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REPORT ON THE READJUSTMENT OF THE FIRST-ORDER TRIANGULATION NET OF THE WESTERN PART OF THE UNITED STATES

By

OSCAR S. ADAMS
Mathematician, Division of Geodesy
U. S. Coast and Geodetic Survey

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REPORT ON THE READJUSTMENT OF THE FIRST-ORDER TRIANGULATION NET OF THE WESTERN PART OF THE UNITED STATES

By OSCAR S. ADAMS, *Mathematician, Division of Geodesy, United States Coast and Geodetic Survey*

In the adjustment of triangulation it is evident that the closure of a loop should be distributed throughout the entire loop. This is a very simple matter when the chain consists of a single loop such as that formed by carrying triangulation around an island or around a lake. However, when the system consists of a number of connected loops, the solution of the equations of condition for all of the loops in a single set presents serious difficulty on account of the amount of computation that would be required in the adjustment.

In the United States the geodetic datum was adopted when the triangulation system consisted of a meager skeleton joining the arc on the Atlantic coast to that on the Pacific coast. As this mere outline was supplemented by new work, the discrepancies in the closure of loops were adjusted into the new arcs, the old work being held fixed. In the earlier work no true geodetic azimuths, derived by applying the Laplace correction to observed astronomic azimuths, were held in the adjustments. As the country became divided up into many closed loops, this method of adjustment became more and more objectionable because often comparatively short arcs were forced to absorb loop closures that were out of all proportion to their lengths and as a result the corrections applied to them were unduly large. In all closures the discrepancies disclosed were such as might be expected from the grade of the work when the whole length of the loop was considered, but they were unduly large for adjustment into the new work with the old work held fixed. It became evident that at some time in the future a readjustment would have to be made of at least some parts of the net to relieve this condition. The opportune time to adjust the western half of the net seems to be found at the present time.

All of the area of the United States to the west of the ninety-eighth meridian is now sufficiently covered by the net of first-order triangulation, except for two short arcs which will be executed during the present year (1926), to permit the determination of the best and final values for the geographic positions of the stations in this area. But to adjust this net as a whole by the old or what may be called the classical method of adjustment would involve more than 3,000 simultaneous equations, and the solution of these in a single net would require many years of computation.

But the question was a most important one, for the arc of first-order triangulation extending from the northwestern part of the United States along the coast of British Columbia and through south-

