REPORT

FROM

THE SECRETARY OF THE TREASURY,

TRANSMITTING

The report of F. R. Hassler, superintendent of the coast survey, and of the fabrication of standard weights and measures.

December 12, 1837.
Laid on the table, and ordered to be printed.

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Ordered, that 886 additional copies be furnished for the use of the Senate.

Treasury Department,
December 6, 1837.

Sir: I have the honor herewith to transmit to you a copy of the report made to this department by Professor Hassler, superintendent of the coast survey, and of the fabrication of standard weights and measures, showing the progress made during the past year in those important works.

I am, very respectfully,
Your obedient servant,
LEVI WOODBURY,
Secretary of the Treasury.

Hon. Richard M. Johnson,
President of the Senate.

Sixth report of F. R. Hassler, as superintendent of the survey of the coast of the United States, and the construction of standards of weights and measures, rendering account of the works of 1837.

1. As soon as the arrangements made last spring for the compensation of all the persons employed in the two works under my charge were completed, I immediately made the organizations of the works for the summer, and proposed such additional appointments as the state of the works required. A number of plane-table parties were organized, and sent out, sufficient to fill up the intermediate parts stated in my last report as necessary to be completed, and to extend them farther easterly and southerly, as much as the time and weather would allow. Thus, nine parties were

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gradually put in activity, from the latter part of April onwards, some of which acted at first jointly, until they were all equally provided with instruments; at the completion of which the mechanicians were engaged in the workshop established for that purpose in the office at Washington.

2. The two parties for secondary triangulations were occupied like last year: the one in New Jersey, towards the Delaware; the other over Long Island sound; in continuation of the plan which I had originally formed to accelerate the work by this distribution of the parties to both sides; both in extension of their preceding work.

3. The two sounding parties were equally continued, upon the plan stated in my last report: the one in the neighborhood of New York, the other in Long Island sound.

4. For a long time, the season was so eminently contrary to the field operations, that the exertions of all these parties were not rewarded with the deserved success; and they uniformly stated that it would be entirely impossible to attempt any thing for the primary triangulation.

5. During this period I employed, therefore, my time to the finishing and adjusting of the system of ounce weights for the mints, which I had the honor to present to the Treasury Department in the middle of June. Upon these works I present, herewith, a special paper, rendering account of the new method which I have employed in their adjustment. Leaving Washington shortly after, I delivered two sets of these weights to the United States Mint in Philadelphia. In New York, I attended to a number of objects relating to the procuring of future necessary means for the construction of standards, and for the forming, and new equipping, of the party for my own field-works, as I had allowed the plane table parties to avail themselves of my former field equipment, in whatever might fit them, in order to establish themselves so much the quicker.

6. During the mechanical operations which this required on the part of the persons in whose duty they lie, I took the first leave of absence on private business, which I have taken since 1829, when my connexions with the Government began again, directing the camp of my next station to be formed during my absence; which was all properly done. But, though I was shortly after at the station, put the instrument in proper order for the work, and attempted to observe, it was not until the 1st of September, exactly, that the weather allowed actual observations; and from thence onwards it has proved very favorable, until now, when, for the plane-table works, winter overtakes us. The temperature of the season is rather below what can be endured in that kind of work, and these parties have broken up, one after the other, in proportion as their work admits of interruption.

7. The results of all these operations have been the following:

8. For the main triangulation those stations have been occupied by me, which bind up the station points of the works of 1833, and 1834, and unite the secondary triangulation of the eastern side of Long Island and Rhode Island on the east, and of the Jerseys, below the bay of New York, on the southwest, with proper extensions.

The new theodolite of thirty inches diameter, which was used for the measurement of these angles, has proved of great advantage for the accuracy, by its stability; in which, I presume, it exceeds any instrument ever constructed for field-work, and its great optical power of telescope and microscopes.

9. Heliotropes, of which I had begun the use last fall, have this year been used for most of the station-points, and for the base-points exclusively. I caused one to be constructed in our shop last winter, after the two received the previous summer from Gottingen, by the kind assistance of Professor Gauss, *the inventor of this instrument; and during my work* this summer I received four more: all (seven) are now in activity. The aim of the instrument is to reflect the sun's image from the station-point, at which they are placed, to the observer on his station, thereby perpetuating, for any time required, or allowed by the sunshine, the reflection of the sun's rays, which is otherwise given by my tin signals, at that certain time, for which the angle of the cone is constructed, as I have heretofore explained.

10. These new instruments require a man of some intelligence to attend to them, and to replace them about every four minutes, according to the motion of the sun; the acceleration of the work, which they procure, compensates this additional expense manifold. They will show a precise luminous point, even through the haze so frequent on our eastern sea shore, when the outline of the hill itself, upon which they stand, cannot be traced. It is probable to me that, without them, almost no distant triangulation would be possible on the farther eastern shores, without a very great loss of time.

