to the direction of the base on copper bolts set into stones. The ends and intermediate sections of the Standard Kilometre were marked by spherical-headed bolts, intended to be used in connection with a so-called cut-off apparatus.

This apparatus consists of an upright cylinder having a conical recess at its foot, which fits over the spherical bolthead. The cylinder can be revolved about its upright axis, and any deviation from verticality is measured by means of an attached spirit level. On top, and at right angles to its vertical axis, the cylinder carries a scale divided into millimetres, to which an object over it can be referred, and which gives its relation to the geometrical center of the bolthead.

In starting, the slide was usually clamped in coincidence with the index, and the knife edge was brought over the starting point by means of a theodolite or transit sector mounted at right angles to the base line.

When stopping for lunch, a line traced on a copper tack in a stub in the ground served as a reference mark.

The record of the measurements of the Holton base line with 5-metre contact slide rods Nos. 13 and 14 is contained in 3 volumes. Volume I, pages 1 to 230, and Volume II, pages 1 to 68, contain the record of the base measurement proper, including two extra measurements of the Standard Kilometre.

Volume II, pages 71–158, contains determinations of the length of the rods in terms of the 100-metre comparator. For description and details of this comparator, see Assistant R. S. Woodward's report.

LENGTH OF THE RODS.

The length of the rods was determined immediately after the base measurement by measuring the distance between the ends of the standard 100-metre comparator. The legs were taken off the trestles and the tripod heads carrying the wedges were screwed on substantial boards, which fitted across the track prepared for the iced bar carriage.

In starting, the slide of the bar was clamped to coincidence. The knife edge was brought under a microscope mounted over one end of the 100-metre comparator, and a pointing was made with the microscope on the knife edge. The axis of the microscope was referred to the end mark by means of the cut-off apparatus.

Measurement of Holton Base with the New Secondary Base Apparatus.

The measurement of the distance was then carried on as usual, except that the tripod heads rested in the track, as above described. On reaching the other end of the comparator, where another microscope and cut-off apparatus were mounted, a spider thread was stretched over the plane surface of the agate of the rod, and the microscope pointing was made on the space, which was never more than about 20 microns in width, between the direct and reflected image of the thread.

For the deductions drawn from these and other determinations of the length, see my report to the Superintendent.

Pages 159 to 162 of Volume II contain the determination of the correction to the zero points of the thermometers attached to the base rods.

Pages 188 and 189 furnish a determination of the error of contact which results from the fact that the agate planes are not at right angles to the axis of the rods.

This error of contact affects the length of the rods when separately determined under microscopes, but does not affect their combined length deduced on the 100-metre comparator.

LAG OF THERMOMETERS.

Volume III, pages 4 to 31, contain a number of measures of the first hundred metres of the Standard Kilometre made in the bright sunshine in order to determine the effect of the sunlight on the lag of the thermometers attached to the rods.

In these measurements the rods were used on the trestles in the usual manner, but at the beginning and end of the 100-metre distance a microscope was mounted and the cut-off apparatus was used in the same manner as on the standard 100-metre comparator, as previously described.

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APPENDIX No. 9—1892.

MEASURE OF THE IRREGULARITY IN ONE TURN OF THE MICROMETER SCREW, AND THE RELATIVE VALUE OF EACH TURN.

A report by GEORGE DAVIDSON, Assistant.
Submitted for publication April 3, 1893.

In 1876 I gave an explanation of the cause, or the principal cause, of the well-known irregularity observed in one turn of the micrometer screws of the zenith telescopes. Many years before I had reported the existence of excessive irregularity in the micrometer screw of an English instrument, but it was explained to arise from the manner in which the screw was cut by the instrument maker.

Nevertheless, the method of determining the arc value of the micrometer screw by observations upon a close circumpolar star, although imperfect on account of the unsteadiness of the image of the star, the fewness of the observations possible in one turn, etc., developed a periodicity in the irregularity that suggested a cause independent of the mere method of cutting the screw.

In my paper on this mechanical defect of the micrometer (1876), two methods of procedure were suggested to determine the character and amount of the irregularity at frequent intervals in one turn. It has happened that neither could be put in practice until lately.

Within the last year two close circumpolar stars have been observed as often as ten times in each turn of the micrometer screw for ten consecutive turns and five times in each turn for twenty other turns. I have not reduced the observations, but am satisfied from the unsteadiness of the object and the comparatively low power used (100 diameters) that the results will be much inferior to those of the second method.

This second method was partly developed, during the observations for variation of latitude at San Francisco in 1891 and 1892, in discussions with T. D. Davidson. It resulted in projecting the image of the micrometer thread by means of the objective of a microscope, upon a

graduated scale, and measuring with the micrometer each division of the scale.

The micrometer box was secured to the stage of a microscope that was turned horizontally with the eye end towards the scale, which was about 2 feet distant. The illumination was by a strong light behind the thread, but was not satisfactory. From the microscope tube to the scale a dark cloth was used to exclude diffused light. One observer was at the scale and the second observer was at the micrometer. After the proper focusing for the best image of the thread, and adjustment for parallelism of the thread with the scale graduations, the observations proceeded. The results were so encouraging that, after the latitude work was done, further experiments were made, with limited means, however. The last observations were as follows:

We had no solid piers upon which to place the scale and the microscope, but the micrometer, somewhat imperfectly secured to the stage of the microscope, was placed upon a library table and the scale upon a crossbar in a door. The eyepiece of the microscope was removed, and the tube carrying a 1½-inch Gundlach objective was turned horizontally towards a wall, upon which, at 15 feet distant, the images of the micrometer rack and thread were fairly well seen by using sunlight illumination through the objective.

T. D. Davidson improvised a means of holding the scale and giving it all required movements except for distance, which was reduced at the microscope to about 12 feet, when the image of the micrometer thread passed over 25 millimetres of the scale for one turn of the micrometer screw. A pasteboard tube 11 feet long and 5 inches in diameter was supported between the microscope tube and the scale, and the intervening spaces at each end were covered with black muslin. The illumination was by lamplight. The image of the thread was weak and at the best did not present a solid line, but naturally appeared as two lines marking the outer edges of the thread with a lighter middle space. The millimetre scale had been purposely cut deeply by the instrument maker, and the surface was afterwards ground by T. D. D. The lines of the scale were, therefore, very strongly marked, but a better scale should be used. A small magnifier, which increased the scale spaces about one and a half times, was conveniently fixed by the scale observer to aid in adjusting the thread image to the millimetre scale, and a hand magnifier was used by the micrometer observer. The micrometer head is 5⅜ inches in circumference, made of black zylonite and finely graduated to one hundred divisions in white. The milled head is the same diameter; both ought to have larger diameters. After the best focus of the image of the micrometer thread had been obtained and the parallelism of thread and scale lines effected, and the instrument parts all lightly tapped to remove strains of parts, the observations commenced.

Measure of the Irregularity in one turn of the Micrometer Screw, etc.

With the micrometer thread at the middle of the rack and reading 2 000, the millimetre scale was moved horizontally until the image of the micrometer thread was between two given graduations. In this position the image was just broad enough to leave a very narrow line of light between the outer edges of the image and the inner edges of the two graduation lines. The relative breadths of these two lines of light enabled the scale observer to judge of the central position of the thread image. The micrometer observer then moved off the image and brought it back to a position finally announced correct by the scale observer. The micrometer reading was recorded. The micrometer was then turned forward to the next scale space and adjusted as before and the reading recorded. So the work proceeded until 25 spaces were observed and recorded, and then the micrometer was run back to the starting space and the reading made to note if any change of position had occurred. With our unstable means the change of position ranged as high as 0·7 of a division in the first series; the average difference in the twenty-four sets was 0·1 of a division. When two sets had been made the observers changed places and two more sets were observed. The scale was then reversed and inverted and four more sets were observed in a similar manner. Each series of four sets occupied one hour at the outset; afterwards less than fifty minutes. The observations were made in the daytime or at night, to suit our convenience.

It was evident from the first set of measures that the character of the screw was above the average that I had hitherto used, and after a few sets had been made there was no doubt to either observer about obtaining the nearest tenth of a micrometer division at the scale.

The independent check of the observer at the micrometer upon the coincidences of contact of scale and image was an interesting check, and each amused himself at times in trying the skill of the scale observer. The greater difficulty was really in reading the micrometer.

It is hardly necessary to give the observations in extenso, although they have been useful in demonstrating that the stability of our outfit was imperfect, and that the graduation of the scale and the ground surface should be better. The means of each series of four sets, and the means of the direct and the reverse scales, and the final means are given to show the possibilities of the method. The results are certainly better than could be gotten by any amount of observations on close circumpolar stars, and they are too plain to require exhibition in graphic form. The actual observing time of twenty-four sets of twenty-six observations each was 5 hours and 22 minutes.

[Micrometer of Zenith Telescope No. 3.]

March 19, 1893; means of 4 sets.		March 21, 1893; means of 4 sets.		March 22, 1893; means of 4 sets.	
Scale D.	Scale R.	Scale D.	Scale R.	Scale D.	Scale R.
2000·66	2000·31	1999·67	2000·44	2000·24	2000·20
4·65	4·36	2003·78	4·46	4·36	4·25
8·64	8·40	7·71	8·46	8·35	8·21
12·65	12·39	11·72	12·40	12·32	12·22
16·59	16·36	15·59	16·35	16·25	16·15
20·61	20·32	19·59	20·36	20·20	20·12
24·61	24·36	23·54	24·38	24·25	24·14
28·61	28·34	27·48	28·35	28·11	28·10
32·65	32·28	31·50	32·30	32·12	32·05
36·61	36·30	35·39	36·28	36·08	36·05
40·62	40·28	39·46	40·30	40·11	40·08
44·64	44·30	43·42	44·37	44·16	44·14
48·69	48·31	47·40	48·36	48·18	48·09
52·74	52·37	51·47	52·36	52·22	52·16
56·75	56·40	55·51	56·45	56·20	56·15
60·76	60·40	59·44	60·41	60·20	60·25
64·77	64·39	63·45	64·42	64·20	64·19
68·78	68·39	67·42	68·44	68·18	68·10
72·80	72·36	71·30	72·40	72·05	72·06
76·78	76·29	75·34	76·32	75·98	75·99
80·72	80·36	79·20	80·34	79·92	80·02
84·82	84·34	83·26	84·37	83·99	84·02
88·87	88·44	87·30	88·40	88·00	88·08
92·87	92·39	91·28	92·44	91·95	92·10
96·89	96·45	95·31	96·44	95·98	96·08
2100·96	2100·54	2099·31	2100·56	2100·04	2100·15
2000·97	2000·40	1999·55	2000·66	2000·35	2000·22
		Checks to first readings.			

Measure of the Irregularity in one turn of the Micrometer Screw, etc.

Means of 3 series.		Means of 6 series, D. and R.	Difference from final mean.
Scale D.	Scale R.		
2000·19	2000·33	2000·26	+·06
4·26	4·36	4·31	+·12
8·23	8·36	8·30—	+·10
12·23	12·34	12·29—	+·10
16·14	16·29	16·22—	+·02
20·13	20·27	20·20	·00
24·13	24·29	24·21	+·01
28·07	28·26	28·17—	—·02
32·09	32·21	32·15	—·05
36·03	36·21	36·12	—·08
40·07	40·22	40·15—	—·04
44·07	44·27	44·17	—·03
48·09	48·25	48·17	—·03
52·14	52·30	52·22	+·02
56·15	56·33	56·24	+·04
60·13	60·35	60·24	+·04
64·14	64·33	64·24—	+·04
68·13	68·31	68·22	+·02
72·05	72·27	72·16	—·04
76·03	76·20	76·12—	—·07
79·95	80·24	80·10—	—·09
84·02	84·24	84·13	—·07
88·06	88·31	88·19	—·01
92·03	92·31	92·17	—·03
96·06	96·36	96·21	+·01
2100·10	2100·42	2100·26	+·06
2000·29	2000·43	2000·36	
Checks to first readings.			
		Final mean, 2050·20—	

Another application of this method was tried last year to determine the relative value of each turn of the micrometer screw.

The reading of the micrometer was noted when the image of the thread was upon any division of the millimetre scale; the micrometer was turned once and the scale reading noted; then the micrometer was bodily moved back by the microscope stage motion to the first scale coincidence, turned once, and the scale reading noted. This method of repetition was carried far enough to test its practicability. It demands a finer motion to the microscope stage than I possessed, but there can be no doubt of the method giving satisfactory results.

I may add that the discussion of the means of getting a better image of the micrometer thread led T. D. Davidson to experiment with the spider thread. He scoured a spider cocoon in boiling water and soap, and then boiled it in a test tube for twenty minutes with black "diamond dye" for silk, according to directions, and produced a jet-black cocoon without injury to the thread. The object was, of course, to obtain a stronger image of the thread at the scale, whereby a stronger magnifier would materially aid the observer. I have compared the

black thread with a natural thread and in color the former is decidedly preferable. In experimenting with this prepared thread for latitude observations we have found that it acquired very marked hygrometric qualities that unfit it for latitude or transit observations.

February 12, 1894.—As this appendix is passing through the press, the Superintendent has authorized the insertion of the following memorandum by Assistant Schott and the graphic illustration of the average inequality of 1 turn of the micrometer screw from 3 series of observations upon 4 close circumpolar stars near elongation at different seasons of the year.

In the figure the broken line exhibits the curve of the inequality of 1 turn as traced through the points determined by the observations through an average of 30 turns. The dotted line exhibits the curve computed from these observations. The heavy broken curve exhibits the results by the microscope-projection method of 24 sets of measures at every 4th division of the 20th turn of the micrometer.*

The expression for the periodic inequality for one turn of the motion of the screw of zenith telescope No. 3, and which may be termed "Eccentricity of the screw head," was derived from 15 series of observations made for the greater part at intervals of $\frac{1}{5}$ of a turn,† with a few at $\frac{1}{4}$ and $\frac{1}{8}$ turns. The transits range over the whole screw (4 to 36 turns) and thus give a fair average value. The particulars of the observations are shown in the following table:

No.	1892.		Star.	Range.	Elongation.	Fractional part of 1 turn.	Value of 1 turn.	Weight.
				t t			″	
1	Feb.	8	α Urs. Min.	5 — 35	W.	1/4	47·668	61
2	Feb.	9	Bradley, 1672.	35 — 9½	E.	1/4	·557	50
3	Feb.	9	α Urs. Min.	5 — 35	W.	1/5	·633	76
4	Feb.	10	Bradley, 1672.	35 — 9½	E.	1/4	·659	49
5	Feb.	10	α Urs. Min.	5 — 35	W.	1/5	·671	73
6	May	4	λ Urs. Min.	35 — 4	E.	1/5	·624	75
7	May	6	δ Urs. Min.	35 — 5	E.	1/2	·656	31
8	May	6	λ Urs. Min.	35 — 5	E.	1/5	·620	76
9	May	7	δ Urs. Min.	35 — 5	E.	1/2	·648	31
10	May	7	λ Urs. Min.	35 — 5	E.	1/5	·621	76
11	Sept.	5	α Urs. Min.	36 — 4	E.	1/5	·638	81
12	Sept.	5	λ Urs. Min.	4 — 36	W.	1/5	·647	81
13	Sept.	6	λ Urs. Min.	4 — 36	W.	1/5	·642	81
14	Sept.	17	α Urs. Min.	36 — 4	E.	1/5	·617	81
15	Sept.	21	λ Urs. Min.	4 — 36	W.	1/5	·644	81

The computation and discussion were made by Mr. H. Farquhar, of the computing division, and resulted in the expression for the eccentricity

$$+ \cdot 079 \cos (3^\circ \cdot 6d) + \cdot 066 \sin (3^\circ \cdot 6d) - \cdot 101 \cos (7^\circ \cdot 2d) - \cdot 089 \sin (7^\circ \cdot 2d),$$

where d = number of divisions read off and the correction for eccentricity is

*In proposing the microscopic method of projection it was not forgotten that the method of using a scale in the eyepiece of a micrometer had been used, but whatever the results thereby obtained may be they are liable to personal equation. In the method here adopted two observers interchange places and the personal equation will, in a measure, be eliminated. Moreover, the method is capable of much development.

†These observations include series of odd and even tenths of a turn.

expressed in divisions. Between zero and 30*d* the correction rises from nearly zero to nearly 0·2*d*; thence to reading 60*d* it diminishes to nearly 0·2*d*, its opposite extreme. The small correction has been applied to the reading of the micrometer for all observations made with this instrument, as given by the following table of limits:

Reading.		Correction.	Reading.	
d	*d*	*d*	*d*	*d*
		+ 0·2		
22.7	33·2			
		+ 0·1		
15·3	38·9		75·5	92·8
		0		
0·0	43·6		70·3	100.0
		— 0·1		
	48·1		65·1	
		— 0·2		

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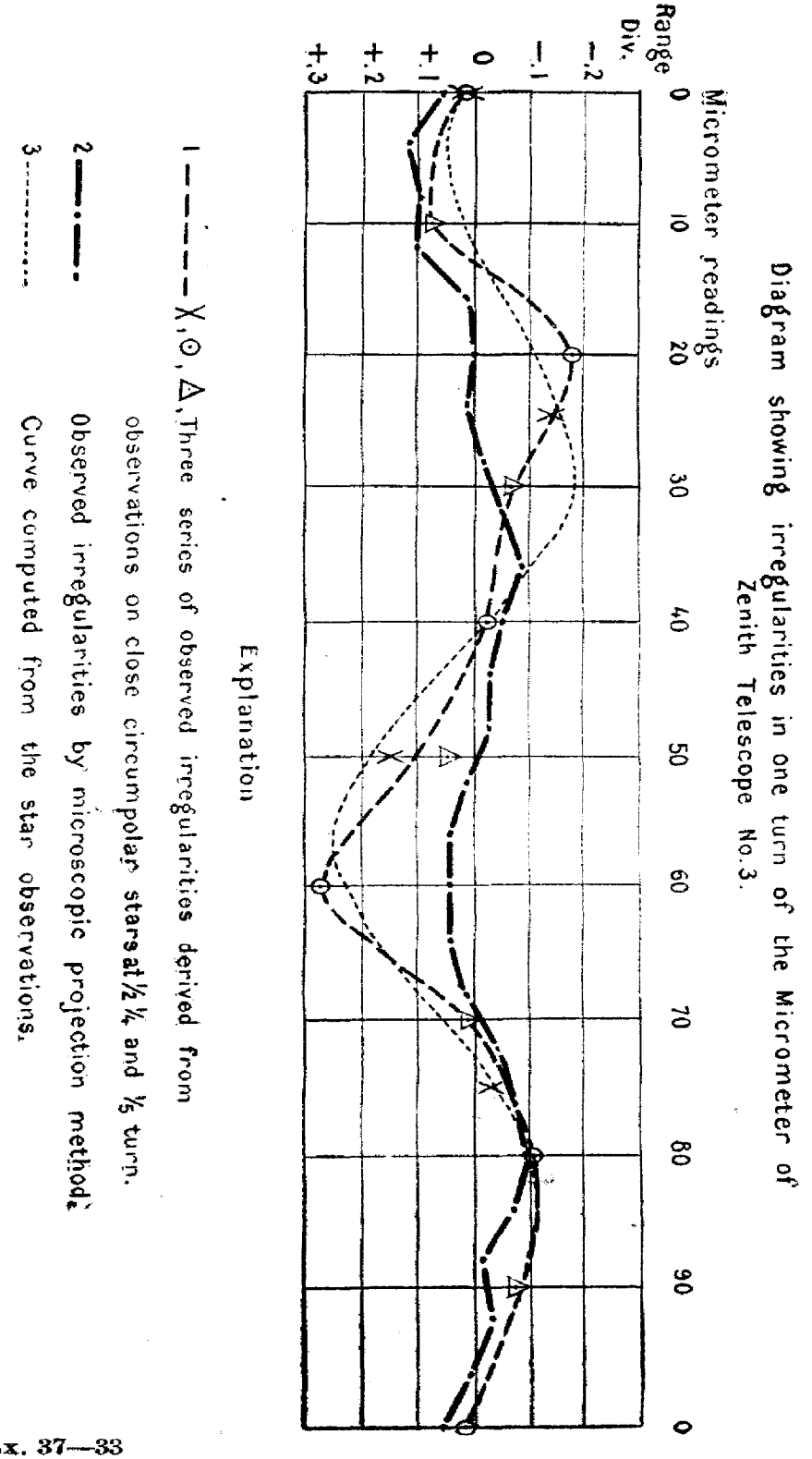
Diagram showing irregularities in one turn of the Micrometer of Zenith Telescope No.3.
Micrometer readings
Range Div.
0
10
20
30
40
50
60
70
80
90
0
-.2
-.1
0
+.1
+.2
+.3
Explanation
1 — — — — X, ⊙, Δ, Three series of observed irregularities derived from observations on close circumpolar stars at ½ ¼ and ⅕ turn.
2 —·—·— Observed irregularities by microscopic projection method.
3 ..... Curve computed from the star observations.