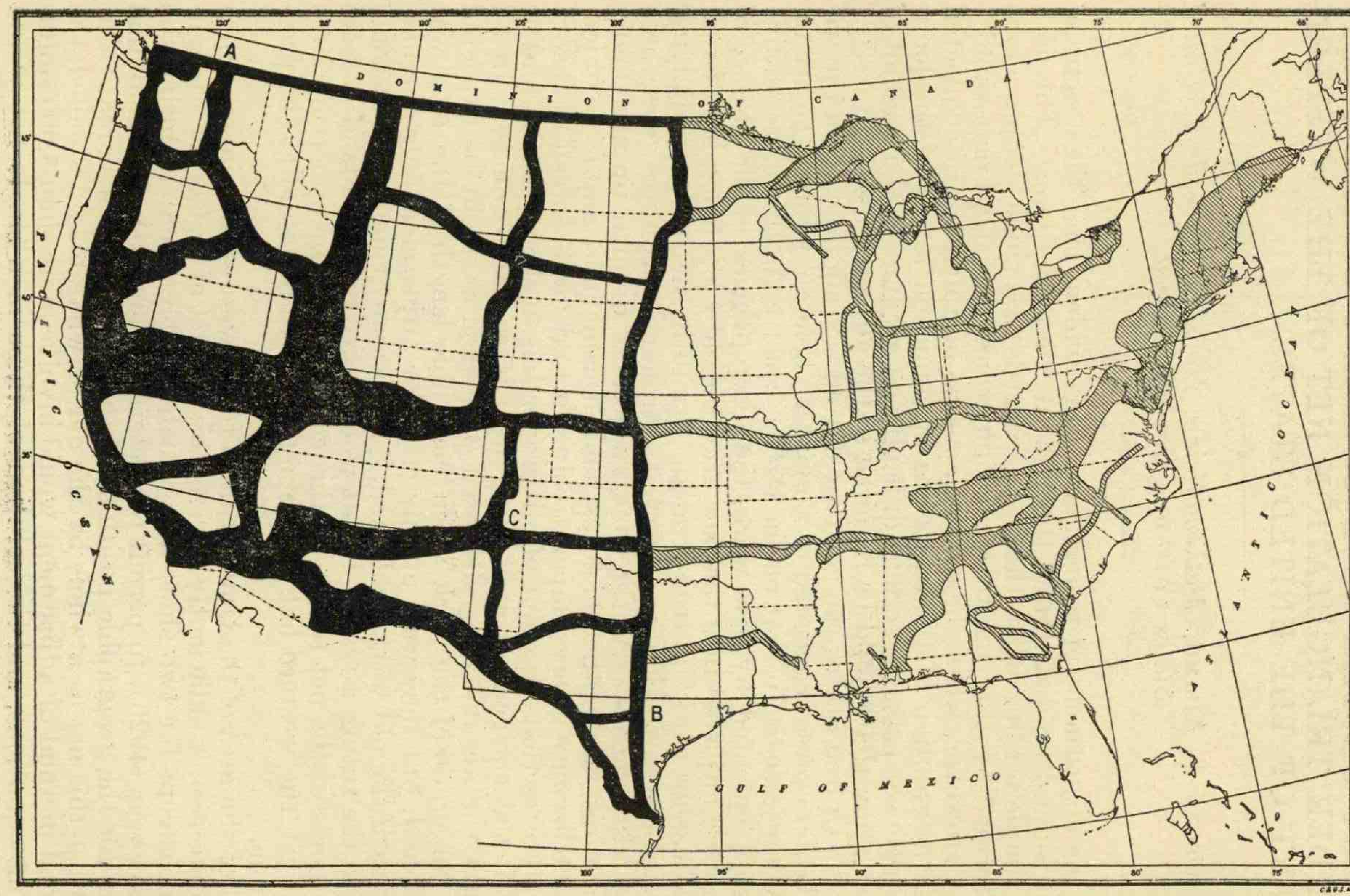

FIG. 1—Arcs of first-order triangulation in the United States. The arcs shown in solid black are those used in the adjustment of the western half of the country, now in progress. It can be seen from the figure that several additional arcs will be required east of the ninety-eighth meridian before the triangulation of the eastern part of the country is adjusted

eastern Alaska was awaiting adjustment, and this could not be undertaken until final values for the geographic positions of the stations from which the new arc would extend could be obtained.

The problem was solved early in 1924 by Dr. William Bowie, chief of the division of geodesy of the United States Coast and Geodetic Survey, who conceived and formulated in a general way a plan by which the adjustment of a large network of triangulation can be executed at small cost of time and money. By the Bowie method the discrepancies in loop closures of a net formed by arcs of triangulation may be distributed in a way somewhat similar to that used in the adjustment of a first-order level net.

The triangulation of the western half of the United States is formed of a number of intersecting arcs. In order to make the readjustment of the net the unit is taken as the section of an arc between junction points, hereafter called simply "section."

The same method of adjustment can also be applied to an area completely covered by triangulation by selecting loops formed by chains of well-shaped triangles, quadrilaterals, or other figures with the intermediate figures or stations omitted which later can be fitted into the adjusted net.