11. The secondary triangulation, east of this, has been continued, joining the parts reported upon heretofore, and extending farther to Block island, and over the shores of Rhode Island, and onwards, grounding upon the eastern side of the main triangulation made for its guide. In a similar manner the secondary triangles south, in New Jersey, have been properly multiplied, between the sea shore and the Delaware, and extending southerly to the neighborhood of Philadelphia, grounding upon the lines of the main triangulation, laid out for that purpose, between Navesink and the neighborhood of Springfield mountain, which had already been a station point of my triangulation in 1817.

12. By the plane table works, all the topography which remained to be filled up between the parts of Jersey, on Navesink, Staten island, &c., and the shore of the sound, on the main land of New York, Long island, and Connecticut, upon which I reported last year, has been entirely filled up. The proper points were furnished all along to guide the operations of the sounding parties. The part about New York has been extended up the North river, to about three miles above King's Bridge. Whether it will be proper to extend it up, till it includes Tappan bay, as I thought at first, I shall nearer investigate this winter. Likewise, the interior of Long island, between the two shore parts, has been filled up. Both parts of the shore of Long Island sound have been further surveyed, as far as Milford, in Connecticut, and to Drowned Meadows, on Long island; the topographical parties furnishing the data to ground the sounding operation upon, as they went along, in the same manner as they had received their guide from the primary and secondary triangulations. They extended more or less in the interior of Connecticut, towards the points of the main triangulation, lying in the rear, as circumstances admitted.

13. In the course of the coming winter, I shall cause all these works, with some of the principal soundings, to be brought together, on a reduced scale, to a sketch map; like I have done in former years, which will show

again the state of the whole work, at this time; and, by its comparison with the similar sketches of the former years, the progress which the work has made in each year. As this can never be done until all the works have been collected in Washington, and reduced for that purpose, these sketches can never be presented with the yearly reports, similar to the present, as they fall in a time when all is yet in full activity otherwise. They can, besides that, in any case, be no more than a register of the work, as it progresses.

14. Already the above shows: that the naval part has kept pace with the topographical parts, as they always work together, in assorted parties. The two hydrographic parties have worked up all the soundings between the shores quoted above, as surveyed by the topographical parties, from which they receive always the points, upon which their determinations must be grounded, in proportion as they progress; for the water itself admits no fixed points, affording any security for the hydrographic works.

15. When I first directed Lieutenant Gedney to take up the soundings of the south side of Long island, and the entrance of New York harbor, I directed his attention to two subjects of importance, then not yet ascertained, but which I had sufficient reason to suppose would be verified; namely: 1st. The existence of a westerly current along the south side of Long island, and another coming, at certain times most likely only, from the Jersey shore of Long branch, and that neighborhood, towards that part of Long island, where I measured the base line, both dangerous to navigation, at very important points. 2d. The existence of a channel into the harbor of New York, north of the one habitually used; though I could form no accurate idea of its depth and stability.

16. The first of these suppositions has not yet been fully investigated, on account of the nature, and extent, of the works hitherto performed, as it will require different operations; but the second has succeeded, even beyond my expectations, as has already become public since my last report, on account of its importance and great value, for the so highly important harbor of New York. Lieutenant Gedney found a channel that admits, even at low water, every size of merchant vessels. This channel has already been buoyed out for service in future; and the passing of the Ohio, 74 gun ship, through it, is a fact of public notoriety.

17. That such a valuable discovery, which appeared to lie so near, was not made earlier, is to be attributed simply to the manner in which nautical surveys have generally been made. Without sufficient accurate fixed points on the shore, which the other works of the coast survey furnished, such a discovery was *impossible;* the most experienced and attentive seamen might have sailed about this channel ever so often, without being able to ascertain the fact; because the place of his vessel, at any time, presents him only an insulated point, disconnected with other parts, and even to a number of such points he is unable to assign a direction sufficiently accurate to *aver* any such facts; such discoveries can only be the result of a systematic work, grounded upon full mathematical principles, as are applied in our works.

18. Lieutenant Blake, commanding the hydrographical party in Long Island sound, has also made discoveries, which, for the same reason just stated, had remained unknown; he discovered ledges of rocks, hitherto unknown, very dangerous to the navigation, particularly in times of low

tide; and the Long Island sound has presented, in various places, a widely different form under water from what it was supposed, though, in other instances, his works have confirmed the former statements of the latest maps. It is very probable that the survey of the eastern part, which has far more accidental variations, will present more novelty; but whether in finding something new, or confirming old statements, these surveys have the essential, and above all other valuable, property of accuracy, and certainty of statements, which it is well known is not credited as the property of the maps now in use.

19. With reference to the appropriation which it is necessary to have disposable for the ensuing year, I have the honor to state: that the nature of the work, in its present situation, requires the continuance of the same number of parties, of each kind, as have been employed the present year, viz: nine topographical parties, two secondary triangulation, two sounding parties, and the establishment for the main triangulation.

20. In my report of 22d November, 1835, I have given some details of estimates of each such party, so that it may be unnecessary here to repeat details. What at that time was stated for the naval parts and instruments, may now be considered as about balancing the increased number of plane-table parties: though the present state of prices, the necessary bettering of the wages and compensations, and other similar circumstances, have increased their expenses.