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APPENDIX No. 10—1892.

ON THE LEAST SQUARE ADJUSTMENT OF WEIGHINGS.

A report prepared by direction of
O. H. TITTMANN,
Assistant in charge of the Office of Standard Weights and Measures,
By JOHN F. HAYFORD.

U. S. COAST AND GEODETIC SURVEY,
OFFICE OF STANDARD WEIGHTS AND MEASURES,
Washington, D. C., August 24, 1892.

SIR: I beg leave to submit for publication the following paper, which, by my direction, was prepared and compiled by Mr. John F. Hayford, of this office.

It consists of formulæ for the least square adjustment of weighings of such series or sets of weights as occur commonly.

If a set of weights of specified nominal dimensions be given, it is required to intercompare them in such a way that a sufficient number of observation equations can be formed to bring out a precise result.

The paper shows by symbols the manner in which the weights should be combined in the weighings.

The result is then worked out symbolically with numerical coefficients for the quantities sought and for their probable errors.

NOTATION.

Let each weight be indicated by its nominal value inclosed in a parenthesis, which has as a subscript the designation of the set to which it belongs, and also, if necessary, as subscript or superscript, a figure to indicate which of two or more similar weights of a set is meant.

Thus: $(10^{gm})^*$, $(1000^{gm})_G$, $(5^{mg})\mathrm{pi}$, $(200^{gm})^{*2}$, $(1^{gm})^3\mathrm{pi}$.

r_a is uniformly used in the following paper to indicate the probable error of a single weighing.

For convenience in computation the nominal value of each weight may be omitted from each absolute term and the weight placed equal to its correction, the nominal value being always understood, though not expressed.

In all the following formulæ it is assumed that all the observations included in a single adjustment have equal weight.

THREE EQUAL WEIGHTS.

To derive the values of two unknown nominally equal weights from the value of a third known weight of the same nominal value.

Let the weights be $(1)_1$, $(1)_2$, and $(1)_3$, of which $(1)_3$ is known with a probable error $r_{(1)_3}$

$$\begin{aligned} \text{Let} \quad x_1 &= (1)_1 - (1)_3 \\ \text{and} \quad x_2 &= (1)_2 - (1)_3 \end{aligned}$$

Let the following weighings be made:

$$\begin{aligned} (1)_1 \qquad\quad - (1)_3 &= a_1 \\ (1)_2 - (1)_3 &= a_2 \\ (1)_1 - (1)_2 \qquad &= a_3 \end{aligned}$$

These observation equations may be written:

$$\begin{aligned} x_1 \qquad &= a_1 \\ x_2 &= a_2 \\ x_1 - x_2 &= a_3 \end{aligned}$$

The normal equations, giving these observation equations equal weight, are:

$$\begin{aligned} 2x_1 - x_2 &= a_1 + a_3 \\ -x_1 + 2x_2 &= a_2 - a_3 \end{aligned}$$

Solving

$$\begin{aligned} x_1 &= \tfrac{1}{3}(2a_1 + a_2 + a_3) \\ x_2 &= \tfrac{1}{3}(a_1 + 2a_2 - a_3) \end{aligned}$$

$$r_a = \tfrac{2}{3}\sqrt{\frac{[v^2]}{1}}$$

$$r_{x_1} = r_{x_2} = 0{\cdot}82\, r_a$$

$$r_{(1)_1} = r_{(1)_2} = \left[(0{\cdot}82\, r_a)^2 + r_{(1)_3}{}^2\right]^{\frac{1}{2}}$$

See Travaux et Memoires, Tome 2, p. D. 73.

FOUR EQUAL WEIGHTS.

To derive the values of three unknown nominally equal weights from the value of a fourth known weight of the same nominal value.

Let the weights be $(1)_1$, $(1)_2$, $(1)_3$, and $(1)_4$, of which $(1)_4$ is known with a probable error $r_{(1)_4}$.

Let

$$\begin{aligned} x_1 &= (1)_1 - (1)_4 \\ x_2 &= (1)_2 - (1)_4 \\ x_3 &= (1)_3 - (1)_4 \end{aligned}$$

On the least square adjustment of weighings.

Let the following weighings be made:

$$\begin{array}{rrrrr} (1)_1 & & & - (1)_4 & = a_1 \\ + (1)_2 & & & - (1)_4 & = a_2 \\ & + (1)_3 & - (1)_4 & = a_3 \\ (1)_1 - (1)_2 & & & & = a_4 \\ + (1)_2 & - (1)_3 & & & = a_5 \\ (1)_1 & - (1)_3 & & & = a_6 \end{array}$$

These observation equations may then be written:

$$\begin{array}{rrrr} x_1 & & & = a_1 \\ + x_2 & & & = a_2 \\ & + x_3 & & = a_3 \\ x_1 - x_2 & & & = a_4 \\ + x_2 & - x_3 & & = a_5 \\ x_1 & - x_3 & & = a_6 \end{array}$$

Giving these observation equations equal weight, the normal equations are:

$$\begin{array}{rrrrrrrrrrr} 3x_1 & - & x_2 & - & x_3 & = a_1 & & + a_4 & & + a_6 \\ - x_1 & + & 3x_2 & - & x_3 & = & + a_2 & - a_4 & + a_5 & \\ - x_1 & - & x_2 & + & 3x_3 & = & + a_3 & & - a_5 & - a_6 \end{array}$$

Solving

$$\begin{array}{l} x_1 = \frac{1}{4}(2a_1 + a_2 + a_3 + a_4 + a_6) \\ x_2 = \frac{1}{4}(a_1 + 2a_2 + a_3 - a_4 + a_5) \\ x_3 = \frac{1}{4}(a_1 + a_2 + 2a_3 - a_5 - a_6) \end{array}$$

$$r_a = \tfrac{2}{3}\sqrt{\frac{[v^2]}{3}}$$

$$r_x = r_{x^2} = r_{x^3} = 0{\cdot}71\, r_a$$

$$r_{(1)_1} = r_{(1)_2} = r_{(1)_3} = \left[(0{\cdot}71\, r_a)^2 + (r_{(1)_4})^2\right]^{\frac{1}{2}}$$

Travaux et Mémoires, Tome 1, p. D. 72.

FIVE EQUAL WEIGHTS.

To derive the values of four unknown nominally equal weights from the value of a fifth known weight of the same nominal value.

Let the weights be $(1)_1$, $(1)_2$, $(1)_3$, $(1)_4$, and $(1)_5$, of which $(1)_5$ is known with a probable error $r_{(1)_5}$.

Let

$$\begin{array}{l} x_1 = (1)_1 - (1)_5 \\ x_2 = (1)_2 - (1)_5 \\ x_3 = (1)_3 - (1)_5 \\ x_4 = (1)_4 - (1)_5 \end{array}$$

Let the following weighings be made:

$$
\begin{array}{rrrrrrrrrrl}
(1)_1 & & & & & & & & - (1)_5 & = & a_1 \\
 & + (1)_2 & & & & & & & - (1)_5 & = & a_2 \\
 & & + (1)_3 & & & & & & - (1)_5 & = & a_3 \\
 & & & & + (1)_4 & & & & - (1)_5 & = & a_4 \\
(1)_1 & & & & & - (1)_4 & & & & = & a_5 \\
 & + (1)_2 & & & & - (1)_4 & & & & = & a_6 \\
 & & + (1)_3 & & - (1)_4 & & & & & = & a_7 \\
(1)_1 & & & - (1)_3 & & & & & & = & a_8 \\
 & + (1)_2 & & - (1)_3 & & & & & & = & a_9 \\
(1)_1 & - (1)_2 & & & & & & & & = & a_{10}
\end{array}
$$

These observation equations may be written:

$$
\begin{array}{rrrrrrrrrrl}
x_1 & & & & & & & & & = & a_1 \\
 & + x_2 & & & & & & & & = & a_2 \\
 & & + x_3 & & & & & & & = & a_3 \\
 & & & & + x_4 & & & & & = & a_4 \\
x_1 & & & & & - x_4 & & & & = & a_5 \\
 & + x_2 & & & & - x_4 & & & & = & a_6 \\
 & & + x_3 & & - x_4 & & & & & = & a_7 \\
x_1 & & & - x_3 & & & & & & = & a_8 \\
 & + x_2 & & - x_3 & & & & & & = & a_9 \\
x_1 & - x_2 & & & & & & & & = & a_{10}
\end{array}
$$

Giving these observation equations equal weight, the normal equations are:

$$
\begin{array}{rrrrrrrrrrrrrrrrrrrrr}
4\,x_1 & - & x_2 & - & x_3 & - & x_4 & = & a_1 & & & & + a_5 & & & & + a_8 & & + a_{10} \\
- & x_1 & + 4\,x_2 & - & x_3 & - & x_4 & = & & + a_2 & & & & + a_6 & & & & + a_9 & - a_{10} \\
- & x_1 & - & x_2 & + 4\,x_3 & - & x_4 & = & & & + a_3 & & & & + a_7 & - a_8 & - a_9 & & \\
- & x_1 & - & x_2 & - & x_3 & + 4\,x_4 & = & & & & + a_4 & - a_5 & - a_6 & - a_7 & & & &
\end{array}
$$

Solving

$$
\begin{array}{rrrrrrrrrrrrrrrrrrrrr}
x_1 = \frac{1}{6}\,(& 2\,a_1 & + & a_2 & + & a_3 & + & a_4 & + a_5 & & & & + a_8 & & & & + a_{10}) \\
x_2 = \frac{1}{6}\,(& a_1 & + & 2\,a_2 & + & a_3 & + & a_4 & & + a_6 & & & & + a_9 & - a_{10}) \\
x_3 = \frac{1}{6}\,(& a_1 & + & a_2 & + & 2\,a_3 & + & a_4 & & & & + a_7 & - a_8 & - a_9 & &) \\
x_4 = \frac{1}{6}\,(& a_1 & + & a_2 & + & a_3 & + & 2\,a_4 & - a_5 & - a_6 & - a_7 & & & & &)
\end{array}
$$

$$
r_a = \tfrac{2}{3}\sqrt{\frac{[v^2]}{6}}
$$

$$
r_{x_1} = r_{x_2} = r_{x_3} = r_{x_4} = 0.63\,r_a
$$

$$
r_{(1)_1} = r_{(1)_2} = r_{(1)_3} = r_{(1)_4} = [(0.63\,r_a)^2 + r^2_{(1)_5}]^{\frac{1}{2}}
$$

SERIES 4, 3, 2, 1, — — —.

Series (10), (4), (3), (2), (1), (0·4), (0·3), (0·2), (0·1), — — —.

Observation equations, supposing (10) to be known with the probable error $r_{(10)}$.

On the least square adjustment of weighings.

$(4) + (3) + (2) + (1) = a_1 =$ Observed difference [combination — (10)] + correction (10).

$(4) + (3) + (2) + \Sigma 1 = a_2 =$ Observed difference [combination — (10)] + correction (10).

$(4) - (3) - (2) + \Sigma 1 = a_3$

$(4) - (2) - (1) - \Sigma 1 = a_4$

$(4) - (3) - (1) = a_5$

$(4) - (3) - \Sigma 1 = a_6$

$+ (3) - (2) - (1) = a_7$

$+ (3) - (2) - \Sigma 1 = a_8$

$+ (2) - (1) - \Sigma 1 = a_9$

$+ (1) - \Sigma 1 = a_{10}$

$\Sigma 1 = (0{\cdot}4) + (0{\cdot}3) + (0{\cdot}2) + (0{\cdot}1).$

Giving these observation equations equal weight, the normal equations are:

$$6(4) - (3) - (1) = a_1 + a_2 + a_3 + a_4 + a_5 + a_6$$

$$-(4) + 7(3) + (2) + (1) = a_1 + a_2 - a_3 - a_5 - a_6 + a_7 + a_8$$

$$+(3) + 7(2) + 2(1) + \Sigma 1 = a_1 + a_2 - a_3 - a_4 - a_7 - a_8 + a_9$$

$$-(4) + (3) + 2(2) + 6(1) + \Sigma 1 = a_1 - a_4 - a_5 - a_7 - a_9 + a_{10}$$

$$+(2) + (1) + 7\Sigma 1 = + a_2 + a_3 - a_4 - a_6 - a_8 - a_9 - a_{10}$$

The values of the unknowns are:

	a_1	a_2	a_3	a_4	a_5
(4) =	+0·216162	+0·183838	+0·161616	+0·159596	+0·123232
(3) =	+ ·139397	+ ·160603	— ·106062	+ ·051516	— ·112123
(2) =	+ ·082828	+ ·117172	— ·171717	— ·107071	+ ·056566
(1) =	+ ·157575	— ·057575	+ ·075758	— ·093940	— ·148485
Σ 1 =	— ·034343	+ ·134343	+ ·156566	— ·114141	+ ·013131

	a_6	a_7	a_8	a_9	a_{10}
(4) =	+0·155556	+0·004040	+0·036364	—0·038384	+0·032323
(3) =	— ·133330	+ ·184844	+ ·163638	— ·006062	— ·021214
(2) =	+ ·022222	— ·129293	— ·163636	+ ·228283	— ·034343
(1) =	+ ·066666	— ·160604	+ ·054546	— ·224242	+ ·215152
Σ 1 =	— ·155556	+ ·041414	— ·127273	— ·143434	— ·168687

$$r_a = \tfrac{2}{3}\sqrt{\frac{[v^2]}{5}}$$

$$r_{(4)} = [(0{\cdot}42\ r_a)^2 + (0{\cdot}4\ r_{(10)})^2]^{\frac{1}{2}}$$

$$r_{(3)} = [(0{\cdot}39\ r_a)^2 + (0{\cdot}3\ r_{(10)})^2]^{\frac{1}{2}}$$

$$r_{(2)} = [(0{\cdot}40\ r_a)^2 + (0{\cdot}2\ r_{(10)})^2]^{\frac{1}{2}}$$

$$r_{(1)} = [(0{\cdot}44\ r_a)^2 + (0{\cdot}1\ r_{(10)})^2]^{\frac{1}{2}}$$

$$r\Sigma_1 = [(0{\cdot}38\ r_a)^2 + (0{\cdot}1\ r_{(10)})^2]^{\frac{1}{2}}$$

(10) may be a single known weight or a combination of known weights.

In proceeding with the next step of the series (0·4), (0·3), (0·2), (0·1), the first observation equation

$$(0\cdot4) + (0\cdot3) + (0\cdot2) + (0\cdot1) = a_1$$

simply expresses the value of $\Sigma 1$ derived above.

The second observation equation

$$(0\cdot4) + (0\cdot3) + (0\cdot2) + \Sigma 0\cdot1 = a_2$$

is to be derived by weighing the indicated combination against (1) and taking the value of (1) from the adjustment just made.

The $r_{(10)}$ of the formulæ for computing the probable errors may then be taken as equal to $r_{\Sigma 1}$.

SERIES 5, 4, 3, 2, 1, —, —, —.

Series (5), (4), (3), (2), (1), (0·5), (0·4), (0·3), (0·2), (0·1), —, —.

(10) Either as a single weight or as a combination of weights is supposed to be known, with a probable error $r_{(10)}$

Observation equations.

(5)	+ (4)					+ (1)	= a_1	= Observed difference [combination — 10] + correction (10)
(5)		+ (3)	+ (2)				= a_2	= Observed difference [combination — 10] + correction (10)
	+ (4)	+ (3)	+ (2)	+ (1)			= a_3	= Observed difference [combination — 10] + correction (10)
(5)	— (4)	— (3)	+ (2)				= a_4	
(5)	— (4)		— (2)	+ (1)			= a_5	
(5)	— (4)			— (1)			= a_6	
(5)		— (3)	— (2)				= a_7	
	+ (4)	— (3)	— (2)	+ (1)			= a_8	
	+ (4)	— (3)		— (1)			= a_9	
		+ (3)	— (2)	— (1)			= a_{10}	

Giving these observation equations equal weight the normal equations are

+ 6 (5)	— 2 (4)	— (3)		+ (1)	= a_1	+ a_2		+ a_4	+ a_5	+ a_6	+ a_7	
— 2 (5)	+ 7 (4)			+ 2 (1)	= a_1		+ a_3	— a_4	— a_5	— a_6		+ a_8 + a_9
— (5)		+ 7 (3)	+ 2 (2)		=	+ a_2	+ a_3	— a_4			— a_7	— a_8 — a_9 + a_{10}
		+ 2 (3)	+ 7 (2)		=	+ a_2	+ a_3	+ a_4	— a_5		— a_7 — a_8	— a_{10}
+ (5)	+ 2 (4)			+ 7 (1)	= a_1		+ a_3		+ a_5	— a_6		+ a_8 — a_9 — a_{10}

On the least square adjustment of weighings.