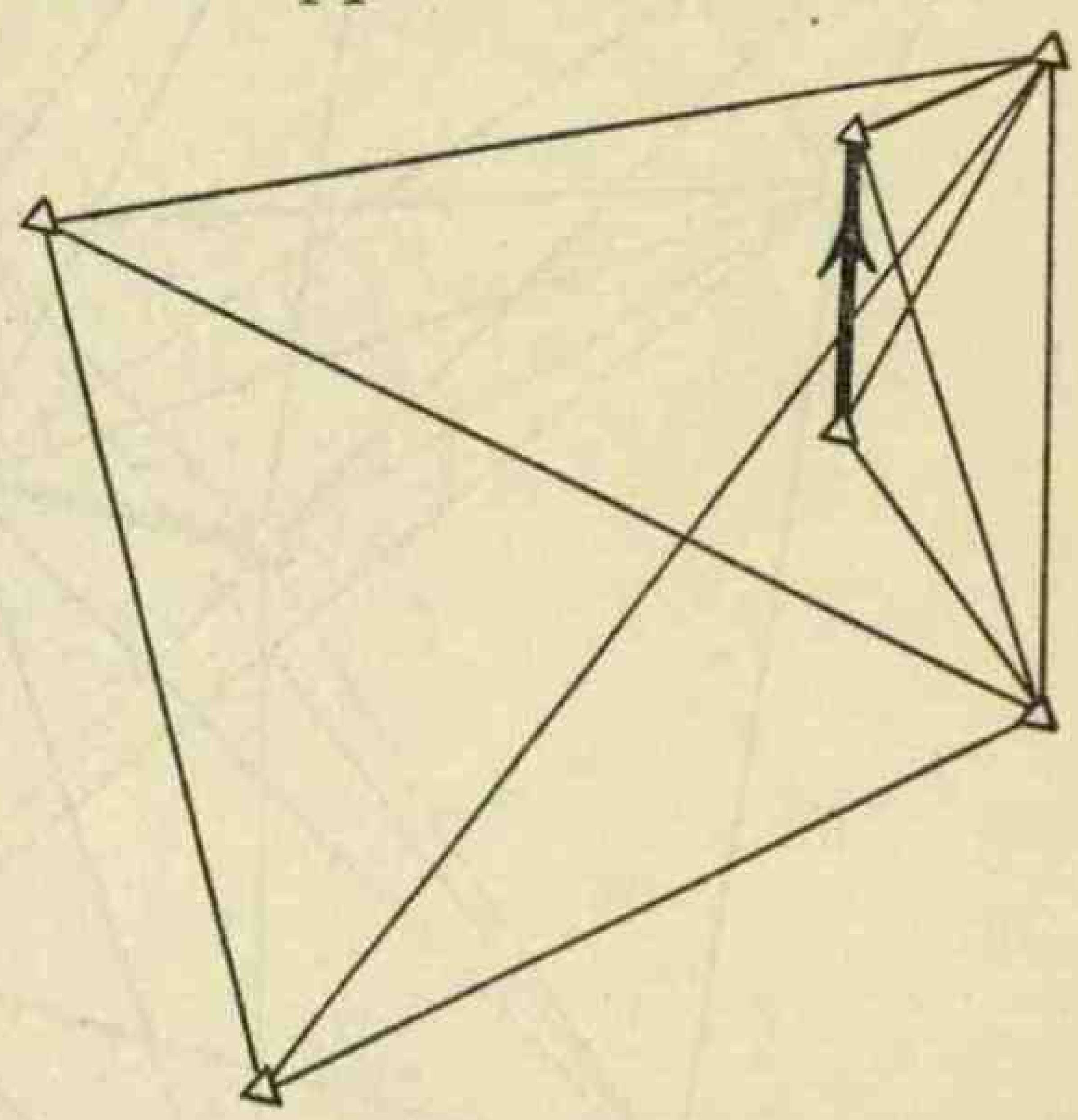

FIG. 2.—A simple junction figure. A base line and a Laplace azimuth are included in the figure. The location is shown at A, Figure 1

After the loop closures have been distributed among the various sections of the arcs of triangulation forming the loops, and in this way the most probable geographic positions of stations used as junction points have been derived, then the sections can be adjusted separately between the junction figures.

At the request of Doctor Bowie the writer worked out a method for forming the equations and making the least-squares adjustments which are necessary to make the plan practicable. On first consideration it was contemplated that the adjustment might be made by the use of interrelated equations for the loop closures but on further investigation it was found that no economical scheme of this kind could be devised. After a careful study a plan was perfected for the use of two independent sets of equations, one set for the closures in latitude and the other set for the longitude closures. This plan was found to be both economical and practical and at the same time it will give a very satisfactory distribution of the discrepancies in loop closures.

