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THE STATE COORDINATE SYSTEMS

(A Manual for Surveyors)

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INTRODUCTION

GENERAL STATEMENT

1. The use of plane-rectangular coordinates in land surveying is by no means a modern development. The ancient Egyptians probably used a form of coordinates when they referred the land corners of the rich valley lands of the Nile, where floods made monumentation impracticable, to permanent marks on higher land. It is known that they constructed right angles on the ground by means of triangles having sides in a 3–4–5 proportion. The land surveyor of today uses plane-rectangular coordinates when he traverses the boundaries of a farm and computes the latitudes and departures of its corners, referred to an arbitrarily-selected origin and meridian of reference.

2. In either case, the utility of the method employed depends in a great measure upon there being recoverable at future times, the monuments at stations from which surveys can be run to determine the ground positions corresponding to the coordinates of points whose monuments have been obliterated or destroyed. But as original survey monuments decay and disappear and the stations they represented are restored by survey methods, these restored stations themselves become the bases from which other restorations are made. And with each subsequent restoration, the accuracy of the result is diminished, since it is affected by the errors of the original survey combined with the errors of the restoration survey or surveys. Eventually, even a good survey of a limited area with only its own monuments to preserve its ground location will, by loss of original monuments and errors of replacement, become little more than a paper record, beyond the power of a surveyor to transform into a ground pattern of monumented lines without the aid of a court decision prescribing a legal method of construing conflicting records and interpreting survey discrepancies.

3. Seeking a remedy for this condition of insecurity, engineers have connected their land surveys with other surveys of greater extent and accuracy and of superior monumentation, whose stations would serve as starting points for resurveys to restore lost corners to their original ground locations. For this purpose, the triangulation stations established by the U. S. Coast and Geodetic Survey are proving satisfactory. Designed to provide control for Federal surveys and maps, the triangulation executed by this Bureau comprises a countrywide network (fig. 1) connecting thousands of marked points whose geodetic positions (latitudes and longitudes) are known with such accuracy and precision that any station, if its marks are destroyed, can be restored closely in its original position on the ground by surveys based on other triangulation stations whose marks have not been disturbed.

4. But such restoration by the usual geodetic survey methods requires a degree of training in the use of those methods with which the land surveyor is ordinarily unacquainted, so means were accordingly sought for making geodetic survey data available

without requiring a knowledge of the tedious and involved procedures of geodetic surveying. This was accomplished to some extent by transforming the latitudes and longitudes of the triangulation stations within a limited area into rectangular coordinates on a plane tangent to the area near its central point. In doing this, some distortion was incurred: one cannot flatten out the curved surface of the Earth, even of a county-size area, without some stretching and compression of the surface. Of course, for a small area, the effect of this distortion on an ordinary land survey may be negligible. But as the geodetic data provided by the national triangulation net became more plentiful, and marked points to which those data pertained became more generally distributed and more closely spaced, their use for referencing land surveys and engineering surveys continued to increase. The advantages of greater accuracy and permanency which resulted from referencing a land survey to the triangulation net of the country naturally developed a demand for a method of obtaining similar advantages for surveys of large areas and for coordinating surveys of widely-separated tracts of land.

ESTABLISHMENT OF THE STATE COORDINATE SYSTEMS

5. This need was submitted to the U. S. Coast and Geodetic Survey by an engineer of a State highway department, who sought a method of utilizing the geodetic data over an entire State which would involve only the formulas of plane surveying. This brought about the establishment in 1933 of the North Carolina Coordinate System, by means of which geodetic positions (latitudes and longitudes) of triangulation stations within the State of North Carolina could be transformed into plane-rectangular (x and y) coordinates on a single grid, and surveys in all parts of the State referenced thereto, so that survey stations and landmarks could be accurately described by stating their coordinates referred to the common origin of the grid.

6. Within a year or so after the establishment of the North Carolina Coordinate System, a similar system had been devised for each of the States of the Union[1] (fig. 2). For some of these, a single origin and meridian of reference were sufficient. Other States, because of their large sizes, were each divided into several belts or zones, each zone having its own origin and reference meridian. It is now the practice of the U. S. Coast and Geodetic Survey to compute and publish the coordinates on the appropriate State system, of all points for which it determines the adjusted positions on the standard geodetic datum (the North American datum of 1927).

STATE SYSTEMS APPROVED BY FEDERAL BUREAUS

7. In a report[2] adopted at the meeting on September 8, 1936, the Federal Board of Surveys and Maps[3] recommended to its member organizations that,

"wherever practicable, they adopt the systems of plane coordinates devised for the various States by the Coast and Geodetic Survey, and use these State systems of plane coordinates as bases for such of their surveys and maps as will not, because of their nature or extent, require the use of some other system of coordinates or method of recording."

[1] See Appendix I, p. 45. The mathematical development of these State systems was the work of Dr. O. S. Adams, mathematician in the Division of Geodesy, U. S. Coast and Geodetic Survey.

[2] This report was published in the GEODETIC LETTER of January, 1937. See Bibliography, p. 44.

[3] The Federal Board of Surveys and Maps was created by Executive Order in 1919, to furnish a means of coordinating the activities of Federal agencies which were directly concerned with surveying and mapping. It had an Advisory Council representing non-Federal organizations having similar interests. At the time of its abolition there were 24 Federal and 22 non-Federal agencies represented on the Board and on its Advisory Council. When the Board was abolished by Executive Order in 1942, its functions were transferred to the Federal Bureau of the Budget. With this transfer to a fiscal organization, the technical committees of the Board, such as the Committee on Control Surveys from whose report the above quotations are taken, ceased to function.

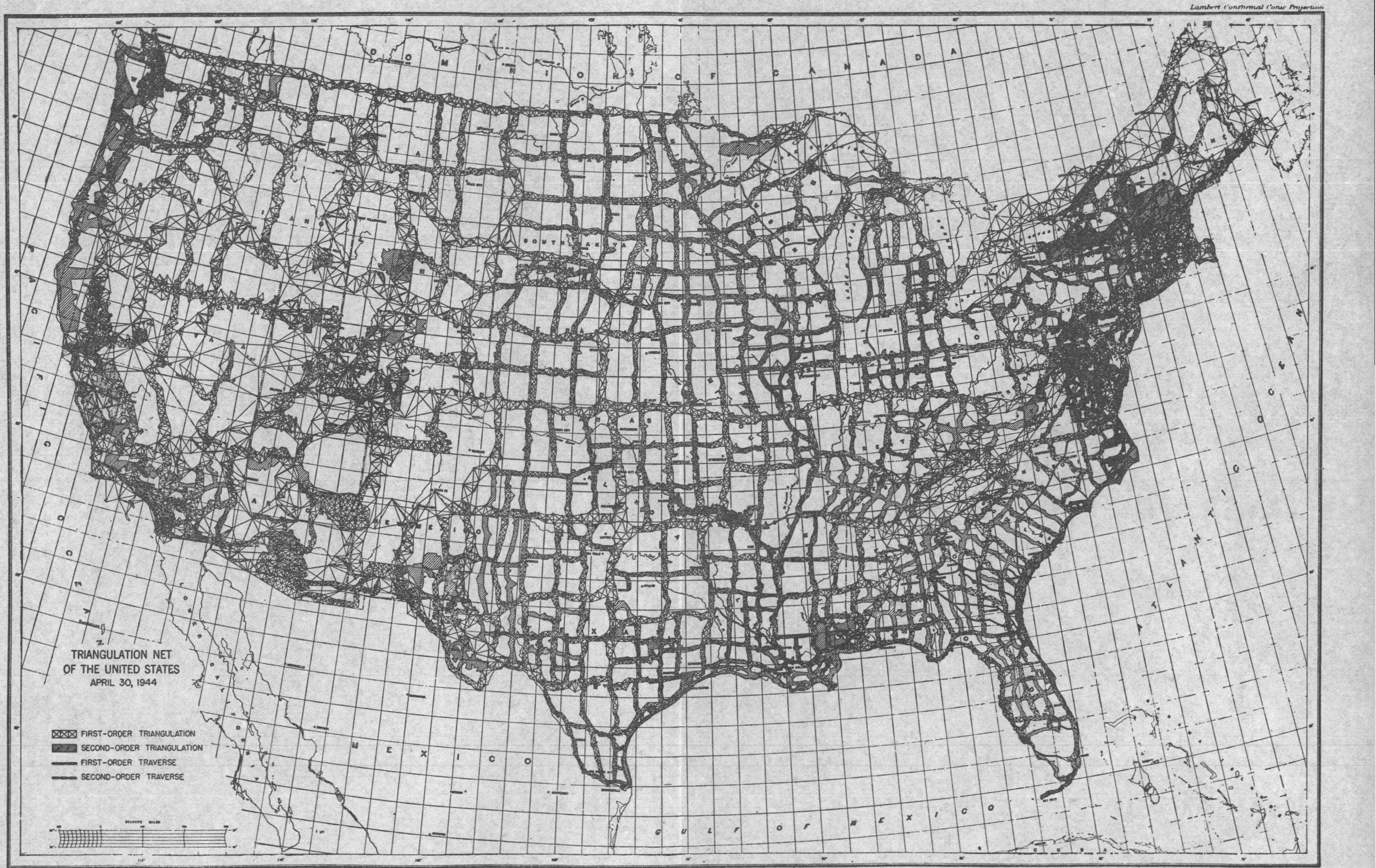

FIGURE 1.—*Triangulation net of the United States*

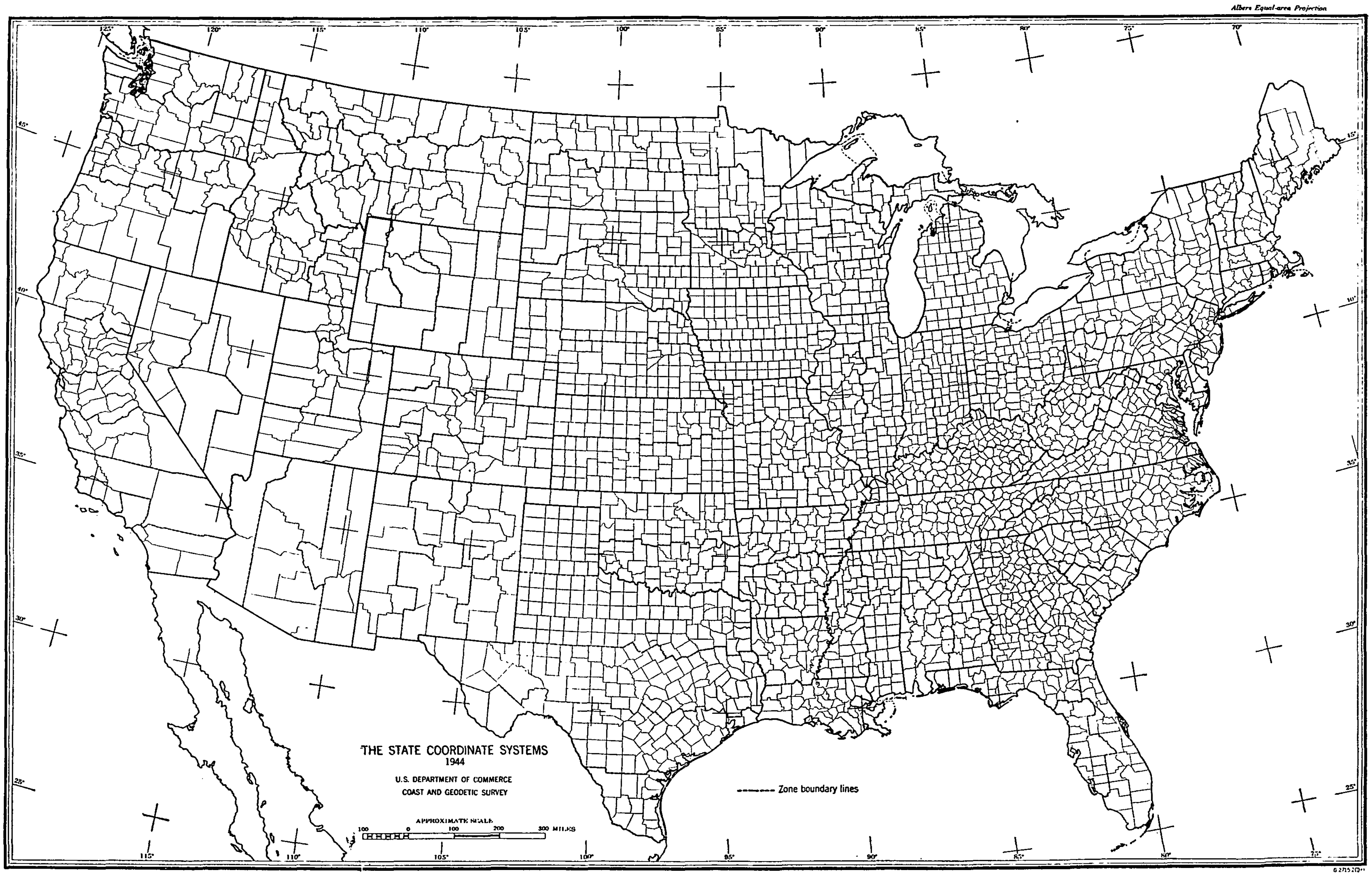

FIGURE 2.—*The State coordinate systems*

The Board further recommended to its member organizations that,

"wherever practicable, they show the appropriate State plane-coordinate systems as supplementary projections on all maps and charts produced by them which may have value and use for engineering purposes, but which because of their nature or extent require a geographic base. Such supplementary projections may be shown on charts and maps by ticks or other symbols, and should be so labeled that a complete and accurate identification of the projection which they represent can be made readily, and no confusion arise even where several projection systems are shown."

In conformity with these recommendations, the State coordinate systems are shown on many Federal maps—particularly on those topographic maps which must be based primarily on geographic projections. These maps thus become useful to the surveyor who desires data for reducing a land survey to a State grid; at the same time the land survey becomes available for transfer to the map. This coordination of topographic and cadastral information is of especial value in the planning of engineering works.

8. One result of the growing use of the State coordinate systems in land surveying is the formal recognition given the systems in a number of State legislatures. Such action has been taken in nine States and is pending in others. The Council of State Governments has approved a model bill for the adoption of the State systems and is including it in a general report on Suggested State War and Postwar Legislation.[4] Particular bills have been prepared for those States in which no formal action has yet been taken.

9. This manual is designed to aid the surveyor who intends to use a State coordinate system for referencing a land or engineering survey so that its results can be placed on a lasting basis and coordinated with surveys in other areas. In accomplishing this, only plane surveying methods and formulas are employed. The State coordinate systems are described, and procedures which their use involves are explained and exemplified by carrying through the computation of a farm survey. The several appendices contain essential information relating to the State systems themselves; a report on their adoption by legislative enactment; and some formulas and tables which will aid in placing a land survey on a State system.

[4] Appendix II, p. 48.

CHAPTER 1. THE STATE COORDINATE SYSTEMS

TERMS AND DEFINITIONS

10. As with any plane-rectangular coordinate system, a projection employed in establishing a State coordinate system may be represented by two sets of parallel straight lines, intersecting at right angles. The network thus formed is termed a *grid*.[5] One set of these lines is parallel to the plane of a meridian passing approximately through the center of the area shown on the grid, and the grid line corresponding to that meridian is the *Axis of Y* of the grid. It is also termed the *central meridian* of the grid. Forming right angles with the *Axis of Y* and to the south of the area shown on the grid is the *Axis of X*. The point of intersection of these axes is the *origin of coordinates*. The position of a point represented on the grid can be defined by stating two distances, termed *coordinates*. One of these distances, known as the *x-coordinate*, gives the position in an east-and-west direction. The other distance, known as the *y-coordinate*, gives the position in a north-and-south direction; this coordinate is always positive. The x-coordinates increase in size, numerically, from west to east; the y-coordinates increase in size from south to north. All x-coordinates in an area represented on a State grid are made positive by assigning the origin the coordinates: $x = 0$ plus a large constant. For any point, then, the x-coordinate equals the value of x adopted for the origin, plus or minus the distance (x') of the point east or west from the central meridian (Axis of Y); and the y-coordinate equals the perpendicular distance to the point from the Axis of X. The linear unit of the State coordinate systems is the foot of 12 inches defined by the equivalence: 1 international meter = 39.37 inches exactly.

11. The linear distance between two points on a State coordinate system, as obtained by computation or scaled from the grid, is termed the *grid length* of the line connecting those points. The angle between a line on the grid and the Axis of Y, reckoned clockwise from the south through 360°, is the *grid azimuth* of the line. The computations involved in obtaining a grid length and a grid azimuth from grid coordinates are performed by means of the formulas of plane trigonometry.

GEODETIC AND PLANE-COORDINATE POSITIONS

12. For more than a century the United States Coast and Geodetic Survey has engaged in geodetic operations which determined the geodetic positions—the latitudes and longitudes—of thousands of monumented points distributed throughout the country (fig. 1). These latitudes and longitudes are on an ideal figure—a spheroid of reference which closely approaches the sea-level surface of the Earth. By mathematical processes, the positions of the grid lines of a State coordinate system are determined with respect to the meridians and parallels on the spheroid of reference. A point that is defined by stating its latitude and longitude on the spheroid of reference may also be defined by stating its x- and y-coordinates on a State grid. If either position is known, the other can be derived by formal mathematical computation. So too with lengths and azimuths: the geodetic length and azimuths between two positions can be transformed into a grid length and azimuth by mathematical operations. Or the process may be reversed when grid values are known and

[5] A system of lines representing geographic parallels and meridians on a map projection is termed a *graticule*.

geodetic values are desired. These computations are beyond the scope and purpose of this manual: they are fully treated in the Special Publications of the Bureau.[6]

13. In general, any survey computations involving the use of geodetic position data can also be accomplished with the corresponding grid data; but with this difference: results obtained with geodetic data are exact, but they require the use of involved and tedious spherical formulas and of special tables. On the other hand, results obtained with grid data are not exact, since they involve certain allowances that must be made in the transfer of survey data from the curved surface of the Earth (spheroid) to the plane surface of a State coordinate system; but the computations with the grid data are quite simple, being made with the ordinary formulas of plane surveying; and with the State coordinate systems, exact correlation of grid values and geodetic values is readily obtained by simple mathematical procedures.

MAP PROJECTIONS AND STATE GRIDS

14. By using a conformal map projection as the base for a State coordinate system and limiting one dimension of the area which is to be covered by a single grid, two things are accomplished: first, on a conformal map projection, angles are preserved. This means that, at a given point, the difference between geodetic and grid azimuths of very short lines is a constant, and angles on the Earth formed by such lines are truly represented on the map. For practical purposes of land surveying, this condition holds for distances up to about ten miles. For longer lines, the difference varies, and the correction to be applied to an observed (geodetic) angle to obtain a corresponding grid angle is the difference of the corrections to the azimuths of the lines, separately derived. Second, the limitation in the width of the projection or grid permits a control of deviations of grid lengths from geodetic lengths. When the width of an area covered by a single grid is 158 statute miles, the extreme difference between geodetic and grid lengths will be 1/10,000 of the length of a line, which is quite satisfactory for most land surveys.

15. Deviations of grid lengths from geodetic lengths will be a maximum along the margins of the longer dimension of the grid and midway between those margins. Along the margins, the grid length of a line will be greater than its geodetic length; along the center line, the geodetic length will be the greater. Between these limits are two lines along which grid and geodetic lengths are equal: these are *lines of exact scale.* The quantity by which a geodetic length is multiplied to obtain the corresponding grid length is termed a *scale factor.* It is greater than unity in areas outside the lines of exact scale; decreases to unity along those lines; and continues to decrease to a minimum about midway between them. The magnitude of a scale factor at any point depends upon the position of the point with respect to the lines of exact scale. It is constant along a line—any line—which is parallel with the lines of exact scale. These *lines of equal scale correction* are grid lines on the transverse Mercator grid, being parallel with the central meridian or Axis of Y; on the Lambert grid they are curved lines, being parallels of latitude. Scale factors admit of simple tabulation. For any given survey line, the scale factor may be readily ascertained and applied to the geodetic length of the line to obtain its grid length; or in an inverse operation, employed in obtaining a geodetic from a grid length. Where the exact length of a line is desired, it is thus easily obtained. It must be remembered that a geodetic length is a distance on the spheroid (sea-level surface of the Earth), whose relationship to the corresponding ground-level distance may be expressed by very simple formulas and accurately

[6] Bibliography, p. 44.

illustrated by a geometrical figure. On the other hand, a grid length is a distance on a plane which is mathematically related to the spheroid, so that the relationship between corresponding lengths on the two surfaces can be expressed by mathematical formulas, but is not susceptible of accurate graphical demonstration. The commonly-used examples of a developed cone for the Lambert grid and a developed cylinder for the transverse Mercator grid are inexact and serve only as illustrations.

16. While a width of 158 statute miles was adopted as a standard in devising the State coordinate systems, departures from that width have been made where geographic conditions permitted or surveying requirements justified the change. If the width of a State is less than 158 miles, the width of the grid was decreased and the effect of the scale factor thereby also decreased. The narrower the strip of the Earth's surface which it is desired to portray on a plane, the smaller will be the distortion involved in the process. The north-south dimension of Connecticut is less than 80 miles: the maximum scale factor for the Connecticut Coordinate System (fig. 3), along the northern and southern boundaries of the State, expressed as a ratio, is around 1:40,000. Midway between the lines of exact scale it is 1:79,000. Where a State is too wide to be covered by a single grid, it is divided into belts, called *zones,* for each of which a separate grid is adopted. The boundary lines between zones follow county lines. The limiting scale factors for the various zones of a State coordinate system need not be the same. For example, the Illinois Coordinate System (fig. 4) comprises two zones. The eastern zone, in which Chicago is located, has much smaller scale factors than the western zone. One thing sought in devising the State coordinate systems was to keep the number of zones in any State to a minimum, consistent with the requirements of scale accuracy. For example, by allowing the scale ratio to go slightly above 1:10,000, the entire State of Texas was divided into five zones.