Solving

$$
\begin{aligned}
(5) &= \tfrac{1}{22}\,(+\ 5\,a_1 + \ \ 5\,a_2 + \ \ \ a_3 + \ \ 2\,a_4 + \ 2\,a_5 + \ 4\,a_6 + \ 4\,a_7 \ \ \ \ + \ \ 2\,a_9 + \ \ 2\,a_{10})\\
(4) &= \tfrac{1}{858}\,(+\ 95\,a_1 + \ 40\,a_2 + \ 63\,a_3 - \ \ 61\,a_4 - \ 83\,a_5 - \ 23\,a_6 + 32\,a_7 + \ \ 55\,a_8 + 115\,a_9 + \ 38\,a_{10})\\
(3) &= \tfrac{1}{3510}\,(+\ 35\,a_1 + 145\,a_2 + 117\,a_3 - 184\,a_4 + \ 58\,a_5 + \ 28\,a_6 - \ 82\,a_7 - \ 110\,a_8 - \ 140\,a_9 + 212\,a_{10})\\
(2) &= \tfrac{1}{1365}\,(-\ \ 5\,a_1 + \ 50\,a_2 + \ 54\,a_3 + \ \ 97\,a_4 - \ 79\,a_5 - \ \ 4\,a_6 - \ 59\,a_7 - \ \ 55\,a_8 + \ \ 20\,a_9 - 101\,a_{10})\\
(1) &= \tfrac{1}{910}\,(+\ \ 5\,a_1 - \ \ 5\,a_2 + \ \ 9\,a_3 + \ \ \ 2\,a_4 + 16\,a_5 - \ 14\,a_6 - \ \ 4\,a_7 + \ 10\,a_8 - \ \ 20\,a_9 - \ 16\,a_{10})
\end{aligned}
$$

$$r_a = \frac{2}{3}\sqrt{\frac{[v^2]}{5}}$$

$$
\begin{aligned}
r_{(5)} &= [(0{\cdot}45\ r_a)^2 + (0{\cdot}5\ r_{(10)})^2]^{\frac{1}{2}}\\
r_{(4)} &= [(0{\cdot}43\ r_a)^2 + (0{\cdot}4\ r_{(10)})^2]^{\frac{1}{2}}\\
r_{(3)} &= [(0{\cdot}40\ r_a)^2 + (0{\cdot}3\ r_{(10)})^2]^{\frac{1}{2}}\\
r_{(2)} &= [(0{\cdot}40\ r_a)^2 + (0{\cdot}2\ r_{(10)})^2]^{\frac{1}{2}}\\
r_{(1)} &= [(0{\cdot}41\ r_a)^2 + (0{\cdot}1\ r_{(10)})^2]^{\frac{1}{2}}
\end{aligned}
$$

SERIES 6, 5, 4, 3, 2, 1, —, —, —,

Series (6), (5), (4), (3), (2), (1), (0·6), (0·5), (0·4), (0·3), (0·2), (0·1), —, —, —.

(10) Either as a single weight or as a combination of weights is supposed to be known, with a probable error $r_{(10)}$

Observation equations.

$$
\begin{aligned}
(6) \quad + (4) &= a_1 = \text{Observed difference [combination} - 10] + \text{correction (10)}\\
+ (5) + (4) \quad + (1) &= a_2 = \text{Observed difference [combination} - 10] + \text{correction (10)}\\
+ (5) \quad + (3) + (2) &= a_3 = \text{Observed difference [combination} - 10] + \text{correction (10)}\\
(6) - (5) - (4) + (3) &= a_4\\
(6) \quad - (4) - (3) + (2) - (1) &= a_5\\
(6) - (5) \quad - (2) + (1) &= a_6\\
(6) - (5) \quad - (1) &= a_7\\
(6) \quad - (4) \quad - (2) &= a_8\\
+ (5) - (4) \quad - (1) &= a_9\\
+ (5) \quad - (3) - (2) &= a_{10}\\
+ (4) - (3) \quad - (1) &= a_{11}\\
+ (3) - (2) - (1) &= a_{12}
\end{aligned}
$$

Giving these observation equations equal weight, the normal equations are

(6)	(5)	(4)	(3)	(2)	(1)		a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}
+ 6	− 3	− 2		− 1	− 1	=	+ 1			+ 1	+ 1	+ 1	+ 1	+ 1				
− 3	+ 7	+ 1	− 1	+ 1		=		+ 1	+ 1	− 1		− 1	− 1		+ 1	+ 1		
− 2	+ 1	+ 7	− 1		+ 2	=	+ 1	+ 1		− 1	− 1			− 1	− 1		+ 1	
	− 1	− 1	+ 6		+ 1	=			+ 1	+ 1	− 1					− 1	− 1	+ 1
− 1	+ 1			+ 6	− 1	=			+ 1		+ 1	− 1		− 1		− 1		− 1
− 1		+ 2	+ 1	− 1	+ 7	=		+ 1			− 1	+ 1	− 1		− 1		− 1	− 1

Solving

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}
(6) =	+0·291087	+0·166330	+0·142574	+0·113861	+0·173269	+0·137628	+0·124747	+0·159408	+0·021784	+0·045546	+0·010886	−0·026739
(5) =	+0·096534	+0·198016	+0·205447	−0·066833	+0·039606	−0·074258	−0·101487	+0·106437	+0·180693	+0·173266	−0·034658	+0·039606
(4) =	+0·228712	+0·130691	+0·040594	−0·089109	−0·113861	+0·000992	+0·098018	−0·124752	−0·125742	−0·035643	+0·187127	+0·086136
(3) =	+0·058910	+0·033662	+0·207427	+0·136138	−0·173267	−0·037623	+0·025247	−0·009406	+0·028218	−0·145545	−0·110892	+0·226732
(2) =	+0·027728	+0·015859	+0·156436	+0·034655	+0·183164	−0·105949	+0·011893	−0·151492	−0·045549	−0·186140	−0·022756	−0·216821
(1) =	−0·028217	+0·126735	+0·001485	+0·027228	−0·034654	+0·152474	−0·154949	+0·038118	−0·114357	+0·010891	−0·182176	−0·234652

On the least square adjustment of weighings.

$$r_a = \tfrac{2}{3}\sqrt{\frac{[v^2]}{6}}$$

$$r_{(6)} = [(0{\cdot}49\ r_a)^2 + (0{\cdot}6\ r_{(10)})^2]^{\frac{1}{2}}$$
$$r_{(5)} = [(0{\cdot}43\ r_a)^2 + (0{\cdot}5\ r_{(10)})^2]^{\frac{1}{2}}$$
$$r_{(4)} = [(0{\cdot}42\ r_a)^2 + (0{\cdot}4\ r_{(10)})^2]^{\frac{1}{2}}$$
$$r_{(3)} = [(0{\cdot}43\ r_a)^2 + (0{\cdot}3\ r_{(10)})^2]^{\frac{1}{2}}$$
$$r_{(2)} = [(0{\cdot}42\ r_a)^2 + (0{\cdot}2\ r_{(10)})^2]^{\frac{1}{2}}$$
$$r_{(1)} = [(0{\cdot}41\ r_a)^2 + (0{\cdot}1\ r_{(10)})^2]^{\frac{1}{2}}$$

SERIES 5, 3, 2, 1, —, —, —.

Series (5), (3), (2), (1), (0·5), (0·3), (0·2), (0·1) —, —, —. (10) either as a single weight or as a combination of weights is supposed to be known with a probable error $r_{(10)}$

Observation equations.

$(5) + (3) + (2) = a_1$ = Observed difference [combination — (10)] + correction (10).

$(5) + (3) + (1) + \Sigma 1 = a_2$ = Observed difference [combination — (10)] + correction (10).

$(5) - (3) - (2) - (1) + \Sigma 1 = a_3$
$(5) - (3) - (2) + (1) - \Sigma 1 = a_4$
$(5) - (3) - (2) = a_5$
$(5) - (3) - (1) - \Sigma 1 = a_6$
$+ (3) - (2) - (1) = a_7$
$+ (3) - (2) - \Sigma 1 = a_8$
$+ (2) - (1) - \Sigma 1 = a_9$
$+ (1) - \Sigma 1 = a_{10}$

Giving these observation equations equal weight the normal equations are—

$+6(5) - 2(3) - 2(2) = a_1 + a_2 + a_3 + a_4 + a_5 + a_6$
$-2(5) + 8(3) + 2(2) + (1) + \Sigma 1 = a_1 + a_2 - a_3 - a_4 - a_5 - a_6 + a_7 + a_8$
$-2(5) + 2(3) + 7(2) = a_1 - a_3 - a_4 - a_5 - a_7 - a_8 + a_9$
$+ (3) + 7(1) = + a_2 - a_3 + a_4 - a_6 - a_7 - a_9 + a_{10}$
$+ (3) + 7\Sigma 1 = + a_2 + a_3 - a_4 - a_6 - a_8 - a_9 - a_{10}$

Solving:

	a_1	a_2	a_3	a_4	a_5
(5) =	+0·277778	+0·222223	+0·111111	+0·111111	+0·111111
(3) =	+0·155556	+0·144445	−0·077778	−0·077778	−0·077778
(2) =	+0·177778	+0·022223	−0·088889	−0·088889	−0·088889
(1) =	−0·022222	+0·122222	−0·131746	+0·153968	+0·011111
Σ 1 =	−0·022222	+0·122222	+0·153968	−0·131746	+0·011111

	a_6	a_7	a_8	a_9	a_{10}
(5) =	+0·166666			+0·055556	
(3) =	−0·066667	+0·200000	+0·200000	+0·011111	
(2) =	+0·066666	−0·200000	−0·200000	+0·155556	
(1) =	−0·133333	−0·171429	−0·028571	−0·144444	+0·142857
Σ 1 =	−0·133333	−0·028571	−0·171428	−0·144444	−0·142857

$$r_a = \tfrac{2}{3}\sqrt{\frac{[v^2]}{5}}$$

$$r_{(5)} = [(0{\cdot}44\ r_a)^2 + (0{\cdot}5\ r_{(10)})^2]^{\frac{1}{2}}$$
$$r_{(3)} = [(0{\cdot}38\ r_a)^2 + (0{\cdot}3\ r_{(10)})^2]^{\frac{1}{2}}$$
$$r_{(2)} = [(0{\cdot}41\ r_a)^2 + (0{\cdot}2\ r_{(10)})^2]^{\frac{1}{2}}$$
$$r_{(1)} = [(0{\cdot}38\ r_a)^2 + (0{\cdot}1\ r_{(10)})^2]^{\frac{1}{2}}$$
$$r_{(21)} = [(0{\cdot}38\ r_a)^2 + (0{\cdot}1\ r_{(10)})^2]^{\frac{1}{2}}$$

SERIES 5, 2, 2, 1 — — —.

Series (5), $(2)_1$, $(2)_2$, (1), (0·5), $(0{\cdot}2)_1$, $(0{\cdot}2)_2$, (0·1) — — —

(10) either as a single weight or a combination of weights is supposed to be known with a probable error, $r_{(10)}$.

First make the weighings—

$$(5) + (2)_1 + (2)_2 + (1) = (10) + \Delta_1$$
$$(5) - (2)_1 - (2)_2 - (1) = \Delta_2$$

Then $$(5) = \tfrac{1}{2}\,[(10) + \Delta_1 + \Delta_2]$$

and $$(2)_1 + (2)_2 + (1) = \tfrac{1}{2}\,[(10) + \Delta_1 - \Delta_2] = W$$

Observation equations.

$$(1) \qquad\qquad - \Sigma 1 = a_1$$
$$(1) - (2)_1 \qquad + \Sigma 1 = a_2 \quad \Sigma 1 = (0{\cdot}5) + (0{\cdot}2)_1 + (0{\cdot}2)_2 + (0{\cdot}1)_1$$
$$(1) \qquad - (2)_2 + \Sigma 1 = a_3$$
$$+ (2)_1 - (2)_2 \qquad = a_4$$
$$(1) + (2)_1 - (2)_2 - \Sigma 1 = a_5$$
$$(1) - (2)_1 + (2)_2 - \Sigma 1 = a_6$$

On the least square adjustment of weighings.

Giving these observation equations equal weight and introducing the rigid condition $(2)_1 + (2)_2 + (1) = W$, the normal equations are:

$$\begin{aligned}
+5(1) - (2)_1 - (2)_2 - \Sigma 1 &= a_1 + a_2 + a_3 + a_5 + a_6 \\
-(1) + 4(2)_1 - 3(2)_2 - \Sigma 1 &= -a_2 + a_4 + a_5 - a_6 \\
-(1) - 3(2)_1 + 4(2)_2 - \Sigma 1 &= -a_3 - a_4 - a_5 + a_6 \\
-(1) - (2)_1 - (2)_2 + 5\Sigma 1 &= -a_1 + a_2 + a_3 - a_5 - a_6
\end{aligned}$$

Solving

$$\begin{aligned}
(1) &= \tfrac{1}{5}\left(\tfrac{2}{3}a_1 + a_2 + a_3 + \tfrac{2}{3}a_5 + \tfrac{2}{3}a_6 + W\right) \\
(2)_1 &= \tfrac{1}{10}\left(-\tfrac{2}{3}a_1 - \tfrac{12}{7}a_2 - \tfrac{2}{7}a_3 + \tfrac{10}{7}a_4 + \tfrac{16}{21}a_5 - \tfrac{44}{21}a_6 + 4W\right) \\
(2)_2 &= \tfrac{1}{10}\left(-\tfrac{2}{3}a_1 - \tfrac{2}{7}a_2 - \tfrac{12}{7}a_3 - \tfrac{10}{7}a_4 - \tfrac{44}{21}a_5 + \tfrac{16}{21}a_6 + 4W\right) \\
\Sigma 1 &= \tfrac{1}{5}\left(-a_1 + a_2 + a_3 - a_5 - a_6 + W\right)
\end{aligned}$$

Or the solution may be put in the form—

$$\begin{aligned}
m_1 &= a_1 + a_2 + a_3 + a_5 + a_6 \\
m_2 &= -a_2 + a_4 + a_5 + a_6 \\
m_3 &= -a_3 - a_4 - a_5 + a_6 \\
S &= \tfrac{1}{8}(m_1 + m_2 + m_3) \\
m'_1 &= m_1 - S \\
m'_2 &= m_2 - S \\
m'_3 &= m_3 - S \\
M &= 4W + m'_2 + m'_3 \\
N &= \tfrac{5}{7}(m'_2 - m'_3) \\
(1) &= \tfrac{1}{5}(m'_1 + W) \\
(2)_1 &= \tfrac{1}{10}(M + N) \\
(2)_2 &= \tfrac{1}{10}(M - N) \\
\Sigma 1 &= \tfrac{1}{5}[(1) - S]
\end{aligned}$$

$$r_a = \tfrac{2}{3}\sqrt{\frac{[v^2]}{3}}$$

r_w must be computed according to the method of deriving the value of W, assuming that the probable error of each weighing is r_a. In continuing the series it should be noted that the value

$$(0{\cdot}5) + (0{\cdot}2)_1 + (0{\cdot}2)_2 + (0{\cdot}1) = \Sigma 1$$

is already known.

$$\begin{aligned}
r_{(1)} &= [(0.37\, r_a)^2 + (0{\cdot}2\, r_w)^2]^{\frac{1}{2}} \\
r_{(2)_1} &= [(0{\cdot}32\, r_a)^2 + (0{\cdot}4\, r_w)^2]^{\frac{1}{2}} \\
r_{(2)_2} &= [(0{\cdot}32\, r_a)^2 + (0{\cdot}4\, r_w)^2]^{\frac{1}{2}} \\
r_{\Sigma 1} &= [(0{\cdot}45\, r_a)^2 + (0{\cdot}2\, r_w)^2]^{\frac{1}{2}}
\end{aligned}$$

See Travaux et Memoirs, Tome III, pp. D 93–D 94.

SERIES 5, 2, 1, 1, — — —

Series (5), (2), $(1)_1$, $(1)_2$, (0·5, (0·2), $(0·1)_1$, $(0·1)_2$, — —.

(10) either as a single weight or a combination of weights is supposed to be known.

First make the weighings—

$$(5) + (2) + (1)_1 + (1)_2 + \Sigma 1 = (10) + \Delta_1$$
$$(5) - (2) - (1)_1 - (1)_2 - \Sigma 1 = \Delta_2$$
$$\Sigma 1 = (0·5) + (0·2) + (01)_1 + 0·1)_2 + \Sigma 0·1$$

Then $$(5) = \tfrac{1}{2}\,[(10) + \Delta_1 + \Delta_2]$$

and $$(2) + (1)_1 + (1)_2 + \Sigma 1 = \tfrac{1}{2}\,[(10) + \Delta_1 - \Delta_2] = K$$

Then make the weighings indicated by the following

Observation equations.

$$\begin{aligned}
(1)_1 - (1)_2 &= a_1\\
(1)_1 - \Sigma 1 &= a_2\\
+ (1)_2 - \Sigma 1 &= a_3\\
(1)_1 + (1)_2 - (2) &= a_4\\
+ (1)_2 + \Sigma 1 - (2) &= a_5\\
(1) + \Sigma 1 - (2) &= a_6
\end{aligned}$$

Giving these observation equations equal weight and introducing the rigid condition $(2) + (1)_1 + (1)_2 + \Sigma 1 = K$, the solution of the resulting normal equations give—

$$\begin{aligned}
(1)_1 &= \tfrac{1}{20}\,(+5a_1 + 5a_2 + 3a_4 - 2a_5 + 3a_6) + \tfrac{1}{5}\,K\\
(1)_2 &= \tfrac{1}{20}\,(-5a_1 + 5a_3 + 3a_4 + 3a_5 - 2a_6) + \tfrac{1}{5}\,K\\
\Sigma 1 &= \tfrac{1}{20}\,(-5a_2 - 5a_3 - 2a_4 + 3a_5 + 3a_6) + \tfrac{1}{5}\,K\\
(2) &= \tfrac{1}{20}\,(-4a_4 - 4a_5 - 4a_6) + \tfrac{3}{5}\,K
\end{aligned}$$

$$r_a = \frac{2}{3}\sqrt{\frac{[v^2]}{3}}$$

$$r_{1_1} = r_{(1)_2} = r_{\Sigma 1} = \left[(0·42r_a)^2 + (0·2r_k)^2\right]^{1/2}$$

$$r_{(2)} = \left[(0·35r_a)^2 + (0·4r_k)^2\right]^{1/2}$$

r_k must be computed according to the method of deriving the value of K, assuming that the probable error of each weighing is r_a. In continuing the series it should be noticed that the value $(0·5) + (0·2) + (0·1)_1 + (0·1)_2 + \Sigma 1 = \Sigma 1$ is already known.