It is much to be doubted whether the remaining balance of this year's appropriation will fully suffice for the equipment of the whole establishment next spring, and the expenses that must be incurred before the next appropriation will likely be disposable.

21. The total estimate for 1836, above quoted, amounted to $87,300; circumstances had occasioned: that a considerable balance of it remained for the use of this year, which, added to the last appropriation, made it possible this year to prosecute the work vigorously, as has been done. At least an equal sum will, therefore, be necessary for the next year, under the supposition that the session of Congress, after the next, will be short, and, therefore, the appropriation be made in March.

22. I take the liberty, upon these grounds, to suggest; that you would please to propose an appropriation of $90,000, to be made by Congress, with a view to prosecute the work upon the same scale, upon which I have made this year the experience: that it is most advantageous to prosecute it; I may even assert, the most economical, proportionally to the result, that can be reached by it.

23. The work appears to gain so much interest and credit, that it must be expected the anxiety of the public to enjoy its results will increase, while, however, it is neither proper, nor possible, to attempt to satisfy it, before such a mass of work will be done as the aim of the greatest public advantage in it requires. With these views, I shall already this winter cause a considerable part of the work to be brought together, to enable me to combine, as early as admissible, plans for the future creditable publication, under the proper authority of the Government.

24. Upon the establishment for the construction of standard weights and measures in general, I have the satisfaction to state: that all the weights for the custom-houses, States, and Territories, to the number stated by you, and implied by the resolution of Congress, are now fully executed as to the work-shop parts. My assistant in that work in Washington is now engaged in ap-

proximating their adjustment; for which purpose he has constructed, in the establishment itself, a lever balance, especially appropriated for the larger weights, which has fully succeeded; there was none of that kind on hand, nor could there have been bought one sufficiently accurate for our purpose. I have also caused glass bulbs and jars to be blown in Brooklyn, where those had been made which served for my first comparison, (rendered account of in 1832,) of such sizes as will serve for the largest mint weights, the gallons, &c. Another lever balance, also for larger weights, is constructing now in the mathematical instrument maker's shop of the office.

25. When I return to Washington, it will be my first assiduous task, during day, to attend to this final adjustment of weights. Last spring, already I began it, and completed a number of the smaller weights, from which I shall continue and proceed on. This will refer those calculations for the coast survey, which fall to my lot, for my work by candle light, when the work of adjusting weights, or any other standards, *cannot be done*. I hope to be able to do justice to both works.

26. The greatest part of the standards of yards are brought to the same approximate state of adjustment. It is desirable that each of them be provided with a divided scale, upon the matrix part of the standard, by which common measures may be adjusted; my assistant has therefore made upon all those yards that are in that state of forwardness, such a scale, divided decimally, as is legally introduced in the custom-houses. It will be necessary to give special charge in every place where such a yard will be deposited, that the actual standard part of it, which is the yard butted between the two projecting parts of a matrix, shall be used only for the most important comparisons; while the yards of the measurers in the custom-houses, the yard-sticks for stores, and all such like measures for common use, are to be adjusted, or compared, upon this divided scale; for this purpose, it is properly inscribed and numbered.

27. The capacity measures for liquids are now under hand in the workshops. They require, of course, more mechanical labor, as well as skill and care, to give them the necessary accuracy. They advance as rapidly as can be expected, and upon their execution, in a mechanical or artistical point of view, evidence is given by the samples delivered to the Treasury Department; like upon the weights and yards by the other standards of each kind that have been delivered.

28. The separate report, upon the methods employed for the establishment and adjustment of ounce weights for the mints, is hereto subjoined.

F. R. HASSLER.

HARROW HILL, NEAR HEMPSTEAD HARBOR, L. I.,
November 18, 1837.

Report upon the establishment of the system of ounce weights for the mints of the United States.

1. The coinage, and other operations, of the mint of the United States are grounded upon the ounce weights, in decimal progression, both for higher and lower weights. They form, therefore, anomalous weights with the pound troy, having only this pound as single reference of their mutual relation. It appears from the sample of the standards in the State Department, received from England, that these ounce weights are generally not made with sufficient accuracy to admit reliance upon them; so much so, that the British mint itself had to announce a depreciation of its old coins, grounded upon the difference between its earlier weights and the newer ones, after the last revision of the British standards. Testimony of this has been communicated to the United States Mint, whose old coins will undoubtedly be equally affected, for the times before the adoption of the new pound, declared standard by Congress in 1828, which excluded, of course, also, the use of the old ounce weights, which bear no simple and commensurate relation towards that pound.

2. The establishment for the construction of standards of weights and measures for the States and custom-houses, presenting the best means for the mint to procure accurate weights, the director of the mint applied to the Treasury Department to direct: that there should be made in that establishment several sets of ounce weights, from one thousand ounces down to the tenths of an ounce, in a decimal scale. The honorable the Secretary of the Treasury directed me, therefore, to construct them, under date of the 13th of May, 1836.