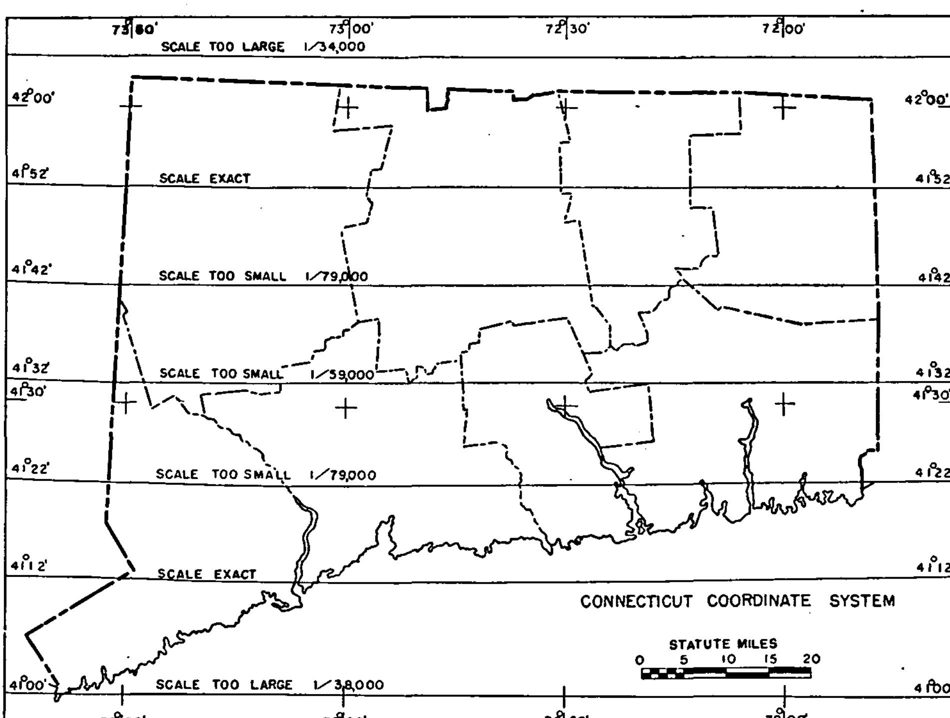

FIGURE 3.—*The Connecticut Coordinate System*

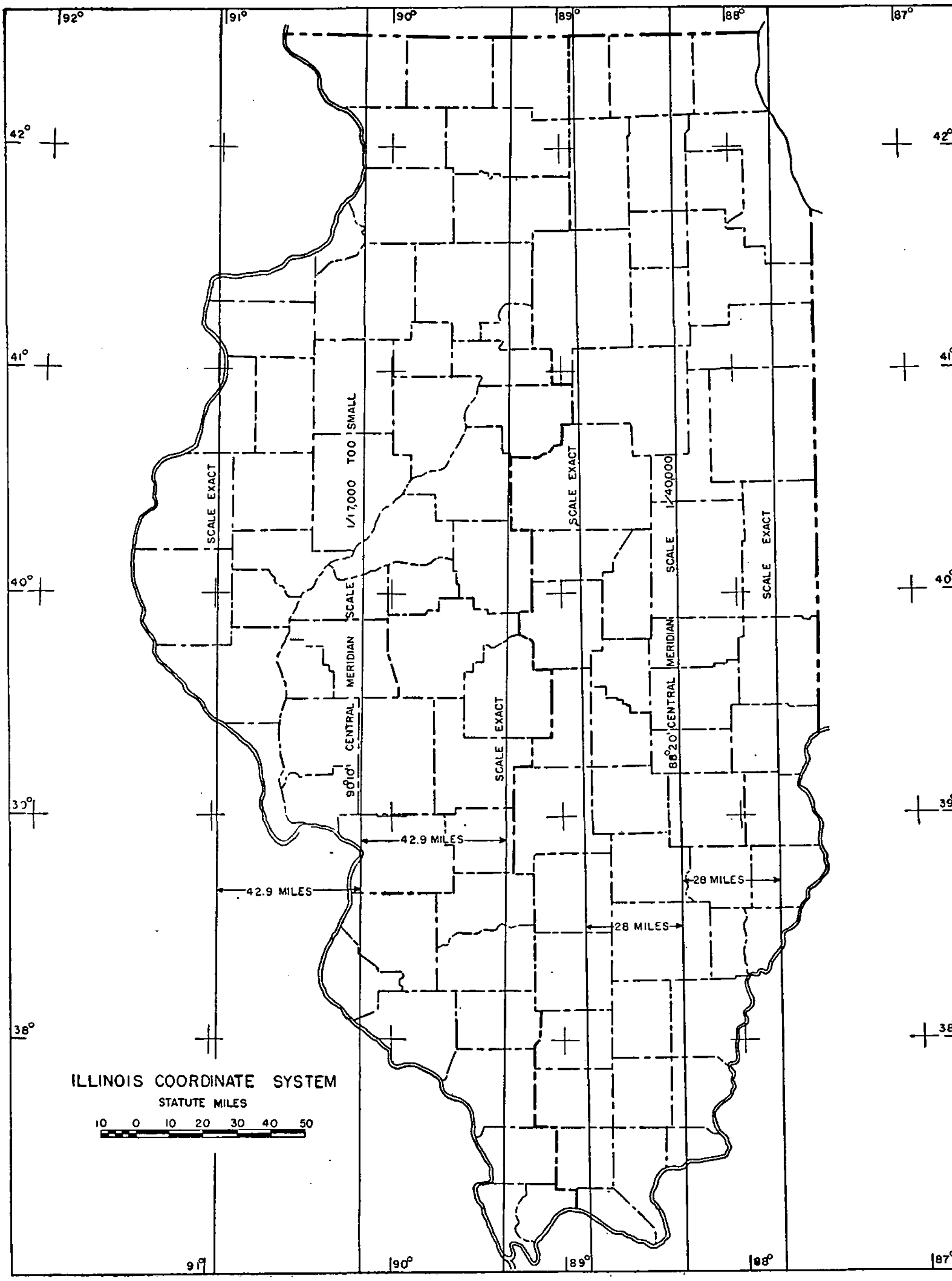

FIGURE 4.—*The Illinois Coordinate System*

17. As some States or zones have a width or smaller dimension extending in a north-and-south direction (Connecticut, for example), while for others the east-and-west dimension is the smaller (example, New Jersey), two forms of geographic map projection were used as the foundations of the State coordinate systems. The mathematical development of the State coordinate systems is described in Coast and Geodetic Survey Special Publica-

tion No. 193.[7] For an area (zone) of small north-and-south dimension, the Lambert conformal map projection with two standard parallels was used. This projection may be illustrated by a cone cutting the surface of the spheroid along two parallels of latitude, called *standard parallels,* situated within the area and about one-sixth the width of the zone from its northern and southern boundaries. The cone is considered as cut along an element and flattened into a plane, carrying with it the parallels and meridians, which have been projected onto it. A grid is then imposed upon this developed surface, its Axis of *Y* coinciding with a selected meridian passing near the center of the area, and the Axis of *X* is placed below the southern limit of the area. The use of a developed cone is for illustration only; the actual relationship of the State grid to the geographical map projection is defined by mathematical formulas.

18. For a zone of limited east-and-west dimension, the transverse Mercator projection was employed. This projection may be illustrated by a cylinder cutting the surface of the spheroid along two small ellipses equidistant from the meridian through the center of the zone. The cylinder is supposed cut along an element and developed into a plane. The State grid is imposed upon the developed cylinder, onto which the parallels and meridians of the Earth have been projected. Here again, as in the case of the Lambert projection, the geometric concept is for purpose of illustration only, exact relationship between the basic map projection and the State grid being obtained by mathematical processes.

19. The popular designation for a State coordinate system based on the Lambert projection is *Lambert grid,* and one based on the transverse Mercator projection is called a *transverse Mercator grid.* The New York and Florida coordinate systems employ both types of projection. In Appendix I is a list of the State coordinate systems giving the designations of the separate zones if there be more than one, the map projection used, and other information which may be desired by a surveyor planning the use of a State coordinate system.

CONTROL SURVEY DATA

20. The mutual relationships which make it practicable to pass with mathematical accuracy from a geodetic to a plane-coordinate system—to transform a geodetic position on the standard datum into plane-rectangular coordinates on a State system—have made it practicable to utilize the precise data of the national triangulation net for the referencing and control of land surveys in many parts of the country. The limitation on such use imposed by lack of stations in many localities is being reduced each year as the Coast and Geodetic Survey extends its triangulation net into new localities, making additional data available for the referencing and control of land surveys. Figure 1 shows the general location of the geodetic surveys forming the triangulation net of the country. The Survey also publishes a letter-size index map (fig. 6) of each State, giving the same general survey information shown on figure 1, but with more geographic detail. Copies of these index maps are furnished upon request and are intended to aid surveyors in formulating requests for control data. A surveyor planning work in a particular area will consult the State index map and if it shows that geodetic surveys have been executed in the locality of the proposed survey, a letter is sent to the Survey,[8] requesting the State coordinates and descriptions of survey stations in that locality. The area for which control data are desired may be indicated on the index map and the map forwarded with the request.

[7] Bibliography, p. 44.

[8] Requests for data should be addressed to the Director, U. S. Coast and Geodetic Survey, Washington 25, D. C.

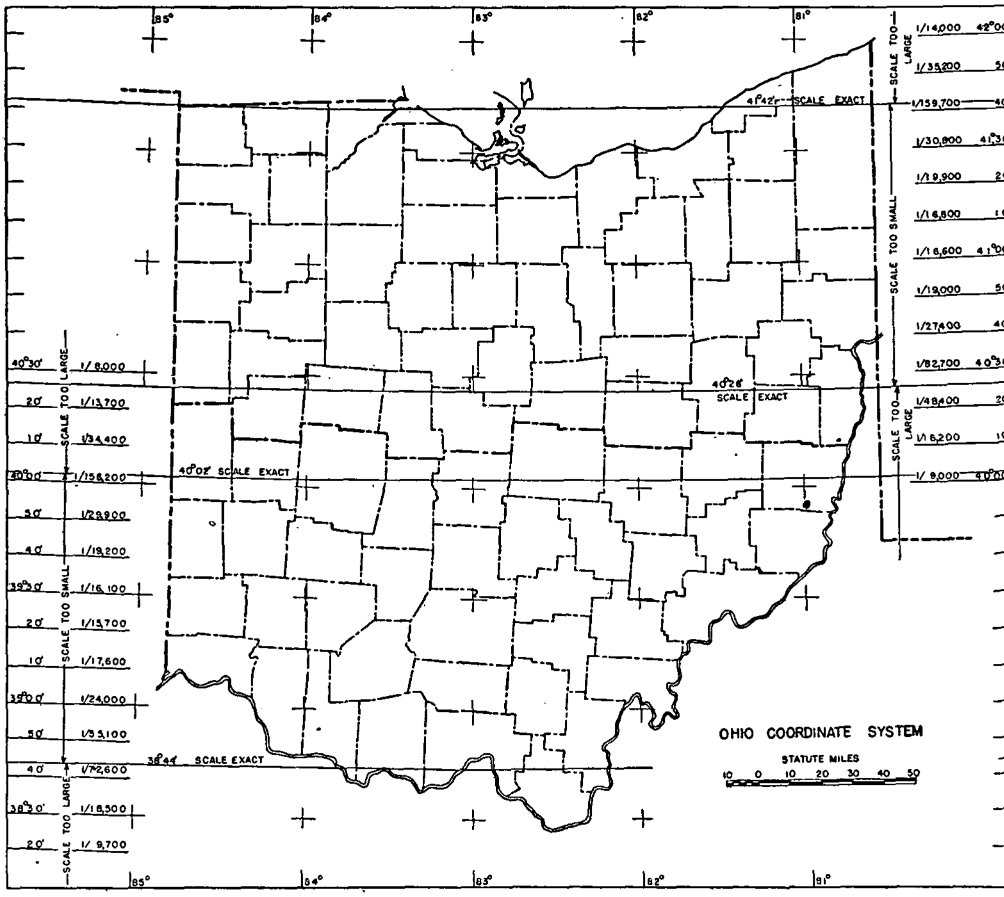

FIGURE 5.—*The Ohio Coordinate System*

USE OF CONTROL SURVEY DATA

21. The advantages derived from referencing a land survey to stations of the national triangulation are readily apparent. Connecting a land survey to a system of survey stations extending throughout the country makes those stations references for the stations and corners of the land survey. Should the monuments which mark the land corners be destroyed, the ground positions occupied by those monuments before they were destroyed may be readily and accurately determined by new surveys, originating at stations of the national survey, and the land corners restored to their original locations. It is most unlikely that the stations of the national survey will ever be destroyed simultaneously over an area so extensive as to render the above procedure impracticable.

22. The value of such restoration of a lost corner by means of its coordinate position is thus described in the GEODETIC LETTER[9] of January, 1937:

> Regardless of the method of determining the plane coordinates of a given corner of the public land surveys, such coordinates cannot overcome or outweigh the physical evidence of the corner monument itself: the corner monument is *the* corner for which it was established, regardless of its position. However, when the corner monument is missing through destruction or decay, it is

[9] Bibliography, p. 44.

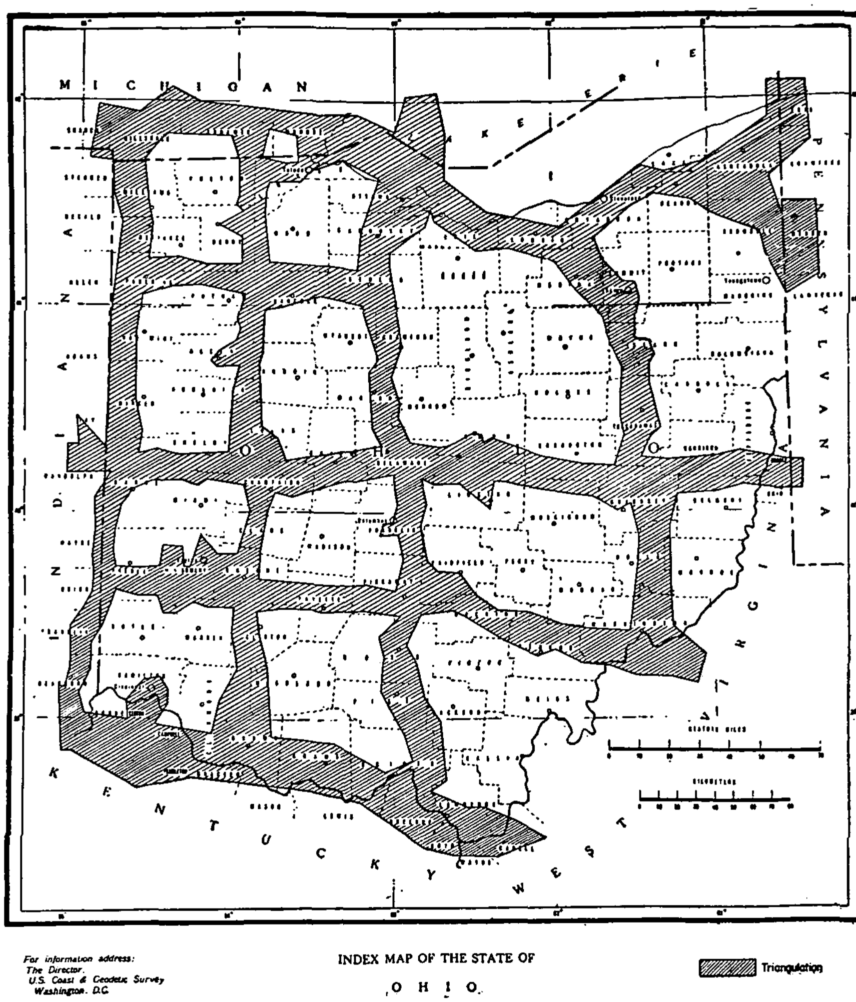

FIGURE 6.—*Index map of the State of Ohio*

apparent that the plane coordinates of that corner, properly and accurately determined, may become the best available evidence of the position of the original corner, and thus afford a ready and accurate means for its restoration.

The above is a quotation from a paper entitled "Coordinate positions as collateral evidence in the restoration of lost corners of the public land surveys," by an engineer of the General Land Office. While specifically referring to public land surveys, it is also applicable to other land surveys, under similar circumstances, wherever located.

23. A second important function served by a survey connection between a land survey and the national triangulation is the increased accuracy of the land survey which such connection makes practicable. While a land survey may be referred to a State coordinate system through a connection with a single station of the national control survey, such connection does not admit of a rigid adjustment of the land survey to the control survey system. But if the land survey is connected to at least two control survey stations, in such manner that computations may be started with the data at one control station and carried through the land survey to a closure on another control station, then systematic errors may

be largely eliminated from the results, and the effects of accidental errors greatly diminished. The method of adjusting a land survey to stations of a control survey is illustrated later in this manual.

24. The use of State coordinates in describing land boundaries is steadily increasing, and where a survey is made to determine boundaries of land which is used in an engineering project, a double purpose is served: an approved method is provided for describing land boundaries for record purposes, and a simple means afforded for correlating cadastral and engineering data. The use of the State coordinate systems for cadastral purposes has been given formal recognition in a number of States by legislative acts which officially adopt and name the systems and give them precise definition.[10]

[10] Appendix II, p. 48.

CHAPTER 2. SURVEYS AND PLANE-RECTANGULAR COORDINATES

25. Any one who has done land surveying has used a plane-rectangular coordinate system in some form or other. He may not have been particularly aware of it or expressed his results in distances east or west and north or south of some single point or origin, but he has assumed in the computation of his results that the work was on a flat or plane surface, and has referred the bearings of all surveyed lines [11] to some meridian. For the survey of one parcel of land, results expressed in bearings and lengths of line are all that is necessary.

26. If, however, the surveyor extends his work beyond the survey of a single parcel of land, adding the surveys of adjoining lands, and continues to do this indefinitely, he will find it convenient to compute for each instrument station and boundary corner, two distances: one giving its position in an east-and-west direction, and the other its position in a north-and-south direction, both referred to some selected point or *origin,* the same *origin* being used for all surveys in a given locality or neighborhood. These two distances are called the coordinates of the point: the east-and-west distance is the x-coordinate; the north-and-south distance, the y-coordinate.

POSITION OF ORIGIN AND SIGNS OF COORDINATES

27. If the origin is located within the area of the surveys, and is given the coordinates 0, 0, some of the x values will be east (+) and others west (−); while some of the y values will be north (+) and others south (−). This occurrence of both positive and negative coordinates in the same computation increases the chances of error in the computations and in the recording of results. It is overcome and all coordinates rendered positive (east or north) by the simple expedient of assigning sufficiently large numerical values to the coordinates of the origin to place the point defined by the coordinates 0, 0 well outside of and to the west and south of the area which is to be represented on the plat. In surveying operations where plane-rectangular coordinates are employed, it is the differences of coordinates rather than their absolute values which enter the computations. Making all coordinates positive does away with taking differences between coordinates of opposite signs and greatly simplifies the work. In the example, the survey is tied to only one point, which is used as an origin of coordinates and given independent (assumed) coordinates. The purpose is to show how all coordinates may be made positive. As stated in section 69, it is desirable that a land survey be based on at least two control stations, so that it may be fixed in both position and scale.

28. Why one is interested in differences of coordinates rather than in their absolute values is easily seen. There is little value in knowing how far a point is north and east of

[11] In many legal and engineering texts, the term *course* is employed with the meaning: bearing or direction of line; in other texts, the term *course* includes both bearing (or azimuth) and length of line. In this manual, confusion will be avoided by not employing the term *course.* Instead the terms *bearing* or *azimuth* will be used to denote the direction of a line, and *length* or *distance* its linear dimension.

an undescribed, unmarked point which is well outside the theater of operations; but it is of great utility to know how far and in what direction a point is from survey stations, landmarks, and other prominent objects. If the coordinates of the various points are known, the differences of the coordinates will provide the desired relationship.

29. Having placed all surveys within a neighborhood on a single system of plane-rectangular coordinates, so that surveyed lines form a harmonious network, the surveyor is in an excellent position to extend the system in any direction: outward, by surveys of additional lands; inward, by subdividing the lands already surveyed.

30. The usefulness of having all coordinates positive may be illustrated by example. Figure 7 represents a parcel of land around whose boundary a traverse has been run. The traverse is considered as balanced or adjusted. Figure 8 shows the usual computation of latitudes and departures referred to an origin with the coordinates 0, 0. Some of the coordinates are positive and some are negative. Bearings are used in this example, and from their qualifying letters, N or S, E or W, it is easy to determine without study the column in which each computed latitude or departure belongs. But it is when one wishes to compute a bearing or distance from coordinates that unusual care must be taken, for a difference of coordinates may involve an arithmetical addition.

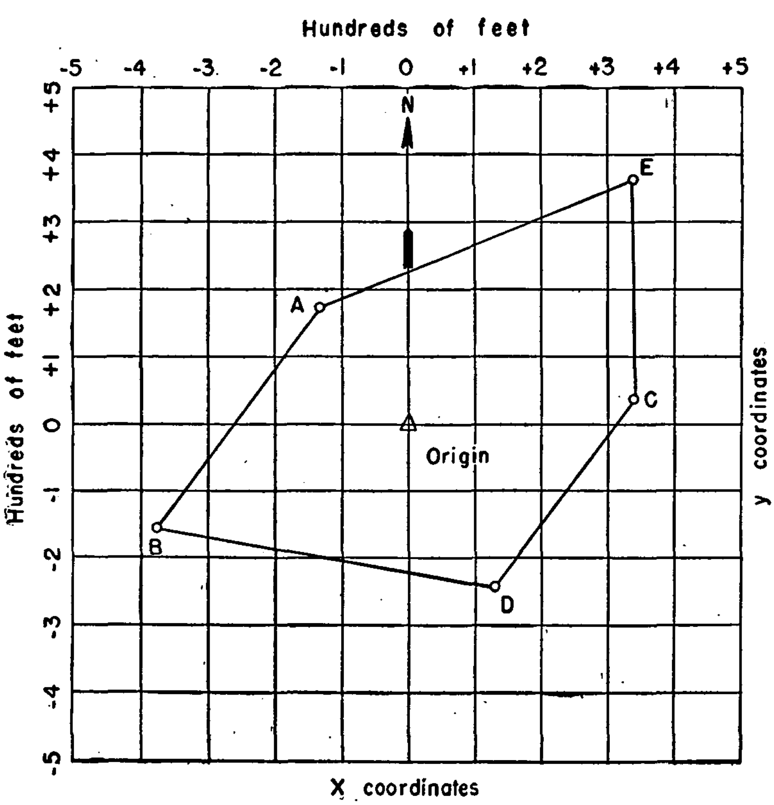

FIGURE 7.—*Positive and negative coordinates (sketch)*

First Method

(Origin = 0,0)

Station	Line (Course) Bearing	Distance	Cos Bear. / Sin Bear.	Latitude N+	S-	Departure E+	W-	Coordinates N S + -	E W + -
Origin		(Feet)						0.0	0.0
A	N37°08'W	215.0	.79723 / .60367	171.4			129.8	+171.4	-129.8
B	S35°53'W	402.8	.81021 / .58614		326.4		236.1	-155.0	-365.9
C	S79°27'E	502.7	.18309 / .98310		92.0	494.2		-247.0	+128.3
D	N36°06'E	361.3	.80799 / .58920	291.9		212.9		+ 44.9	+341.2
E	N 0°07'W	319.7	1.00000 / .00204	319.7			0.7	+364.6	+340.5
A	S67°40'W	508.4	.37999 / .92499		193.2		470.3	+171.4	-129.8

FIGURE 8.—*Positive and negative coordinates (computation)*

AZIMUTHS PREFERRED TO BEARINGS

31. In the second example, Figures 9 and 10, azimuths are employed and the origin is arbitrarily given the coordinates 500, 500. The azimuth of a line is usually defined as the angle between the line and the geographic meridian, reckoned from the south clockwise through 360°. This gives the following azimuths for the cardinal points: south, 0° or 360°; west, 90°; north, 180°; and east, 270°. The principal advantage in using azimuths in place of bearings is that the difference of the azimuths is the angle between the lines. The observed angle between two lines can be applied directly to the azimuth of one of the lines to obtain the azimuth of the other line. In plane surveying, the azimuth of a line at one end is exactly 180° different from its azimuth at the other end. In this example, departures are placed before latitudes; this is done to bring the coordinates into the usual order of expression, *x*, *y* (not *y*, *x*). The signs of the departures and latitudes are determined by the signs of the trignonometric functions through which they are obtained, and these depend upon the quadrant in which the azimuth lies. This association of functions and signs is shown at the tops of the columns marked "Latitude" and "Departure." For example, the azimuth of the first line is 142° 52′, placing the line in the northwest quadrant, where all cosines are negative and all sines are positive. The departure of this line is therefore negative, and its latitude positive.

32. While azimuths and bearings produce identical results, azimuths are used exclusively in the national control survey, and in the published data derived from that survey.

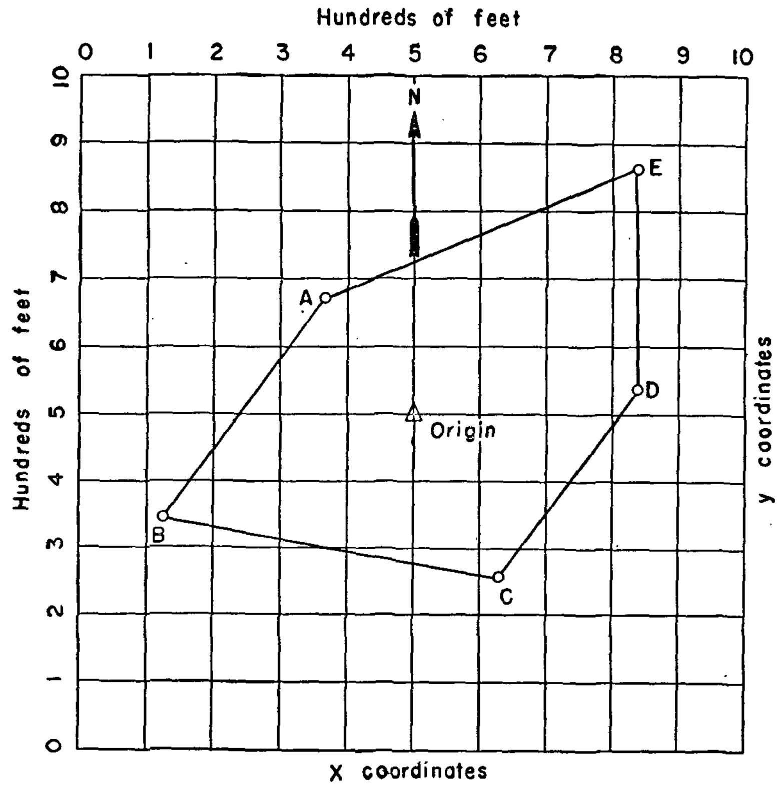

FIGURE 9.—*All coordinates positive (sketch)*

Second Method

(Origin = 500,500)

Station	Line (Course) Azimuth	Distance (Feet)	Sin Az. Cos Az.	Departure 180°–360° + / 0°–180° −	Latitude 90°–270° + / 270°–90° −	Coordinates x	y
Origin						500.0	500.0
A	142°52'	215.0	.60367 .79723	−129.8	+171.4	370.2	671.4
B	35°53'	402.8	.58614 .81021	−236.1	−326.4	134.1	345.0
C	280°33'	502.7	.98310 .18309	+494.2	− 92.0	628.3	253.0
D	216°06'	361.3	.58920 .80799	+212.9	+291.9	841.2	544.9
E	179°53'	319.7	.00204 1.00000	− 0.7	+319.7	840.5	864.6
A	67°40'	508.4	.92499 .37999	−470.3	−193.2	370.2	671.4

FIGURE 10.—*All coordinates positive (computation)*

This makes it desirable to use azimuths in any dependent survey which is based on the national control survey. It is recommended that azimuths also be used on isolated surveys, where the accuracy is sufficient to justify their being placed later on a State coordinate system, when and if the connection to the State system becomes practicable.