These formulæ may be applied to the office sets of weights as follows:

For the platinum-iridium metric weights pi, series 5, 2, 1, 1, p. 526. The third 1-gramme weight may take the place of $\Sigma 1$ in the formula. In closing the series one extra half milligramme with that which belongs to the set will furnish $\Sigma 0·001^{gm}$.

On the least square adjustment of weighings.

For the star metric weights ( )*.

Weights 1000^{gm}, 500^{gm}, $200_1{}^{gm}$, $200_2{}^{gm}$, $100_1{}^{gm}$, $100_2{}^{gm}$, use formulæ of pages 524 and 525, putting $100_2{}^{gm}$ in the place of $\Sigma 1$.

Weights 50^{gm}, 30^{gm}, 20^{gm}, 10^{gm}, use formulæ of pages 523 and 524.

Weights 5^{gm}, 4^{gm}, 3^{gm}, 2^{gm}, 1^{gm}, use formulæ of pages 520 and 521.

For the remainder of the set use formulæ of pages 518 and 519.

For star grain weights use formulæ of pages 520 and 521.

For star troy ounce weights: Weights 10^{oz}, 6^{oz}, 5^{oz}, 4^{oz}, 3^{oz}, 2^{oz}, 1^{oz}, use formulæ of pages 521 and 522.

For the remainder of the set use formulæ of pages 520 and 521.

For the new gilded metric weights ( )G use formulæ of pages 520 and 521.

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APPENDIX No. 11—1892.

RESULTS OF MAGNETIC OBSERVATIONS AT STATIONS IN ALASKA AND IN THE NORTHWEST TERRITORY OF THE DOMINION OF CANADA.

Observations at five stations in Alaska by J. E. McGRATH and J. HENRY TURNER, Assistants, U. S. Coast and Geodetic Survey, in the years 1889, 1890, and 1891.

Discussion of results and comparison with other stations by CHARLES A. SCHOTT, Assistant.

Submitted for publication June 22, 1892.

The report which I have the honor to submit herewith relates only to the magnetic results reached by Assistant J. E. McGrath and Assistant J. H. Turner in connection with their survey of the eastern boundary of Alaska, between Mount St. Elias and the Arctic Ocean, during 1889–'90–'91.

The magnetic records and observers' computations are the only returns made to date; hence they were taken up first. The office computation is by Mr. L. A. Bauer. The instrumental outfit of Mr. McGrath consisted of Magnetometer No. 7 and Barrow Dip Circle No. 12; Mr. Turner had Theodolite Magnetometer No. III and Barrow Dip Circle No. 8; also a prismatic compass.

(1) *Observations on Amaknak Island, Iliuliuk Harbor (Unalaska).*

[$\phi = 53° 52'.9$. $\lambda = 166° 31'.6$ W.]

			° ′	° ′		
1889	June 28, 29	McGrath	$D = -17\ 46.0$	$\theta = 67\ 13.2$	$H = 0.2058$	$F = 0.5314$
	28, 29	Turner	$-18\ 12.4$	$67\ 06.8$	0.2052	0.5278
1891	July 15, 16, 17	Turner and Edmonds	$-18\ 06.9$	$67\ 17.2$	0.2057	0.5328

The predicted annual decrease of declination (App. 7, Rep. 1888), epoch 1890, was 2'.4; it is probable that 3.0 is nearer the truth. A new discussion of the 14 values, now available, resulted in the expression $D = -17.65 + 2.26 \sin (1.3\,m - 69°)$.

(2) *Observations at St. Michael, on tundra back of the village.*

[Stations A and B close together. $\phi = 63° 29'.1$. $\lambda = 162° 05'.2$ W.]

1889	July 7, 8, 9	McGrath	$D = -23\ 15.0$	$\theta = 74\ 57.8$	$H = 0.1479$	$F = 0.5702$
	7, 8, 9, 10	Turner	$-23\ 09.8$	$75\ 06.5$	0.1471	0.5723
1890	Oct. 26, 27, 28, 29, 30 / Nov. 26, 27, 28	Turner	$-23\ 07.0$	$75\ 12.1$	0.1474	0.5770
1891	Mar. 7, 8, 9, 10 / June 16 to 23	Turner	$-23\ 02.4$	$75\ 05.0$	0.1475	0.5728

This is not a secular-change station, but may become one hereafter. In the summer of 1827 Admiral Lütke found 30° 30′ E. in Norton Sound near this place, and in June, 1874, L. M. Turner found 23° 0′ E., this last probably a rough and unreliable value. We infer from other stations that the declination has been diminishing since Lütke's time; hence an annual rate of 7′, which seems to fall in well with results at stations in the vicinity.

(3) *Observations at Fort Yukon (now abandoned).*

[$\phi = 66° 33'.8$. $\lambda = 145° 17'.8$ W.]

1889	Aug. 5, 6, 7, 8	McGrath	$D = -34\ 46.9$	$\theta = 79\ 34.3$	$H = 0.1070$	$F = 0.5911$
	8, 9, 10, 11	Turner	$-35\ 12.0$	$79\ 37.8$	0.1070	0.5944

The only other observations known to me at this place are those of Capt. C. W. Raymond, U. S. E. He found: 1869, August 14, 16 and 26, $D = -36° 32'.7$, $\theta = 79° 49'.5$, but his intensity results $h = 2.7321$ and $f = 15.472$, or when converted into C. G. S. terms $H = 0.1260$ and $F = 0.7134$, appear to be too large. The average annual diminution of declination becomes 4′.6, a smaller value than had been supposed heretofore. Probably the truth lies between 4′.6 and 10′.0, particularly since no extreme value was known to have been reached in the interval of twenty years. The isogonic chart for 1890, Appendix No. 11, Report for 1889, p. 396, gives the declination $-33°$, or about 2° too small, due to the cause just mentioned.

(4) *Camp Davidson, eastern boundary, on Yukon River.*—$\varphi = 64° 41'$, $\lambda = 140° 57'$. At this place magnetic observations were made by McGrath generally on three or more days each month, between September, 1889, and May, 1891 (inclusive). The results are as follows:

Mean Date.	Decl'n E.	Dip.	Horiz. Int.	Total Int.
	° ′	° ′		
1889.8	35 47.3	78 36.6	0.1171	0.5929
1890.5	44.1	32.6	.1173	.5903
1891.2	43.0	33.9	.1173	.5918

Results of Magnetic Observations at stations in Alaska and in the Northwest Territory.

No reliable value for annual change of declination can be deduced, the series being too short, but here also the indications point to a smaller value than had been inferred previously. A diminution of about 5′ or 6′ is probable.

(5) *Camp Colonna, Porcupine River.*—$\varphi = 67° 25'{\cdot}1$, $\lambda = 140° 58'{\cdot}8$. At this station we have absolute measures as well as differential ones by J. H. Turner and party. The results of the former are as follows:

From daily observations between June 13 and 25, inclusive, 1890.

$$D = -\ 38° 06'{\cdot}8$$
$$\theta = \quad 80° 36'{\cdot}9$$
$$H = \quad 0{\cdot}0972 \text{ and } F = 0{\cdot}5961$$

H. W. Edmonds, observer. The differential measures form quite an extensive series and admitted of some of the usual deductions respecting the variations of declination. These measures are confined to variations in declination and were made every five minutes through twenty-four hours on one day (Tuesday) of every week between November, 1889, and June, 1890; in all, covering thirty days of observation. In this work Mr. Turner was assisted by H. W. Edmonds, J. Meade, W. E. Lingard, and C. P. Harkort. The five-minute intervals were considered as unnecessarily close, i. e., the same results were deducible from the hourly values, hence the latter only were reduced to a uniform zero by applying the correction for the various circle readings; that is, they were converted into absolute measure. The position of the magnetic axis of the long or intensity magnet was determined in June, 1890, also at Iliuliuk and at St. Michael in June and July, 1889; a weighted mean value has been used. In the hourly tabulation of results the letter "a" indicates the appearance of an aurora. Between December 9 and 16 the sun disappeared and re-appeared above the horizon between January 20 and 27. The adopted scale reading of the axis was $32^{d}{\cdot}1$, and the value of one division of the scale is $1'{\cdot}60$; there are many readings connecting the zero of the horizontal circle of the declinometer with the azimuth mark. The tabulation is herewith appended. A result differing more than 13′ from its hourly mean has been marked as a disturbance and is indicated by a star attached to it. The character of the diurnal variation remains the same for the total and the normal diurnal variations, the latter showing less amplitude. The accompanying table has been prepared to show the accord of the diurnal variation of the declination at this camp with the variations observed at several surrounding stations, though widely separated geographically. They are (a) to the northward and westward, Uglaamie, Point Barrow, Lieut. P. H. Ray, $\varphi = 71° 17'{\cdot}7$, $\lambda = 156° 39'{\cdot}8$ W. (see Appendix No. 13, Coast and Geodetic Survey Report for 1883), results from hourly series, October, 1882, to March, 1883, inclusive. (b) Plover Point, Point Barrow, about 16 kilometres to the north and east of Uglaamie, $\varphi = 71° 21'{\cdot}4$, $\lambda = 156° 16'{\cdot}1$

W., series by Commander R. Maguire, R. N., in 1852–'53–'54; Phil. Trans. Roy. Soc., 1857; results from November, 1852, to June, 1853, inclusive, and from October, 1853, to June, 1854, inclusive. (c) To the southward and eastward, Sitka (Japonsky Island); Russian magnetic observatory in $\varphi = 57°\ 02'{\cdot}9$, λ 135° 20′·3 W. This series extends from 1848 to 1864, but is much broken up and very irregular as to hours of observation, and the variation could not be made out for each month separately (see table at foot of p. 347, Appendix No. 13, Report for 1883). (d) To the eastward and southward, Fort Rae, Great Slave Lake, in $\varphi = 62°\ 38'{\cdot}9$, $\lambda = 115°\ 43'{\cdot}8$ W., Capt. H. P. Dawson, R. A., 1882–'83 (see observations of the International Polar Expeditions, London, 1886); the results given refer to the time, October, 1882, to March, 1883, inclusive. And (e) Fort Chipewyan, Lake Athabasca, to the southward of Fort Rae, and in $\varphi = 58°\ 43'$, $\lambda = 111°\ 18'{\cdot}7$ (see magnetical and meterological observations at Lake Athabasca by Capt. J. H. Lefroy, London, 1885, series from October 16, 1843, to February 29, 1844, inclusive).*

Local hour.	Camp Colonna, Porcupine River,	Uglaamie, Point Barrow.	Plover Point, Point Barrow.	Sitka, Japonsky Island.	Fort Rae, Great Slave Lake.	Fort Chipewyan, Lake Athabasca.†
	′	′	′	′	′	′
Midnight.	— 7·7	—15·2	—10·8	— 0·6	—10·5	— 1·9
1	— 4·2	—13·6	— 8·0	+ 0·2	+12·3	— 1·6
2	— 5·4	— 4·9	— 1.9	+ 1·0	— 7·3	+ 0·9
3	+ 3·1	+ 6·5	+ 3·6	+ 1·4	+ 1·5	+ 1·6
4	+ 4·0	+ 6·9	+10·9	+ 2·0	+ 8·7	+ 5·8
5	+11·5	+15·7	+16·6	+ 2·9	+15·6	+ 8·2*
6	+11·5	+19·2	+19·3	+ 4·2	+19·9	+ 5·8
7	+11·3	+21·8	+27·1*	+ 5·3	+20·8	+ 5·5
8	+16·4*	+24·7*	+27·0	+ 6·0*	+24·7*	+ 3·9
9	+12·0	+17·8	+19·9	+ 5·3	+24·1	+ 3·7
10	+ 5·6	+ 8·3	+ 9·3	+ 3·0	+15·5	+ 1·3
11	+ 1·1	— 0·6	— 0·4	+ 0·6	+ 7·7	— 3·0
Noon.	— 5·3	— 6·8	— 8·2	— 2·1	— 2·0	— 4·0
13	— 5·2	— 7·1	—10·7†	— 3·2	— 5·5	— 4·9
14	— 7·3†	— 8·0†	— 9·8	— 4·2	— 8·5	— 5·2†
15	— 6·5	— 6·1	— 9·9	— 4·6	—10·5	— 4·2
16	— 6·4	— 7·6	— 9·8	— 4·6†	—10·7†	— 3·3
17	— 5·6	— 7·7	—10·2	— 3·8	—10·5	— 2·9
18	— 5·0	— 8·2	— 9·7	— 3·2	—10·7	— 1·8
19	— 3·6	— 6·1	— 8·4	— 2·4	— 9·9	— 1·4
20	— 3·0	— 6·5	— 9·0	— 1·4	—10·6	— 1·2
21	— 2·0	— 6·5	— 7·5	— 0·8	— 9·0	— 1·0
22	— 2·9	—11·5	— 7·9	— 0·4	—10·3	— 0·3
23	— 5·2	— 4·2	—11·5	— 0·6	—10·3	— 0·1
Midnight.	— 7·7	—15·2	—10·8	— 0·6	—10·5	— 1·9

A + sign signifies a deflection of the north end of the needle to the *east*, a — sign the contrary motion. For other North American stations compare table given in Appendix 9, Report for 1890, p. 263. Owing to the sun spot or eleven year inequality and the circumstance that the tabular values do not strictly refer to the same months, the ranges of the diurnal variation are not directly comparable, but the hours of extreme range, marked thus, * and †, as well as the hours of average daily value, indicated by change of sign and a bar —— may be so compared.

*** Figures slightly altered to refer to mean of 24 hours.**

Results of Magnetic Observations at stations in Alaska and in the Northwest Territory.

Analysis of the larger disturbances.—This was roughly done by Sabine's method* (as fully explained in Appendix No. 9, Report for 1890, p. 269, and following), the adopted limit of ± 13′ separated 68 out of a total of 720 hourly values, or about 1 in 11. The predominance of the easterly over the westerly disturbances, both in number and amount, as heretofore noted for North American stations, is again confirmed at Camp Colonna, as shown in the following table:

	Number.		Amount.	
	E.	W.	E.	W.
			′	′
Nov.	7	2	163	74
Dec.	6	2	143	64
Jan.	7	5	150	92
Feb.	6	1	139	31
Mch.	7	3	183	96
Apr.	8	2	193	36
May	9	3	322	55
Total.	50	18	1 293	448

Ratio of easterly to westerly disturbances, 1 to 2·8 for number and 1 to 2·9 for amount. The average disturbance is about 26′, and the time of greatest activity (for easterly deflections) is between 5^h and 9^h and of least activity between 14^h and 20^h. However, the total number is too small and the series too incomplete to place much reliance on this deduction. The extremes of the series are 37° + 2° 42′ and 37° + 6′.

(*6*) *Station in valley of Three Rivers*, in $\varphi = 68° 36'·8$, $\lambda = 141° 00'$ W. Turner, observer. Declination, April, ?, 1890, — 40° 33′.

(*7*) *Station mouth of Firth River*, in $\varphi = 69° 23'·5$, $\lambda = 140° 00'$ W. Turner, observer. Declination, April, ?, 1890, — 43° 22′. This last station is not far from the Arctic Ocean, between Demarcation Point and Herschel Island; an observation of August 16, 1889, at Herschel Island, by officers of the *Thetis*, agrees well with it. By means of these observations the isogonic curves may now be extended as far east as the Mackenzie Bay.†

There may be other observations for declination taken on the Yukon and Porcupine rivers "en route" of which I have no information.

* Here simplified.

† See accompanying isogonic chart of Alaska for 1890, second edition (illustration No. 35), containing this latest information.

ISOGONIC CHART OF ALASKA AND ADJACENT REGIONS FOR THE EPOCH 1890

Scale 1:7 700 000

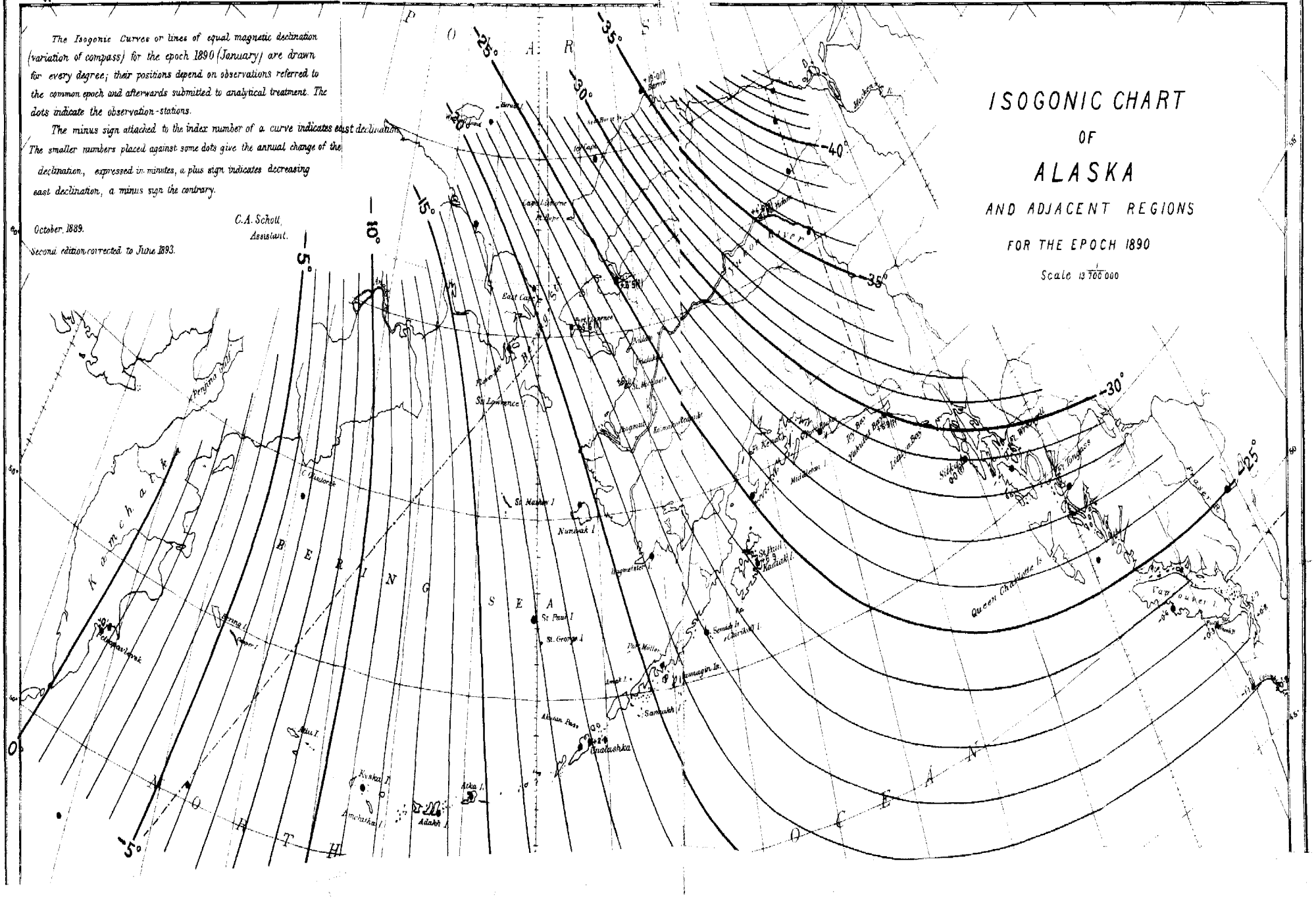
The Isogonic Curves or lines of equal magnetic declination (variation of compass) for the epoch 1890 (January) are drawn for every degree; their positions depend on observations referred to the common epoch and afterwards submitted to analytical treatment. The dots indicate the observation-stations.