3. It is of course desirable for the public as well as for Government, that a public account be rendered of the means and methods, by which the weights, which were constructed in conformity to this direction, have been adjusted to their proper standards. This consideration induces me to present, herewith, such a report upon the operations, by which I obtained the aim, as may serve to judge upon the accuracy aimed at and attained in that operation; as well as upon the confidence, that can be placed upon them, and upon the operations that may be performed with them.

4. The description of the actual mechanical construction of these weights cannot be the object here; it belongs to the general account upon all the operations performed for the general aim of procuring uniform standards of weights and measures for the States and custom-houses, which will, of course, be rendered in proper time. Only the mathematical principles applied to obtain accuracy in this individual case, and the manner in which they were used, are here to be presented.

5. It is evident that the task implied the establishment of a full new system of weights, for which no other guide was given but the troy pound, to correspond to twelve ounces of that system; from which, therefore, all the parts of the new dicimal system were to be deduced. It is well known, by all the accounts of similar operations in Europe, that they present scope for labor of great attention and assiduity, and requiring various combinations of means.

6. It is easy to *multiply* a weight, the unit of which is given; but it is not

so with the division of it into any parts without the assistance of accessory weights, upon which it is not allowed to rely in such an operation; thence the operation must be carried on in such a manner, as to render the ultimate results independent of any such medium, as it may have been necessary to make a temporary use of at the first outset. The ultimate result, in the case under view, must always be; that such a number of ounces, or parts of ounces, must again represent that pound, when their nominal amount is the twelve ounces, which it represents; and also the coinciding multiples of the pound, and of the new system, must again be equal in their commensurate numbers.

7. These considerations led me to the application of a system of combined weighing, and more particularly by binary and ternary combinations of equal weights, to a certain given or averred weight, equal to their sum. It must, of course, be extended to such a number of individuals as may be found most advantageous, according to the disposable means, to produce a certain number of results, to take a mean of them for each individual; to this the number of weighings is proportional, and naturally determined. These combined weighings, keeping account of the sum of the errors of the two or three individual weights employed in each weighing, give, therefore, the means to determine such small differences even, as the balance would otherwise not be able to show, their sum being determinable in their varied combinations; and the value of each individual weight becomes evidently determined by the solving of the obtained results of the weighings into those of the individual values; it gives, at the same time, by the differences which these results may show, the measure of the accuracy of each weighing operation.

8. It appears proper to show here, in one continued system, the different schemes of combination, and the manner of applying them, which I have put into practice. I had already made use of a combinatory method in determining the length of a number of French metres, of which I have rendered account, first in my papers upon the coast survey, and afterwards in my report upon the comparisons of standard weights and measures, rendered in 1832. I have always found it equally valuable and successful.

9. In the binary combination it is evident: that three quantities combined, 2 by 2, will give in result a value for each, by means of the three sums which they present; suppose them a, b, c, the binary weighings which they give for our use are

$$\begin{array}{l} 1\text{st},\ a+b \\ 2\text{d},\ a+c \\ 3\text{d},\ b+c \end{array}$$

and when any two of these are added, and the third subtracted from the sum, each such operation will give in remainder the double value of that one of them, which is placed twice in the sum; thus, evidently

$$\begin{array}{l} (a+b+a+c)-(b+c)=2\ a,\ \text{or}=1\text{st}+2\text{d}-3\text{d}\ \text{weighing.} \\ (a+b+b+c)-(a+c)=2\ b,\ \text{or}=1\text{st}+3\text{d}-2\text{d}\quad\ \text{“} \\ (b+c+a+c)-(b+a)=2\ c,\ \text{or}=2\text{d}+3\text{d}-1\text{st}\quad\ \text{“} \end{array}$$

so that each individual is determined by dividing by 2 the remainder obtained. Four quantities thus combined, give, by means of 6 weighings, two results of each one of them, &c.

10. The advantage gained by increasing the number of objects (here

weights) increases very rapidly; but I found it most advantageous, for reasons which will appear, habitually to limit that number to 5 weights; their binary combination gave me 10 weighings, and these furnished 6 results for each of the individual weight. If the weight is represented by W, and the several individuals are distinguished by successive accentuation, they will present the following scheme of weighings:

	Weights combined.	Complementary weight.
No.	$1 = W + W^1 =$	—0,010
"	$2 = W + W^2 =$	—0,010
"	$3 = W + W^3 =$	—0,014
"	$4 = W + W^4 =$	—0,018
"	$5 = W^1 + W^3 =$	—0,005
"	$6 = W^1 + W^3 =$	—0,000
"	$7 = W^1 + W^4 =$	—0,005
"	$8 = W^2 + W^3 =$	—0,010
"	$9 = W^2 + W^4 =$	—0,018
"	$10 = W^3 + W^4 =$	—0,000

The weight, representing such a sum of two weights, being equilibrated, each of these weighings will present a certain small difference, expressed in decimals of the grain, as I have here represented by the example of one operation. The additions and subtractions of the result, (the absolute weight that is aimed at in the weighings evidently disappearing in the calculation,) in the manner above stated, furnish the different results for each. The weighings being expressed by their successive numbers, the following series of them contains their combination, and the sequel of results which they present for each individual weight, thus:

No. of the weighings.	Weights engaged.	
(1+2)— 5	$= (W + W^1 + W + W^2) - (W^1 + W^2) =$	
(1+3)— 6	$= (W + W^1 + W + W^3) - (W^1 + W^3) =$	
(1+4)— 7	$= (W + W^1 + W + W^4) - (W^1 + W^4) =$	$2.W$
(2+3)— 8	$= (W + W^2 + W + W^3) - (W^2 + W^3) =$	
(2+4)— 9	$= (W + W^2 + W + W^4) - (W^2 + W^4) =$	
(3+4)—10	$= (W + W^3 + W + W^4) - (W^3 + W^4) =$	

In exactly similar manner as here represented for the first weight = W results for all the others. W^1, W^2, &c., are obtained, by the combination of the following weighings, which I will quote only by their numbers, thus:

(1st + 5th)— 2d		(2d + 5th)— 1st	
(1 + 6)— 3		(2 + 8)— 3	
(1 + 7)— 4	$= 2.W^1$	(2 + 9)— 4	$= 2.W^2$
(5 + 6)— 8		(5 + 8)— 6	
(5 + 7)— 9		(5 + 9)— 7	
(6 + 7)—10		(8 + 9)—10	
(3 + 6)— 1		(4 + 7)— 1	
(3 + 8)— 2		(4 + 9)— 2	
(3 + 10)— 4	$= 2.W^3$	(4 + 10)— 3	$= 2.W^4$
(6 + 8)— 5		(7 + 9)— 5	
(6 + 10)— 7		(7 + 10)— 6	
(8 + 10)— 9		(9 + 10)— 8	

If, by way of example, the weighings quoted above are taken into calculation, according to this scheme, the following will be the six individual results for each weight, and their ultimate resulting means:

$$W = \left\{ \begin{matrix} -0{,}0075 \\ -0{,}0120 \\ -0{,}0115 \\ -0{,}0070 \\ -0{,}0050 \\ -0{,}0160 \end{matrix} \right\} \text{mean.} = -0{,}0098 \qquad W^1 = \left\{ \begin{matrix} -0{,}0025 \\ +0{,}0020 \\ +0{,}0015 \\ +0{,}0025 \\ +0{,}0040 \\ -0{,}0025 \end{matrix} \right\} \text{mean.} = +0{,}0009$$

$$W^2 = \left\{ \begin{matrix} -0{,}0025 \\ -0{,}0030 \\ -0{,}0050 \\ -0{,}0075 \\ -0{,}0090 \\ -0{,}0140 \end{matrix} \right\} = -0{,}0067 \qquad W^3 = \left\{ \begin{matrix} -0{,}0020 \\ -0{,}0070 \\ +0{,}0020 \\ -0{,}0025 \\ +0{,}0025 \\ +0{,}0040 \end{matrix} \right\} = -0{,}0005$$

$$W^4 = \left\{ \begin{matrix} -0{,}0065 \\ -0{,}0130 \\ -0{,}0020 \\ -0{,}0090 \\ -0{,}0025 \\ -0{,}0040 \end{matrix} \right\} = -0{,}0061$$

Excess of the 5 weights over the true sum.

$$\left\{ \begin{matrix} W & -0{,}0098 \\ W^1 & +0{,}0009 \\ W^2 & -0{,}0067 \\ W^3 & -0{,}0005 \\ W^4 & -0{,}0061 \end{matrix} \right\} \text{grain.} = -0{,}0212$$

I found it most advantageous to use the signs so: that the weight aimed at should be represented by the weight compared, when the difference is applied with the sign given to it; thus, the sign — indicates the individual weight to be too large, and the sign + would designate it too small, for the amount following such sign.

11. The above example is taken from the 6 oz. weights, which required, therefore, always one pound troy in each basin of the balance. Their sum represents, therefore, 30 oz. and shows —0,0212 gr. total overweight. When weighed together, comparatively with another authentically made out 30 oz. weight, or $2\frac{1}{2}$ pounds, the result found was = —0,0222, therefore only 0,0010 gr. different, (less than one fourteen-millionth part of the weight,) the sum itself deviating less than one seven-millionth part of its weight from the standard, and the mean of them would present for each yet an excess of = —0,0035 grain, or about $\frac{1}{\text{0 0 0 0 0 0 0}}$ of the weight. That such proportions are only calculable, and not physically discernible, will be clear to every man in the least acquainted with such subjects; in fact, the simple wiping with an oiled leather, as is often done for a pretended cleaning, would take off a much greater weight. (Still W, which showed the largest, was still somewhat reduced, though it is in general rather too precarious to be advisable to do so when weights are so near.)

12. The ternary combination by five pieces, or weights, which I have made use of, presents the following scheme, when presented like the binary, except denominating, for more ease, the five pieces successively by a, b, c, d, e. They present the following scheme of

WEIGHINGS.