DIFFERENCE BETWEEN GEODETIC AND GRID AZIMUTHS

33. In placing a survey on a system of plane-rectangular coordinates, all bearings or azimuths are referred to the same meridian. This produces a condition which some surveyors and many others who are interested in land-boundary descriptions have difficulty in accepting: except at the origin and other points on the meridian of reference, there is a difference between the *grid azimuth* or *bearing* of a line and its *geographic azimuth* or *bearing*. Everyone is familiar with the fact that the geographic meridians converge towards the poles, where they meet and intersect. If one is making a survey of a piece of land which extends a considerable distance in an east-and-west direction, he knows that he will not obtain true azimuths (Section 36) for the traverse lines by applying the traverse angles successively to the true azimuth of the starting line. As the survey proceeds further and further east or west and the distance from the meridian through the origin increases, the difference in the directions of that reference meridian and of the local meridian will steadily increase, and there will result a corresponding increase in the difference between true and grid azimuths.

34. It is sometimes important for the surveyor to know the difference between the geodetic and grid azimuths of a survey line, for purposes of obtaining a check on a computed value or to provide a starting azimuth for a survey. In land surveys, for lines up to or about one mile in length, it is satisfactory to use the angle between the geodetic and grid meridians. On the Lambert grid this angle is known as the theta angle, designated by θ, and listed in the State projection tables. On the transverse Mercator grid, the angle is the delta alpha angle, designated by $\Delta\alpha$, employed in geodetic computations to represent the convergence of the meridians. In current publications of State coordinate position data, the appropriate value of θ or $\Delta\alpha$ is listed for each station. For either grid, precise values of the difference between geodetic and grid azimuths may be obtained by the application of small corrections to the θ or $\Delta\alpha$ values. These corrections are too small to be noticed in this publication.

CONVERGENCE OF MERIDIANS

35. The convergence of meridians and its effect on a true azimuth or bearing is illustrated in Figure 11. The azimuth angle A increases from west to east. The azimuth angle at A_1 differs from the azimuth angle at A_3 by the amount the meridian at A_1 fails to be parallel with the meridian at A_3. For the survey of a small parcel of land, this difference is negligible, and the surveyor who uses a system of plane coordinates, and states the resulting bearings are true bearings, when only the bearing of lines converging at the origin may be accurately called true bearings, is technically wrong, but sufficiently accurate for practical purposes. But as a survey reaches further and further from the initial meridian, these differences become important, and the use of the term "true" to designate a bearing that is really a grid bearing may become a blunder of such magnitude as to develop costly consequences.

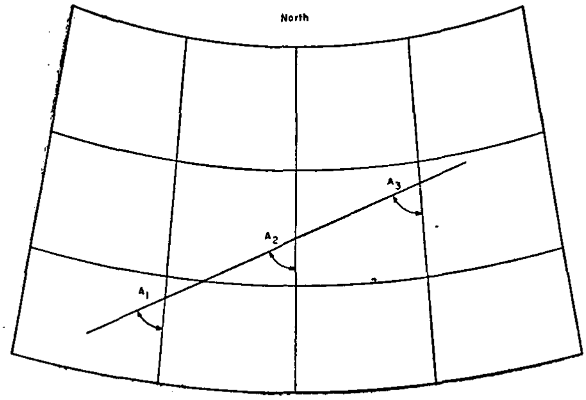

FIGURE 11.—*Convergence of meridians*

36. The use of the term "true" to indicate geographic values in surveying and mapping in this country has long been established. It has been employed to distinguish between bearings which depended upon astronomical observations and those which were obtained with the magnetic compass. The terms "magnetic north," "magnetic meridian" and "true north," "true meridian" are of considerable significance in surveying literature. The term "true" is now become an overworked word, and it is hoped that its use may be restricted to interpreting records and papers already in existence, and that where in future work it would by custom be used, the more definitive term "astronomic" will be employed. This becomes more important as the use of geodetic values increases. The term "geographic" includes both astronomic and geodetic values, and while in land surveying there is usually only a slight difference between astronomic and geodetic azimuths, there is a difference, sometimes numerically significant, and no misunderstanding is possible if *astronomic* is used where *true* would have been used at an earlier time, but where the present use of *true* might be interpreted to mean *geodetic*.

37. Taking account of constantly changing azimuths or bearings is so objectionable that surveyors avoid it by using some form of plane-rectangular coordinates, ignoring the consequences described in preceding sections of this manual. The form exemplified in figure 10 is simple and for a small area presents no difficulties in practical application. But as the surveys extend into new areas, the surveyor may wish to continue the use of the same origin of coordinates and the same reference meridian. If he does this, he will have to accept the errors that are inherent in the use of a local system of plane coordinates and when those errors become too large to be tenable, the standard procedure would be to adopt a new origin and meridian, and make a new start. At least this was the standard procedure until the State coordinate systems made it possible to extend surveys on a single grid over large areas without encountering insuperable scale difficulties.

PLANE COORDINATES AND CITY SURVEYS

38. Even in city surveys, where areas are somewhat limited, a required accuracy of plane-coordinate values could be obtained only by definitely limiting the distance from the origin for which the plane system could be used. For example, in the triangulation of Greater New York City, 1903-1908, three local systems of plane coordinates were employed in reporting the results of that survey. Today, the most precise survey of New York City and of all of Long Island could be referred to a single grid, the Long Island

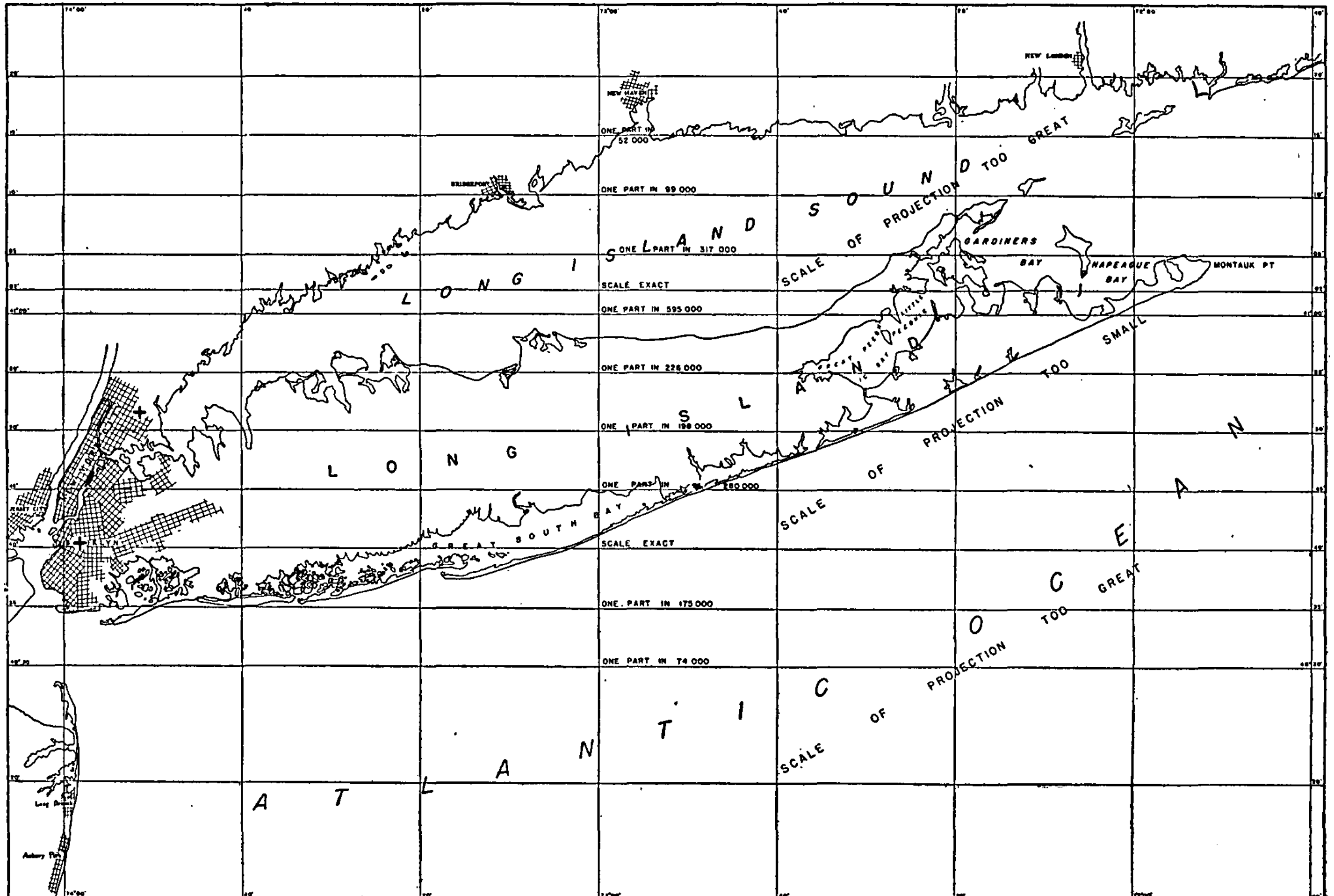

FIGURE 12.—*New York Coordinate System, Long Island Zone showing origins (+) of three local systems of New York City Survey (1903-08)*

Zone (fig. 12) of the New York Coordinate System, without in any place incurring significant errors due to the use of a plane in representing a portion of the Earth's surface. In 1936, a control survey of Denver, Colo., was executed under the sponsorship of the city department of improvement and parks. The work was under the direction of the United States Geological Survey. Consideration was given to four different systems of plane-rectangular coordinates before decision was made to use the Colorado Coordinate System, devised by the Coast and Geodetic Survey, as the base upon which to record the position data of the city survey. As reported in the GEODETIC LETTER [12] of January, 1937:

> It was adopted as a basis for the Denver map because it satisfies local and general needs and, at the same time, represents the most progressive ideas in modern large-scale map construction.

39. As the coordinates on the appropriate State system are known for all stations of the national survey for which adjusted positions on the standard geodetic datum are available, it follows that one may often connect a land survey to several such control stations and in so doing accomplish two things. If the connection is to two or more control stations and is so made as to include several land corners, it will be possible to make a good adjustment of the land survey to the national control survey; this should to a great extent eliminate the effects of systematic errors from the survey results and make it possible to at least estimate their accuracy. If the survey connection is adequate, the accuracy can be closely determined. The connection to stations of the national control survey also makes the stations of that net—all of them—reference marks of the land survey. Not just the stations to which the connection is made, but all of the stations of the national control survey become not witness marks, but reference marks, any of which, within practical surveying distance, may be used in reestablishing a point on a land survey or in proving its recovery.

40. While the State coordinate systems will be described in other parts of this manual, and their use exemplified by the computation of a typical survey, it may be well to point out here some of the characteristics which distinguish the State systems, and anticipate some parts of the computation just mentioned by an explanation of fundamental qualities.

STATE GRID ZONES

41. One of the important characteristics of the State coordinate systems is the small number of separate grids required to cover a State; or, to put it differently, the large area that is served by a single origin and reference meridian. For example, New Jersey and Tennessee are each covered by a single grid, and Texas by five. In New Jersey or Tennessee, a survey in any part of the State can be fully coordinated with a survey in any other part of the State; in Texas, a survey close to the Sabine River can be coordinated with a survey in about the same latitude on the banks of the Rio Grande. There are 21 counties in New Jersey, 95 in Tennessee, and 254 in Texas. The simplicity of the State systems should be evident from these figures. If a separate system of coordinates were established for each county, of the kind employed in processing the 1903-1908 triangulation of New York City (Section 38), New Jersey would have 21 origins of coordinates and reference meridians where it now has only one; Tennessee would have 95 local or county systems in place of a single State grid; and Texas, in place of a State system comprising 5 origins and meridians, would have 254 origins and meridians. In the above comparisons, any of the States could have been used as illustrations in place of the ones chosen, without changing the conclusions.

[12] Bibliography, p. 44.

LOCAL SYSTEMS OF COORDINATES

42. Both State and local systems of plane coordinates may be based directly on stations of the national geodetic control survey and the benefits of accuracy of results and permanency of ground position may thus be obtained. The employment of conformal map projections, however, in establishing the State systems, makes practicable the use of much larger zones than is possible with a local system based on a tangent plane, a method of projection that was standard for transforming geodetic positions into plane coordinates for a number of years. An example of the advantage of the State systems is the simplicity with which the scale distortion in any part of the grid may be determined over a wide area and applied to survey values. In the local system, employed for city and county surveys over a period of years, this correction was sufficiently difficult to determine as to unduly limit the size of the zone.

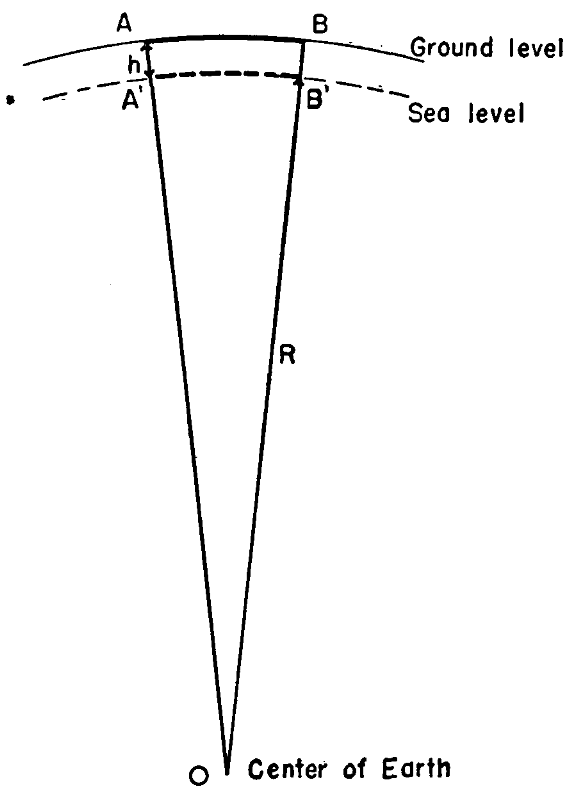

FIGURE 13.—*Reduction to sea level*

GROUND LENGTHS REDUCED TO SEA LEVEL

43. Since the geodetic data determined by the national control survey—the latitudes and longitudes of points, and the lengths and azimuths of lines—are sea-level data, it follows that surveys which are to be adjusted to stations of the national survey must first be reduced to a sea-level base. And as the State coordinate systems are developed directly from geodetic values, the use of those systems requires the further reduction of the sea-level values to grid values.

44. Consider first the reduction to sea level, and for illustration use a sphere having a radius equal to the mean radius of curvature of the spheroid (Earth) over the United States, 20,906,000 feet.

45. Few people and not many surveyors are concerned with the fact that the distance between plumb lines passing through two points on the surface of the Earth depends in a small way upon the elevation of those points above sea level—sea level being the surface of the sea if it were free to flow under the surface of the land. Consider a level surface one mile above sea level: in figure 13, $A'B'$ represents the sea-level surface and AB the surface of the land. The plumb-line directions or verticals through A and B are not parallel but converge toward O, the center of the sphere. The sea-level distance $A'B'$ will be shorter than the measured length AB by an amount about proportional to the elevation of the ground divided by the radius of the sphere. From the figure

$$AB : A'B' : : AO : A'O$$

whence

$$AB - A'B' : A'B' : : AO - A'O : A'O$$

If C is the correction to reduce the ground-level distance AB to the sea-level distance $A'B'$; S the measured length, AB; R the radius of the sphere, $A'O$; and h the elevation of the ground above sea level:

$$C = (S - C)\ \frac{h}{R} = S\ \frac{h}{R+h}$$

or, for practical purposes

$$C = S\ \frac{h}{R}$$

In the illustration, the ground is shown above sea level, and the sea-level distance $A'B'$ equals the ground-level distance AB minus the correction, that is $A'B' = AB - C$. In one small area in the United States (Death Valley, Calif.), the ground is below sea level and the correction is additive, but so small as to be negligible. If the elevation of the ground is 1 mile and the radius of the Earth is 4,000 miles, the line $A'B'$ will be shorter than the line AB by about 1/4,000 part of AB. In other words, if the line AB is found by measurement to be exactly 4,000 feet long, and it were possible to plumb down to sea level and measure the distance $A'B'$, this new distance would be found to be 3,999 feet. The distance between two points at sea level which are directly under two points on the surface of the Earth is the sea-level or geodetic length of the surface line. The difference between ground-level and sea-level lengths is small, but for many parts of the country the elevation of the ground is sufficiently great to require that it be considered in accurate and extensive work. Using the mean radius of the Earth for this country, the approximate formula becomes

$$C = S\frac{h}{20{,}906{,}000}$$

As the value for R is in feet, h must also be in feet. C will then be in the same unit as S. In developing this formula, no account is taken of the Earth being a spheroid instead of a sphere, but this will produce no significant errors in land surveys where lengths are short and areas comparatively small. In this publication, for convenience in computing, the sea-level distance is obtained, not by applying a correction as shown above, but by multiplying the ground-level distance by a factor, $(1 - \frac{h}{20{,}906{,}000})$. Values of this sea-level factor for various elevations are given in Appendix III.

46. In land surveying, actual ground-level distances are measured and used in the computation. If a local plane-rectangular coordinate system is used, the ground-level distances may be used in computing the coordinates, but if the survey is to be based upon

control stations of the national survey, the measured distances must be reduced to sea level, regardless of the kind of coordinate system employed. This reduction presents no difficulties as the average elevation of even a large area can be used, and the reduction is easy to apply. This average elevation may be obtained from a number of sources such as the topographic maps and leveling data issued by various governmental and private organizations. It might also be obtained by means of barometric observations. The accuracy with which the elevation of the ground must be known to obtain a prescribed accuracy in the reduction to sea level may be determined by consulting the table of comparative errors in section 58.

SEA-LEVEL LENGTHS REDUCED TO GRID LENGTHS

47. The reduction of a sea-level length to a grid length on a local coordinate system is not a simple procedure, but as such local systems have almost completely been superseded by the State systems, the transformation of sea-level lengths to grid lengths on a local system will not be discussed here beyond saying that it is impracticable of achievement and never made. On the other hand, the reduction of a sea-level (geodetic) length to a grid length on a State system is simple and is easily made. A geodetic length is reduced to a grid length on a State system by the application of a *scale factor* which is taken from a table with a single argument—the position of the line with respect to a central control line of the grid. This will be considered with sufficient detail for the purposes of this manual in connection with the actual examples of use (secs. 52, 55). If one desires to go even further into the study of scale factors, Special Publication No. 193 [13] of the Coast and Geodetic Survey should be consulted.

COMBINATION FACTORS: REDUCTION TO SEA LEVEL AND GRID

48. In reducing a ground-level length to its corresponding grid length on a State coordinate system, the two processes involved—reduction to sea level and thence to the grid—may, for most land surveys, be performed in a single operation, employing a factor which is a combination of the sea-level and scale factors. The usual land survey is of such limited extent and has such small variation in relief that this rarely offers any difficulty. In studying the effect of using a State system over an extensive area, it is the rate at which the scale factor changes over the area rather than its absolute values which should be considered. For example, it is near the center of the belt limited by the lines of zero scale error of a State grid that one maximum value of the scale factor occurs, but it is also there that the factor changes most slowly, so that a mean scale factor may be employed there for much wider areas than where the scale factors are much smaller but are rapidly changing. Results obtained with large scale factors are as accurate as results obtained with small scale factors. The principal advantage of small factors is that they may be neglected in all but the most precise surveying.

49. We now come to the case where a higher degree of accuracy is obtained in the measurement of the traverse, and it is desired to obtain a consistent accuracy in the computations. The general location of the work within the State is known from maps, while its relation to the State grid is determined from the control stations to which it is tied. Also, the general elevation of the land above sea level is obtained from available sources (sec. 46).

[13] Bibliography, p. 44.

50. Let us assume that a surveyor plans to make an extensive survey in Cobb County, Ga., and that the work will be done with greater accuracy than is employed on an ordinary land survey—not the highest accuracy, but with that indicated by closing errors of between 1 : 5,000 and 1 : 10,000—probably nearer the latter figure. This means that the surveyor will use a standardized tape, that is, a tape whose length in standard units is known at a given temperature, tension and method of support. The tape will be used at the standard tension and method of support, but correction for temperature will be made (sec. 89), and as the distance measured will be on the slope of the ground, there will also be a correction for grade, to reduce the slope distance to the corresponding horizontal distance. The grade is determined by leveling or by observed vertical angles. In a traverse of this quality, the measured or ground-level lengths are also reduced to grid lengths.

CHARACTERISTICS AND LENGTH REDUCTIONS

TRANSVERSE MERCATOR GRID

51. The Georgia Coordinate System is formed of two zones, the line of separation being approximately north and south, following county boundaries. Cobb County is in the West Zone. As these zones are of limited east-west dimension, and extend north and south the entire length of the State, the State coordinate system is based on the transverse Mercator map projection. The name of the base projection is of no special importance to the surveyor who starts with grid coordinates and azimuths. It does appear in connection with the technical definition of the State system and is useful in establishing the legal identity of the system. The information which the surveyor obtains from the technical definition may be summarized as follows:

(a) The central meridian of the West Zone is the geodetic meridian 84° 10′ west of Greenwich, and it is to this meridian that the grid azimuths in this zone are referred. The x-coordinate of the central meridian of this zone is arbitrarily given the value 500,000 feet, so that the distance from the central meridian to a point whose x-coordinate is known may be obtained by simple subtraction. This distance, east or west, is called x'. $x' = x - 500{,}000$.

(b) The scale of the projection may be expressed in several ways. It may be given as a ratio of the grid length to the sea-level (geodetic) length, or it may be expressed as a numerical correction to be applied to the measured length reduced to sea level, to further reduce it to the grid length. This correction may be given as so many feet per thousand feet in length.

(c) In the normal form of this type of projection, the scale error and therefore the scale factor increases with distance from the central meridian. If the scale along that meridian is exact, a 1 to 1 correspondence, then the grid lengths not on that meridian will be longer than the sea-level lengths. As the scale errors increase as the square of the distance from the central meridian, those along the outer edge of the grid will not only be larger but will change much more rapidly than those near the central meridian. In order that the errors along the edges of the grid will not be excessively large, a scale factor is adopted for the central meridian which will make grid distances smaller than sea-level distances by an amount which will be closely balanced by resulting scale errors at the grid edges, where the grid distances will be the larger. In the case of the Georgia Coordinate System, West Zone, the scale along the central meridian was set at 1 part in 10,000 too small, so that a sea-level distance at that meridian is reduced by 1/10,000 of its length to obtain the corresponding grid length. At a distance of about 56 miles from the central meridian, the scale becomes exact: grid lengths agreeing exactly with geodetic lengths.

And at 79 miles from the central meridian, the grid length is 1 part in 10,000 too large. Thus, for a zone 158 miles in width, the scale error is kept to 1 part in 10,000 and under, too small for the inner part of the zone, too large for the outer parts, by amounts varying from 0 to the stated maximum. This scale may be expressed in the following forms, using the scale along the central meridian for illustration: 1/10,000 too small; 0.1 foot per thousand too small; or 0.9999. The last figure is known as the scale factor, and is usually tabulated for the argument x', the distance from the central meridian.