The minus sign attached to the index number of a curve indicates east declination. The smaller numbers placed against some dots give the annual change of the declination, expressed in minutes, a plus sign indicates decreasing east declination, a minus sign the contrary.

October, 1889.

C. A. Schott, Assistant.

Second edition corrected to June 1893.



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APPENDIX No. 12—1892.

ON THE DIRECT SYNTHETICAL METHOD OF ADJUSTING A TRIANGULATION.

By CHAS. H. KUMMELL, Computing Division, U. S. Coast and Geodetic Survey.
Submitted for publication July 7, 1893.

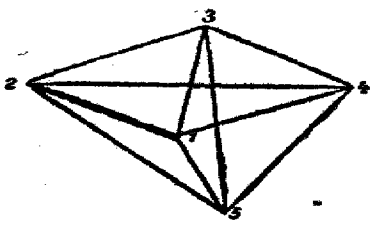
The method in general use for adjusting a triangulation is the indirect one. At each station a station adjustment is made, and then, in order not to disturb the station relations, the further corrections to the angles are expressed in the form of a difference of corrections to its two directions; and the rigorous conditions of the triangulation being thus expressed and multiplied by indeterminate factors, usually called correlates, are then added to the minimum function, i. e., the weighted sum of squares of corrections to the observed directions, before the conditions for the minimum are found by differentiation. This gives, then, the corrections to each direction in terms of the correlates, and by substituting these into the rigorous conditions we obtain the normal equations, which, being solved, give the numerical values of the correlates, and thence also the corrections to the directions, and finally those of the angles.

There are two principal difficulties in the application of this method. One of these is the correct establishment of the rigorous conditions; the inexperienced computer may easily omit an essential condition and use one instead which is derivable from the others. Another difficulty is experienced sometimes by the presence of small angles, which require an extremely precise solution of the normal equations, and it may happen, nevertheless, that the conditions are not as exactly fulfilled as desired. The latter difficulty would be avoided if, instead of using correlates, we eliminate as many corrections as there are rigorous conditions, and then make the weighted sum of the squares of independent and dependent corrections a minimum in the usual manner. The normal equations thus obtained require neither such precision in their formation nor in their solution, because they fulfill only the minimum condition. But it has the difficulty in the establishment of conditions in common with the indirect method and a new, even greater, difficulty besides, viz, the elimination of dependent corrections of directions,

especially from a very large number of conditions. This method, which I call the direct analytical method, to distinguish it from a method free of these three difficulties, the direct synthetical, is given in some books (see, for instance, Jordan's Handbuch der Vermessungskunde, Vol. I, p. 115) as a possible method, not to be used on account of its great complexity for a great number of conditions.

The direct synthetical method, which is the subject of this paper, has an object similar to that of the direct analytical method, viz, to express the corrections to the dependent angles in terms of those to the independent ones; but it does that without establishing the conditions in toto. Whether only one station or a whole triangulation is considered, or a number of new points joined to already fixed ones, a set of fundamental angles is selected, and in the work of expressing the corrections to the dependent or supernumerary angles the proper conditions for this purpose present themselves. A condition thus used needs no further consideration, and by thus removing condition after condition the work takes a strictly prescribed course, differing, however, more or less for a different set of fundamental angles. Thus we avoid completely the danger of duplication or omission of conditions.

As to the number of normal equations to be solved, either mode of the direct method has the advantage generally. It happens only in undeveloped figures that the indirect method leads to fewer normal equations; for, in order to establish n points, $2(n-2)$ angles are sufficient, which number is therefore the number of normal equations required by either mode of the direct methods of adjustment.* If the number of supernumerary angles is $<2(n-2)$ there will be fewer normal equations by the indirect method; but then the triangulation will also be weak. If, however, every possible angle is measured between n points, we have $(n-2)^2$ normal equations to solve by the indirect method, so that the direct method becomes the more advantageous in this respect, the more perfect the triangulation is. It was already stated that in such a triangulation, particularly if n is large, the work of expressing the corrections to the dependent angles in terms of those to independent ones is considered impracticable by the direct analytical method. This may be so; but I shall now show that by the synthetical direct method it is the more systematic the more complete the triangulation is, and take the case of a complete pentagon, in which I suppose at first that the angles at each station are already adjusted. Number the stations 1, 2, 3, 4, 5; denote the measured angles $1_{2\cdot3}$, $1_{2\cdot4}$, $1_{2\cdot5}$, $1_{3\cdot4}$, $1_{3\cdot5}$, $1_{4\cdot5}$, $2_{3\cdot1}$, etc., the corresponding adjusted angles by $[1_{2\cdot3}]$, $[1_{2\cdot4}]$, etc. Each of these angles receives a correction, which may be very conveniently expressed as the difference of the corrections to its two directions, so that we have for example $[3_{4\cdot5}] = 3_{4\cdot5} - 3_4$



* In case n points are joined to a system of m fixed points, then there are $2n$ equations to be solved.

On the direct synthetical method of adjusting a triangulation.

$+ 3_5$, where in an actual numerical instance $3_{4\cdot5}$ is the measured value in arc measure; and if this were a fundamental angle, then this would be also the final form of the correction to be used in the formation of the normal equations.

I shall assume here the angles at 1 and 2 for fundamental angles; then the work may be done in the following form:

$$
\begin{aligned}
[1_{2\cdot3}] &= 1_{2\cdot3} - 1_2 + 1_3 \\
[2_{3\cdot1}] &= 2_{3\cdot1} - 2_3 + 2_1 \\
[3_{1\cdot2}] &= 3_{1\cdot2} + 1_2 - 1_3 + 2_3 - 2_1 + \Delta_{1\cdot2\cdot3} \\
[1_{2\cdot4}] &= 1_{2\cdot4} - 1_2 + 1_4 \\
[2_{4\cdot1}] &= 2_{4\cdot1} - 2_4 + 2_1 \\
[4_{1\cdot2}] &= 4_{1\cdot2} + 1_2 - 1_4 + 2_4 - 2_1 + \Delta_{1\cdot2\cdot4}
\end{aligned}
$$

These two triangles are contained in the quadrilateral 1. 2. 3. 4. and have each two fundamental angles, so that the corrections to the third angles, $3_{1\cdot2}$, $4_{1\cdot2}$, are easily expressed in terms of fundamental angles by closing these triangles, and the numerical quantities $\Delta_{1\cdot2\cdot3}$, $\Delta_{1\cdot2\cdot4}$ are the deficiencies of the measured angles $3^{1\cdot2}$, $4_{1\cdot2}$. The remaining two triangles, 1. 3. 4 and 2. 3. 4, have each only one fundamental angle; and we may now proceed in four different ways, according as we express the corrections to the remaining angles in terms of those of the fundamental angles and of the angles $3_{4\cdot1}$ or $4_{1\cdot3}$ or $3_{4\cdot2}$ or $4_{2\cdot3}$. In the first case we have,

$$
\begin{aligned}
[1_{3\cdot4}] &= 1_{3\cdot4} - 1_3 + 1_4 \\
[3_{4\cdot1}] &= 3_{4\cdot1} - 3_4 + 3_1 \\
[4_{1\cdot3}] &= 4_{1\cdot3} + 1_3 - 1_4 + 3_4 - 3_1 + \Delta_{1\cdot3\cdot4} \\
[2_{3\cdot4}] &= 2_{3\cdot4} - 2_3 + 2_4 \\
[3_{4\cdot2}] &= 3_{4\cdot2} + 1_2 - 1_3 + 2_3 - 2_1 - 3_4 + 3_1 + \Delta_{1\cdot2\cdot3} \\
[4_{2\cdot3}] &= 4_{2\cdot3} - 1_2 + 1_2 - 2_4 + 2_1 + 3_4 - 3_1 + \Delta_{1\cdot3\cdot4} - \Delta_{1\cdot2\cdot4}
\end{aligned}
$$

It remains now to express also the correction to $3_{4\cdot1}$ by corrections to fundamental angles. For this purpose we form any one of the four side equations, preferably* that at pole 4, viz:

$$
1 = \frac{\sin [1_{3\cdot4}]}{\sin [3_{4\cdot1}]} \cdot \frac{\sin [2_{4\cdot1}]}{\sin [1_{2\cdot4}]} \cdot \frac{\sin [3_{4\cdot2}]}{\sin [2_{3\cdot4}]}
$$

We have, then, using above forms and denoting the changes for $1''$ of the logarithmic sines in terms of the seventh decimal place, by placing δ before the angle:

$$
\begin{aligned}
\log \sin [1_{3\cdot4}] &= \log \sin 1_{3\cdot4} - \delta 1_{3\cdot4}(1_3 - 1_4) \\
-\log \sin [3_{4\cdot1}] &= -\log \sin 3_{4\cdot1} + \delta 3_{4\cdot1}(3_4 - 3_1) \\
\log \sin [2_{4\cdot1}] &= \log \sin 2_{4\cdot1} - \delta 2_{4\cdot1}(2_4 - 2_1)
\end{aligned}
$$

* Though theoretically indifferent, it is better as pointed out by Mr. Doolittle to choose the pole opposite the smallest angles.

$$
\begin{aligned}
-\log\sin[1_{2\cdot4}] &= -\log\sin 1_{2\cdot4} + \delta 1_{2\cdot4}(1_2 - 1_4)\\
\log\sin[3_{4\cdot2}] &= \log\sin(3_{4\cdot2} + \Delta_{1\cdot2\cdot3}) + \delta 3_{4\cdot2}(1_2 - 1_3 + 2_3 - 2_1 - 3_4 + 3_1)\\
-\log\sin[2_{3\cdot4}] &= -\log\sin 2_{3\cdot4} + \delta 2_{3\cdot4}(2_3 - 2_4)
\end{aligned}
$$

$$
\begin{aligned}
\therefore \quad 0 = {} & 4_{1\cdot2\cdot3} + (\delta 1_{2\cdot4} + \delta 3_{4\cdot2})1_2 - (\delta 1_{3\cdot4} + \delta 3_{4\cdot2})1_3 + (\delta 1_{3\cdot4} - \delta 1_{2\cdot4})1_4\\
& + (\delta 2_{4\cdot1} - \delta 3_{4\cdot2})2_1 + (\delta 2_{3\cdot4} + \delta 3_{4\cdot2})2_3 - (\delta 2_{4\cdot1} + \delta 2_{3\cdot4})2_4\\
& + (\delta 3_{4\cdot1} - \delta 3_{4\cdot2})(3_4 - 3_1)
\end{aligned}
$$

where $4_{1\cdot2\cdot3}$ denotes the small discrepancy of the side equation in terms of the seventh decimal place. Dividing by $\delta 3_{4\cdot1} - \delta 3_{4\cdot2}$ and transposing we obtain the required correction to $3_{4\cdot1}$, but it is unnecessary to do this analytically, since it would be more compact in a numerical instance. Having obtained this correction, those of $3_{4\cdot2}$, $4_{1\cdot3}$, $4_{2\cdot3}$ easily follow.

We take now the quadrilateral 1. 2. 3. 5, and we need only to go through three of its triangles, thus:

$$
\begin{aligned}
[1_{5\cdot2}] &= 1_{5\cdot2} - 1_5 + 1_2\\
[5_{2\cdot1}] &= 5_{2\cdot1} + 1_5 - 1_2 + 2_1 - 2_5 + \Delta_{1\cdot2\cdot5}\\
[2_{1\cdot5}] &= 2_{1\cdot5} - 2_1 + 2_5
\end{aligned}
$$

$$
\begin{aligned}
[1_{3\cdot5}] &= 1_{3\cdot5} - 1_3 + 1_5\\
[3_{5\cdot1}] &= 3_{5\cdot1} - 3_5 + 3_1\\
[5_{1\cdot3}] &= 5_{1\cdot3} + 1_3 - 1_5 + 3_5 - 3_1 + \Delta_{1\cdot3\cdot5}
\end{aligned}
$$

$$
\begin{aligned}
[2_{3\cdot5}] &= 2_{3\cdot5} - 2_3 + 2_5\\
[3_{5\cdot2}] &= 3_{5\cdot2} + 1_2 - 1_3 + 2_3 - 2_1 - 3_5 + 3_1 + \Delta_{1\cdot2\cdot3}\\
[5_{2\cdot3}] &= 5_{2\cdot3} - 1_2 + 1_3 - 2_5 + 2_1 + 3_5 - 3_1 + \Delta_{1\cdot2\cdot5} + \Delta_{1\cdot3\cdot5}
\end{aligned}
$$

We have now to express the correction to $3_{5\cdot1}$ by means of a side equation, and we take that at pole 5 as most convenient, viz:

$$
1 = \frac{\sin[1_{5\cdot2}]}{\sin[2_{1\cdot5}]} \quad \frac{\sin[3_{5\cdot1}]}{\sin[1_{3\cdot5}]} \quad \frac{\sin[2_{3\cdot5}]}{\sin[3_{5\cdot2}]}
$$

Using the forms already given, we have

$$
\begin{aligned}
\log\sin[1_{5\cdot2}] &= \log\sin 1_{5\cdot2} - \delta 1_{5\cdot2}(1_5 - 1_2)\\
-\log\sin[2_{1\cdot5}] &= -\log\sin 2_{1\cdot5} + \delta 2_{1\cdot5}(2_1 - 2_5)\\
\log\sin[3_{5\cdot1}] &= \log\sin 3_{5\cdot1} - \delta 3_{5\cdot1}(3_5 - 3_1)\\
-\log\sin[1_{3\cdot5}] &= -\log\sin 1_{3\cdot5} + \delta 1_{3\cdot5}(1_3 - 1_5)\\
\log\sin[2_{3\cdot5}] &= \log\sin 2_{3\cdot5} - \delta 2_{3\cdot5}(2_3 - 2_5)\\
-\log\sin[3_{5\cdot2}] &= -\log\sin(3_{5\cdot2} + \Delta_{1\cdot2\cdot3}) - \delta 3_{5\cdot2}(1_3 - 1_2 + 2_3 - 2_1 - 3_5 + 3_1)
\end{aligned}
$$

$$
\begin{aligned}
0 = {} & 5_{1\cdot2\cdot3} + (\delta 1_{5\cdot2} - \delta 3_{5\cdot2})1_2 + (\delta 1_{3\cdot5} + \delta 3_{5\cdot2})1_3 - (\delta 1_{5\cdot2} + \delta 1_{3\cdot5})1_5\\
& + (\delta 2_{1\cdot5} + \delta 3_{5\cdot2})2_1 - (\delta 2_{3\cdot5} + \delta 3_{5\cdot2})2_3 - (\delta 2_{1\cdot5} - \delta 2_{3\cdot5})2_5\\
& - (\delta 3_{5\cdot1} - \delta 3_{5\cdot2})(3_5 - 3_1)
\end{aligned}
$$

This gives, then, the correction to $3_{5\cdot1}$, and thence those to $3_{5\cdot2}$, $5_{1\cdot3}$, and $5_{2\cdot3}$.

On the direct synthetical method of adjusting a triangulation.

Passing now to the quadrilateral 1. 2. 4. 5, we need only go through two of its triangles, thus:

$$[1_{4\cdot5}] = 1_{4\cdot5} - 1_4 + 1_5$$
$$[4_{5\cdot1}] = 4_{5\cdot1} - 4_5 + 4_1$$
$$[5_{1\cdot4}] = 5_{1\cdot4} + 1_4 - 1_5 + 4_5 - 4_1 + \Delta_{1\cdot4\cdot5}$$
$$[2_{4\cdot5}] = 2_{4\cdot5} - 2_4 + 2_5$$
$$[4_{5\cdot2}] = 4_{5\cdot2} + 1_2 - 1_4 + 2_4 - 2_1 - 4_5 + 4_1 + \Delta_{1\cdot2\cdot4}$$
$$[5_{2\cdot4}] = 5_{2\cdot4} - 1_2 + 1_4 - 2_5 + 2_1 + 4_5 - 4_1 + \Delta_{1\cdot2\cdot5} + \Delta_{1\cdot4\cdot5}$$

and to express the correction to $4_{5\cdot1}$ we use the side equation

$$1 = \frac{\sin [1_{5\cdot2}]}{\sin [2_{1\cdot5}]} \cdot \frac{\sin [4_{5\cdot1}]}{\sin [1_{4\cdot5}]} \cdot \frac{\sin [2_{4\cdot5}]}{\sin [4_{5\cdot2}]}$$

which being computed, gives:

$$\begin{aligned}
\log\sin[1_{5\cdot2}] &= \log\sin 1_{5\cdot2} - \delta 1_{5\cdot2}(1_5 - 1_2)\\
-\log\sin[2_{1\cdot5}] &= -\log\sin 2_{1\cdot5} + \delta 2_{1\cdot5}(2_1 - 2_5)\\
\log\sin[4_{5\cdot1}] &= \log\sin 4_{5\cdot1} - \delta 4_{5\cdot1}(4_5 - 4_1)\\
-\log\sin[1_{4\cdot5}] &= -\log\sin 1_{4\cdot5} + \delta 1_{5\cdot4}(1_4 - 1_5)\\
\log\sin[2_{4\cdot5}] &= \log\sin 2_{4\cdot5} - \delta 2_{4\cdot5}(2_4 - 2_5)\\
-\log\sin[4_{5\cdot2}] &= -\log\sin(4_{5\cdot2} + \Delta_{1\cdot2\cdot4}) - \delta 4_{5\cdot2}(1_2 - 1_4 + 2_4 - 2_1 - 4_5 + 4_1)
\end{aligned}$$

$$\begin{aligned}
0 = 5_{1\cdot2\cdot4} &+ (\delta 1_{5\cdot2} - \delta 4_{5\cdot2})1_2 + (\delta 1_{4\cdot5} + \delta 4_{5\cdot2})1_4 - (\delta 1_{5\cdot2} + \delta 1_{4\cdot5})1_5\\
&+ (\delta 2_{1\cdot5} + \delta 4_{5\cdot2})2_1 - (\delta 2_{4\cdot5} + \delta 4_{5\cdot2})2_4 - (\delta 2_{1\cdot5} - \delta 2_{4\cdot5})2_5\\
&- (\delta 4_{5\cdot1} - \delta 4_{5\cdot2})(4_5 - 4_1)
\end{aligned}$$

and thus we obtain the correction to $4_{5\cdot1}$, and then to $4_{5\cdot2}$, $5_{1\cdot4}$, $5_{2\cdot4}$.