No. of the weighing.		Weight combined.
1.	=	a+b+c
2.	=	a+b+d
3.	=	a+b+e
4.	=	a+c+d
5.	=	a+c+e
6.	=	a+d+e
7.	=	b+c+d
8.	=	b+c+e
9.	=	b+d+e
10.	=	c+d+e

To solve this combination, for the obtaining of the four results, which it furnishes for each individual, it needs the addition of three of these weighings, and the subtraction of the double of a fourth, which are so selected as to give in their remainder the triple of that individual, which it is intended to determine; this will stand thus, under the same form as the preceding:

No. of the weighing.	Value of the combination.	
(1+2+4)—2.7	=(3.a+2.b+2.c+2.d)—2.(b+c+d)=	3.a
(2+3+6)—2.8	=(3.a+2.b+2.d+2.e)—2.(b+d+e)=	
(1+3+5)—2.9	=(3.a+2.b+2.c+2.e)—2.(b+c+e)=	
(4+5+6)—2.10	=(3.a+2.c+2.d+2.e)—2.(c+d+e)=	

The other results are obtained in the same manner, by the solution of the following combined weighings:

(1+2+7)—2.4 =	3.b	(2+4+ 7)—2.1=	3.d
(1+3+8)—2.5 =		(2+6+ 9)—2.3=	
(2+3+9)—2.6 =		(4+6+10)—2.5=	
(7+8+9)—2.10=		(7+9+10)—2.9=	
(1+4+ 7)—2.2=	3.c	(3+5+ 8)—2.1=	3.e
(1+5+ 8)—2.3=		(3+6+ 9)—2.2=	
(4+5+10)—2.6=		(5+6+10)—2.4=	
(7+8+10)—2.9=		(8+9+10)—2.7=	

The room for giving an example may be spared here, as the whole operation is evident from the preceding one of the binary combination.

13. With the augmentation of the number of the weights, the number of combined weighings must of course increase, and so will the results; six weights produce fifteen binary weighings, and render ten results for each individual; the ternary combination of six weights requires twenty weighings, and produces equally ten results for each individual, &c. But to make so many, or more, weighings in succession, with the adjustments of the balance, the temperature, &c., all equal throughout, is too precarious, and this equality of all circumstances is the requisite of accuracy in the results. Upon proper experience, I have, therefore, generally limited myself to the number of five weights for each combination, where I could perform the ten weighings required in one uninterrupted series, unless some adverse accidental circumstances, as a sudden change of temperature, or others, occurred; the nicety of all adjustments, the necessary equality of all circumstances, and the close and minute attention, which such accurate weighings exact from the operator, indicate evidently a limit which it is no more advantageous to exceed.

In the first preparatory beginnings of bringing a weight near to adjustment, I, of course, sometimes even limited myself to a less number of combinations, though never in the final adjustment, even up to the thirty ounce weights inclusive, which requires to load each basin of the balance with the weight of five pounds, which it admitted yet to unexpected satisfaction, the results not varying sensibly more than those of the six ounce quoted as example.

14. The order in which the weights below the pound were to be determined, was of course prescribed by their ratio to the pound, and to each other, thus:

The 6 ounces by binary combination upon the 1 pound.

The 4 ounces by ternary combination upon the 1 pound.

The 3 ounces by binary combination upon 6 ounces.

The 2 ounces were compared by both the binary combination upon four ounces, and the ternary upon six ounces. They were evidently of great importance, as the ten ounce weights, the most important of the set, had to be made by the deduction of two ounces from the pound, to prepare them for the ternary combination, through which they were carried by the total sum of 2 lb. + 6 oz. The five ounce weights were combined binarily upon ten ounces. For the one ounce, and the inferior weights, the mode of proceeding can easily be imagined from this statement; the sensibility of the balance increasing with the decrease of the weights; I aided it still, by having a series of basins made lighter, in proportion to the weight they had to be used with.

To go here through the details of the results would be too long, without adequate marking interest; it may suffice to say, that, by successive reductions, the weights were all brought to a degree of accuracy never less than that quoted, until the thirty ounce inclusive, which required the balance to be laden with sixty ounces. Above thirty ounces, the combined weighing exceeded the limit of the balance, so that the adjustments had to be made single by the sum of inferior ones. The new weights used in each of the subsequent comparisons were always taken into account for that corrected amount, which had resulted from the combined weighing, when they were themselves adjusted.

15. It would be of no special interest to quote of each weight its ultimate value, at which it was left: it may suffice to say, that even the greatest were not suffered to deviate as much as the hundredth part of a grain; no such difference was even found when six weights, of ten ounces each, were compared at once, with a five pound troy weight of Gilbert, accompanying the balance procured from him, originally for the Virginia University. It was this balance which I used for the combined weighings, and determinations of the ounce weights, up to the fifty ounce, it being constructed for the extreme weight of sixty ounces, or five pounds troy. Every series of results of a combined weighing, furnished naturally the direction by which each individual weight was gradually reduced to its proper standard. Weights falling below, or getting too large a quantity (or, as may be said, requiring too much addition,) were of course discarded, as for their ultimate rank in the sets, and not delivered, though they were still near enough to serve in the combinatory weighing, for comparison, to adjust the others. I was so lucky, by dint of attention, as to have hardly any to discard.