52. Coming back to the case of Cobb County, we find by scaling a map showing both county lines and the geographic projection (graticule) that the most westerly point of this county is 33.1 miles or 175,000 feet from the central meridian, and the easternmost point approximately 12.0 miles or 63,000 feet also west of the meridian. These are values of x', with which as arguments the Georgia projection tables are entered and scale factors obtained of 0.9999351 and 0.9999045, the mean of which is 0.9999198. This average factor may be put in the form of a correction of 0.0802 foot per thousand feet, by which sea-level lengths in the county are reduced to obtain grid lengths. The largest error that may be caused by using a mean scale factor for the entire county will be 0.9999351 minus 0.9999198, or 0.0000153, corresponding to 1 part in 65,000, which is well within the accuracy of the best traverse work.

53. A fair estimate of the average elevation above sea level of Cobb County, exclusive of the mountains, is 1,000 feet. This is obtained from various sources, such as maps, lines of leveling, etc. Using the formula in section 45, a value for the sea-level factor is obtained of 0.9999522, which is equivalent to 0.0478 foot per thousand. For use over the entire county the two factors, sea-level and scale, can be combined into a single factor and a ground-level length reduced to a grid length by a single multiplication. This combination factor will be $0.9999198 \times 0.9999522 = 0.999872$. Put in the form of a correction, this is $-0.0802-0.0478=-0.128$ foot per thousand. This can also be derived directly from the factor 0.999872. A traverse line in Cobb County which is exactly 1,000 feet by measurement will have a grid length of 999.872 feet. While the reduction of a ground-level length to the corresponding grid length has been described above in some detail, it is a simple operation, easily and readily performed.

LAMBERT GRID

54. To exemplify a State coordinate system based on a Lambert projection, let us take Salt Lake County, Utah, which lies in the area covered by the Utah Coordinate System, Central Zone. On a Lambert grid, scale errors are constant along parallels of latitude and scale factors are therefore tabulated with latitude as the argument. By the same methods as were employed with the transverse Mercator grid in keeping scale errors within a prescribed maximum, the scale errors on a Lambert grid are also controlled. The 1 to 1 correspondence is established along two selected parallels, known as *standard parallels.* Between these parallels the scale is too small, outside them it is too large.

55. From a good map we find that Salt Lake County is between the parallels 40° 54′ and 40°26′.5 north latitudes, for which the corresponding scale factors, taken from the projection tables for the Utah Coordinate System, Central Zone, are 1.0000718 and 0.9999548 respectively. The average of these two factors is 1.0000133 or 0.0133 foot per thousand feet too large. Since this county is considerably larger than Cobb County, Ga., the use of this average scale factor can result in an error at the north and south borders of the county of 1: 17,000, which is still within the limits of accuracy for the type of work being discussed here.

56. The adoption of a mean elevation for the entire county presents difficulties which did not exist for Cobb County (above) where the range of elevation was probably a matter of only two or three hundred feet. A good portion of Salt Lake County is a valley floor with an average elevation of around 4,300 feet, but along the western border of the county are mountains up to 9,000 feet in elevation, while near its eastern and southeastern borders the mountains rise to 11,000 feet. From the somewhat level portion of the county with an average elevation of 4,300 feet, a sea-level factor is obtained of 0.9997944, or 0.2056 foot per thousand, the amount by which the sea-level length is smaller than the ground-level length. The combination factor, sea-level and grid, is 0.9998077 or 0.1923 foot per thousand too small. A ground-level length of 1,000 feet will have a grid length of 1,000−0.1923= 999.8077 feet, which is the same as obtained by multiplying 1,000 by the combination factor, 0.9998077. For areas of greater elevation, the elevation factor will have to be determined for the locality. A difference of elevation of 1,000 feet will change the elevation factor by about 1 part in 21,000.

57. In the western part of the country there are counties of such size and so mountainous that a single factor can not be used for the entire county if it is desired to obtain results of the order of 1 : 7,500 accuracy or better. Such a county can be subdivided into areas for which mean factors as described above can be used. In taking out the scale factor for a transverse Mercator grid, the area can extend indefinitely in a north-and-south direction; and for a Lambert grid, in an east-and-west direction.

ACCURACY OF SEA-LEVEL REDUCTION

58. The area for which elevation factors can be averaged may be selected by examination from a suitable map. It takes considerable change in elevation to produce a significant effect on the reduced lengths, as shown by the following table, which will aid a surveyor in deciding when special precautions must be taken in making sea-level reductions.

Effects of Errors in Elevation on Accuracy of Lengths Reduced to Sea Level

(Based on a radius of curvature of the Earth of 20,906,000 feet)

Error in elevation	Proportional error in length	Error in elevation	Proportional error in length
(feet)		*(feet)*	
100	1: 209,000	2,091	1: 10,000
500	1: 41,800	2,787	1: 7,500
836	1: 25,000	3,000	1: 7,000
1,000	1: 20,900	4,000	1: 5,200
1,500	1: 13,900	4,181	1: 5,000
2,000	1: 10,500	5,000	1: 4,200

These are also the sea-level factors expressed as ratios, for elevations represented by the first-column arguments.

59. In another part of this manual is a practical example of a land survey, commencing with the observed angles and measured lengths, and going through the various processes of a complete computation, and concluding with the adjusted coordinates on a State coordinate system. There naturally will occur some repetitions of matters covered in earlier sections of this work, which should prove helpful to those making their first use of the State systems, and it is largely to help such beginners that this manual is prepared.

CHAPTER 3. THE LAND SURVEY

60. In this chapter, the actual procedures involved in placing a land survey on a State coordinate system will be considered. Many of the steps involved are standard operations in which land surveyors are well trained, and for which detailed instructions and needed formulas may be found in any modern text on surveying. Mention of such steps will therefore be brief, and made largely to furnish a complete outline of the program as an aid to those who are just beginning their work in this field of engineering or to remind the more experienced of certain details which through lack of use may have been forgotten, but which should be considered in surveys described in this manual.

PLANNING THE SURVEY

61. The planning and execution of an engineering project usually involves several consecutive steps, if it is to be accomplished in an orderly and economical manner. Making a survey of a parcel of land is an engineering project, the first step in which is the collection of certain basic data required in planning the work and in its execution. This is the *reconnaissance*. Then follows the survey proper, the traverse of the land boundary and associated lines: this is termed the *field work,* in contrast to the *office work,* which is the third and final step, in which observations made in the field are transformed into data describing the land and used in computing its area. These three steps are considered in the following pages: the first and second somewhat briefly, emphasis being placed only on those matters with which the land surveyor is usually not well acquainted; the third step, when it is desired to place a land survey on a State coordinate system, is described in detail, using practical examples to illustrate the procedures involved.

THE RECONNAISSANCE

62. It is assumed that the surveyor has attended to certain matters relating to the land itself: the records have been examined and a deed description of the land to be surveyed has been obtained, as well as information about other lands in the vicinity which may be required in recovering or re-establishing the corners of the land which is to be surveyed. Until the ground position of a land corner is known, its State coordinate position can not be determined. This manual offers nothing new about the recovery or restoration of land corners by the usual methods of land surveying. Its purpose is to describe the methods by which points on the ground can be referenced to a State system of coordinates, so that future restorations can be effected with accuracy and facility.

63. Placing a land survey on a State coordinate system requires a *survey connection* to an already established station, termed a *control station,* whose coordinates on a State system are known. As shown in Figure 18 and demonstrated in the practical example, it is very desirable that the connection be to more than one control station. The national triangulation net executed by the Coast and Geodetic Survey comprises thousands of control stations distributed throughout the country. In some areas, the spacing of these control stations is satisfactory for land survey purposes; in other areas, the spacing is so wide that

the survey connection for remote localities would not be practicable. This condition is being steadily corrected as the geodetic surveys are extended into new areas. Information relating to control stations established by the U. S. Coast and Geodetic Survey may be obtained from the Director of that Bureau, Washington 25, D. C. In requesting such information, the location of the land to be surveyed should be described by giving its approximate geographic position or its relative position from some town or other feature which appears on a general map. This information is needed in selecting control stations which are in the same locality.

Station Marks

64. The control stations, for which the Coast and Geodetic Survey supplies State coordinate data and descriptions, are marked in a permanent manner. For the older stations, the marks are of various kinds: stones or bottles buried beneath the surface of the

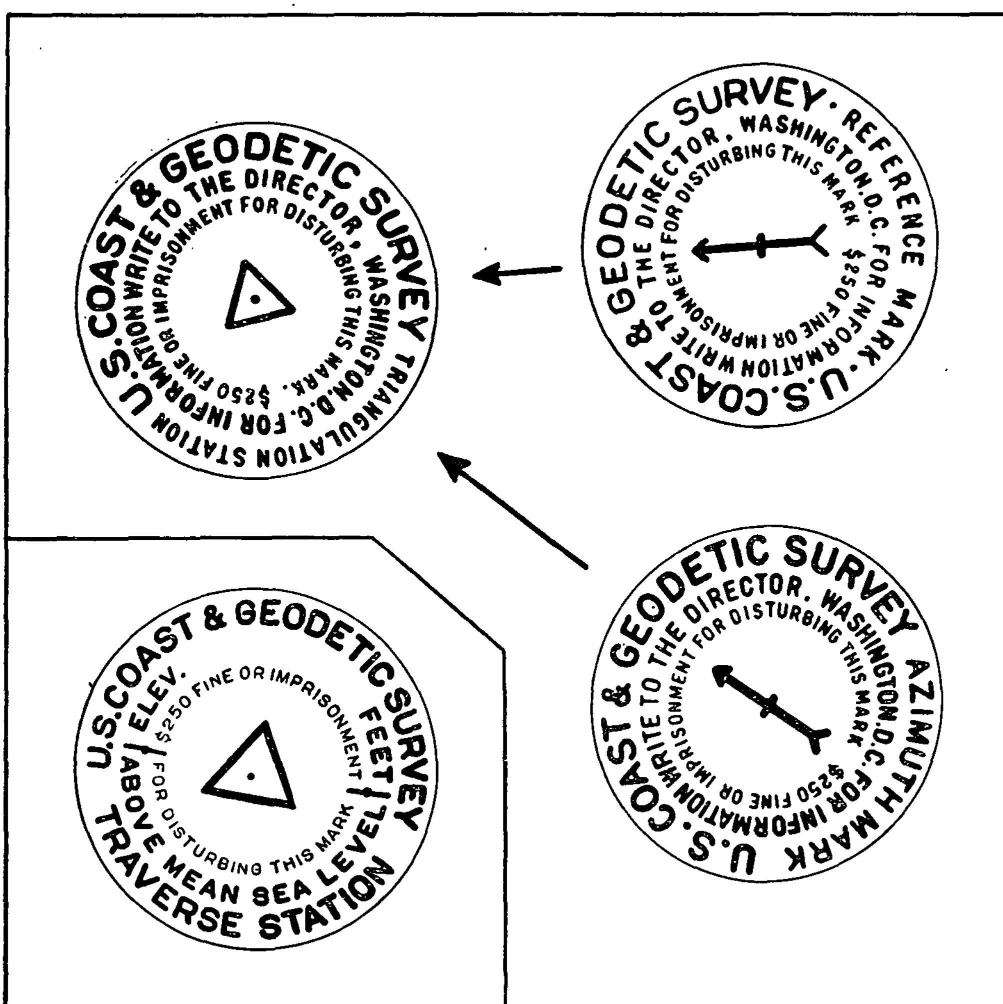

Figure 14.—*Station, reference, and azimuth marks*

ground; drill holes in stone posts or boulders; bolts in stone posts; pipes set in concrete; etc. At some stations there are single marks; at other stations there are double marks, one at the surface of the ground, and another directly beneath it to be resorted to only when the upper mark has been disturbed. But for stations established in recent years (and this includes a very large portion of all stations established by the Bureau), a standard type of marking is employed (fig. 14). At each main-scheme station there are placed a station mark, two reference marks, and an azimuth mark. These are metal disks (bronze) set in the top surfaces of posts of concrete or stone, and suitably inscribed to identify the mark as a station or center mark, a reference mark, or an azimuth mark. If the ground conditions permit, there are two such marks placed at the center or station: the surface mark may be flush with or extend slightly above the ground; the underground mark is directly beneath but independent of the surface mark; it is sufficiently deep to be free from ordinary surface disturbances, and should be resorted to only when there is evidence that the upper mark has been disturbed. It occasionally happens that a station is located in a cultivated field: in such case only an underground mark is set; it is placed well below the reach of the deepest cultivation.

Reference Marks

65. Two, sometimes three, reference marks are established near each main-scheme control station. These are disks in concrete posts, rock ledges, or other suitable objects, in protected spots, clearly described and easily found. It is the function of the reference marks to aid in the recovery of the station mark and to establish the integrity of that mark when it has been recovered. This is accomplished by checking the directions and distances of the reference marks from the station marks; these data are furnished with the description of the station. A failure to check these data exactly does not necessarily mean that the station mark has been disturbed, as it is possible that the reference marks have been disturbed, or that the reference data may contain an error. Failure to obtain a good check should be reported to the Washington Office of the Bureau, and instructions requested for proceeding with the work. Obtaining a satisfactory check indicates that the marks are in their original positions. All of the evidence of ground location given in the description of a station (sec. 67) should be carefully examined and only when there is positive evidence that the surface mark has been disturbed should recourse be had to the underground mark.

Azimuth Marks

66. Near each main-scheme station established by the Coast and Geodetic Survey since about 1928 there has been placed an *azimuth mark,* consisting of a disk set in concrete or stone. At first, standard reference disks were employed, but after a few years these gave way to disks bearing the words "Azimuth Mark" (fig. 14). As with reference marks, the name of the station with which the azimuth mark is associated is usually stamped on the disk. The azimuth mark is placed at a greater distance from the station than are the reference marks; usually at a distance of a half mile or more. The function of the azimuth mark is to provide a marked line of known azimuth which the surveyor can use in establishing a reference meridian for his survey. The distance of the azimuth mark from its primary is such as to render insignificant the angular error in observing on it which might come from permissible errors in centering either instrument or target (rod). For convenience in use, the site of the azimuth mark is such that a rod held

on it can be viewed with an instrument at the station, without the use of special supports for elevating either to obtain a clear line of sight. Time may modify a condition which existed at the time the station was established: trees may grow up or a building be erected which will make unduly difficult or completely prevent the use of the azimuth mark. However, at many triangulation stations, azimuths are determined to such objects as water tanks, church spires, flag poles, house cupolas, etc., and these may be employed in determining the direction of the meridian for the control of the land survey. The reconnaissance should make sure that the object used is the same as that for which data are furnished. Tanks are destroyed and rebuilt, but not always in exactly the same spot; buildings are remodeled, and spires and ornamental parts replaced on the remodeled structure but not in the same location as before being remodeled; and a flagpole destroyed by storm or other cause is very seldom replaced in its original location.

Descriptions of Stations

67. These are designed to facilitate the recovery of the physical monuments marking the described stations and proving the recovery. They also contain some significant data that may be required in computing the results of a survey for which they supply control. A typical description (fig. 15) opens with the name of the station and the date of its establishment; some descriptions also give dates of subsequent visits, with a note as to what was found—whether it was recovered or not, new marks placed, if any, etc. The State and county in which the station is located are given, then a general description of the locality with reference to a nearby town, if it be near a town, and to well-known landmarks, such as bridges, crossroads, etc. Often the description will give the route along which the

Sundre (Marinette County, Wis., C. A. Schanck, 1934).—About 11 miles southeast of Crivitz, about 10 miles west by north of Marinette, and about 10 miles east by north of Pound, in NE¼ of sec. 25, T. 31 N., R. 21 E., on land owned and occupied by R. E. Sundre. To reach from intersection of U. S. Route 41 and State Route 64 in Marinette, follow Route 64 west 9.15 miles to side gravel road to north, turn north onto gravel road and go 1.6 miles to Mr. Sundre's house on left and station site. To reach from Pound, follow State Route 64 (also U. S. Route 141) north 1 mile to their junction, turn east onto Route 64 and go 10 miles to gravel road to north, then proceed to station as above. Station is in northeast corner of Mr. Sundre's yard, 75 yards east-northeast of house, 35 feet west of center line of road, 18 feet west of fence line, and 12 feet south of fence line. Surface and underground marks are standard station disks in concrete, notes 1b and 7a; upper mark is 3 inches below surface of ground. Reference and azimuth marks are standard reference disks in concrete, note 11b. No. 1 projects 7 inches above ground, is 18 feet east of center line of road, 1 foot west of fence line, and 73.86 feet from station in azimuth 239°29′. No. 2 projects 6 inches above ground, is 25 feet west of center line of road, 15 feet northwest of 12-inch birch tree, and 79.04 feet from station in azimuth 348°54′. Distance between reference marks is 124.85 feet. Azimuth mark projects 5 inches above ground, is on east edge of field, 200 yards south of farmhouse, 30 feet west of center line of gravel road, 1 foot east of fence line, and approximately 0.4 mile from station in azimuth 250°12′20″.

Plane coordinates (W), $x=739{,}375.55$ feet, $y=1{,}324{,}276.07$ feet; grid azimuth to azimuth mark $=249°32'53''$.

FIGURE 15.—*Description of triangulation station*

Notes 1b, 7a, and 11b describe the concrete blocks in which the disks are set. Copies of these notes are sent with descriptions of stations.

station may be reached by automobile, with speedometer readings at identifiable points along the route. The description of the general locality is followed by a description of the particular locality in which are given a series of clues which will bring the surveyor closer and closer to the station; either to the reference marks or to the station itself. In making a survey connection from a control station to a land corner, care must be taken not to confuse a reference mark with the station mark, and make the connection to the reference mark. This is not an uncommon blunder. It can be avoided by a careful study of the description, an examination of the physical marks which are recovered, and checking the reference and witness data contained in the description.

68. Figure 15 illustrates a typical description of a control station. In addition to information required in the search for the station marks, many descriptions contain also its coordinates on the State system, the grid azimuths to marked points, and other useful information.

Selection of Traverse Station Sites

69. Having received the material requested from the Coast and Geodetic Survey, the surveyor makes a search for those stations which are best located for the control of the land survey, and having made sufficient positive recoveries for his purpose, he determines the route along which the survey connection is to be run, and proceeds to select sites for stations along that route. It is desirable that the land survey be connected to at least two control stations for which State coordinate data are available, and that these be so situated with respect to the land which is to be surveyed that a traverse starting at one control station and ending at another will include points on or close to the boundary of the land, preferably several of the land corners themselves. Basing a land survey on two or more control stations, as described above, gives position to the survey and permits its adjustment in scale and azimuth. The traverse which brings the State coordinate system into contact with the land survey should be of a higher order of accuracy than the traverse which determines the State coordinates of the remainder of the boundary, except where the latter may at a future time serve as control for other surveys, and then the higher order of accuracy should be maintained throughout the survey.

70. In making the reconnaissance for the survey connection, regard must be had for several conditions which, if neglected, may result in sub-standard accuracy for the connection. One such condition is that stations through which the azimuth is carried must be sufficiently far apart to minimize angle errors resulting from eccentricities of the instrument and target which are always present to some degree. This is referred to in more detail in section 88. Where the topography makes it necessary to carry length measures through a short line or series of short lines, the reconnaissance should provide a second route along which the azimuth can be carried satisfactorily.

71. A second condition is that no line of sight shall pass close to a structure or natural object which might be the cause of excessive lateral refraction. A line of sight passing close to the side of a road cut or an embankment upon which the sun has been shining may suffer refraction because of the disturbed condition of the atmosphere through which the line passes, and there will result a corresponding error in the observed direction of the line. One of the purposes of the reconnaissance is to avoid the inclusion of such lines in the traverse, for lateral refraction may produce a closing error in azimuth which, when distributed by any of the usual methods of adjustment, will place corrections where they do not belong, and thereby reduce the quality of the results.

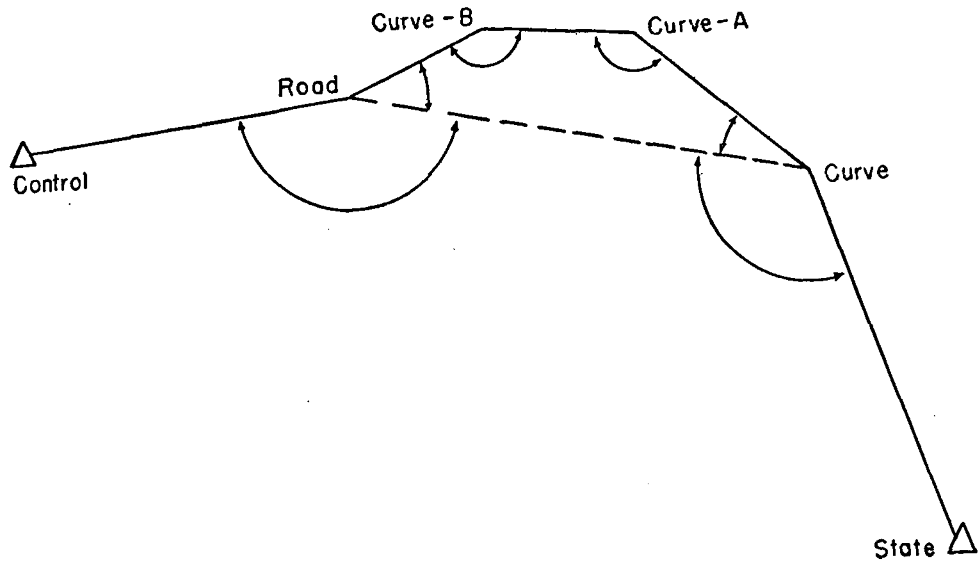

FIGURE 16.—*Azimuth carried through long lines*

In the above sketch, measured lines are represented by full lines, and observed angles by circular arcs. The computation of azimuth is carried through the line Curve-Road, whose length is obtained by projection from the measured lengths through Curve-A and Curve-B. The computed line Curve-Road is then used as a main line in the computation, and after the positions of Curve and Road are fixed, the positions of Curve-A and Curve-B are adjusted to this line.

THE FIELD WORK

LENGTH MEASURES

72. The traverse proper comprises two distinct and separate operations: the measuring of distances and the observing of angles. What is sought is the horizontal distance between contiguous points (survey stations and land corners), and the horizontal angles formed by adjacent measured lines. These quantities may not be directly measurable at and between points on the land boundary: it may not always be practicable to center an instrument over a land corner or stretch a tape along a boundary line. But the usual methods of overcoming such difficulties by means of eccentric setups and offset lines are described in text books and will not be considered here. Nor will there be any discussion of the relative merits of the two methods of using a tape: holding it in a horizontal position and plumbing its fiducial marks over the ground stations; or measuring distances along the slope of the ground, and obtaining the horizontal distance by a computation which requires that the inclination of the tape be known, either by direct observation or by leveling operations. It will be stated as an accepted conclusion that it takes experienced chainmen to obtain an accuracy of 1: 5,000 using the tape in a horizontal position. On the other hand measuring a slope distance and reducing it by computation to the corresponding horizontal distance can be accomplished with any prescribed accuracy, which is limited only by the instruments and methods employed and the care taken in making the observations.

73. Since the survey connecting the State system and the land survey should be run with second-order accuracy,[14] it follows that the length measures in this connection should

[14] Second-order accuracy in traverse work is achieved with a closing error in position of 1 part in 10,000 or better, and in azimuth of 4″ or less per main station.

employ the slope method. For the remainder of the boundary survey, if experienced chainmen are available, measures may be made with the tape horizontal, provided, of course, the survey is not to be used at some later time to control other surveys. If it is not practicable to tie the land survey to at least two control stations, but one such station is available, the survey connection should be made with requisite checks to obtain second-order accuracy.

ANGLE MEASURES

74. There are also two methods of observing the horizontal angles in a traverse line, using a surveyor's or engineer's transit. In one method, the deflection angle is measured. This is the angle between the line from the last station projected beyond the instrument station and the line to the next station, measured to the right or left as the case may be. A deflection angle to the right is positive and is added to the azimuth of the preceding line to obtain the azimuth of the new line. Similarly, a deflection angle to the left is negative and is subtracted from the azimuth of the preceding line. In the other method (fig. 17), the horizontal angle between backsight and foresight is measured directly, always in a clockwise order, backsight to foresight. With this method, the angle is usually measured by repetitions, the sum of six measures being accumulated on the circle, the telescope being reversed after three measures are made. While a deflection angle may also be repeated, the other method is preferred where more than a double (direct and reverse) measure of the angle is desired. In the practical example given later in this chapter, the method of repetitions was employed for angles of the survey connection, while deflection angles were measured along that part of the boundary for which lesser accuracy is considered satisfactory.