There is now one more triangle, 3. 4. 5, which is outside the base 1. 2. Its angles are expressed by means of station relations, thus:

$$[3_{4\cdot5}] = 3_{4\cdot5} - 3_4 + 3_5$$
$$[4_{5\cdot3}] = 4_{5\cdot3} + 1_3 - 1_4 + 3_4 - 3_1 - 4_5 + 4_1 + \Delta_{1\cdot3\cdot4}$$
$$[5_{3\cdot4}] = 5_{3\cdot4} - 1_3 + 1_4 - 3_5 + 3_1 + 4_5 - 4_1 + \Delta_{1\cdot4\cdot5} - \Delta_{1\cdot3\cdot5}$$

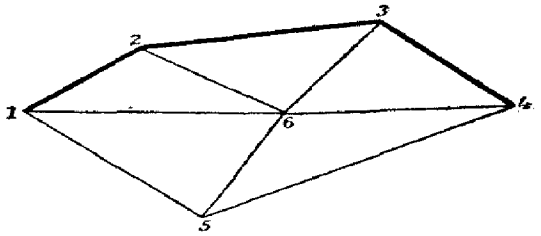
and these are readily expressed in terms of fundamental angles alone.

After having performed the eliminations as indicated, and which are far more simple in a numerical instance, we have the corrections to each angle in the figure in terms of the symbols 1_2, 1_3, 1_4, 1_5, 2_1, 2_3, 2_4, 2_5, apparently eight independent quantities, but in reality six, since only their differences have an absolute value; and to reduce the problem to its essential form we may put, for instance, $1_2 = 2_1 = 0$, but it is advantageous for checking the formation and solution of the normal equations to retain them.

Since we have supposed for the present that at each of the stations a local adjustment has been made, we must derive the weights of each angle from that previous work. The attentive reader will observe, however, that the process, just shown for the complete pentagon, to express the angle corrections in terms of those of fundamental angles, may be applied just as well to the unadjusted station angles, so that it is not at all necessary to make a separate station adjustment. The

details of the work, if no station adjustment is made, I reserve for a future paper.

There is no difficulty whatever of applying the process to a greater number of points; for we have seen that the work on the quadrilaterals 1. 2. 3. 4; 1. 2. 3. 5; 1. 2. 4. 5, far from interfering with each other had, on the contrary, parts in common. For a sixth point we should go through the additional quadrilaterals 1. 2. 3. 6; 1. 2. 4. 6; 1. 2. 5. 6. in the same manner. Then there will be three new extra base triangles 3. 4. 6; 3. 5. 6; 4. 5. 6, which are filled out by regarding station relations only. If some directions have been observed but one way, this makes no difference in the process, except that angles not observed are not expressed, provided there are at least two stations which see all the other stations; for we can then use the angles at these stations as fundamental angles. If, however, this condition is not fulfilled, we can not find the required number of fundamental angles at any two stations, and it may thus be necessary to select them at three or more points. It seems preferable (though for lack of experience this may be subject to future modifications) to select these angles so that all measured angles at the stations make up the required number. If directions are entirely wanting, the process requires some modification, particularly as to the form of side equations, which then often involve more than four points. I select a somewhat typical case in applying the process to the following polygon:



Here we take the angles at 1, 2, 3, 4 for fundamental angles (a good selection would also be the angles at 1, 6, 4), and the work proceeds as follows:

$$[1_{2\cdot6}] = 1_{2\cdot6} - 1_2 + 1_6 \qquad [2_{3\cdot6}] = 2_{3\cdot6} - 2_3 + 2_6$$

$$[2_{6\cdot1}] = 2_{6\cdot1} - 2_6 + 2_1 \qquad [3_{6\cdot2}] = 3_{6\cdot2} - 3_6 + 3^2$$

$$[6_{1\cdot2}] = 6_{1\cdot2} + 1_2 - 1_6 + 2_6 - 2_1 + \Delta_{1\cdot2\cdot6} \qquad [6_{2\cdot3}] = 6_{2\cdot3} + 2_3 - 2_6 + 3_6 - 3_2 + \Delta_{2\cdot3\cdot6}$$

$$[3_{4\cdot6}] = 3_{4\cdot6} - 3_4 + 3_6 \qquad [4_{5\cdot6}] = 4_{5\cdot6} - 4_5 + 4_6$$

$$[4_{6\cdot3}] = 4_{6\cdot3} - 4^6 + 4_3 \qquad [5_{6\cdot4}] = 5_{6\cdot4} - 5_6 + 5_4$$

$$[6_{3\cdot4}] = 6_{3\cdot4} - 3_4 - 3_6 + 4_6 - 4_3 + \Delta_{3\cdot4\cdot6} \qquad [6_{4\cdot5}] = 6_{4\cdot5} + 4_5 - 4_6 + 5_6 - 5_4 \cdot \Delta_{4\cdot5\cdot6}$$

$$[1_{6\cdot5}] = 1_{6\cdot5} - 1_6 + 1_5$$

$$[6_{5\cdot1}] = 6_{5\cdot1} - 1_2 + 1_6 - 2_3 + 2_1 - 3_4 + 3_2 - 4_5 + 4_3 - 5_6 + 5_4 - \Delta_{1\cdot2\cdot6} - \Delta_{2\cdot3\cdot6} - \Delta_{3\cdot4\cdot6} - \Delta_{4\cdot5\cdot6}$$

$$[5_{1\cdot6}] = 5_{1\cdot6} + 1_2 - 1_5 + 2_3 - 2_1 + 3_4 - 3_2 + 4_5 - 4_3 + 5_6 - 5_4 + \Delta_{1\cdot2\cdot6} + \Delta_{2\cdot3\cdot6} + \Delta_{3\cdot4\cdot6} + \Delta_{4\cdot5\cdot6} + \Delta_{1\cdot6\cdot5}$$

On the direct synthetical method of adjusting a triangulation.

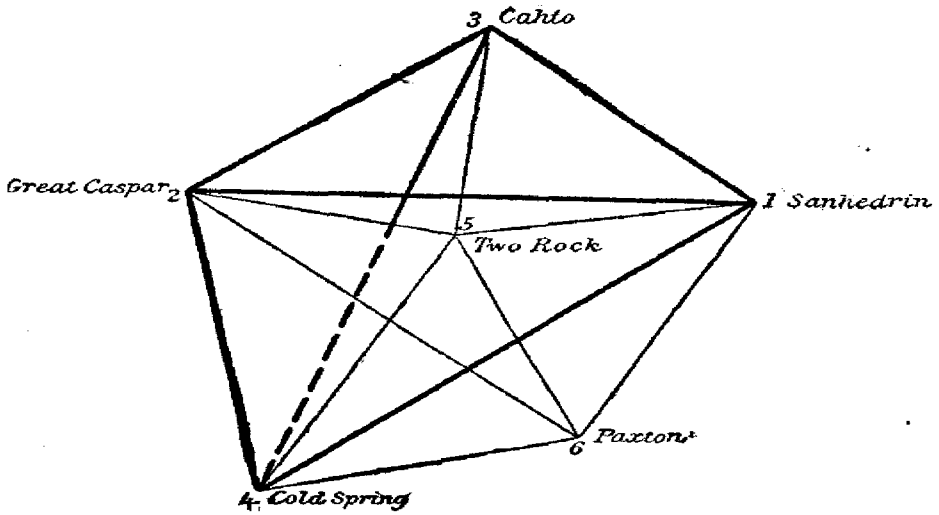
The correction to the non-fundamental angle $5_{6\cdot4}$ is now to be eliminated from these forms by means of the side equation,

$$1 = \frac{\sin [1_{2\cdot6}]}{\sin [2_{6\cdot1}]} \cdot \frac{\sin [2_{3\cdot6}]}{\sin [3_{6\cdot2}]} \cdot \frac{\sin [3_{4\cdot6}]}{\sin [4_{6\cdot3}]} \cdot \frac{\sin [4_{5\cdot6}]}{\sin [5_{6\cdot4}]} \cdot \frac{\sin [5_{1\cdot6}]}{\sin [1_{6\cdot5}]}$$

in which we have to use the forms just given. The expression for the required correction of $5_{6\cdot4}$ is then easily obtained in the manner shown. The forms for the corrections of $6_{4\cdot5}$, $6_{5\cdot1}$, $5_{1\cdot6}$ then follow, and all angles that form triangles have then their corrections expressed in terms of those to the fundamental angles at 1, 2, 3, 4. The expressions for corrections of angles that do not form triangles, such as $1_{2\cdot5}$, $6_{5\cdot3}$, etc., are easily formed. In the minimum sum of squares of corrections we may either exclude or include these latter, and there will always be something arbitrary unless we use angles not locally adjusted. In that case we know not only which angles are measured, but also their precise weight, and everything arbitrary disappears. However, for secondary triangulations such questions have very little importance, and it would hardly be objected to by anyone to include in the minimum sum of squares only the corrections of angles which form triangles. It is easily seen that by using this principle we shall have for a single chain of triangles without any diagonals the old common-sense method, viz, to distribute the closing error equally. These questions, however, form no part of my method, whose object is simply to obtain expressions for the corrections of angles in terms of those of the fundamental angles.

A problem of great importance and frequent occurrence is to join one or more new stations to a system of points already fixed. Since n new points are established by $2n$ angles, we can express the correction of every measured angle in terms of the corrections to $2n$ fundamental angles, and thus we have to form and solve $2n$ normal equations, whatever may be the number of fixed points. In this case the direct synthetical method leads to a certain number of normal equations for any number of fixed points, while for the indirect method the number of normal equations increases with that number.

For a numerical example I shall adjust the following figure, in which the new stations Two Rock and Paxton are joined to the rigid system Cahto, Sanhedrin, Cold Spring, Great Caspar:



The following tables give the station adjusted directions between these stations:

	Observed.	Adjusted.
CAHTO (3).	° ′ ″	″
King Peak.	0 00 00·000	00·00
Sanhedrin.	155 56 24·824	23.51
Two Rock.	203 52 30·678	
Cold Spring.	217 46 28·975	27·53
Great Caspar.	239 10 40·137	38·90
SANHEDRIN (1).		
Reference Mark.	0 00 00·000	00·00
Paxton.	73 54 03·121	
Cold Spring.	84 01 14·479	14·51
Two Rock.	110 28 13·942	
Great Caspar.	120 19 54·734	54·63
Cahto.	164 40 04·538	05·36
TWO ROCK (5).		
Paxton.	0 00 00·000	
Cold Spring.	34 41 51·041	
Great Caspar.	109 13 52·678	
Cahto.	187 03 20·732	
Sanhedrin.	264 55 28·589	

	Observed.	Adjusted.
GREAT CASPAR (2).	° ′ ″	′
King Peak.	0 00 00·000	00·00
Cahto.	38 18 05·97	07·79
Sanhedrin.	90 43 45·79	46·30
Two Rock.	105 10 31·91	
Paxton.	143 35 42·42	
Cold Spring.	174 58 03·10	04·53
COLD SPRING (4).		
Great Caspar.	0 00 00·000	00·00
Two Rock.	35 40 27·72	
Sanhedrin.	59 27 06·62	07·32
Paxton.	80 58 04·52	
Cahto.	21 55	54·64
PAXTON (6).		
Sanel Mt.	0 00 00·000	
Cold Spring.	77 06 48·60	
Great Caspar.	144 46 24·62	
Two Rock.	177 07 21·08	
Sanhedrin.	225 28 40·16	

On the direct synthetical method of adjusting a triangulation.

Assuming the angles at Sanhedrin 1, and Great Caspar 2, as fundamental, we have going through the quadrilateral 1. 2. 3. 5 the following work:

$$\begin{aligned} [1_{2\cdot3}] &= 44\ 20\ 10{\cdot}73 \\ [2_{3\cdot1}] &= 52\ 25\ 38{\cdot}51 \\ [3_{1\cdot2}] &= 83\ 14\ 15{\cdot}39 \\ \text{Excess} &= 4{\cdot}63 \end{aligned}$$

$$\begin{aligned} [1_{5\cdot2}] &= 9\ 51\ 40{\cdot}69 - 1_5 \\ [5_{2\cdot1}] &= 155\ 41\ 35{\cdot}91 + 1_5 + 2_5 - 1.''35 \\ [2_{1\cdot5}] &= 14\ 26\ 45{\cdot}61 + 2_5 \\ \text{Sum} &= 2{\cdot}21 \\ \text{Excess} &= 0{\cdot}86 \end{aligned}$$

$$\begin{aligned} [1_{5\cdot3}] &= 54\ 11\ 51{\cdot}42 - 1_5 \\ [5_{3\cdot1}] &= 77\ 52\ 07{\cdot}86 + 1_5 - 3_5 - 3.''20 \\ [3_{1\cdot5}] &= 47\ 56\ 07{\cdot}17 + 3_5 \\ \text{Sum} &= 6{\cdot}45 \\ \text{Excess} &= 3{\cdot}25 \end{aligned}$$

$$\begin{aligned} [2_{3\cdot5}] &= 66\ 52\ 24{\cdot}12 + 2_5 \\ [3_{5\cdot2}] &= 35\ 18\ 08{\cdot}22 - 3_5 \\ [5_{2\cdot3}] &= 77\ 49\ 28{\cdot}05 - 2_5 + 3_5 + 1.''85 \\ \text{Sum} &= 0{\cdot}39 \\ \text{Excess} &= 2{\cdot}24 \end{aligned}$$

We now take the side equation at pole 5 and in logarithms we have

$$\begin{aligned} \log\sin[1_{5\cdot3}] &= 9{\cdot}9090420 - 15\,(1_5) \\ -\log\sin[3_{1\cdot5}] &= -9{\cdot}8706315 - 19\,(3_5) \\ \log\sin[2_{1\cdot5}] &= 9{\cdot}3970140 + 82\,(2_5) \\ -\log\sin[1_{5\cdot2}] &= -9{\cdot}2336653 + 121\,(1_5) \\ \log\sin[3_{5\cdot2}] &= 9{\cdot}7618453 - 30\,(3_5) \\ -\log\sin[2_{3\cdot5}] &= -9{\cdot}9636174 - 9\,(2_5) \\ 0 &= -129 + 106\,(1_5) + 73\,(2_5) - 49\,(3_5) \end{aligned}$$

$$\begin{aligned} 3_5 &= 2{\cdot}163\,(1_5) + 1{\cdot}490\,(2_5) - 2{\cdot}633 = v\,3_{1\cdot5} = -v\,3_{5\cdot2} \\ v\,5_{3\cdot1} &= -1{\cdot}163\,(1_5) - 1{\cdot}490\,(2_5) - 0{\cdot}567 \\ v\,5_{2\cdot3} &= 2{\cdot}163\,(1_5) + 0{\cdot}490\,(2_5) - 0{\cdot}783 \end{aligned}$$

We next go through the quadrilateral 1. 2. 4. 5:

$$\begin{aligned} [1_{4\cdot2}] &= 36\ 18\ 40{\cdot}12 \\ [4_{2\cdot1}] &= 59\ 27\ 07{\cdot}32 \\ [2_{1\cdot4}] &= 84\ 14\ 18{\cdot}23 \\ \text{Excess} &= 5{\cdot}67 \end{aligned}$$

$$\begin{aligned} [1_{4\cdot5}] &= 26\ 26\ 59{\cdot}43 + 1_5 \\ [4_{5\cdot1}] &= 23\ 46\ 39{\cdot}60 - 4_5 \\ [5_{1\cdot4}] &= 129\ 46\ 22{\cdot}45 - 1_5 + 4_5 + 1{\cdot}11 \\ \text{Sum} &= 1{\cdot}48 \\ \text{Excess} &= 2{\cdot}59 \end{aligned}$$

$$\begin{aligned} [2_{5\cdot4}] &= 69\ 47\ 32{\cdot}62 - 2_5 \\ [5_{4\cdot2}] &= 74\ 32\ 01{\cdot}64 + 2_5 - 4_5 + 0{\cdot}24 \\ [4_{2\cdot5}] &= 35\ 40\ 27{\cdot}72 + 4_5 \\ \text{Sum} &= 1{\cdot}98 \\ \text{Excess} &= 2{\cdot}22 \end{aligned}$$

To express the correction 4_5 we use the side equation at pole 5:

$$\begin{aligned}
\log \sin [1_{5\cdot2}] &= \quad 9{\cdot}2336653 - 121\,(1_5)\\
-\log \sin [2_{1\cdot5}] &= -\,9{\cdot}3970140 - \quad 82\,(2_5)\\
\log \sin [4_{5\cdot1}] &= \quad 9{\cdot}6055083 - \quad 48\,(4_5)\\
-\log \sin [1_{4\cdot5}] &= -\,9{\cdot}6487641 - \quad 42\,(1_5)\\
\log \sin [2_{5\cdot4}] &= \quad 9{\cdot}9724099 - \quad 8\,(2_5)\\
-\log \sin [4_{2\cdot5}] &= -\,9{\cdot}7658011 - \quad 29\,(4_5)\\
\hline
0 &= \quad + 43 - 163\,(1_5) - 90\,(2_5) - 77\,(4_5)
\end{aligned}$$

$$\begin{aligned}
\therefore (4_5) = v4_{2\cdot5} = -\,v4_{5\cdot1} &= -\,2{\cdot}117\,(1_5) - 1{\cdot}170\,(2_5) + 0{\cdot}558\\
v5_{1\cdot4} &= -\,3{\cdot}117\,(1_5) - 1{\cdot}170\,(2_5) + 1{\cdot}668\\
v5_{4\cdot2} &= +\,2{\cdot}117\,(1_5) + 2{\cdot}170\,(2_5) - 0{\cdot}318
\end{aligned}$$

The corrections to the extra base angles in triangle 3. 4. 5 are now easily expressed:

$$\begin{aligned}
[3_{5\cdot4}] &= 13\ 53\ 56{\cdot}85 - 3_5\\
[5_{4\cdot3}] &= 152\ 21\ 29{\cdot}69 + 3_5 - 4_5 + 2{\cdot}09\\
[4_{3\cdot5}] &= 13\ 44\ 33{\cdot}08 + 4_5
\end{aligned}$$

$$\begin{aligned}
\therefore v3_{5\cdot4} &= -\,2{\cdot}163\,(1_5) - 1{\cdot}490\,(2_5) + 2{\cdot}633\\
v5_{4\cdot3} &= +\,4{\cdot}280\,(1_5) + 2{\cdot}660\,(2_5) - 1{\cdot}101\\
v4_{3\cdot5} &= -\,2{\cdot}117\,(1_5) - 1{\cdot}170\,(2_5) + 0{\cdot}558
\end{aligned}$$

The work done so far relates only to the introduction of station Two Rock, 5, into the rigid system and involves only the two unknowns 1_5, 2_5, and thus we accomplish this by only two normal equations, while the indirect method requires six.