16. When once the value of the weights is brought so near that the variations from their intended accurate weight is considerably below what the balance is capable to show by single weighing, a result which is unattainable otherwise than by the combined weighing which I have used, it

is no more prudent to touch them. Not only can their value, as determined in that case, be fully accepted, but it cannot be expected that in the length of time of their use they can preserve that accuracy; thence arises the habit, except in the unique standard of a country, of allowing a certain *sufferance*, for which the weight is left too heavy at the first making, the quantity of which is made proportional to the intended use of the weight.

17. The following table shows a collection of results for each kind of the weights, and their total amount for the five sets, (and some over) that were made to answer the purpose of the work, from 1 ounce up to 30 ounces, the limit of the combined weighing, which I may assert is also the limit within which such nice determinations are possible or necessary. The 40-ounce, 50-ounce, and the 100-ounce weights were made accurate to the limit which single weighing is able to show, viz: about three or five thousandths of a grain, as would appear by a calculation that could be established upon the different weighings. They were the most extreme limit, to which I submitted the balance, reluctantly, but still found it not affected by it; needing, however, extreme caution in all respects.

TABLE.

Pieces.	Value.	Their sum in ounces.	Variation from absolute weight.
10.	1.oz.	10.oz.	+0,0063
7.	2.	14.	0,00895
9.	3.	27.	—0,0004
5.	4.	20.	—0,0067
5.	5.	25.	—0,0407
5.	6.	30.	—0,0123
6.	10.	60.	—0,0221
5.	0,5	2,5	+0,0074
6.	0,4	2,4	—0,01256
5.	0,3	1,5	—0,0100
5.	0,2	1,0	—0,0083
5.	20.	100.	—0,0081
5.	30.	150.	—0,0052
Sums 78.		443,4	—0,05381

18. This shows that from the 0,2-ounce till 30-ounce inclusive, there were 78 weights made, comprehending 443,4 ounces. The result of their deviation showed the total excess or deviation of —0,0538. Taking the means of these results, as is otherwise habitual, in estimating the proportional accuracy, the following would be the resulting means, under different references; it exhibits the following approximate means, in decimal parts of the grain—

$$\text{For each piece} = \frac{0{,}05381}{78} = 0{,}0007 \text{ of the grain.}$$

$$\text{per ounce} = \frac{0{,}05381}{443{,}4} = 0{,}00012$$

$$\text{per grain} = \frac{0{,}05381}{443{,}4 \times 480} = 0{,}0000002$$

proportional to the whole sum, the $\frac{1}{5000000}$ part of the whole.

This must, however, not be considered otherwise: but as a proof of the fully satisfactory accuracy obtained in the operation, as it is evident that even in the smallest weights, where the sensibility of the balances is the greatest, such a proportional weight and accuracy could not possibly be physically represented; still, however, a few of the weights were slightly retouched afterwards; and for the 3 sets sent to the mint and to the Treasury Department, a proper selection was made of the most accurate of each kind.

19. All the weighings were, of course, made by substitution; but notwithstanding that by this process the same state of equilibrium is intended to be produced, by placing the two weights in the identical position towards the counterpoise and the balance, this very identical state of the balance is not very easily produced. It is necessary that the points of reference of the balance be exactly horizontal, and the temperature of both arms to remain the same, though they may not be at the same temperature, because the variation of the momentum of the weight is very great, and increases very rapidly with the weight, and with a very small change of temperature. The simple change of air that takes place by (for instance) opening the glass case, to feed the balance, more frequently on one side than on the other, will affect the temperature. It is evidently difficult to ascertain this fact, because any thermometer can only exhibit the temperature of the near ambient air, which may be, or may not be, that of the arm itself. It is, therefore, sooner discovered in its effect upon the weighing, than on the thermometer itself. It is necessary to lay up the weights lightly and cautiously, in the middle of the basin, and not to give any jar to the balance. The basin should not come in an oscillatory motion, when put in action. The initial momentum, given by the suddenly setting the balance in action, produces an excess of oscillation, which is not good; it is even not always easy to prevent changes in the points of suspension. The estimation of the coincidence should always be judged only within the first division, nearest to the horizontal points; the estimates between farther distances will seldom fully agree with those found afterwards, when near coincidence. To esteem any weight by the difference of the oscillation, is applicable only within these narrow limits, and there only it will be found good; for weights less than the balance can show in a direct manner.

20. In general, as simple an operation as weighing appears to be, in its daily common life application, it becomes extremely delicate, and therefore very difficult, when mathematical accuracy is aimed at. It is extremely difficult to keep the balance exactly in equal adjustment for a number of successive weighings; and to produce the same circumstances, in all respects, by another adjustment, is depending too much on what may be called a lucky chance, which it is not always easy to produce as completely as required. The delicacy, and strained attention required for that task, proved to me extremely fatiguing.

21. The individual weight used to derive the system of ounce weights from it, was principally a troy pound newly procured, by way of verification of the determinations made in my former comparisons. It was compared at the United States Mint in Philadelphia, with the weight there preserved as standard, and presented a perfect equipoise. As an assistant in the combinatory weighings, I could also make use of the troy pound weight, belonging to the balance procured from the University of Virginia, which proved to have suffered very little, though the box containing them

all arrived in a very broken state. For the first preliminary adjustment of the weight, I used the subdivisions of the French kilogrammes, which I had procured in 1813, for the collection of the coast survey, which were superior in accuracy to the corresponding English small weights.