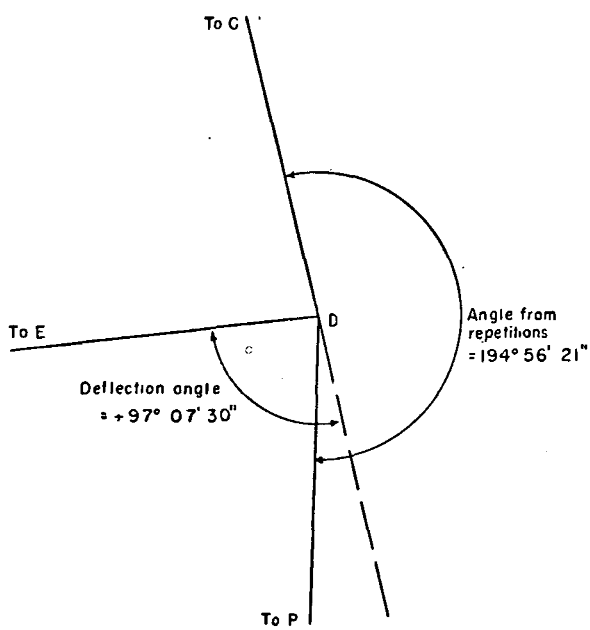

FIGURE 17.—*Two methods of observing horizontal angles*

THE OFFICE WORK

GENERAL PROCEDURE

75. The procedures used in computing a land survey on a State coordinate system are essentially the same as the usual methods familiar to all land surveyors. Ordinarily, the surveyor adopts some point on the boundary as an initial at which to start computations, and a line through that point as a meridian to which bearings are referred. With this start, the bearings of the lines around the boundary are computed, an angular closure is developed, and the bearings corrected accordingly. Latitudes and departures are next computed, a closure in position obtained and distributed back through the computation, obtaining final values for the coordinates of the land corners. Finally, a computation is made to bring the bearings and distances into accord with the adjusted positions. The result is a survey record which is complete in itself, and probably satisfactory in the interrelationship of its parts. However, being isolated from and independent of all other surveys save possibly a few local ones, it depends for the perpetuation of its ground position on the permanency of a few physical marks at its corners.

76. If the surveyor commences work at one control station and proceeds as described in a previous section to make a survey connection with the land boundary, and then closes on another control station, he will first compute the azimuth of the lines of the survey connection, and distribute any closure in azimuth which is developed. Then the latitudes and departures are computed, and the position closure obtained and distributed. The azimuths will be grid azimuths on the State coordinate system, since the computation started with a grid position and azimuth at one control station and closed with similar data at a second control station. And now, instead of an isolated survey dependent for preservation of its ground position on the physical monuments at its corners, for each such corner there is a coordinate position which is accurately related to corresponding positions of a great number of marked points from any of which surveys could be run to re-establish the ground positions of the land corners. Under such conditions, a corner whose position on a State coordinate system is known can never become a *lost corner* within the legal meaning of the term.

A LAND SURVEY BASED ON A STATE GRID

77. There follows an exemplification of a land survey computed on a State coordinate system for the purpose of obtaining accurate values of the coordinates of the land corners for use in deeds and public registers.[15]

Abstract of Angle Observations

78. Before commencing the computation proper, the actual measures made in the field should be tabulated in convenient form and a sketch prepared showing the angles and lines measured, and the control stations to which the land survey is connected. Figure 18 is such a sketch. The correction for closing the horizon at a station is applied to the observed angles in the record book, after which an abstract is prepared showing the corrected angles. If required, corrections for eccentricity of instrument or target are applied to the angles in the abstract. Where a number of lines are observed over at a station, it may be found convenient to prepare a List of Directions, in which the directions of the various stations

[15] See "Description by Coordinates," p. 42.

are referred to a single initial; the angle between any two lines will then be the difference of their listed directions. The angles and directions are always reckoned in a clockwise order.

Reduction of Measured Lengths

79. The lengths of lines as measured with the tape, whether horizontal or on the slope of the ground, are corrected for the difference between the nominal length of the tape and its standard length. This *length correction* of the tape is given for standard conditions of temperature, tension, and support. For example, the tape described in section 89 was graduated to an even 100 feet, but had a standard length of 100.003 feet when supported throughout its length, at a temperature of 68° Fahrenheit, and under a tension of 20 pounds. Its effective length in use will depend upon its temperature, method of support, and tension. The quantity 0.003 foot is the *length correction*; it is a constant and is applied to each tape length measured. The surveyor can control the tension and method of support and, using those for which the standard length is known, avoids having to make corrections for those conditions. He can not control the temperature in use, but must accept conditions as he finds them, and make suitable corrections. To do this, he must know the coefficient of thermal expansion of the tape, that is, the proportional change in length which results from a change of 1° in its temperature. And, of course, he must also know the temperature of the tape in use. The temperature generally used in stating the standard length of a tape is 68° Fahrenheit. The temperature correction will be the length of the tape times the difference in temperatures of use and 68°, times the coefficient of thermal expansion. For temperatures below standard, the tape will be shorter than standard length, and longer for temperatures above standard. The temperature of the tape may be obtained with a thermometer attached to the front end of the tape, or when the tape is stretched on the ground, a thermometer protected against breakage by a metal case may be placed on the ground while the tape is in use.

80. The standard length of the tape should be known for at least two methods of support: supported throughout its length and supported at its end points only; or for a long tape, supported also at its middle point. As a tape can generally be used in one of these positions, with this information no correction for method of support will be required. The length correction will be different for different methods of support, even under the same conditions of temperature and tension. The tape should be used with the tension for which its standard length is known, so that no correction for tension is required. If unusual conditions encountered in the field make it necessary to depart from standard conditions, the surveyor can usually obtain the correction required by measuring a short base line using both the standard methods and the unusual methods for which corrections are to be made.

81. After corrections have been applied for length and temperature, the corrections for slope, if measures were made on the slope, are next computed and applied. In practice, these various corrections are tabulated and combined before being added (algebraically) to a taped distance. The corrections for slope may be taken from the tables "Inclination Correction for 100-Foot Tape," Appendix III, page 58. It is advisable to have one tape fully standardized by an official agency and kept at headquarters for use in determining standard lengths of tapes used in the field. In making a comparison of the standard and field tapes, working conditions should be simulated. The length of a taped line, after the corrections noted above have been applied, will be the horizontal distance between the plumb lines through the end stations of the line. This distance will be along a level line at about the average elevation of the various tape lengths above a horizontal datum surface. The

elevation datum surface for the control survey of the country is mean sea level, so a correction is applied to the line as a whole to reduce it to a sea-level distance (sec. 45). A further correction is required to reduce the sea-level distance to the corresponding State grid distance (sec. 47). As areas involved in most land surveys are relatively small, and topographic relief moderate, it is usually convenient to combine the factors for reduction to sea-level and grid, and reduce the ground-level distance to the corresponding grid distance by a single operation (sec. 53).

Computation of Land Survey

82. We are now ready to consider in a concrete way some of the problems relating to traverse surveys and their processing on a State coordinate system, utilizing for the pur-

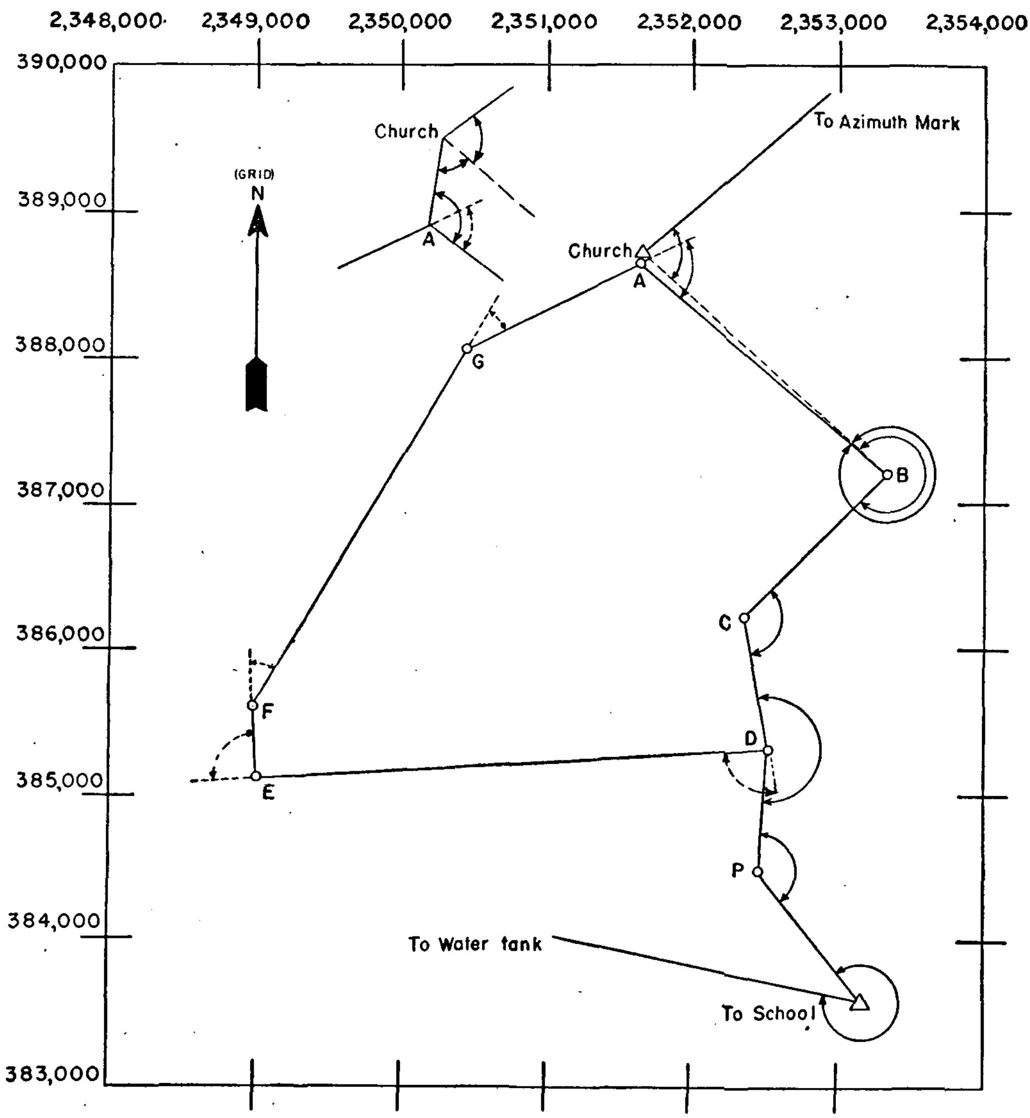

FIGURE 18.—*Plat of farm survey*

pose the field notes of a hypothetical survey which, though fictitious, will serve as a basis for the explanation of many points of interest to the surveyor.

83. The problem in the example is to survey the boundaries of a farm, A–B–C–D–E–F–G–A in figure 18, and place the results on a State coordinate system, in this case the Virginia Coordinate System, North Zone. The search for corners has been made, and these have either been found or re-established, and where necessary re-marked on the ground with durable monuments. Search, too, has been made for control stations whose coordinates on the State system are available. The descriptions of these stations and their coordinate data have been obtained from an official source, as a Federal bureau or a State office.[16]

84. In the example given, control stations are located in convenient positions with respect to the land which is to be surveyed. In some parts of the country, there are areas where distances between control stations are too great for the use of the State coordinate systems to be practicable. In such areas there will be land so remote from a control station that it will be difficult to justify the added expense of the survey connection. But if there are a sufficient number of parcels of land for which surveys will be required in the near future, the cost of even an extensive traverse can be distributed among the various surveys over a period of years without becoming burdensome to the individual land owner. A county engineer who ties his road and drainage surveys to the State system will eventually cover the county with a network of survey stations on that system which will be of great value not only to his own department but of great usefulness to other county offices which are interested in land boundaries and areas, and through those offices most useful to the citizens of the county.

Connection to State System

85. In the example given, two control stations have been recovered, and also an azimuth mark at each of these stations. One control station, CHURCH, is less than 100 feet from Corner A of the land; the other station, SCHOOL, is about 1,900 feet south of the southeast corner of the land. The importance of having at least two control stations to which to tie the land survey must be stressed. The illustration shows a good relationship between the control stations and the land to be surveyed. A survey connection commences at one control station and closes on the other, thus providing a check on the work. At the same time, the connection itself is made quite strong by having several stations (A, B, C, and D) common to both the survey connection and to the land survey. If but one control station had been available, the advantage of obtaining checks on the work by position and azimuth closures at the second station would be lacking, and it would be necessary to make additional measures over a spur-line connection to insure it against the occurrence of blunders and untenable errors.

86. The survey connection has been planned to start at CHURCH and end at SCHOOL; it is to be a second-order traverse, of which land corners at A, B, C, and D will be stations. The line CHURCH—AZIMUTH MARK will provide the initial azimuth; SCHOOL—WATER TANK the closing azimuth. The first step in the computation is to carry a grid azimuth through the traverse to a closure at SCHOOL, then distribute this closure back through the traverse. Next, departures and latitudes are computed, and the State coordinates of each point, derived from the known coordinates of CHURCH, are

[16] There is a growing recognition of the usefulness of a State Survey and Map Office, where data and information relating to control surveys are filed, and made available to surveyors and others. The statute adopting the Maryland coordinate system provided for a Bureau of Surveys and Maps, one function of which is a service such as indicated above.

computed. This gives a computed value for the coordinates of SCHOOL, which are compared with the fixed coordinates of that point, and the closure obtained is distributed back through the traverse by an accepted method of adjustment. In this example, the method known as the Compass Rule was used. Finally, the azimuths and lengths of the lines of the traverse are revised to agree with the adjusted positions.

87. The traverse along the remainder of the land boundary is run, commencing at D, through E, F, and G, to A. This traverse is of a lower order of accuracy than the survey connection, third-order standards being considered satisfactory. But if the surveyor thinks there is a possibility of its later serving as a base for the extension of surveys, he will do well to maintain the higher order of accuracy for it. This traverse is computed in the same order as described for the survey connection, and is controlled by the coordinates and azimuths at D and A, which are held fixed as determined by the survey connection.

Computation of Azimuths

88. At the very start of making the survey connection, there arises the problem of accurately determining the azimuth of the line A—B. The line CHURCH—A is only about 75 feet in length, which is altogether too small for second-order angle measurement. An

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 250

3

HORIZONTAL ANGLES

STATION: C   STATE: Virginia   ISLAND OR COUNTY: DATE:

OBSERVER: INSTRUMENT:

11—737   11—737

OBJECTS OBSERVED	TIME h. m.	TEL. D OR R	REP'S	ANGLE ° '	A "	B "	MEAN OF VERNIERS	ANGLE MEAN D AND R ° ' "	REMARKS
B to D		D	0	0 00	30	60	45		
		D	1	125 59					
		D	6	35 49	00	30	15	125 58 05.0	° ' " "
		R	6	0 00	30	30	30	125 58 07.5	125° 58' 06.2" (07.5)
D to B		R	0	0 00	30	30	30		
		R	1	234 02					
		R	6	324 11	30	60	45	234 01 52.5	
		D	6	0 00	30	60	45	234 01 50.0	234 01 51.2 (52.5)
									359 59 57.4 (60.0)
B to D		D	0	90 00	30	00	15		
		D	6	125 49	00	00	00	125 58 07.5	
		R	6	90 00	00	00	00	125 58 10.0	125 58 08.8 (06.9)
D to B		R	0	90 00	00	00	00		
		R	6	54 11	30	60	45	234 01 57.5	
		D	6	90 00	30	30	30	234 01 52.5	234 01 55.0 (53.1)
									360 00 03.8 (00.0)

Do not write in this margin

Do not write in this margin

Mean Angle B to D 125° 58' 07"

FIGURE 19.—*Observation of horizontal angles, repetitions*

eccentricity of ¼ inch of either instrument or target (rod) for a line of this length could introduce an error of 1′ into the measured angle and thence into the computed azimuths. This difficulty was overcome by observing the three angles of the triangle CHURCH—B—A; the angle at B was observed with unusual care, the other angles serving as checks by testing the triangle closure. This closure is divided equally between the angles at CHURCH and A, by means of which the azimuth of the line CHURCH—A is obtained and used in computing the position of A from the position of CHURCH. The azimuth is introduced into the traverse by using the angles AZIMUTH MARK—CHURCH—B and CHURCH—B—A and obtaining the azimuth of the line B–A. This computation is given on the "List of Preliminary Grid Azimuths" (fig. 21), which shows the azimuth carried through the survey connection to SCHOOL, where a closure in azimuth of 13″ is obtained. This is distributed uniformly back through the traverse angles to obtain corrected azimuths. In the computations of the azimuths for the supplementary survey, D–E–F–G–A, the azimuth closure was 25″.

Form 760
DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY

LIST OF TRAVERSE ANGLES

(FOR USE IN CONNECTION WITH PLANE-COORDINATE COMPUTATIONS)

State Virginia Zone North Locality

Date	Station	From Station—	To Station—	Angle °	′	″	Remarks
	Church	Az. Mark	A	150	04	30	
	Church	Az. Mark	B	83	09	40	
	A	Church	B	111	17	00	
	B	Church	A	358	11	29	
	B	Church	C	271	34	51	
	C	B	D	125	58	07	
	D	C	P	194	56	21	
	P	D	School	136	09	08	
	School	P	Water Tank	320	41	51	
	D	C	E	+97	07	30	Deflection Angle
	E	D	F	+90	40	45	" "
	F	E	G	+33	09	30	" "
	G	F	A	+33	34	45	" "
	A	G	B	+66	06	30	" "

FIGURE 20.—*List of traverse angles*

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 758

LIST OF PRELIMINARY GRID AZIMUTHS

State Virginia (North Zone) Locality .... Line ΔChurch - ΔSchool

From station—	To station—	Preliminary azimuth (Clockwise from south) °	′	″	Correction for closure ″	Corrected azimuth °	′	″
Δ Church	Azimuth Mark	229	01	06	(Fixed)	229	01	06
∠ Azimuth Mk.	A	150	04	30				
Δ Church	A	19	05	36		19	05	36
A	Δ Church	199	05	36				
∠ Δ Church	B	111	17	00				
A	B	310	22	36	(Approx. check only)			
∠								
Δ Church	Azimuth Mark	229	01	06	(Fixed)	229	01	06
∠ Azimuth Mk.	B	83	09	40				
Δ Church	B	312	10	46	+ 2	312	10	48
B	Δ Church	132	10	46				
∠ Δ Church	A	358	11	29				
B	A	130	22	15	+ 2	130	22	17
B	Δ Church	132	10	46				
∠ Δ Church	C	271	34	51				
B	C	43	45	37	+ 4	43	45	41
C	B	223	45	37				
∠ B	D	125	58	07				
C	D	349	43	44	+ 7	349	43	51
D	C	169	43	44				
∠ C	P	194	56	21				
D	P	4	40	05	+ 9	4	40	14
P	D	184	40	05				
∠ D	Δ School	136	09	08				
P	Δ School	320	49	13	+11	320	49	24
Δ School	P	140	49	13				
∠ P	Water Tank	320	41	51				
Δ School	Water Tank	101	31	04	+13	101	31	17
		101	31	17	(Fixed)			
∠	Corr. =	+ 13/6 or +2.17″ per angle						

FIGURE 21.—*List of preliminary grid azimuths, survey connection*

COMPUTATION OF LENGTHS

89. The taping was done with a 100-foot steel tape, whose length at temperature 68° F. was determined by comparison with an office standard to be 100.003 feet, when supported throughout at a tension of 20 pounds. This tension and method of support were used throughout the survey connection. The mean temperatures were recorded for each line. For the survey connection, length measures were made on the slope, which was obtained either by levels over the tape ends or, where the slope was uniform for a considerable distance, the vertical angle it made with the plane of the horizon was observed. This was easily done by sighting a point on the rod at the height of the instrument above the ground. As an example of how the various corrections are tabulated and the horizontal length of a line at ground elevation obtained, the following data for the line P—SCHOOL are given:

Section	Tape reads	Difference of elevation, slope	Correction for slope	Temperature
	(feet)		*(foot)*	F.°
P-1	100	4.4 feet	0.097	82
1-2	100	....	....	..
2-3	100	....	....	81
3-4	100	....	....	..
4-5	100	0° 53′	0.100	78
5-6	100	....	....	..
6-7	100	....	....	76
7-8	100	....	....	..
8-9	100	2.7 feet	0.036	75
9-10	100	2.6 feet	0.034	..
10-11	100	2.2 feet	0.024	76
11-School	22.77	0.8 foot	0.014	..
	Sum 1,122.77		Sum 0.305	Mean 78°

The corrections for slope, which are always negative, were taken from the table of such corrections for a 100-foot tape (page 58). The difference of elevation of the tape ends is the argument for this table. Where the vertical angle was observed, the slope (inclination) correction was computed by the formula

$$\text{Correction for inclination} = -L(1-\cos V)$$

where L is the slope distance (tape reading) and V is the vertical angle. The slope correction for the odd length (22.77 feet) was obtained by first computing the difference of elevation for 100 feet on the same slope

$$(100/22.77)\times 0.8 = 3.5 \text{ feet}$$

The slope correction for a difference of elevation of 3.5 feet and a length of 100 feet as taken from the table is 0.061 foot. The correction for 22.77 feet will be

$$22.77/100 \text{ of } 0.061 = -0.014 \text{ foot}$$

The length of the tape at temperature 68° F. and under standard tension (20 pounds) is 100.003 feet. The length correction for one tape length is 0.003 foot, and for the line P—SCHOOL is

$$11.23(+0.003) = +0.034 \text{ foot}$$

The mean temperature of the tape during the measurement of this line was 78° F., which is 10° above standard temperature. Assuming a coefficient of thermal expansion for a steel tape to be 0.00000645 per degree Fahrenheit, the temperature correction for this line is

$$10\times 0.00000645\times 1123 = 0.072 \text{ foot}$$

To determine the sign of this correction, it is better to reason the problem out than to depend upon a formula which may not be at hand when needed. Steel, like most materials, expands when heated. At a temperature of 78° the tape is longer than it is at a temperature of 68°. In measuring the distance between fixed points on the ground, the tape readings will be too small, and the correction to the measured length due to temperature of the tape will be positive. When the tape temperature is below standard, the reverse will be true: the tape will be short, the observed distance too long, and the correction negative. The corrected length of the line P—SCHOOL will therefore be

	Feet
Nominal (observed) tape length	1,122.77
Length correction	+0.034
Slope correction	−0.305
Temperature correction	+0.072
Corrected length, P—SCHOOL	1,122.57

90. The corrected lengths, obtained as described in the preceding section, are listed as "Tape Lengths (corrected)" on the sheet "Computation of Preliminary Grid Lengths" (fig. 22), and as this survey is to be placed on the Virginia Coordinate System, two more corrections are required before proceeding with the computation of the grid coordinates: first, the corrected tape lengths, which are at ground-level elevation, are reduced to sea-level lengths; these sea-level lengths are next reduced to grid lengths on the State system. In the survey which we are discussing, the mean elevation of the farm is taken as 520 feet and the mean latitude as 38°43′.6. These data give an elevation factor of 0.9999751 and a scale factor of 0.9999502. The sea-level factor is taken from the table in Appendix C; the scale factor from the projection tables for the Virginia Coordinate System, North Zone; it can also be obtained with the formula in Appendix III. These two factors are combined by multiplication into a single factor, 0.9999253, and the corrected tape lengths reduced to grid lengths by the single operation:

$$\text{Tape length (corrected)} \times 0.9999253 = \text{grid length}$$

The factor might also be written as $(1-.0000747)$ and interpreted as a correction to the tape lengths, -0.0747 foot per thousand feet. Either method of expression will give the tabulated grid lengths.