Introducing station Paxton 6, the work proceeds as follows:

$$\begin{aligned}
[1_{6\cdot2}] &= 46\ 25\ 51{\cdot}51 - 1_6\\
[6_{2\cdot1}] &= 80\ 42\ 15{\cdot}54 + 1_6 - 2_6 + 1{\cdot}68\\
[2_{1\cdot6}] &= 52\ 51\ 56{\cdot}12 + 2_6\\
\text{Sum} &= 3{\cdot}17\\
\text{Excess} &= 4.85
\end{aligned}$$

$$\begin{aligned}
[2_{6\cdot4}] &= 31\ 22\ 22{\cdot}11 - 2_6\\
[6_{4\cdot2}] &= 67\ 39\ 36{\cdot}02 + 2_6 - 4_6 - 0{\cdot}47\\
[4_{2\cdot6}] &= 80\ 58\ 04{\cdot}52 + 4_6\\
\text{Sum} &= 2{\cdot}65\\
\text{Excess} &= 2{\cdot}18
\end{aligned}$$

$$\begin{aligned}
[1_{6\cdot4}] &= 10\ 07\ 11{\cdot}39 - 1_6\\
[6_{4\cdot1}] &= 148\ 21\ 51{\cdot}56 + 1_6 - 4_6 + 1{\cdot}21\\
[4_{1\cdot6}] &= 21\ 30\ 57{\cdot}20 + 4_6
\end{aligned}$$

On the direct synthetical method of adjusting a triangulation.

The side equation at pole 6 is:

$$\begin{aligned}
\log \sin [1_{6\cdot2}] &= \quad 9{\cdot}8606651 - \quad 20\,(1_6)\\
-\log \sin [2_{1\cdot6}] &= -\,9{\cdot}9015790 - \quad 16\,(2_6)\\
\log \sin [4_{1\cdot6}] &= \quad 9{\cdot}5643811 + \quad 53\,(4_6)\\
-\log \sin [1_{6\cdot4}] &= -\,9{\cdot}2447914 + 118\,(1_6)\\
\log \sin [2_{6\cdot4}] &= \quad 9{\cdot}7165078 - \quad 34\,(2_6)\\
-\log \sin [4_{2\cdot6}] &= -\,9{\cdot}9945814 - \quad 5\,(4_6)\\
\hline
0 &= \quad + 22 + \quad 98\,(1_6) - 50\,(2_6) + 48\,(4_6)
\end{aligned}$$

$$\begin{aligned}
\therefore 4_6 = v4_{2\cdot6} = v4_{1\cdot6} &= -\,2{\cdot}042\,(1_6) + 1{\cdot}042\,(2_6) - 0{\cdot}458\\
v6_{4\cdot2} &= \quad 2{\cdot}042\,(1_6) - 0{\cdot}042\,(2_6) - 0{\cdot}012\\
v6_{4\cdot1} &= \quad 3{\cdot}042\,(1_6) - 1{\cdot}042\,(2_6) + 1{\cdot}668
\end{aligned}$$

$$\begin{aligned}
&\qquad\quad {}^{\circ}\ \ {}'\ \ {}''\\
[1_{6\cdot5}] &= 36\ 34\ 10{\cdot}82 - 1_6 + 1_5\\
[6_{5\cdot1}] &= 48\ 21\ 19{\cdot}08 + 1_6 - 1_5 + 5_1 - 5_6 + 1{\cdot}11\\
[5_{1\cdot6}] &= 95\ 94\ 31{\cdot}41 - 5_1 + 5_6\\
&\qquad\quad \text{Sum} = 1{\cdot}31\\
&\qquad\quad \text{Excess} = 2{\cdot}42
\end{aligned}$$

$$\begin{aligned}
&\qquad\quad {}^{\circ}\ \ {}'\ \ {}''\\
[2_{5\cdot6}] &= 38\ 25\ 10{\cdot}51 - 2_5 + 2_6\\
[5_{6\cdot2}] &= 109\ 13\ 52{\cdot}68 - 1_5 + 2_5 + 5_1 - 5_6 + 1{\cdot}35\\
[6_{2\cdot5}] &= 32\ 20\ 56{\cdot}46 + 1_5 - 2_6 - 5_1 + 5_6 + 0{\cdot}57\\
&\qquad\quad \text{Sum} = 59{\cdot}65\\
&\qquad\quad \text{Excess} = 1{\cdot}57
\end{aligned}$$

The side equation at pole 6 is:

$$\begin{aligned}
\log \sin [1_{6\cdot5}] &= \quad 9{\cdot}7751005 - 28\,(1_6 - 1_5)\\
-\log \sin [5_{1\cdot6}] &= -\,9{\cdot}9982938 + \quad 2\,(5_6 - 5_1)\\
\log \sin [2_{1\cdot6}] &= \quad 9{\cdot}9015790 + 16\,(2_6)\\
-\log \sin [1_{6\cdot2}] &= -\,9{\cdot}8600651 + 20\,(1_6)\\
\log \sin [5_{6\cdot2}] &= \quad 9{\cdot}9750613 - \quad 8\,(-1_5 + 2_5 - 5_1 + 5_6)\\
-\log \sin [2_{5\cdot6}] &= -\,9{\cdot}7933821 + 27\,(2_5 - 2_6)\\
\hline
0 &= -\,2 + 36\,(1_5) - 8\,(1_6) + 19\,(2_5) - 11\,(2_6) + 10\,(5_6 - 5_1)
\end{aligned}$$

$$\begin{aligned}
\therefore 5_6 - 5_1 = v\,5_{1\cdot6} &= -\,3{\cdot}60\,(1_5) + 0{\cdot}80\,(1_6) - 1{\cdot}90\,(2_5) + 1{\cdot}10\,(2_6) + 0{\cdot}20\\
v\,5_{6\cdot2} &= +\,2{\cdot}60\,(1_5) - 0{\cdot}80\,(1_6) + 2{\cdot}90\,(2_5) - 1{\cdot}10\,(2_6) + 1{\cdot}15\\
v\,6_{5\cdot1} &= +\,2{\cdot}60\,(1_5) + 0{\cdot}20\,(1_6) + 1{\cdot}90\,(2_5) - 1{\cdot}10\,(2_6) + 0{\cdot}91\\
v\,6_{2\cdot5} &= -\,2{\cdot}60\,(1_5) + 0{\cdot}80\,(1_6) + 1{\cdot}90\,(2_5) + 0{\cdot}10\,(2_6) + 0{\cdot}77
\end{aligned}$$

There remains now the extra base triangle 4.5.6 to be gone through, the expressions for the corrections of its angles being obtained by station relations only. We have then

$$[4_{5\cdot6}] = [4_{2\cdot6}] - [4_{2\cdot5}] = 45\ 17\ 36{\cdot}80 + 2{\cdot}117\,(1_5) + 1{\cdot}170\,(2_5) - 2{\cdot}042\,(1_6) + 1{\cdot}042\,(2_6) - 1{\cdot}016$$

$$[5_{6\cdot4}] = [5_{1\cdot4}] - [5_{1\cdot6}] = 34\ 41\ 51{\cdot}04 + 0{\cdot}483\,(1_5) + 0{\cdot}730\,(2_5) - 0{\cdot}800\,(1_6) - 1{\cdot}100\,(2_6) + 1{\cdot}468$$

$$[6_{4\cdot5}] = [6_{4\cdot2}] + [6_{2\cdot5}] = 100\ 00\ 32{\cdot}48 - 2{\cdot}600\,(1_5) - 1{\cdot}900\,(2_5) + 2{\cdot}842\,(1_6) + 0{\cdot}058\,(2_6) + 0{\cdot}758$$

Sum $= 0{\cdot}32$
Excess $= 1{\cdot}53$

We now form the following table of observation equations, giving weight 1 to all angles with one new direction and weight 0·5 to those with two new directions.

p	v	1_5	2_5	1_6	2_6	
1	$1_{5\cdot2}$	−1				
1	$1_{5\cdot3}$	−1				
1	$1_{4\cdot5}$	+1				
1	$2_{3\cdot5}$		+1			
1	$2_{1\cdot5}$		+1			
1	$2_{5\cdot4}$		−1			
1	$3_{1\cdot5}$	+2·163	+1·490			−2.633
1	$3_{5\cdot4}$	−2·163	−1·490			+2·633
1	$3_{5\cdot2}$	−2·163	−1·490			+2·633
1	$4_{5\cdot1}$	+2·117	+1·170			−0·558
1	$4_{3\cdot5}$	−2·117	−1·170			+0·558
1	$4_{2\cdot5}$	−2·117	−1·170			+0·558
0·5	$5_{4\cdot2}$	+2·117	+2·170			−0·318
0·5	$5_{4\cdot3}$	+4·280	+2·660			−1·101
0·5	$5_{1\cdot4}$	−3·117	−1·170			+1·668
0·5	$5_{2\cdot3}$	+2·163	+0·490			−0·783
0·5	$5_{2\cdot1}$	+1	−1			−1·35
0·5	$5_{3\cdot1}$	−1·163	−1·490			−0·567
1	$1_{6\cdot2}$			−1		
1	$1_{6\cdot4}$			−1		
0·5	$1_{6\cdot5}$	+1		−1		
1	$2_{1\cdot6}$				+1	
1	$2_{6\cdot4}$				−1	
0·5	$2_{5\cdot6}$		−1		+1	
0·5	$4_{5\cdot6}$	+2·117	+1·170	−2·042	+1·042	−1·016
1	$4_{1\cdot6}$			−2·042	+1·042	−0·458
1	$4_{2\cdot6}$			−2·042	+1·042	−0·458
0·5	$5_{1\cdot6}$	−3·600	−1·900	+0·800	+1·100	+0·200
0·5	$5_{6\cdot2}$	+2·600	+2·900	−0·800	−1·100	+1·150
0·5	$5_{6\cdot4}$	+0·483	+0·730	−0·800	−1·100	+1·468
0·5	$6_{2\cdot1}$			+1	−1	+1·68
0·5	$6_{4\cdot1}$			+3·042	−1·042	+1·668
0·5	$6_{5\cdot1}$	+2·600	+1·900	+0·200	−1·100	+0·910
0·5	$6_{4\cdot2}$			+2·042	−0·042	−0·012
0·5	$6_{2\cdot5}$	−2·600	−1·900	+0·800	+0·100	+0·770
0·5	$6_{4\cdot5}$	−2·600	−1·900	+2·842	+0·058	+0·758

Combining the first eighteen of these equations we obtain

$$0 = 50{\cdot}22\,(1_5) + 27{\cdot}82\,(2_5) - 27{\cdot}08$$
$$0 = 27{\cdot}82\,(1_5) + 22{\cdot}07\,(2_5) - 15{\cdot}57$$

whence

$$(1_5) = +0{\cdot}49$$
$$(2_5) = +0{\cdot}09$$

On the direct synthetical method of adjusting a triangulation.

If we intended to join only the new point Two Rock 5 to the rigid system 1.2.3.4, disregarding for the present Paxton 6, these corrections being applied effect this, and from triangle 1.2.5 we can compute at once geographical positions consistent with every point except Paxton 6; and the filling out of the other triangles is not necessary except for checking or for an estimate of accuracy. Including, however, Paxton 6, we have the normal equations

$$\begin{aligned}
0 = &\quad 73{\cdot}08\,(1_5) + 43{\cdot}84\,(2_5) - 9{\cdot}80\,(1_6) - 4{\cdot}22\,(2_6) - 27{\cdot}47\\
0 = &\quad 43{\cdot}84\,(1_5) + 34{\cdot}92\,(2_5) - 6{\cdot}67\,(1_6) - 4{\cdot}13\,(2_6) - 14{\cdot}73\\
0 = &- 9{\cdot}80\,(1_5) - 6{\cdot}67\,(2_5) + 25{\cdot}13\,(1_6) - 6{\cdot}53\,(2_6) + 7{\cdot}26\\
0 = &- 4{\cdot}22\,(1_5) - 4{\cdot}13\,(2_5) - 6{\cdot}53\,(1_6) + 8{\cdot}68\,(2_6) - 4{\cdot}97
\end{aligned}$$

Solving we obtain

$$\begin{aligned}
(1_5) &= +0{\cdot}46; \quad (1_6) = +0{\cdot}10\\
(2_5) &= -0{\cdot}03; \quad (2_6) = +0{\cdot}85
\end{aligned}$$

These corrections being applied will completely establish the two new points consistent with the rigid system, and it is not necessary, except for checking and an estimate of precision, to compute the other corrections. This will, however, be done, and these are the results:

v	$+0{\cdot}46 = (1_5)$	$-0{\cdot}03 = (2_5)$	$+0{\cdot}10 = (1_6)$	$+0{\cdot}85 = (2_6)$		Corrections.
$1_{5\cdot2}$	—0·46					= —0·46
$1_{5\cdot3}$	—0·46					= —0·46
$1_{4\cdot5}$	+0·46					= +0·46
$2_{3\cdot5}$		—0·03				= —0·03
$2_{1\cdot5}$		—0·03				= —0·03
$2_{5\cdot4}$		+0·03				= +0·03
$3_{1\cdot5}$	+0·99	—0·04			—2·63	= —1·68
$3_{5\cdot4}$	—0·99	+0·04			+2·63	= +1·68
$3_{5\cdot2}$	—0·99	+0·04			+2·63	= +1·68
$4_{5\cdot1}$	+0·98	—0·04			—0·56	= +0·38
$4_{3\cdot5}$	—0·98	+0·04			+0·56	= —0·38
$4_{2\cdot5}$	—0·98	+0·04			+0·56	= —0·38
$5_{4\cdot2}$	+0·98	—0·07			—0·32	= +0·59
$5_{4\cdot3}$	+1·97	—0·08			—1·10	= +0·79
$5_{1\cdot4}$	—1·44	+0·04			+1·67	= +0·27
$5_{2\cdot3}$	+0·99	—0·01			—0·78	= +0·20
$5_{2\cdot1}$	+0.46	+0·03			—1·35	= —0·86
$5_{3\cdot1}$	—0·53	+0·04			—0·57	= —1·06
$1_{6\cdot2}$			—0.10			= —0·10
$1_{6\cdot4}$			—0·10			= —0·10
$1_{6\cdot5}$	+0.46		—0·10			= +0·36
$2_{1\cdot6}$				+0·85		= +0.85
$2_{5\cdot4}$				—0·85		= —0·85
$2_{5\cdot6}$		+0·03		+0·85		= +0·88
$4_{5\cdot6}$	+0·98	—0·04	—0·20	+0·88	—1·02	= +0·60
$4_{1\cdot6}$			—0·20	+0·88	—0·46	= +0·22
$4_{2\cdot6}$			—0·20	+0·88	—0·46	= +0·22
$5_{1\cdot6}$	—1·66	+0·06	+0·08	+0·94	+0·20	= —0·38
$5_{6\cdot2}$	+1·20	—0·09	—0·08	—0·94	+1·15	= +1·24
$5_{6\cdot4}$	+0·22	—0·02	—0·08	—0·94	+1·47	= +0·65
$6_{3\cdot1}$			+0·10	—0·85	+1·68	= +0·93
$6_{4\cdot1}$			+0·30	—0·88	+1·67	= +1·09
$6_{5\cdot1}$	+1·20	—0·06	+0·02	—0·94	+0·91	= +1·13
$6_{4\cdot2}$			+0·20	—0·03	—0·01	= +0·16
$6_{3\cdot5}$	—1·20	+0·06	+0·08	+0·09	+0·77	= —0·20
$6_{4\cdot5}$	—1·20	+0·06	+0·29	+0·05	+0·76	= —0·04