22. All the weights, from 1 to 10 ounces inclusive, were solid brass cylinders, of the same diameter, and in height proportional to their weight; those below 10 ounces having light lines around them, indicating the number of ounces. The decimals of ounces have, of course, a diameter reduced proportionally, but all again equal, and equally marked, for their weight, by fine lines around them, like the whole ounce weights. All the weights are excavated at the bottom, so as to stand upon a rim of proper proportional breadth, which at the same time that it greatly preserves the weights from rubbing, furnished the means for the nicest adjustment, which was made ultimately by rubbing them upon a fine hone, until they were reduced to their desired weight. The weights above 10 ounces were made cylinders with knobs, exactly similar to those made for the general series of standard weights, and proportioned, equally like those, in height and diameter. They were of 20, 30, 40, 50, and 100 ounces.

23. It is a bad habit, still too much prevailing, to make the weights in part hollow, to introduce into the cavity such pieces of wire, or other pieces of metal, (often lead, the worst of all,) to produce the weight desired: this is very objectionable on all considerations. The hollow part is not enough under the command of the workman to be so well worked as not, by the rolling of the pieces introduced into it, to lose weight. When a different metal is introduced, the galvanic effect occasions oxidation, which immediately destroys the original accuray. It may easily happen that, out of mere curiosity, the weight is opened, and then *certainly* its accuracy will have changed. It is besides evident, that the determination of the specific gravity of such a weight is not possible, without altering it. This it is, however, sometimes desirable to ascertain. A weight must be a solid mass of compact, uniform metal; the selection of the metal depends on the circumstances of the case, of its application for use, &c. The reason for my selection of the brass, I have already stated in its proper place.

24. The different buoyancy of different metals in the air, according to their specific gravity, is well known, as well as its unavoidable variation with the state of the atmosphere; but the accurate scale of its variation is less so, and escapes still, in a great measure, the efforts towards its accurate determination, on account of its minuteness; it has, therefore, been taken into account only since the establishment of the French metric system, which may be considered as the first establishment of standards, made with the scientific means of modern times. The keeping account of these considerations in the present case was evidently not necessary; and not answering any special aim, it was, therefore, omitted entirely. Having brass standards to make other brass standards from, and always placing both exactly under the same physical influences, it would have been more than idle to attempt to keep account of any possible differences; the differences between the observations would likely have exceeded the difference searched after.

25. That different proportions of the alloy of copper with other metals must present different proportions in these influences, is evident, though their quantity is not yet within our reach to determine precisely; and in the English standards, from which ours are derived, it does not appear that

ever any particular reference to that was taken into account. In the copy of the British standards in the State Department, the troy weight set appears to have but a slight addition, apparently, of tin, to the copper, only sufficient to prevent its taking impressions too easily. The avoirdupois weights are of the color of cannon metal, (often, in England, improperly called bell metal.)

26. *To state the manner and the measures taken to procure brass of the greatest perfection*, which has been employed for the weights, belongs more properly to the description of all the means employed for the whole establishment of all the standards, for the States and custom-houses. The mint weights have, of course, partaken of these, in all respects. It may suffice here to say, that to obtain the proper result in the quality of the metal, it has been necessary to establish for ourselves the reduction of the zinc from the ore, on account of the zinc of commerce being too impure, and particularly containing iron, which will always produce a porous and very unequal brass.

27. As desirable as it may be, for ease in using the weights, and to prove their authenticity; it is, however, not admissible, with the proper accuracy, to have any marks engraved upon weights. They will always collect dust, and thereby alter the weight. All that is admissible, for the purpose of authenticity only, is a light stamp, which can easily be made so as to present no edges, and no depth, in which dust can collect. The stamp used was purposely made so as to have all the delineating different depths rounded off, and it is put on very lightly, so that the impression can be felt only slightly. The indication of the quantity of the weight must be left to the judgment, upon their size, and the marks, as well as places in their packing boxes, where they are lodged in exactly fitting cavities, lined with velvet.

28. For the manipulation of the weights, the boxes are furnished with forks, pincers, and forceps, appropriated to each size of weights—for they must never be touched with the naked hand, after adjustment. It is difficult, even impossible, to prevent entirely that they tarnish, notwithstanding that, in the manipulation necessary for their adjustment, after coming from the workmen, pieces of silk were always used over them, and that they were almost constantly covered with silk stuff in the intermediate times. But the tarnish is, of course, an effect of the atmosphere; and, if the brass in general, principally when polished, is suffered to take this tarnish, at first from the dry atmosphere alone, so as to present what is called in numismatics "*ærugo nobilis*," it appears that it becomes less liable to take afterwards the tarnish by touching.

F. R. HASSLER.

HARROW HILL, NEAR HEMPSTEAD HARBOR, L. I.
November 18, 1837.