Form 7■■
DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY

COMPUTATION OF PRELIMINARY GRID LENGTHS

State Virginia (North Zone) Locality .... ~~Line~~ Scale factor .9999 502

Mean latitude 38° 43′.6 Average elevation 520 Elevation factor .9999 751

Stations	Taped Length (corrected) Feet	Geodetic Length Feet	Grid Factor (Elev. and Scale)	Grid Length Feet
Δ Church - A	75.84		.9999 253	75.83
A - B	2210.92		"	2210.75
B - C	1386.36		"	1386.26
C - D	910.21		"	910.14
D - P	846.03		"	845.97
P - Δ School	1122.57		"	1122.49
			-0.0747 foot per thousand	
D - E	3538.79			
E - F	486.06			
F - G	2853.72			
G - A	1342.33			

FIGURE 22.—*Computation of preliminary grid lengths*

COMPUTATION OF COORDINATES

91. The computation is now carried ahead as described in section 76, all operations being shown on a standard form, "Computation of Coordinates" (fig. 23), and the continuous sum of the grid distances is shown in parentheses in the column marked "Grid Distance." In this computation the Compass Rule was used in distributing the closures in departure and latitude, the corrections to the preliminary values of the coordinates being in the proportion of the accumulated lengths, commencing at CHURCH, to the total length of the traverse. While the Transit Rule, which distributes closures according to

latitudes and departures rather than according to lengths of line, is preferred by some surveyors, it is believed that the Compass Rulė will usually give better results for both azimuth and length, and is less difficult to apply.

92. As the application of closure corrections to the coordinates makes the adjusted coordinates inconsistent with the azimuths and lengths used in the computation, the azimuths and lengths are changed accordingly to bring all data into accord. These corrections are quite small, and in this example have been made only to the azimuths of lines which are to be used in the control of the supplementary survey, D–E–F–G–A. The change in the azimuth of the line A—B is negligible; the azimuth of C—D is changed by 9″.

93. The traverse from D, through E, F, and G, to A, is next computed (figs. 24 and 25). For this part of the survey, deflection angles are employed, principally to exemplify their use, as described in section 74. The length measures for this part of the survey were

COMPUTATION OF COORDINATES

TRAVERSE LINE NO.
STATE Virginia (North Zone) COUNTY INITIAL STATION Δ Church
YEAR MONTH CLOSING STATION Δ School

Station	Plane azimuth Clockwise from south ° ′ ″	Grid distance Feet	Log departure – Log sin azimuth / Log distance – Log cos azimuth / Log latitude	Departure Feet	Latitude Feet	Grid coordinates x Feet	Grid coordinates y Feet
Δ Church					(Fixed)	2,351,676.19	388,706.19
A	19 05 36	75.83 (76)	.327 1079 .944 9870	- 24.80	- 71.66	2,351,651.39 0.00 2,351,651.39	388,634.53 0.00 388,634.53
B	310 22 17	2210.75 (2287)	.761 8619 .647 7395	+1684.29	-1431.99	2,353,335.68 + 0.11 2,353,335.79	387,202.54 - 0.09 387,202.45
C	43 45 41	1386.26 (3673)	.691 6566 .722 2265	- 958.82	-1001.19	2,352,376.86 + 0.17 2,352,377.03	386,201.35 - 0.15 386,201.20
D	349 43 51	910.14 (4583)	.178 2727 .983 9811	+ 162.25	- 895.56	2,352,539.11 + 0.22 2,352,539.33	385,305.79 - 0.19 385,305.60
P	4 40 14	845.97 (5429)	.081 4263 .996 6794	- 68.88	-843.16	2,352,470.23 + 0.26 2,352,470.49	384,462.63 - 0.22 384,462.41
Δ School	320 49 24	1122.49 (6551)	.631 7137 .775 2018	+709.09	-870.16 (Fixed)	2,353,179.32 + 0.31 2,353,179.63	383,592.47 - 0.27 383,592.20

x corr. $= + \frac{0.31}{6.551}$ or .0473 foot per thousand

Closure $= \sqrt{0.31^2 + 0.27^2} = 0.41$ or 1:16,000

y corr. $= - \frac{0.27}{6.551}$ or .0412 foot per thousand

FIGURE 23.—*Computation of coordinates*

made with tape held horizontal. The computation was made as described for the survey connection, and the closing error of 1 : 7,300 distributed according to the Compass Rule. The computation sheet shows grid bearings of the lines; these are in parentheses, directly beneath the grid azimuths. The computation of the two control azimuths, C–D and A–B, is shown at the bottom of the computation sheet, "List of Preliminary Grid Azimuths" (fig. 24). The computation of this part of the survey was made with logarithms, again for the purposes of demonstration. The surveys described in this publication are based on control stations for which State coordinate data are already available. In computing such surveys, 7-place logarithmic or 8-place natural tables are used. Since this Bureau will furnish, upon request, the State coordinates of stations for which it determines standard geodetic positions, it is not contemplated that the land surveyor will find occasion for making the transformation of geodetic positions into State coordinates, a computation which requires more extensive tables, not usually available. Information about suitable tables can be obtained by writing this Office. Address: Director, U. S. Coast and Geodetic Survey, Washington 25, D. C.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 758

LIST OF PRELIMINARY GRID AZIMUTHS

State Virginia (North Zone) Locality .... Line D to A

U. S. GOVERNMENT PRINTING OFFICE 11—11719

	From station—	To station—		Preliminary azimuth °	′	″	Correction for closure ′	″	Corrected azimuth °	′	″
	C	D		349	43	42	(Fixed)		349	43	42
Defl.	∠ to	E	+	97	07	30					
	D	E		86	51	12		- 5	86	51	07
Defl.	∠ to	F	+	90	40	45					
	E	F		177	31	57		-10	177	31	47
Defl.	∠ to	G	+	33	09	30					
	F	G		210	41	27		-15	210	41	12
Defl.	∠ to	A	+	33	34	45					
	G	A		244	16	12		-20	244	15	52
Defl.	∠ to	B	+	66	06	30					
	A	B		310	22	42		-25	310	22	17
				310	22	17	(Fixed)				
	∠	Corr. = $\frac{-25}{5}$ or - 5″ per angle									
	∠										

Computation of Azimuths

		x	y
∠	C	2,352,377.03	386,201.20
	D	2,352,539.33	385,305.60
∠		+ 162.30	- 895.60

Tangent Azimuth = $\frac{162.30}{895.60}$ = 0.181 219 Azimuth C - D = 349°43′42″

∠

		x	y
	A	2,351,651.39	388,634.53
∠	B	2,353,335.79	387,202.45
		+ 1,684.40	- 1,432.08

Tangent Azimuth = $\frac{1684.40}{1432.08}$ = 1.176 191 Azimuth A - B = 310°22′17″

FIGURE 24.—*List of preliminary grid azimuths, supplementary traverse*

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 738
Ed. May 1935

COMPUTATION OF COORDINATES

TRAVERSE LINE NO
STATE Virginia (North Zone) COUNTY INITIAL STATION D
YEAR MONTH CLOSING STATION A

U. S. GOVERNMENT PRINTING OFFICE 13—11307

Station	Plane azimuth (Grid Bearing)	Grid distance	Log departure / Log sin azimuth / Log distance / Log cos azimuth / Log latitude	Departure	Latitude	Grid coordinates x	Grid coordinates y
		Feet		Feet	Feet	Feet	Feet
D			3.548 1989	-3533.45	(Fixed)	2,352,539.33	385,305.60
	86 51 07		9.999 3441				
	(S 86 51 07 W)	3538.79	3.548 8548			2,349,005.88	385,111.26
			8.739 7012			+ 0.39	+ 0.29
E		(3539)	2.288 5560		- 194.34	2,349,006.27	385,111.55
			1.321 1785	- 20.95			
	177 31 47		8.634 4886				
	(N 2 28 13 W)	486.06	2.686 6899			2,348,984.93	385,596.87
			9.999 5962			+ 0.44	+ 0.33
F		(4025)	2.686 2861		+ 485.61	2,348,985.37	385,597.20
			3.163 2733	+1456.38			
	210 41 12		9.707 8620				
	(N 30 41 12 E)	2853.72	3.455 4113			2,350,441.31	388,050.99
			9.934 4838			+ 0.75	+ 0.57
G		(6879)	3.389 8951		+2454.12	2,350,442.06	388,051.56
			3.082 4914	+1209.18			
	244 15 52		9.954 6321				
	(N 64 15 52 E)	1342.33	3.127 8593			2,351,650.49	388,633.85
			9.637 7080			+ 0.90	+ 0.68
		(8221)	2.765 5673		+ 582.86	2,351,651.39	388,634.53

x corr. =+0.90/8.221 or 0.109 ft. per thousand. y corr. =+0.68/8.221 or 0.083 ft. per thousand.
Closure = 1.13 or 1:7,300

FIGURE 25.—*Computation of coordinates, supplementary traverse*

LAND AREA

94. The area of a parcel of land at ground elevation may be obtained by first computing its grid area, using the State coordinates of its corners, and then dividing this grid area by the *square* of the combination factor which was employed in reducing its ground-level lengths to grid lengths. The ground-level area is greater or less than the grid area, according as the combination factor is less or greater than unity.

LOST AND OBLITERATED CORNERS

95. If the physical marks at a point whose coordinates on a State system are known, can not be found by the usual methods of search based on its description, its ground position can be recovered by determining the ground position corresponding to its known coordinates. This is accomplished by a survey starting at other stations for which State coordinate data are available and whose marks have been recovered, and running to a new point in the immediate vicinity of the sought-for point. The coordinates of the new point and of the sought-for point are compared, and the distance and azimuth from the new to the old laid out on the ground. The point thus established is the ground position corresponding to the coordinates of the sought-for point. If any remnants of the original station marks remain in position, they may be found by a careful and detailed examination of the ground within a small distance of the restored position. The restoration survey is conducted and computed as described in this manual.

96. As the accuracy of the restored position reflects the errors of both the original and the restoration surveys, care must be taken to maintain a high order of accuracy in the restoration survey: this is especially important if the restoration survey is more extensive—longer—than the original survey. While it is not necessary to employ the same stations in the resurvey that were used in the original survey, it is desirable to use stations which were in the same traverse or survey net. A station on some other traverse might not be so well related in position to the lost mark, due to methods used in the previous adjustment.

97. The above method has been employed by the Coast and Geodetic Survey in searching for triangulation stations for which all surface marks have disappeared and for which the descriptions, references, etc., proved of no help. It sometimes happens that the underground mark at such a station will remain undisturbed for many years after surface evidences of location have become obliterated. A new survey establishes ground position of the old triangulation station with reference to recovered stations, and very frequently, upon excavating, the old underground mark is recovered. This method of searching for old stations has proved so satisfactory that, when it fails to find the mark, the station is considered lost for geodetic purposes, though the point indicated by the coordinate position is usually acceptable for purposes of general surveying. While the Coast and Geodetic Survey uses geodetic data and methods in these recovery surveys, State coordinates will be equally satisfactory within the limited areas for which they are employed.

98. Marks which are otherwise difficult to find, and if found, apt to be considered as of doubtful origin, may be recovered and proved authentic by the use of the State coordinate systems. A fragment of stone, a piece of decayed wood, or even a bit of windblown soil may constitute a satisfactory recovery if found in the ground position corresponding to the coordinates of a landmark which was marked by a buried stone, a wooden stake, or small pit or hole in the ground. And if no marks are found in the position indicated by the coordinates, it may safely be concluded that they have been destroyed, and the coordinate position then accepted as primary evidence of ground location.

DESCRIPTION BY COORDINATES

99. The description of a parcel of land for record purposes should have two aims:

(a) It should positively identify the land for title purposes.

(b) It should provide all necessary information for locating the land on the ground. These two aims are not identical: a description which satisfies the first may be wholly inadequate for the second. However, a single description to accomplish both aims can be prepared by adding to the description through which descent of title is traced, a technical description which includes the coordinates on the State system of one or more of the corners of the land.

100. One of the simplest standard forms of land description for title purposes is afforded by the public-land method of surveying, where a description by reference to township, range, and section can apply to one and only one parcel of land on the Earth. In its Second Progress Report,[17] the Joint Committee on Land Surveys and Titles, American Bar Association and the American Society of Civil Engineers, gives the following description of a tract of land in Iowa:

[17] Bibliography, p. 44.

"The northwest quarter of the southeast quarter (NW¼SE¼) of Section Ten (10), Township Seventy-nine North (T79N), Range Six West (R6W) of the Fifth Principal Meridian."

and comments as follows:

"This description, where practicable, is convenient, easy to express, easy to understand, and entirely satisfactory."

"For the sake of perpetuating the corners that control these subdivisions, the statement of their positions in terms of the State coordinate systems is very desirable."

According to the "Specifications for Descriptions of Tracts of Land for Use in Executive Orders and Proclamations," [18] the above description would be put in the form:

Fifth Principal Meridian

T. 79 N., R. 6 W., sec. 10, NW¼SE¼

101. *It is fully agreed that, for purposes of tracing title, this description is "entirely satisfactory."* Furthermore, as long as there remain in position the physical monuments marking the corners of the public-land subdivision of which the parcel is a fraction, and those marks are susceptible of recovery and identification, the description is adequate for survey purposes. Regardless of how much the ground positions of the corner monuments disagree with the field notes of the original survey, the monuments mark the true corners. Where the coordinates of a corner monument have been determined on a State system, and the monument is later obliterated, those coordinates will provide an effective means of recovering and identifying any part of the monument that may remain in position. If the monument is totally destroyed, its coordinates become primary evidence of where it stood. Since its coordinates can be determined only while a monument is still in existence, it is evident that if they are accurately determined, there can be no conflict [19] between the position they define and the position which was defined by the monument.

102. For a description of a public-land subdivision, an acceptable use of a State coordinate system would be the placing of the coordinates of the marked corners on an official plat, to which specific reference would be made in the written description. The plat should also, if practicable, show the coordinates of the control stations from which the coordinates of the subdivision corners were derived. If the use of a plat is not practicable, the description suggested above might continue with a statement of the coordinates of a corner or corners to which the land is directly related. In the example given above it would be helpful if something like the following were added:

"The coordinates of the center of Section 10, referred to above, on the Iowa Coordinate System, South Zone, are: $x=$_______________feet and $y=$ _______________feet."

If to this were added the coordinates of the south and east quarter corners of section 10, and of its southeast corner, this would be a nearly perfect description.

103. In other than public-land States, the inclusion of State coordinates in any form of description is also very desirable, except where there is an official plat on file showing the State coordinates, in which case a reference to the plat will make it part of the legal description. A carefully-prepared plat of an area embracing a number of land parcels and showing the State coordinates of the land corners and of control stations which define the State coordinate system is a very satisfactory description for survey purposes. A suitable

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[18] Bibliography, p. 44.

[19] Appendix II, p. 54, sec. 8

reference to the plat in the description prepared for title purposes will make that description satisfactory for all purposes.

BIBLIOGRAPHY

The following manuals have been prepared by the U. S. Coast and Geodetic Survey for the guidance of its field and office employees. They may be obtained from the Superintendent of Documents, Government Printing Office, Washington, D. C., at the prices listed.

(A) MANUAL OF SECOND- AND THIRD-ORDER TRIANGULATION AND TRAVERSE. Special Publication No. 145. Price, 45 cents. This publication contains a description of the specifications and criteria for second- and third-order triangulation, traverse, and base measurement, with detailed instructions for field and office operations and specimens of field records and office computations. The section on instrumental adjustments and errors, and the discussion of the sources of error in triangulation and traverse are written in considerable detail.

(B) MANUAL OF PLANE COORDINATE COMPUTATION. Special Publication No. 193. Price, 35 cents. This publication describes two types of conformal projections that have been used as bases for the State systems of plane coordinates. Examples are given of the various applications of the coordinate systems to geodetic measurements.

(C) MANUAL OF TRAVERSE COMPUTATION ON THE LAMBERT GRID. Special Publication No. 194. Price, 45 cents. This manual supplements Special Publication No. 193. It contains a more complete account of the system of plane coordinates which is based on the Lambert projection. A number of traverses in Nebraska that were measured under the Civil Works Administration are computed and adjusted as samples of computation on this system of coordinates. Very full explanation of all steps of the work is given as an aid to the proper understanding of the principles involved. Traverses in other States having this type of grid should be computed in a similar manner.

(D) MANUAL OF TRAVERSE COMPUTATION ON THE TRANSVERSE MERCATOR GRID. Special Publication No. 195. Price, 25 cents. This manual is a companion volume to Special Publication No. 194 and serves the same purpose for the transverse Mercator grid. Sample computations of traverses executed under the Civil Works Administration are included. These traverses are located in New York, in the vicinity of Rochester, and in New Jersey. Traverses in other States for which the Mercator grid has been adopted should be computed in a similar manner.

Other publications to which reference is made in this manual are as follows:

(E) LAND SURVEYS AND TITLES. First and Second Progress Reports of the Joint Committee of the Real Property Division, American Bar Association, and the Surveying and Mapping Division, American Society of Civil Engineers. Published in the PROCEEDINGS of the American Society of Civil Engineers, November, 1938, page 1879, and June, 1941, page 1065.

(F) SPECIFICATIONS FOR DESCRIPTIONS OF TRACTS OF LAND FOR USE IN EXECUTIVE ORDERS AND PROCLAMATIONS, Revision of 1942. Prepared by the Committee on Cadastral Surveys and Maps of the Federal Board of Surveys and Maps,[20] and published at the request of the Bureau of the Budget, 1943.

(G) GEODETIC LETTER. An informal journal of the Division of Geodesy, Coast and Geodetic Survey, published from March, 1934, to January, 1937. Two numbers of this journal were devoted to the consideration of the control of land and engineering surveys. The May, 1935, number presented a symposium on "The Use of Geodetic Control for Boundary Surveys." The January, 1937, number considered the application of the State coordinate systems to that use.

The following should also be mentioned:

(H) A TREATISE ON THE LAW OF SURVEYING AND BOUNDARIES, by Frank Emerson Clark of the Minnesota Bar. Second Edition, 1939. The Bobbs-Merrill Company, Indianapolis, Indiana. A general treatise on surveys and boundaries, by an author experienced in both the legal and surveying aspects of the subject, and having particular reference to conditions in the public-land States. First published in 1922, the new (1939) edition contains a new chapter, "Co-ordinate Surveying," in which the use of the State coordinate systems is considered.

[20] Note 3, p. VI.

APPENDIX I

THE STATE COORDINATE SYSTEMS

State / Zone	Grid	Central meridian or parallel: Lat. or Long.	Central meridian or parallel: Scale ratio	Central meridian or parallel: R* (minus)	Origin: Lat.	Origin: Long.	Origin: x	Origin: y	Standard parallels; x[1] of lines of exact scale (feet)
							(ft.)	(ft.)	
ALABAMA:									
East	tr.M.	85°50'	1:25,000	.0400	30°30'	85°50'	500,000	0	186,000
West	tr.M.	87 30	1:15,000	.0667	30 00	87 30	500,000	0	241,300
ARIZONA:									
East	tr.M.	110°10'	1:10,000	.1000	31°00'	110°10'	500,000	0	295,600
Central	tr.M.	111 55	1:10,000	.1000	31 00	111 55	500,000	0	295,600
West	tr.M.	113 45	1:15,000	.0667	31 00	113 45	500,000	0	241,300
ARKANSAS:									
North	L.	35°35'	1:15,600	.0641	34°20'	92°00'	2,000,000	0	34°56' & 36°14'
South	L.	34 02	1:12,300	.0815	32 40	92 00	2,000,000	0	33 18 34 46
CALIFORNIA:									
Zone 1	L.	40°50'	1: 9,500	.1054	39°20'	122°00'	2,000,000	0	40°00' & 41°40'
Zone 2	L.	39 05	1:11,700	.0853	37 40	122 00	2,000,000	0	38 20 39 50
Zone 3	L.	37 45	1:14,100	.0708	36 30	120 30	2,000,000	0	37 04 38 26
Zone 4	L.	36 37.5	1:16,900	.0592	35 20	119 00	2,000,000	0	36 00 37 15
Zone 5	L.	34 45	1:12,800	.0779	33 30	118 00	2,000,000	0	34 02 35 28
Zone 6	L.	33 20	1:21,800	.0459	32 10	116 15	2,000,000	0	32 47 33 53
Zone 7	L.	34 08.5	1:87,300	.01146	34 08	118 20	x = 4,186,692.58		33 52 34 25
							y = 4,160,926.74		
COLORADO:									
North	L.	40°15'	1:23,200	.0431	39°20'	105°30'	2,000,000	0	39°43' & 40°47'
Central	L.	39 06	1:15,600	.0641	37 50	105 30	2,000,000	0	38 27 39 45
South	L.	37 50	1:18,300	.0546	36 40	105 30	2,000,000	0	37 14 38 26
CONNECTICUT:	L.	41°32'	1:59,300	.0169	40°50'	72°45'	600,000	0	41°12' & 41°52'
DELAWARE:	tr.M.	75°25'	1:200,000	.0050	38°00'	75°25'	500,000	0	66,100
FLORIDA:									
East	tr.M.	81°00'	1:17,000	.0588	24°20'	81°00'	500,000	0	226,500
West	tr.M.	82 00	1:17,000	.0588	24 20	82 00	500,000	0	226,500
North	L.	30 10	1:19,400	.0516	29 00	84 30	2,000,000	0	29°35' & 30°45'
GEORGIA:									
East	tr.M.	82°10'	1:10,000	.1000	30°00'	82°10'	500,000	0	295,400
West	tr.M.	84 10	1:10,000	.1000	30 00	84 10	500,000	0	295,400
IDAHO:									
East	tr.M.	112°10'	1:19,000	.0526	41°40'	112°10'	500,000	0	214,700
Central	tr.M.	114 00	1:19,000	.0526	41 40	114 00	500,000	0	214,700
West	tr.M.	115 45	1:15,000	.0667	41 40	115 45	500,000	0	241,600
ILLINOIS:									
East	tr.M.	88°20'	1:40,000	.0250	36°40'	88°20'	500,000	0	147,900
West	tr.M.	90 10	1:17,000	.0588	36 40	90 10	500,000	0	226,800
INDIANA:									
East	tr.M.	85°40'	1:30,000	.0333	37°30'	85°40'	500,000	0	170,800
West	tr.M.	87 05	1:30,000	.0333	37 30	87 05	500,000	0	170,800
IOWA:									
North	L.	42°40'	1:18,300	.0546	41°30'	93°30'	2,000,000	0	42°04' & 43°16'
South	L.	41 12	1:19,400	.0516	40 00	93 30	2,000,000	0	40 37 41 47
KANSAS:									
North	L.	39°15'	1:23,200	.0432	38°20'	98°00'	2,000,000	0	38°43' & 39°47'
South	L.	37 55	1:15,600	.0641	36 40	98 30	2,000,000	0	37 16 38 34
KENTUCKY:									
North	L.	38°28'	1:26,400	.0379	37°30'	84°15'	2,000,000	0	37°58' & 38°58'
South	L.	37 20	1:18,300	.0546	36 20	85 45	2,000,000	0	36 44 37 56
LOUISIANA:									
North	L.	31°55'	1:11,700	.0853	30°40'	92°30'	2,000,000	0	31°10' & 32°40'
South	L.	30 00	1:13,500	.0743	28 40	91 20	2,000,000	0	29 18 30 42