Applying these corrections and computing the triangles we have

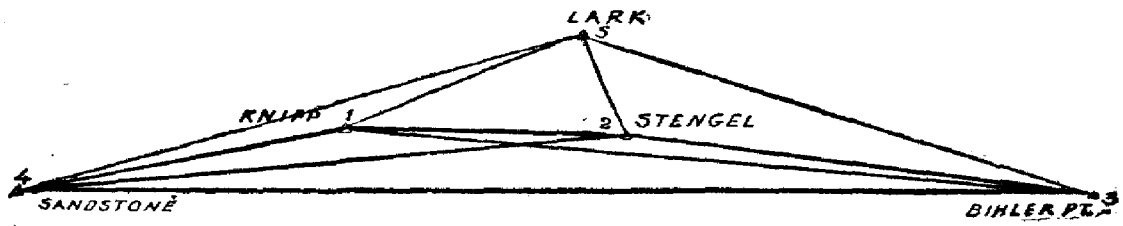
No.	Stations.	△ No.	Observed angles.	Correction.	Spher'l angles.	Spher'l excess.	Plane angles.	Logarithms.
	Great Caspar to Cahto.							4·604548
1	Sanhedrin.	1	44 20 10·73			1·54	44 20 09.19	0·155608
	Great Casper.	2	52 25 38·51			1·54	52 25 36·97	9·899041
	Cahto.	3	83 14 15·39			1·55	83 14 13·84	9·996968
			4·63			4·63		
	Sanhedrin to Cahto.							4·659196
	Great Caspar.							4·757123
	Great Caspar to Sanhedrin.							4·757123
2	Cold Spring.	4	59 27 07.32			1·89	59 27 05·43	0·064896
	Great Caspar.	2	84 14 18·23			1·89	84 14 16·34	9·997800
	Sanhedrin.	1	36 18 40·12			1·89	36 18 38·23	9·772441
			5·67			5·67		
	Cold Spring to Sanhedrin.							4·819819
	Great Caspar.							4·594460
	Great Caspar to Cahto.							4.604548
3	Cold Spring.	4	21 55 54·64			0·92	21 55 53·72	0·427710
	Great Caspar.	2	136 39 56·74			0·91	136 39 55·83	9·836484
	Cahto.	3	21 24 11·37			0·92	21 24 10·45	9·562202
			2·75			2·75		
	Cold Spring to Cahto.							4·868742
	Great Caspar.							4·594460
	Cahto to Sanhedrin.							4·659196
4	Cold Spring.	4	37 31 12·68			2·52	37 31 10·16	0·215360
	Cahto.	3	61 50 04·02			2·52	61 50 01·50	9·945263
	Sanhedrin.	1	80 38 50·85			2·51	80 38 48·34	9·994187
			7·55			7·55		
	Cold Spring to Sanhedrin.							4·819819
	Cahto.							4·868743
	Great Caspar to Sanhedrin.							4·757123
5	Two Rock.	5	155 41 35·91	−0·86	35·05	0·28	155 41 34·77	0 385497
	Great Caspar.	2	14 26 45·61	−0·03	45·58	0·29	14 26 45·29	9·397011
	Sanhedrin.	1	9 51 40·69	−0·46	40·23	0·29	9 51 39·94	9·233656
			2·21			0·86		
	Two Rock to Sanhedrin.							4·539631
	Great Caspar.							4·376276
	Cahto to Sanhedrin.							4.659196
6	Two Rock.	5	77 52 07·86	−1·06	06·80	1·09	77 52 05·71	0·009809
	Cahto.	3	47 56 07·17	−1·68	05·49	1·08	47 56 04·41	9·870626
	Sanhedrin.	1	54 11 51·42	−0·46	50·96	1·08	54 11 49·88	9·909040
			6·45			3·25		
	Two Rock to Sanhedrin.							4·539631
	Cahto.							4·578045
	Great Caspar to Cahto.							4·604548
7	Two Rock.	5	77 49 28·05	+0·20	28·25	0·74	77 49 27.51	0·009881
	Great Caspar.	2	66 52 24·12	−0·03	24·09	0·75	66 52 23·34	9·963617
	Cahto.	3	35 18 08·22	+1·68	09·90	0·75	35 18 09·15	9.761848
			0·39			2·24		
	Two Rock to Cahto.							4·578046
	Great Caspar.							4·376277
	Sanhedrin to Cold Spring.							4·819819
8	Two Rock.	5	129 46 22·45	+0·27	22·72	0·87	129 46 21·85	0·114306
	Sanhedrin.	1	26 26 59·43	+0·46	59·89	0·86	26 26 59·03	9·648762
	Cold Spring.	4	23 46 39·60	+0·38	39·98	0·86	23 46 39·12	9·605506
			1·48			2·59		
	Two Rock to Cold Spring.							4·582887
	Sanhedrin.							4·539631

On the direct synthetical method of adjusting a triangulation.

No.	Stations.	△ No.	Observed angles.	Correction.	Spher'l angles.	Spher'l excess.	Plane angles.	Logarithms.
	Cold Spring to Great Caspar.							4·594460
9	Two Rock.	5	74 32 01·64	+0·59	02·23	0·74	74 32 01·49	0·016020
	Cold Spring.	4	35 40 27·72	−0·38	27·34	0·74	35 40 26·60	9·765798
	Great Caspar.	2	69 47 32·62	+0·03	32·65	0·74	69 47 31·91	9·972409
			1·98			2·22		
	Two Rock to Great Caspar.							4·376278
	Cold Spring.							4·582889
	Cold Spring to Cahto.							4·868743
10	Two Rock.	5	152 21 29·69	+0·79	30·48	0·57	152 21 29·91	0·333538
	Cold Spring.	4	13 44 33 08	−0 38	32·70	0·57	13 44 32·13	9·375763
	Cahto.	3	13 53 56 85	+1·68	58·53	0·57	13 53 57·96	9·380606
			59·62			1·71		
	Two Rock to Cahto.							4·578044
	Cold Spring.							4·582887
	Great Caspar to Sanhedrin.							4·757123
11	Paxton.	6	80 42 15·54	+0·93	16·47	1·61	80 42 14·86	0·005741
	Great Caspar.	2	52 51 56·12	+0·85	56·97	1·62	52 51 55·35	9·901578
	Sanhedrin.	1	46 25 51·51	−0·10	51·41	1·62	46 25 49·79	9·860062
			3·17			4·85		
	Paxton to Sanhedrin.							4·664442
	Great Caspar.							4·622926
	Cold Spring to Sanhedrin.							4·819819
12	Paxton.	6	148 21 51·56	+1·09	52·65	0·46	148 21 52·19	0·280243
	Cold Spring.	4	21 30 57·20	+0·22	57·42	0·45	21 30 56·97	9·564380
	Sandhedrin.	1	10 07 11·39	−0·10	11·29	0·45	10 07 10·84	9·244784
			0·15			1·36		
	Paxton to Sanhedrin.							4·664442
	Cold Spring.							4·344846
	Two Rock to Sanhedrin.							4·530631
13	Paxton.	6	48 21 19·08	+1·13	20·21	0·81	48 21 19·40	0·126516
	Two Rock.	5	95 04 31·41	−0·38	31·03	0·80	95 04 30·23	9·998294
	Sanhedrin.	1	36 34 10·82	+0 36	11·18	0·81	36 34 10·37	9·775099
			1·31			2·42		
	Paxton to Sanhedrin.							4·664441
	Two Rock.							4·441246
	Cold Spring to Great Caspar.							4·594460
14	Paxton.	6	67 39 36·02	+0·16	36·18	0·73	67 39 35·45	0·033885
	Cold Spring.	4	80 58 04·52	+0·22	04·74	0·72	80 58 04·02	9·994581
	Great Caspar.	2	31 22 22·11	−0·85	21·26	0 73	31 22 20·53	9·716502
			2·65			2·18		
	Paxton to Great Caspar.							4·622926
	Cold Spring.							4·344847
	Great Caspar to Two Rock.							4·376277
15	Paxton.	6	32 20 56·46	−0·20	56·26	0·52	32 20 55·74	0·271588
	Great Caspar.	2	38 25 10·51	+0·88	11·39	0·52	38 25 10·87	9·793383
	Two Rock.	5	109 13 52·68	+1·24	53·92	0·53	109 13 53·39	9·975062
			59·65			1·57		
	Paxton to Two Rock.							4·441248
	Great Caspar.							4·622927
	Cold Spring to Two Rock.							4·582888
16	Paxton.	6	100 00 32·48	−0·94	32·44	0·51	100 00 31·93	0·006660
	Cold Spring.	4	45 17 36·80	+0·60	37·40	0·51	45 17 36·89	9·851699
	Two Rock.	5	34 41 51·04	+0·65	51·69	0·51	34 41 51·18	9·755299
			0·32			1·53		
	Paxton to Two Rock.							4·441247
	Cold Spring.							4·344847

It is to be noticed that the indirect method leads to ten normal equations, while we had only four.

A second example is the following:



N. B.—Heavy lines fixed. Base angles 2_5 and $5_{2\cdot1}$

1. Quadrilateral . 2·5·3·1

$$
\begin{array}{lrrrl}
 & \circ & ' & '' & \\
v5_{2\cdot1} = [5_{2\cdot1}] - & 88 & 05 & 45\cdot9 & = -5_2 + 5_1 \\
v2_{1\cdot5} = [2_{1\cdot5}] - & 72 & 50 & 07\cdot1 & = \quad 2_5 \\
v1_{5\cdot2} = [1_{5\cdot2}] - & 19 & 03 & 56\cdot6 & = -2_5 + 5_2 - 5_1 + 10''\cdot4 \\
 & & & 49\cdot6 &
\end{array}
$$

$$
\begin{array}{lrrrl}
 & \circ & ' & '' & \\
v2_{5\cdot3} = [2_{5\cdot3}] - & 128 & 34 & 52\cdot3 & = -2_5 \\
v5_{3\cdot2} = [5_{3\cdot2}] - & 36 & 56 & 55\cdot9 & = -5_3 + 5_2 \\
v3_{2\cdot5} = [3_{2\cdot5}] - & 14 & 28 & 19\cdot1 & = \quad 2_5 + 5_3 - 5_2 - 7''\cdot3 \\
 & & & 07\cdot3 &
\end{array}
$$

$$
\begin{array}{lrrrl}
 & \circ & ' & '' & \\
v5_{3\cdot1} = [5_{3\cdot1}] - & 125 & 02 & 41.8 & = -5_3 + 5_2 + (-5_2 + 5_1) \\
v3_{1\cdot5} = [3_{1\cdot5}] - & 26 & 28 & 31\cdot9 & = \quad 2_5 + 5_3 - 5_2 - \ 7''\cdot3 \\
v1_{5\cdot3} = [1_{5\cdot3}] - & 28 & 28 & 43\cdot2 & = -2_5 + 5_2 - 5_1 + 10''\cdot4 \\
 & & & 56\cdot9 &
\end{array}
$$

Side equation (2). 1. 5. 3.

$$
\begin{array}{lll}
\log 2\cdot3 & = 3\cdot4315930 & \\
\log \sin [5_{2\cdot1}] & = 9\cdot9997630 + \ 0\cdot7\ (-5_2 + 5_1) & \\
\log \sin [3_{2\cdot5}] & \quad 9\cdot3977178 + 81\cdot6\ (\quad 2_5 + 5_3 - 5_2) & \\
 & 2\cdot8290738 &
\end{array}
$$

$$
\begin{array}{lll}
\log 1\cdot2 & = 3\cdot5359530 & \\
\log \sin [1_{5\cdot2}] & = 9\cdot5141493 + 60\cdot9\ (-2_5 + 5_2 - 5_1) & \\
\log \sin [5_{3\cdot2}] & = 9\cdot7789481 + 28\cdot0\ (-5_3 + 5_2) & \\
 & 2\cdot8290504 &
\end{array}
$$

$$
\begin{array}{rl}
0 = & +234 \quad + 142\cdot6\ (2_5) + 61\cdot6\ \ (-5_2 + 5_1) - 109\cdot6\ (-5_3 + 5_2) \\
-5_3 + 5_2 = & v5_{3\cdot2} = +1\cdot301\ (2_5) + 0\cdot562\ (-5_2 + 5_1) + 2\cdot135 \\
 & v5_{3\cdot1} = +1\cdot301\ (2_5) + 1\cdot562\ (-5_2 + 5_1) + 2\cdot135 \\
 & v3_{2\cdot5} = -0\cdot301\ (2_5) - 0\cdot562\ (-5_2 + 5_1) - 9\cdot435 \\
 & v3_{1\cdot5} = -0\cdot301\ (2_5) - 0\cdot562\ (-5_2 + 5_1) - 9\cdot435
\end{array}
$$

On the direct synthetical method of adjusting a triangulation.

2. Quadrilateral . 2. 5. 4. 1

$$\begin{aligned} v5_{2\cdot4} &= [5_{2\cdot4}] - 93\ 21\ 23{\cdot}6 = -5_2 + 5_4 \\ v2_{4\cdot5} &= [2_{4\cdot5}] - 78\ 02\ 57{\cdot}2 = \quad 2_5 \\ v4_{5\cdot2} &= [4_{5\cdot2}] - \quad 8\ 35\ 34{\cdot}1 = -2_5 + 5_2 - 5_4 + 5{\cdot}1 \\ &\qquad\qquad\qquad\quad 54{\cdot}9 \end{aligned}$$

$$\begin{aligned} v5_{1\cdot4} &= [5_{1\cdot4}] - \quad 5\ 15\ 37{\cdot}7 = -5_1 + 5_4 \\ v1_{4\cdot5} &= [1_{4\cdot5}] - 170\ 31\ 18{\cdot}2 = \quad 2_5 - 5_2 + 5_1 - 10{\cdot}4 \\ v4_{5\cdot1} &= [4_{5\cdot1}] - \quad 4\ 13\ 09{\cdot}4 = -2_5 + 5_2 - 5_4 + \quad 5{\cdot}1 \\ &\qquad\qquad\qquad\quad 05{\cdot}3 \end{aligned}$$

Side equation ①. 2. 4. 5

$$\begin{aligned} \log 4{\cdot}1 &= 3{\cdot}6121090 \\ \log \sin [5_{2\cdot1}] &= 9{\cdot}9997630 + 0{\cdot}7\ (-5_2 + 5_1) \\ \log \sin [4_{5\cdot1}] &= 8{\cdot}8668684 + 285{\cdot}3\ (-2_5 + 5_2 - 5_4) \\ &\qquad\qquad 2{\cdot}4787404 \end{aligned}$$

$$\begin{aligned} \log 1{\cdot}2 &= 3{\cdot}5359530 \\ \log \sin [2_{1\cdot5}] &= 9{\cdot}9802127 + 6{\cdot}5\ (2_5) \\ \log \sin [5_{1\cdot4}] &= 8{\cdot}9622918 + 228{\cdot}7\ (-5_1 + 5_4) \\ &\qquad\qquad 2{\cdot}4784575 \end{aligned}$$

$$\begin{aligned} 0 &= +2829 \quad - 292{\cdot}0\ (2_5) + 229{\cdot}4\ (5_1) + 284{\cdot}8\ (5_2) - 514{\cdot}2\ (5_4) \\ &= +2829 \quad - 292{\cdot}0\ (2_5) + 229{\cdot}4\ (-5_2 + 5_1) - 514{\cdot}2\ (-5_2 + 5_4) \end{aligned}$$

$$\begin{aligned} -5_2 + 5_4 = v5_{2\cdot4} &= -0{\cdot}568\ (2_5) + 0{\cdot}446\ (-5_2 + 5_1) + 5{\cdot}517 \\ v5_{1\cdot4} &= -0{\cdot}568\ (2_5) - 0{\cdot}554\ (-5_2 + 5_1) + 5{\cdot}517 \\ v4_{5\cdot2} &= -0{\cdot}432\ (2_5) - 0{\cdot}446\ (-5_2 + 5_1) - 0{\cdot}417 \\ v4_{5\cdot1} &= -0{\cdot}432\ (2_5) - 0{\cdot}446\ (-5_2 + 5_1) - 0{\cdot}417 \end{aligned}$$

$$\begin{aligned} v5_{3\cdot4} &= [5_{3\cdot4}] - 130\ 18\ 19{\cdot}5 = -5_3 + 5_4 \\ v3_{4\cdot5} &= [3_{4\cdot5}] - \quad 34\ 08\ 29{\cdot}0 = \quad 2_5 + 5_3 - 5_2 - 7{\cdot}3 \\ v4_{5\cdot3} &= [4_{5\cdot3}] - \quad 15\ 33\ 13{\cdot}7 = -2_5 + 5_2 - 5_4 + 5{\cdot}1 \\ &\qquad\qquad\qquad\quad 02{\cdot}2 \end{aligned}$$

$$\begin{aligned} v5_{3\cdot4} &= +0{\cdot}733\ (2_5) + 1{\cdot}008\ (-5_2 + 5_1) + 7{\cdot}652 \\ v3_{4\cdot5} &= -0{\cdot}301\ (2_5) - 0{\cdot}562\ (-5_2 + 5_1) - 9{\cdot}435 \\ v4_{5\cdot3} &= -0{\cdot}432\ (2_5) - 0{\cdot}446\ (-5_2 + 5_1) - 0{\cdot}417 \end{aligned}$$

Observation equations.

p	v	2_5	-5_2+5_1		p	v	2_5	-5_2+5_1	
1	$1_{5\cdot2}$	−1	−1	+10·4	1	$4_{5\cdot1}$	−0·432	−0·446	−0·417
1	$1_{5\cdot3}$	−1	−1	+10·4	1	$4_{5\cdot2}$	−0·432	−0·446	−0·417
1	$1_{4\cdot5}$	+1	+1	−10·4	1	$4_{5\cdot3}$	−0·432	−0·446	−0·417
1	$2_{1\cdot5}$	+1			0·5	$5_{2\cdot1}$		+1	
1	$2_{5\cdot3}$	−1			0·5	$5_{3\cdot2}$	+1·301	+0·562	+2·135
1	$2_{4\cdot5}$	+1			0·5	$5_{3\cdot1}$	+1·301	+1·562	+2·135
1	$3_{2\cdot5}$	−0·301	−0·562	−9.435	0·5	$5_{1\cdot4}$	−0.568	−0·554	+5·517
1	$3_{1\cdot5}$	−0·301	−0·562	−9.435	0·5	$5_{2\cdot4}$	−0·568	+0·446	+5·517
1	$3_{4\cdot5}$	−0·301	−0·562	−9.435	0·5	$5_{3\cdot4}$	+0·733	+1·008	+7·652

Normal equations.

$$0 = -19{\cdot}74 + 9{\cdot}12\,(2_5) + 5{\cdot}86\,(-5_2+5_1)$$
$$0 = -\ 8{\cdot}91 + 5{\cdot}86\,(2_5) + 7{\cdot}20\,(-5_2+5_1)$$

$$2_5 = +2{\cdot}87 \qquad\qquad -5_2+5_1 = -1{\cdot}00$$

Comparison of the results by the synthetic method and by the method of correlates.

	By synthetic method.	By correlates.		By synthetic method.	By correlates.
	″	″		″	″
$1_{5\cdot2}$	+8·5	+8·9	$4_{5\cdot1}$	−1·2	−0·9
$1_{5\cdot3}$	+8·5	+8·9	$4_{5\cdot2}$	−1·2	−0·9
$1_{4\cdot5}$	−8·5	−8·9	$4_{5\cdot3}$	−1·2	−0·9
$1_{1\cdot5}$	+2·9	+2·8	$5_{2\cdot1}$	−1·0	−1·3
$2_{5\cdot3}$	−2·9	−2·8	$5_{3\cdot2}$	+5·3	+4·8
$2_{4\cdot5}$	+2·9	+2·8	$5_{3\cdot1}$	+4·3	+3·4
$3_{2\cdot5}$	−9·7	−9·2	$5_{1\cdot4}$	+4·4	+4·6
$3_{1\cdot5}$	−9·7	−9·2	$5_{2\cdot4}$	+3·4	+3·2
$3_{4\cdot5}$	−9·7	−9·2	$5_{3\cdot4}$	+8·7	+7·9

I conclude by giving a summary of the claims in favor of the direct synthetical method of adjustment:

1. It is impossible to omit or duplicate conditions.

2. It becomes the more systematic the more complete the triangulation.

3. It leads to fewer normal equations in strong triangulations.

4. These require far less precision in their formation and solution than those for the indirect method.

5. Joining new points to a number of fixed points, the number of normal equations is independent of the number of fixed points.

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Annual Report of the Superintendent of the Coast Survey

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