*Appendix III

THE STATE COORDINATE SYSTEMS

State Zone	Grid	Central meridian or parallel			Origin				Standard parallels; x' of lines of exact scale (feet)
		Lat. or Long.	Scale ratio	R* (minus)	Lat.	Long.	x	y	
							(ft.)	(ft.)	
MAINE:									
East	tr.M.	68°30'	1:10,000	.1000	43°50'	68°30'	500,000	0	295,900
West	tr.M.	70 10	1:30,000	.0333	42 50	70 10	500,000	0	170,900
MARYLAND:	L.	38°52'.5	1:19,900	.0501	37°50'	77°00'	800,000	0	38°18' & 39°27'
MASSACHUSETTS:									
Mainland	L.	42°12'	1:28,200	.0354	41°00'	71°30'	600,000	0	41°43' & 42°41'
Island	L.	41 23	1:660,000	.0015	41 00	70 30	200,000	0	41 17 41 29
MICHIGAN:									
East	tr.M.	83°40'	1:17,500	.0572	41°30'	83°40'	500,000	0	223,700
Central	tr.M.	85 45	1:11,000	.0909	41 30	85 45	500,000	0	282,100
West	tr.M.	88 45	1:11,000	.0909	41 30	88 45	500,000	0	282,100
MINNESOTA:									
North	L.	47°50'	1:10,300	.0972	46°30'	93°06'	2,000,000	0	47°02' & 48°38'
Central	L.	46 20	1:12,800	.0780	45 00	94 15	2,000,000	0	45 37 47 03
South	L.	44 30	1:12,800	.0780	43 00	94 00	2,000,000	0	43 47 45 13
MISSISSIPPI									
East	tr.M.	88°50'	1:25,000	.0400	29°40'	88°50'	500,000	0	186,800
West	tr.M.	90 20	1:17,000	.0588	30 30	90 20	500,000	0	226,700
MISSOURI:									
East	tr.M.	90°30'	1:15,000	.0667	35°50'	90°30'	500,000	0	241,400
Central	tr.M.	92 30	1:15,000	.0667	35 50	92 30	500,000	0	241,400
West	tr.M.	94 30	1:17,000	.0588	36 10	94 30	500,000	0	226,800
MONTANA:									
North	L.	48°17'	1:35,100	.0285	47°00'	109°30'	2,000,000	0	47°51' & 48°43'
Central	L.	47 10	1:12,800	.0780	45 50	109 30	2,000,000	0	46 27 47 53
South	L.	45 38	1:11,200	.0892	44 00	109 30	2,000,000	0	44 52 46 24
NEBRASKA:									
North	L.	42°20'	1:28,200	.0355	41°20'	100°00'	2,000,000	0	41°51' & 42°49'
South	L.	41 00	1:12,800	.0779	39 40	99 30	2,000,000	0	40 17 41 43
NEVADA:									
East	tr.M.	115°35'	1:10,000	.1000	34°45'	115°35'	500,000	0	295,700
Central	tr.M.	116 40	1:10,000	.1000	34 45	116 40	500,000	0	295,700
West	tr.M.	118 35	1:10,000	.1000	34 45	118 35	500,000	0	295,700
NEW HAMPSHIRE:	tr.M.	71°40'	1:30,000	.0333	42°30'	71°40'	500,000	0	170,900
NEW JERSEY:	tr.M.	74°40'	1:40,000	.0250	38°50'	74°40'	2,000,000	0	147,900
NEW MEXICO:									
East	tr.M.	104°20'	1:11,000	.0909	31°00'	104°20'	500,000	0	281,800
Central	tr.M.	106 15	1:10,000	.1000	31 00	106 15	500,000	0	295,600
West	tr.M.	107 50	1:12,000	.0833	31 00	107 50	500,000	0	269,800
NEW YORK:									
Long Island	L.	40°51'	1:196,500	.0051	40°30'	74°00'	2,000,000	100,000	40°40' & 41°02'
East	tr.M.	74 20	1:30,000	.0333	40 00	74 20	500,000	0	170,800
Central	tr.M.	76 35	1:16,000	.0625	40 00	76 35	500,000	0	233,900
West	tr.M.	78 35	1:16,000	.0625	40 00	78 35	500,000	0	233,900
NORTH CAROLINA:	L.	35°15'	1: 7,900	.127	33°45'	79°00'	2,000,000	0	34°20' & 36°10'
NORTH DAKOTA:									
North	L.	48°05'	1:15,600	.0642	47°00'	100°30'	2,000,000	0	47°26' & 48°44'
South	L.	46 50	1:15,600	.0642	45 40	100 30	2,000,000	0	46 11 47 29
OHIO:									
North	L.	41°04'	1:16,400	.0609	39°40'	82°30'	2,000,000	0	40°26' & 41°42'
South	L.	39 23	1:15,600	.0641	38 00	82 30	2,000,000	0	38 44 40 02
OKLAHOMA:									
North	L.	36°10'	1:18,300	.0546	35°00'	98°00'	2,000,000	0	35°34' & 36°46'
South	L.	34 35	1:15,600	.0641	33 20	98 00	2,000,000	0	33 56 35 14

THE STATE COORDINATE SYSTEMS

State Zone	Grid	Central meridian or parallel: Lat. or Long.	Central meridian or parallel: Scale ratio	R* (minus)	Origin: Lat.	Origin: Long.	Origin: x (ft.)	Origin: y (ft.)	Standard parallels; x' of lines of exact scale (feet)
OREGON:									
North	L.	45°10'	1: 9,500	.1054	43°40'	120°30'	2,000,000	0	44°20' & 46°00'
South	L.	43 10	1: 9,500	.1054	41 40	120 30	2,000,000	0	42 20 44 00
PENNSYLVANIA:									
North	L.	41°25'	1:23,200	.0432	40°10'	77°45'	2,000,000	0	40°53' & 41°57'
South	L.	40 27	1:24,700	.0405	39 20	77 45	2,000,000	0	39 56 40 58
RHODE ISLAND:	tr.M.	71°30'	1:160,000	.0062	41°05'	71°30'	500,000	0	73,900
SOUTH CAROLINA:									
North	L.	34°22'	1:18,300	.0546	33°00'	81°00'	2,000,000	0	33°46' & 34°58'
South	L.	33 00	1:14,800	.0674	31 50	81 00	2,000,000	0	32 20 33 40
SOUTH DAKOTA:									
North	L.	45°03'	1:16,400	.0609	43°50'	100°00'	2,000,000	0	44°25' & 45°41'
South	L.	43 37	1:10,700	.0931	42 20	100 20	2,000,000	0	42 50 44 24
TENNESSEE:	L.	35 50	1:19,400	.0516	34°40'	86°00'	2,000,000	100,000	35°15' & 36°25'
TEXAS:									
North	L.	35°25'	1:11,200	.0891	34°00'	101°30'	2,000,000	0	34°39' & 36°11'
North Central	L.	33 03	1: 7,800	.1274	31 40	97 30	2,000,000	0	32 08 33 58
Central	L.	31 00	1: 8,500	.1183	29 40	100 20	2,000,000	0	30 07 31 53
South Central	L.	29 20	1: 7,300	.1368	27 50	99 00	2,000,000	0	28 23 30 17
South	L.	27 00	1: 9,500	.1052	25 40	98 30	2,000,000	0	26 10 27 50
UTAH:									
North	L.	41°15'	1:23,200	.0432	40°20'	111°30'	2,000,000	0	40°43' & 41°47'
Central	L.	39 50	1: 9,900	.1012	38 20	111 30	2,000,000	0	39 01 40 39
South	L.	37 47	1:20,500	.0487	36 40	111 30	2,000,000	0	37 13 38 21
VERMONT:	tr.M.	72°30'	1:28,000	.0357	42°30'	72°30'	500,000	0	176,800
VIRGINIA:									
North	L.	38°37'	1:19,400	.0516	37°40'	78°30'	2,000,000	0	38°02' & 39°12'
South	L.	37 22	1:18,300	.0546	36 20	78 30	2,000,000	0	36 46 37 58
WASHINGTON:									
North	L.	48°07'	1:17,300	.0577	47°00'	120°50'	2,000,000	0	47°30' & 48°44'
South	L.	46 35	1:11,700	.0854	45 20	120 30	2,000,000	0	45 50 47 20
WEST VIRGINIA:									
North	L.	39°37'.5	1:16,900	.0592	38°30'	79°30'	2,000,000	0	39°00' & 40°15'
South	L.	38 11	1:13,500	.0743	37 00	81 00	2,000,000	0	37 29 38 53
WISCONSIN:									
North	L.	46°10'	1:18,300	.0547	45°10'	90°00'	2,000,000	0	45°34' & 46°46'
Central	L.	44 52.5	1:16,900	.0593	43 50	90 00	2,000,000	0	44 15 45 30
South	L.	43 24	1:14,800	.0674	42 00	90 00	2,000,000	0	42 44 44 04
WYOMING:									
Zone I	tr.M.	105°10'	1:17,000	.0588	40°40'	105°10'	500,000	0	226,900
Zone II	tr.M.	107 20	1:17,000	.0588	40 40	107 20	500,000	0	226,900
Zone III	tr.M.	108 45	1:17,000	.0588	40 40	108 45	500,000	0	226,900
Zone IV	tr.M.	110 05	1:17,000	.0588	40 40	110 05	500,000	0	226,900

APPENDIX II, THE STATE COORDINATE SYSTEMS

REPORT OF THE COUNCIL OF STATE GOVERNMENTS

The Council of State Governments,[21] in its General Report on Suggested State War and Postwar Legislation for 1945, offered the following for the consideration of the various State legislatures.

THE STATE PLANE COORDINATE SYSTEMS

INTERPRETATIVE STATEMENT

Purpose and Effect of the Proposed Legislation

For over one hundred years the United States Coast and Geodetic Survey has been engaged in determining precise geographic positions of thousands of monumented points distributed throughout the country. In 1933 these geographic data were made available to land surveyors and engineers generally by the establishment of the State coordinate systems—a system for each State—the geographic positions being transformed by mathematical processes into plane coordinates—positions on plane surfaces—so that their use involved only the usual methods and formulas of land (plane) surveying.

The State coordinate systems provide a means for the scientific location and description of land boundaries. Their use for that purpose has developed an urgent need for legislation designed to give each system an official designation and a legal definition. The proposed legislation is intended

(a) to establish the legal status of the State systems;

(b) to insure uniformity and definiteness in terms used; and

(c) to impose reasonable standards in the use of the systems when the State coordinates are to become part of the public records.

In recognition of these needs, enabling legislation has been adopted in Louisiana, Maryland, Massachusetts, New Jersey, New York, North Carolina, Pennsylvania, Texas, and in Georgia, for one county.[22] The formal adoption of the State coordinate systems has been urged by the American Bar Association and the American Society of Civil Engineers.[23]

Considerable use of the State coordinate systems is already being made by many Federal bureaus by whom the proposed legislation is endorsed. The list of Federal bureaus includes: the Soil Conservation Service; the Bureau of Plant Industry, Soils, and Agricultural Engineering; the Bureau of Agricultural Economics; the Agricultural Adjustment Agency; the Office of Land Use Coordination; the Bureau of Reclamation; the National Park Service; the Geological Survey; the International Boundary Commission, United States, Alaska, and Canada; the Public Roads Administration; the Office of the Chief of Engineers, the United States Army; the Bureau of Yards and Docks, United States Navy Department. It has been given the official approval of the War Department, of the Department of Agriculture, and of the Department of the Interior, which includes the General Land Office.

The legislation proposed is similar in form for all States, except for differences in the technical descriptions of the separate systems; these have been prepared by the United States Coast and Geodetic Survey.

The legislation proposed provides that the use of the State coordinate systems shall be permissive, not mandatory. Used as bases for control surveys, the State systems will effect a tremendous saving in time and money in preparing specifications for and putting into effect many postwar plans relating to

[21] "The Council of State Governments is a joint governmental agency established by the states, serving the states, and supported by the states."

"It is the Secretariat for the Governors' Conference, the National Association of Attorneys General, the National Association of Secretaries of State, and the American Legislator's Association, and it works in close cooperation with the National Conference of Commissioners on Uniform State Laws. It serves as a clearing house and research center for legislators, legislative reference bureaus, and for the above organizations affiliated with it. The Council maintains a central office in Chicago, an office in Washington, D. C., and regional offices in New York and San Francisco." From the Book of the States, 1943-44.

[22] See Appendix B, p. 53.

[23] See Appendix C, p. 53.

urban and rural land development, in the extension of private business facilities, and in public works construction.

Adoption of the proposed State coordinate legislation does not entail any new expenditures. It does provide a simple control to insure a proper use of those systems.

Background

The United States Coast and Geodetic Survey has been in operation for over a century, surveying and measuring the territory of the United States. It is carrying on a continual process of filling the many unsurveyed gaps.

Through its system of triangulation—which, in brief, is a method of surveying founded in the trigonometrical proposition that if three parts of a triangle are known (such as the measurements of one side and two angles) the other three parts may be obtained by computation—the United States Coast and Geodetic Survey has established thousands of well-monumented points (physical marks on the ground) for which position data have been established. In any survey of great extent such as of the United States, it is necessary to consider the size and spheroidal shape of the earth. Consequently, geodetic surveying is highly specialized and, in addition to the use of special expensive equipment, requires mathematical calculations and reductions in which the ordinary land surveyor is not trained.

To make the geodetic data of the national survey readily available to land surveyors and engineers, the Coast and Geodetic Survey by 1935 established what are known as the State coordinate systems, a separate system for each State of the Union. In these systems representations of the surface of the earth were projected mathematically upon cones or cylinders which were then flattened into planes. In this way a strip of the earth's surface, 158 miles wide and of indefinite length, can be represented on a single plane projection, and errors of scale caused by representing the curved surface of the earth in a plane will not exceed 1 part in 10,000 or about 1 foot every 2 miles—which is accurate surveying.[24]

The Coast and Geodetic Survey computes and publishes the plane coordinates, on the appropriate State coordinate system, of all points for which it determines geographic positions (latitudes and longitudes). Just as there is but one point on the surface of the earth corresponding to a geographical position stated by latitude and longitude, so too, there is but one point corresponding to a given pair of plane coordinates, stated as x (east-west) and y (north-south) on a definite plane projection. As an example, when all surface marks at a landmark or station have disappeared it has been very effective practice to reproduce on the ground by surveys from other stations the coordinate position of the sought-for point, and on digging there, to recover the underground mark. In this way old stations and lost points are retrieved and the plane coordinate system has afforded a simple means of utilizing geodetic data.

The results of the national triangulation in which surveyors generally will be interested consists of accurately-determined coordinates of points on the ground which are marked with substantial material monuments, and so described that these monuments may be readily recovered and used as starting points for local surveys. The monuments or points or stations, however they are referred to, usually consist of concrete shafts, four or five feet long, set nearly flush with the ground, with bronze disks in their tops to identify them and a small hole representing the exact position. Buried under each shaft is a short cylinder of concrete with another disk set in direct vertical line with the surface disk. Usually, three reference monuments with disks are established near each station, and tied thereto by accurate measures of distance and bearing. Current publications of geodetic data contain both geographic positions and State coordinates of the survey stations reported on. They also include the descriptions of the locations and markings of those stations with the lengths and azimuths of the lines between contiguous stations. The land surveyor who ties his work into stations of the national triangulation net, and defines the positions of his land corners by giving their plane coordinates on a State system, provides indisputable evidence for their accurate restoration should they ever be destroyed.

Today, for every geographic position (stated in latitude and longitude) determined by the national geodetic survey in any of the States, there is a corresponding plane-coordinate position, which is obtained, not by scaling from a map, but by accurate and precise mathematical procedure. The United States Coast and Geodetic Survey computes and makes generally available the State coordinates of all stations for which it determines standard geographic positions. The surveyor who desires to make use

[24] See Appendix A, p. 51.

of those stations may do so through the medium of their plane coordinates on the State systems. He need know nothing of the geodetic processes by which the geographic values were obtained or of the mathematics used in converting the geographic positions into State coordinates. He starts with plane-coordinate data and uses plane-surveying methods throughout the work. The value of basing his surveys on such data is obvious. A given geographic position, and its equivalent State-coordinate position, can indicate one and only one point on the earth.

In practice, the land surveyor uses one of the metal-disk-topped monuments or other station, as the initial of his survey. (Such point not only provides a position on the State system, but usually the direction or bearing to a second marked point.) From this point he runs a traverse to the nearest or most conveniently located corner of the land which he is to survey, and continues around the land to its initial corner. He makes his computations in much the same way as he would have done had there been no station with State coordinates available, but with this difference: he starts with a known, not assumed, position and bearing and he takes account of each extra or additional traverse which was needed to connect the land survey to the State system. But when he has completed the work and its computation, he has a State-coordinate position for each of the corners of the land, and he has in effect keyed his survey to all the stations of the national geodetic survey, not simply with the one which was used as an initial point. He might have used any conveniently located station of the national survey for his initial point and obtained the same coordinates for his land corners.

As a result, all stations of the national geodetic survey become witnesses to the positions of land corners whose coordinates on a State system are known. The material marks of the land corners, such as trees, stones, fence posts, stakes, or any sort of artificial marker, may be destroyed, yet the positions they occupied on the ground can be closely reproduced from any recoverable stations of the national survey which are within practical surveying distances of those corners. In this manner, a survey station or land corner which is described in terms of a State coordinate system is practically indestructible.

Why Legislation?

The permanence of marks of land boundary, or "corners," the finding of such marks or monuments when they are no longer readily visible, the replacing of them when they have been removed, are matters which have long been of serious concern to land owners and to others—engineers, lawyers, abstracters—who have of necessity been interested in the occupation, use or ownership of land. No such marks are truly permanent. All are subject to removal or obliteration, willful or accidental. The only feasible means yet devised to insure permanence of a position of a land corner, as contrasted with the permanence of the physical object which may mark that position, is that for which the Egyptians long ago developed the science of geometry; namely, relating this position to the position of other monuments, by mathematical means, so that if not all are simultaneously destroyed, those which remain may be satisfactorily used to recover the missing positions.

The plane-rectangular coordinates of a point offer a most satisfactory means to this end. The chief bar to their use heretofore was the lack of any established system of general availability, and the consequent absence of any means of giving permanent or legal significance to their use in a recorded land description. There is nothing new about the use of plane coordinates for surveying field work, mapping or computations. The only new item was the establishment by 1935 of general State systems, so tied to the national triangulation net as to be available for use anywhere in a State within reach of a properly-located triangulation or traverse station. Loss of monuments will not prevent the accurate reestablishment of boundary corners whose positions are known in terms of a State system. The work of reestablishment will involve nothing more than ordinary surveying, unless all monuments over a very large area disappear at one time.

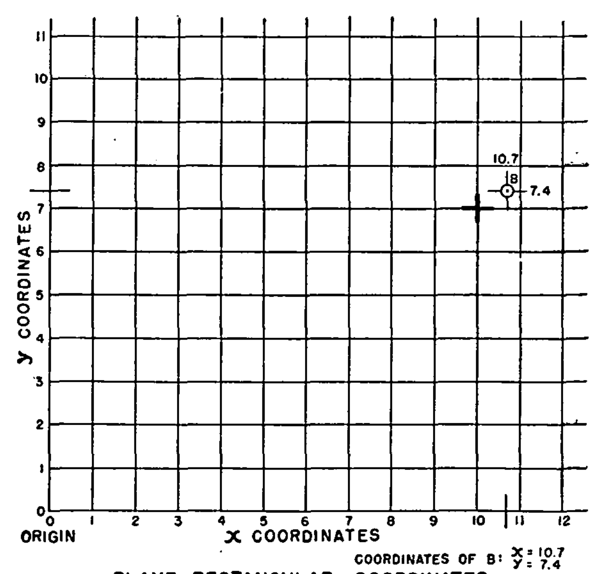
An important step toward facilitating the use of this convenient method of recording the location of property corners is officially to define the name of the State system, so that when reference to it is made there can be no doubt as to what is meant, and so as to preclude improper use of it, or ambiguous reference to it. This should be done by legislative enactment, and has already been done in New Jersey (1935), Pennsylvania (1937), New York (1938), Maryland (1939), North Carolina (1939), Massachusetts (1941), Texas (1943), Louisiana (1944), Georgia (1939—applies to one county only). Except for differences in mathematics which are obvious and easily handled, the matter of legislative definition is much the same in all States.

The fact which should be clearly understood is this: The use of State coordinates in land descriptions does not mean the abandonment of the older methods of description, either by platted subdivision or fraction of subdivisions, or by metes and bounds. It is intended to supplement either of these methods, and to facilitate the finding, or the checking of the location of, any of the corners implied or described in the older forms of descriptions. It is simply a means of increasing the certainty and facilitating the future field use of the old descriptions, not of superseding them. So, while description by fraction of section is excellent from the point of view of the examiner of title, it does not, without outside help from some source, give any assurance that the parcel can be located on the ground. And, after all, that is where the land will finally have to be found. Introducing the coordinates into a deed description may increase the bulk of the description, but the data are worth including. Extra care will be required in transcribing such description to avoid an error in stating a number, but again, the data are worth the extra care required.

The extent to which the State coordinate systems can be used for property description, now or in the early future, is still limited. Control stations, with coordinates determined, are available within reach of only part of the total area of land to be described. However, as the national survey continues to fill the gaps, it is very desirable that an act defining and naming the system for a State, should be a part of the statute law of the State well in advance of any such use. This would insure from the very beginning, uniformity of terminology and certainty of the meaning of the terms used. It would also impose reasonable standards for, and limitations upon, the use of coordinates for descriptions of boundary when those descriptions are to become a part of the public records.

APPENDIX A. COORDINATES AND STATE COORDINATE SYSTEMS

In non-technical and simple terms, what are plane-rectangular coordinates? Assume a sheet of blank paper is laid flat and on it is drawn a system of lines, two sets of equally-spaced straight lines, one set vertical and one set horizontal, intersecting at right angles. Number the lines in each set consecutively, beginning with 0 in the lower left-hand corner of the sheet. (See diagram.)



PLANE-RECTANGULAR COORDINATES

Any one of the squares formed by the two sets of intersecting lines can be identified by giving the numbers of the two lines whose intersection forms that square's lower left-hand corner. Now place a button in one of the squares, say the one formed by vertical lines 10 and 11 and horizontal lines 7 and 8. The position of the button on the sheet can be described by giving the two numbers, 10 and 7, which identify the square it occupies.

In doing this, use is made of a system of plane-rectangular coordinates. The system of intersecting lines forms what is called a "grid." The point -0- at the lower left-hand corner of the sheet from which the lines are numbered is the "origin" of the system. The numbers are "coordinates," 10 and 7 being coordinates of a corner of the square occupied by the button. Those numbers which progress horizontally across the sheet are the "x coordinates," those indicating distance up the sheet are the "y coordinates."

By drawing a vertical and a horizontal line through the center of the button and using fractional numbers to indicate the positions of those lines in the system, the coordinates of the button itself are determined and not simply those of a corner of the square occupied by the button.

Now suppose that instead of the grid being drawn on a sheet of blank paper, it is drawn on a map. In place of a button in a selected square, every square of the grid is filled with map information. In the square whose lower left-hand corner is identified as "$x = 10$, $y = 7$," there is now the map symbol for a church, or a street intersection, or some other local feature. The position on the map of any feature shown may be described by simply stating its coordinates. And as each point on the map represents a definite point on the ground, its coordinates are likewise descriptive of the ground position of the point. By reducing the size of the squares of the grid until quite small, and using decimals to express the fractional parts of a square, a close position on the map of any point can be given by simply stating its coordinates; namely, two numbers which indicate distances east and north of the origin.

But while coordinates scaled from a map accurately describe the position of a point on the map, they are not completely satisfactory for describing the position occupied by the point on the ground. A map is a small-scale representation of a particular part of the earth's surface, and even where coordinates (distances) are measured on the map to the one-hundredth part of an inch, the error in a ground position represented by the one-hundredth of an inch on a map is too great to be tolerated in a land survey. If the map is constructed on the scale of 1 : 10,000, that is, 1 inch on the map represents 10,000 inches on the ground, an error of 0.01 inch on the map will represent a distance on the ground of 100 inches, or 8⅓ feet, which is altogether too large for surveying purposes.

In practice, therefore, it is the coordinates determined by accurate surveys made on the ground which are used to record in a document the ground position of a survey station or land corner. Such coordinates also form the data upon which a map is based. In other words, the map follows the survey and is but a form of recording survey data.

Unfortunately in transforming the data obtained from large scale surveying to local use, a practical problem is encountered. The surface of the earth is not flat. It can be referred to a regular mathematical figure whose curvature and dimensions can be and have been determined. The function of geodetic surveying is to determine distances and directions on this mathematical figure between points which represent actual points on the earth. By the use of special instruments and methods this is accomplished with a high degree of precision and accuracy. With distances and directions, and a selected starting point, geographical positions are determined. These are stated in latitudes, distances from the equator; and longitudes, distances from some selected meridian, which for this country is the meridian of Greenwich.

The work involved in computing a geodetic survey is difficult and tedious, requiring particular training, and special mathematical tables and equipment. On the other hand, the computations involved in the use of plane coordinates are simple and are familiar to every land surveyor. He uses plane coordinates in the simplest land survey, and so is well prepared to make use of the data of the geodetic survey, provided the geographic positions can be converted into plane coordinates. This has been done. Each geographic position determined by the geodetic survey has been stated in terms of a plane-coordinate position on the appropriate State coordinate system.

How this is done can be explained by a simple analogy. It is apparent that one cannot flatten out a large part of a spherical surface, for example an orange peel, without tearing it in some places and compressing or folding it in other places. It is also apparent that a small piece of the spherical surface —small both in width and length—can be flattened out with very little distortion. A similar result can be obtained by simply increasing one dimension of the small piece. Thus a long, narrow strip of a spherical surface can be flattened out with very little difficulty, keeping the distortion within controlled limits. Transposing this principle to the problem of surveying and mapping Statewide areas as if they were planes, the U. S. Coast and Geodetic Survey considered each State as composed of one or more long narrow strips, some with the long dimensions extending north and south, others with the east-west dimension the greater. For example, Texas is divided into five east-west strips or "zones," as they are called. New York is covered by three north-south zones and one east-west zone, the latter covering all of Long Island. The amount of distortion which results when a strip is developed as a plane is so small that it has little effect on the survey data.

APPENDIX B. SUMMARY OF LAWS ESTABLISHING STATE COORDINATE SYSTEMS

Louisiana—Laws 1944, Act 226 (S. B. 159).
Maryland—Laws 1939, c. 628. Flack's Ann. Code (1939) Art. 91, Sec. 25-42.
Massachusetts—Laws 1941, c. 47. General Laws Ann. (1932) c. 97, Sec. 8-13.
New Jersey—Laws 1935, c. 116. Revised Stats. 51:3-7 to 51:3-10. See also Revised Stats. 51:3-11, 51:3-12.
New York—Laws 1938, c. 545. McKinney's Consolidated Laws, Book 65, Unconsolidated Laws sec. 8501-8508.
North Carolina—Laws 1939, c. 163. General Statutes (1943) sec. 102-1 to 102-11.
Pennsylvania—Laws 1937, Act. No. 310, p. 1208. Purdons Statutes, Title 76, sec. 145-152.
Texas—Laws 1943, c. 354. Vernon's Ann. Civil Statutes, Art. 5300a.

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Georgia—Laws 1939, p. 609, No. 68 (Local Laws—For Glynn County).

APPENDIX C. BIBLIOGRAPHY

"A Treatise on the Law of Surveying and Boundaries" (2d Ed., Bobbs-Merrill Co.) by Frank E. Clark. See c. 29, "Coordinate Surveying."

"Geodetic Letter." Issues May, 1935, January, 1937. Published by Division of Geodesy, U. S. Coast and Geodetic Survey, Department of Commerce, Washington, D. C.

"Land Surveys and Titles." First and Second Progress Reports, from reprints of proceedings November, 1938, and June, 1941, Joint Committee of Real Property Division, American Bar Association and Surveying and Mapping Division, American Society of Civil Engineers.

"Specifications for Descriptions of Tracts of Land for Use in Executive Orders and Proclamations," (Revision 1942). Published, through cooperation of General Land Office, United States Department of the Interior, by United States Government Printing Office, Washington, D. C. See c. 6, "Description by Coordinates."

INTERPRETATIVE SUMMARY OF THE PROVISIONS OF THE BILL

Section 1—Adopts a name for the State Coordinate System, and geographically groups the counties of the State into zones, in order to control the size of any one area and thereby maintain a prescribed degree of accuracy. (The name is particularized in section 2, by zone designation, and its use protected in section 7.)

The line of demarcation between zones follows present county boundaries. This is regarded desirable because county areas or names may change in the future, by consolidation or otherwise. At the date of adoption of the act the county boundaries and names are known, and the areas of the zones become stabilized as of that date. Hence the language—the area "now included" in certain counties "shall constitute" a given zone—is a safeguard against the uncertainty which change might bring.

Section 2—Assigns a name for each of the zones of the State. The terms "north," "south," "central," as the case may be, are descriptive, and hence useful.

Section 3—Tells how the coordinates define the position of a point and gives their common names: x-coordinate and y-coordinate. This section also ties the State Coordinate System to the precise geodetic surveys executed by the United States Coast and Geodetic Survey.

Section 4—Provides that where a tract of land, covered by a single survey, lies in more than one zone of the State Coordinate System, a single description of it may be given, using the coordinates of any one zone. This overcomes the impracticality of dividing the tract and preparing a separate description for the part in each zone.

Section 5a—Is a technical definition of the State Coordinate System, prepared by the United States Coast and Geodetic Survey, by whom the State systems were originated. Such technical definition accurately relates the State Coordinate System to the mathematical figure of the earth (Clarke Spheroid of 1866) which has been adopted as the surface of reference for the geodetic survey of this country. It is this definite relationship which forms the basis for the mathematical transformation of geographic positions (latitudes and longitudes) of the national Geodetic Survey into plane (x and y) coordinates on a State system.

Section 5b—Establishes the ground location of the State coordinate system, without which its use by land surveyors and engineers would not be possible. This section also states the standards, namely, first- and second-order work, which must characterize the coordinate positions of those locations to make them acceptable for the given purpose. It should be noted that the requirement is for stations established in conformity with prescribed standards. There is no prescription as to who may establish these stations; this may be done by any competent agency. Although it is contemplated that all stations of the United States Coast and Geodetic Survey will be used, the section does not require that the stations be established by the United States Coast and Geodetic Survey.

Section 6—The intent of this section is to prevent official records of land boundaries, such as recorded deeds, being made a repository of inaccurate data. By prohibiting the *presentation* of unacceptable data for record, section 6 places upon the surveyor, or other person presenting the data, the responsibility for its being acceptable.

A rule for judging the acceptability of data offered for record is provided by limiting to one-half mile the maximum length of a survey connecting a land corner with a station established in conformity with the standard prescribed in section 5b. This rule rests upon a reasonable presumption that no competent surveyor will make any intolerable errors in so short a survey, but rather, because of its importance, will exercise extreme care in making the connection and thereby obtain better than usual accuracy. The section therefore requires that, before coordinates can be obtained for a point located outside this limit, a station must be established, upon the standard of at least second-order work, within the fixed limits. While this may be done by any competent person, it will often require the services of a State or federal organization. In this regard, the United States Coast and Geodetic Survey is steadily extending its control surveys to areas now devoid of triangulation and traverse stations, making State coordinates available in those areas.

The effect of section 6 is to require that *each* point whose coordinates are offered for record shall be within one-half mile distance of the standard station to which it is tied. However, section 6 does not require that coordinates for all corners of a land boundary be available before use can be made of the State coordinate system for record purposes. Where coordinates of some, but not all, corners of a tract of land are available, they may be included in a deed or other document presented for record, as additional informative data of lasting benefit.

The concluding clause of section 6, authorizing a modification of the one-half mile limitation, recognizes that standards of surveying will vary with localities: coordinates in city districts must be determined with a precision and accuracy which would be unwarranted in remote country areas. To prescribe a scale of standards for surveys under such conditions requires the use of suitable rules and regulations, such as could be prepared and promulgated by a competent authority.

Section 7—Safeguards the use of the formal name of the State system by requiring that where State coordinates are used in any documents, such use shall be properly identified on the documents; and, furthermore, that the name of the State system shall never be applied to coordinates on any system other than the one defined in the act.

Section 8—Eliminates any conflict which might arise should a land description by coordinates not coincide with a co-existing description based upon reference to the legal subdivisions of the United States public land surveys.

Section 9—Expresses the intent of the act to permit, but not to require, the use of the State coordinate system for title purposes. The use of the coordinate system is permissive and not mandatory.

Section 10—Is a severability clause to provide against partial invalidity voiding the entire act.

Section 11—The act may be made effective at the time of its passage or at some designated future date. Designation of a future date will permit the preparation and promulgation, by an authorized agency, of rules and regulations for its operation, or if there be no agency charged with its administration, comparable action, advisory in character, by a competent private organization, such as a State association of surveyors and engineers.

PROPOSED MODEL ACT FOR STATE COORDINATE SYSTEM

An Act to describe, define, and officially adopt a system of coordinates for designating the position of points on the surface of the earth within the State of ....

BE IT ENACTED BY THE LEGISLATURE OF THE STATE OF ....

Section 1. The system of plane coordinates which has been established by the United States Coast and Geodetic Survey for defining and stating the positions or locations of points on the surface of the earth within the State of .... is hereafter to be known and designated as the ".......... (name of State) Coordinate System."

For the purpose of the use of this system the State is divided into a "North Zone" and a "South Zone," (or such other zones as may be required).[25]

The area now included in the following counties shall constitute the North Zone: (here enumerate the names of the counties to be included).

The area now included in the following counties shall constitute the South Zone: (here enumerate the counties to be included).

Section 2. As established for use in the North Zone, the .... (name of State) Coordinate System shall be named, and in any land description in which it is used it shall be designated, the ".......... (name of State) Coordinate System, North Zone."

As established for use in the South Zone the .... (name of State) Coordinate System shall be named, and in any land description in which it is used it shall be designated, the ".......... (name of State) Coordinate System, South Zone."

Section 3. The plane coordinates of a point on the earth's surface, to be used in expressing the position or location of such point in the appropriate zone of this system, shall consist of two distances, expressed in feet and decimals of a foot. One of these distances, to be known as the "x-coordinate," shall give the position in an east-and-west direction; the other, to be known as the "y-coordinate," shall give the position in a north-and-south direction. These coordinates shall be made to depend upon and conform to the coordinates, on the .... (name of State) Coordinate System, of the triangulation and traverse stations of the United States Coast and Geodetic Survey within the State of ...., as those coordinates have been determined by the said Survey.

Section 4. When any tract of land to be defined by a single description extends from one into the other of the above coordinate zones, the positions of all points on its boundaries may be referred to either of the two zones, the zone which is used being specifically named in the description.

Section 5. (a) For purposes of more precisely defining the .... (name of State) Coordinate System the following definition by the United States Coast and Geodetic Survey is adopted:

The .... (name of State) Coordinate System, North Zone, is a Lambert conformal projection of the Clarke spheroid of 1866, having standard parallels at north latitudes (specify in degrees and minutes), along which parallels the scale shall be exact.[26]

The origin of coordinates is at the intersection of the meridian (specify in degrees and minutes) west of Greenwich and the parallel (specify in degrees and minutes) north latitude. This origin is given the coordinates: x=2,000,000 feet and y=0 feet.[27]

The .... (name of State) Coordinate System, South Zone, is a Lambert conformal projection of the Clarke spheroid of 1866, having standard parallels at north latitudes (specify in degrees and minutes), along which parallels the scale shall be exact. The origin of coordinates is at the intersection of the meridian (specify in degrees and

[25] A small State may be covered by a single zone, thus requiring no additional designation or listing of counties. Where a State is divided into several zones, these are given specific designation, either by number, as Zone 1, Zone 2, etc.; or by characteristic of relative position, as North Zone, South Zone, etc., or East Zone, West Zone, etc.

[26] Where the longer dimension of the area covered by a zone is north and south rather than east-west, the definition would read: "The .... (name of State) Coordinate System, ... (designate) Zone, is a transverse Mercator projection of the Clarke spheroid of 1866, having a central meridian (specify in degrees and minutes) west of Greenwich, on which meridian the scale is set at one part in .... (specify amount) too small."

[27] Where the transverse Mercator projection is used, the origin is given the coordinates: x=500,000 feet and y=0 feet.

minutes) west of Greenwich and the parallel (specify in degrees and minutes) north latitude. This origin is given the coordinates: x=2,000,000 feet and y=0 feet.

(b) The position of .... (name of State) Coordinate System shall be as marked on the ground by triangulation or traverse stations established in conformity with standards adopted by the United States Coast and Geodetic Survey for first-order and second-order work, whose geodetic positions have been rigidly adjusted on the North American datum of 1927, and whose coordinates have been computed on the system herein defined. Any such station may be used for establishing a survey connection with the .... (name of State) Coordinate System.

Section 6. No coordinates based on the .... (name of State) Coordinate System, purporting to define the position of a point on a land boundary, shall be presented to be recorded in any public land records or deed records unless such point is within one-half mile of a triangulation or traverse station established in conformity with the standards prescribed in Section 5 of this Act; provided that said one-half mile limitation may be modified by a duly authorized State agency to meet local conditions.

Section 7. The use of the term ".......... (name of State) Coordinate System" on any map, report of survey, or other document, shall be limited to coordinates based on the .... (name of State) Coordinate System as defined in this Act.

(The following Section 8 should be included only for a State where the United States public land system surveys are established.)

Section 8. Wherever coordinates based on the .... (name of State) Coordinate System are used to describe any tract of land which in the same document is also described by reference to any subdivision, line, or corner of the United States public land surveys, the description by coordinates shall be construed as supplemental to the basic description of such subdivision, line, or corner contained in the official plats and field notes filed of record, and in the event of any conflict the description by reference to the subdivision, line, or corner of the United States public land surveys shall prevail over the description by coordinates.

Section 9. Nothing contained in this Act shall require any purchaser or mortgagee to rely on a description, any part of which depends exclusively upon the .... (name of State) Coordinate System.

Section 10. If any provision of this Act shall be declared invalid, such invalidity shall not affect any other portion of this Act which can be given effect without the invalid provision, and to this end the provisions of this Act are declared to be severable.

Section 11. This Act shall take effect ....

APPENDIX III, FORMULAS AND TABLES

TEMPERATURE CORRECTION FOR STEEL TAPES

The effective length of a steel tape depends in part on its temperature (sec. 79). Steel tapes are standardized at a temperature of 68° Fahrenheit, which is called the standard temperature. At temperatures above 68°, the tape is longer, below 68° shorter, than it is at standard temperature. The amount the tape is longer or shorter is given by the second term on the right hand side of the equation:

$$L_1 = L_0 + L_0(t_1 - t_0)C_t$$

in which L_1 is the effective length of the tape, L_0 its standard length, t_1 its temperature in use, t_0 the standard temperature (usually 68° F.), and C_t the tape's coefficient of thermal expansion. Of course, L_1 and L_0 must be in the same unit of length, and t_1, t_0, and C_t must employ the same thermometric scale. The table which follows is based on the Fahrenheit scale; L_1 and L_0 may be in any linear unit, but it must be the same unit for both.

Correction for Temperature

Values of $L_0\ (t_1 - t_0) C_t$
[$t_0 = 68°$ Fahrenheit; $C_t = 0.00000645$ per 1° F.]

t_1 F°	L_0 10	20	30	40	50	60	70	80	90	100	t_1 F°
68	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	68
69	.000	.000	.000	.000	.000	.000	.000	.001	.001	.001	67
70	.000	.000	.000	.001	.001	.001	.001	.001	.001	.001	66
71	.000	.000	.001	.001	.001	.001	.001	.002	.002	.002	65
72	.000	.001	.001	.001	.001	.002	.002	.002	.002	.003	64
73	.000	.001	.001	.001	.002	.002	.002	.003	.003	.003	63
74	.000	.001	.001	.002	.002	.002	.003	.003	.003	.004	62
75	.000	.001	.001	.002	.002	.003	.003	.004	.004	.005	61
76	.001	.001	.002	.002	.003	.003	.004	.004	.005	.005	60
77	.001	.001	.002	.002	.003	.003	.004	.005	.005	.006	59
78	.001	.001	.002	.003	.003	.004	.005	.005	.006	.006	58
79	.001	.001	.002	.003	.004	.004	.005	.006	.006	.007	57
80	.001	.002	.002	.003	.004	.005	.005	.006	.007	.008	56
81	.001	.002	.003	.003	.004	.005	.006	.007	.008	.008	55
82	.001	.002	.003	.004	.005	.005	.006	.007	.008	.009	54
83	.001	.002	.003	.004	.005	.006	.007	.008	.009	.010	53
84	.001	.002	.003	.004	.005	.006	.007	.008	.009	.010	52
85	.001	.002	.003	.004	.005	.007	.008	.009	.010	.011	51
86	.001	.002	.003	.005	.006	.007	.008	.009	.010	.012	50
87	.001	.002	.004	.005	.006	.007	.009	.010	.011	.012	49
88	.001	.003	.004	.005	.006	.008	.009	.011	.012	.013	48
89	.001	.003	.004	.005	.007	.008	.009	.011	.013	.014	47
90	.001	.003	.004	.006	.007	.009	.010	.011	.013	.014	46
91	.001	.003	.004	.006	.007	.009	.010	.012	.014	.015	45
92	.002	.003	.005	.006	.008	.009	.011	.012	.015	.015	44
93	.002	.003	.005	.006	.008	.010	.011	.013	.015	.016	43
94	.002	.003	.005	.007	.008	.010	.012	.013	.016	.017	42
95	.002	.003	.005	.007	.009	.010	.012	.014	.016	.017	41
96	.002	.004	.005	.007	.009	.011	.013	.014	.017	.018	40
97	.002	.004	.006	.007	.009	.011	.013	.015	.017	.019	39
98	.002	.004	.006	.008	.010	.012	.014	.015	.018	.019	38
99	.002	.004	.006	.008	.010	.012	.014	.016	.019	.020	37
100	.002	.004	.006	.008	.010	.012	.014	.017	.019	.021	36
101	.002	.004	.006	.009	.011	.013	.015	.017	.020	.021	35
102	.002	.004	.007	.009	.011	.013	.015	.018	.020	.022	34
103	.002	.005	.007	.009	.011	.014	.016	.018	.020	.022	33
104	.002	.005	.007	.009	.012	.014	.017	.019	.021	.023	32
105	.002	.005	.007	.010	.012	.014	.017	.019	.021	.023	31
106	.002	.005	.007	.010	.012	.015	.017	.020	.022	.024	30

(Left side, t_1 68–106:) Tape too long; taped distance too small; correction +.
(Right side, t_1 68–30:) Tape too short; taped distance too long; correction —.

CORRECTION FOR INCLINATION OF TAPE

A length L measured along a slope is reduced to the corresponding horizontal distance by the application of a correction C_i (correction for inclination). This correction may be expressed in terms of L and of h, the difference in elevation of the ends of the line; or in terms of L and a function of V, the angle of inclination of the line as measured.

If the slope of the measured line is given in terms of L and h:

$$C_i = -(L - \sqrt{L^2 - h^2}) = -h^2/2L - h^4/8L^3 - h^6/16L^5 - \ldots.$$

The following table gives values of C_i for a 100-foot tape ($L = 100$ feet) and for various values of h.

Inclination Correction for 100-Foot Tape

[Difference of elevation (h) in feet and tenths of a foot.]

Correction in feet; all values negative.

h	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.002	0.003	0.004
1	.005	.006	.007	.008	.010	.011	.013	.014	.016	.018
2	.020	.022	.024	.026	.029	.031	.034	.036	.039	.042
3	.045	.048	.051	.054	.058	.061	.065	.068	.072	.076
4	.080	.084	.088	.092	.097	.101	.106	.110	.115	.120
5	.125	.130	.135	.141	.146	.151	.157	.163	.168	.174
6	.180	.186	.192	.199	.205	.211	.218	.225	.231	.238
7	.245	.252	.260	.267	.274	.282	.289	.297	.305	.313
8	.321	.329	.337	.345	.353	.362	.370	.379	.388	.397
9	.406	.415	.424	.433	.443	.452	.462	.472	.481	.491
10	.501	.511	.522	.532	.542	.553	.563	.574	.585	.596
11	.607	.618	.629	.641	.652	.663	.675	.687	.699	.711
12	.723	.735	.747	.759	.772	.784	.797	.810	.823	.836
13	.849	.862	.875	.888	.902	.915	.929	.943	.957	.971
14	.985	.999	1.013	1.028	1.042	1.057	1.072	1.086	1.101	1.116
15	1.131	1.147	1.162	1.177	1.193	1.209	1.224	1.240	1.256	1.272
16	1.288	1.305	1.321	1.337	1.354	1.371	1.387	1.404	1.421	1.438
17	1.456	1.473	1.490	1.508	1.525	1.543	1.561	1.579	1.597	1.615
18	1.633	1.652	1.670	1.689	1.707	1.726	1.745	1.764	1.783	1.802
19	1.822	1.841	1.861	1.880	1.900	1.920	1.940	1.960	1.980	2.000
20	2.020	...	...	...	...	...	...	...	...	...

If the slope of the measured line is given in terms of L and V:

$$C_i = -L(1-\cos V)$$

The following table gives values of C_i for a 100-foot tape ($L=100$ feet) and for various values of V.

Inclination Correction for 100-Foot Tape

[Slope of line given at ten minute intervals]

Correction in feet; all values negative.

V	00′	10′	20′	30′	40′	50′
0°	0.000	0.000	0.002	0.004	0.007	0.011
1	.015	.021	.027	.034	.042	.051
2	.061	.071	.083	.095	.108	.122
3	.137	.153	.169	.187	.205	.224
4	.244	.264	.286	.308	.332	.356
5	.381	.406	.433	.460	.489	.518
6	.548	.579	.610	.643	.676	.710
7	.745	.781	.818	.856	.894	.933
8	.973	1.014	1.056	1.098	1.142	1.186
9	1.231	1.277	1.324	1.371	1.420	1.469
10	1.519	1.570	1.622	1.675	1.728	1.782
11	1.837	1.893	1.950	2.008	2.066	2.125
12	2.185	2.246	2.308	2.370	2.434	2.498
13	2.563	2.629	2.696	2.763	2.831	2.900
14	2.970	3.041	3.113	3.185	3.258	3.333
15	3.407	3.483	3.560	3.637	3.715	3.794

REDUCTION TO SEA LEVEL

In reducing ground-level lengths to sea-level, the following table will be found convenient. It is based on the formula in Section 45:

$$C = S\frac{h}{20{,}906{,}000}$$

from which the sea-level length becomes

$$S - C = S\left(1 - \frac{h}{20{,}906{,}000}\right)$$

The quantity in parentheses represents the sea-level factor. Since 20,906,000 is in feet, h must also be in feet: S may then be in any linear unit. From this table the factor needed for reducing a ground-level length to its corresponding sea-level length is obtained by direct interpolation. In the example in section 56, the sea-level factor for 4,300 feet is .999809−3/5 of .000024=.999795.

Table of Sea-Level Factors

Elevation h	Sea-level factor	Elevation h	Sea-level factor
(feet)		*(feet)*	
500	0.999976	5,500	0.999737
1,000	.999952	6,000	.999713
1,500	.999928	6,500	.999689
2,000	.999904	7,000	.999665
2,500	.999880	7,500	.999641
3,000	.999857	8,000	.999617
3,500	.999833	8,500	.999593
4,000	.999809	9,000	.999570
4,500	.999785	9,500	.999546
5,000	.999761	10,000	.999522

REDUCTION OF SEA-LEVEL LENGTHS TO GRID LENGTHS

In transforming a measured length into a grid length; two operations are involved: the reduction of the measured (ground-level) length to a sea-level (geodetic) length, and then to a grid length. As described in section 48, these two operations may be accomplished in a single computation, using a combination of the sea-level and grid-scale factors. But these two factors must be obtained sepatately before the combination factor can be had. A table of sea-level factors is given immediately preceding this. Values of scale factors are given in the projection tables for the various State grids.

The land surveyor is not apt to have a copy of the projection tables for his State, since their general purpose is to supply data required in transforming geodetic positions into State coordinates, and any State coordinates of control stations he may require are usually already available without his having to compute them. He must, however, have some means of obtaining values of the scale factor for the locality of the survey, and for this purpose the following formulas will prove convenient. While these formulas are empirical and do not give as exact values as may be obtained from the projection tables, the accuracy obtained with them is satisfactory for most traverse work.

For a *Lambert grid,* the formula is:

$$\text{Correction (feet per thousand)} = R + 0.0000421(\Delta\phi')^2$$

in which R is the reduction in feet per thousand on the central parallel and $\Delta\phi'$ is the difference in latitude, expressed in minutes, between the station or area being considered and the central parallel. Values of R and of the central parallels for various State grids are given in Appendix I.

Example. A piece of land in Virginia is to be surveyed. The mean latitude of the land is 36°37′. The survey will be placed on the Virginia Coordinate System, South Zone, a Lambert grid having a value of $R=-0.0546$, and a central parallel at north latitude 37°22′. Using these values in the above formula we obtain

Correction$=-0.0546+0.0000421(45)^2=+0.031$ foot per thousand. The corresponding scale factor in the projection tables is 1.0000306, or a correction of +0.0306 foot per thousand.

For a *transverse Mercator grid* the formula is:

$$\text{Correction (feet per thousand)} = R + 0.0001144\left(\frac{x'}{10{,}000}\right)^2$$

This may also be written in the form $R+1.144(x')^2(10^{-12})$ in which R is the reduction in feet per thousand on the central meridian and x' is the distance in feet from that meridian.

EXAMPLE: A farm in Indiana is to be surveyed and results placed on the Indiana Coordinate System, East Zone, a transverse Mercator grid. The value of x for the general location of the farm, obtained from the grid positions of the control stations to which the survey is to be tied, is about 220,000 feet. From the tables (Appendix I) we find that R is -0.0333, and from the x coordinate, we determine x' to be 280,000, the distance from the central meridian. Dividing this by 10,000 we get 28.00, and substituting these values in the formula for the transverse Mercator grid, we obtain

Scale correction $= -0.0333 + 0.0001144(28.00)^2 = +0.056$ foot per thousand. The corresponding factor obtained from the projection tables is 1.0000563 or +0.0563 foot per thousand.

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