A preliminary statement of the plan was prepared by the writer, which was presented by Doctor Bowie at the meeting of the Section of Geodesy of the International Geodetic and Geophysical Union at Madrid in 1924.

The adjustment of the network of triangulation covering, in skeleton form, the western half of the area of the United States has been under way under the direction of the writer since the summer of 1925.

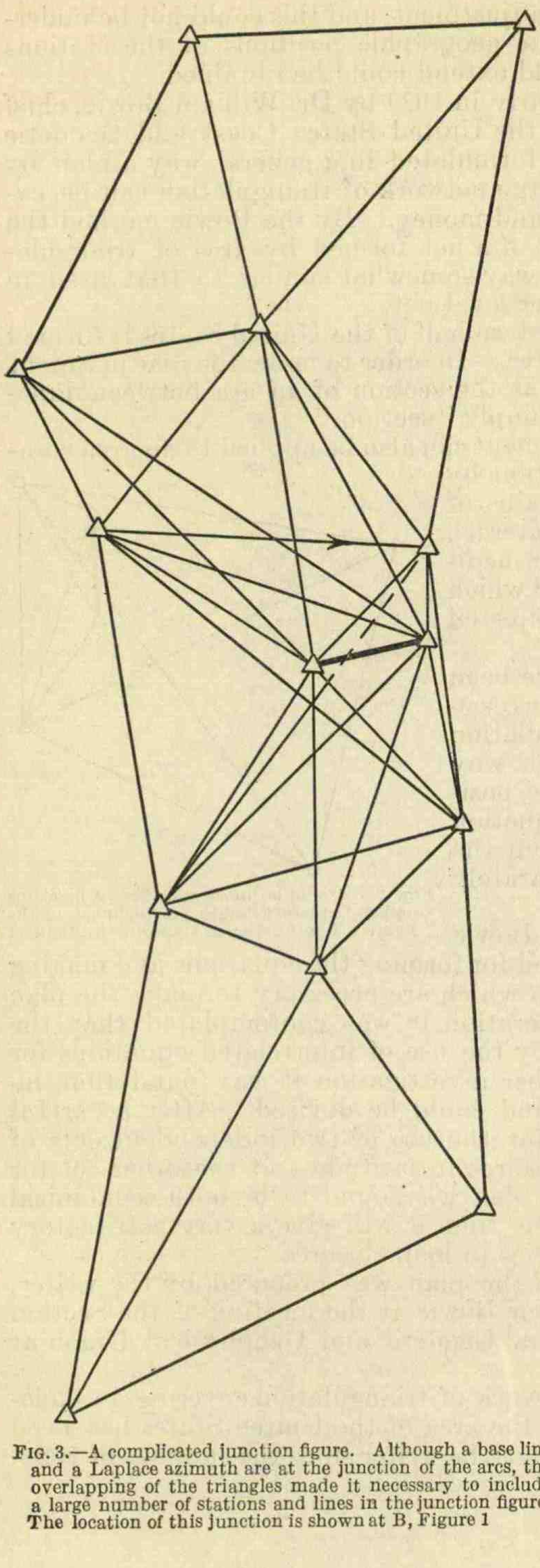

FIG. 3.—A complicated junction figure. Although a base line and a Laplace azimuth are at the junction of the arcs, the overlapping of the triangles made it necessary to include a large number of stations and lines in the junction figure. The location of this junction is shown at B, Figure 1

This includes the arc along the ninety-eighth meridian and the other triangulation to the westward. (See fig. 1.) After the network of arcs of triangulation in the eastern half of this country has been sufficiently strengthened by additional arcs, an adjustment of it also will be made by the Bowie method.

The remainder of this paper gives an account of the way in which the new plan is being carried out in the actual adjustment of the net.

Before the sections between junction points of the various arcs of triangulation are readjusted, it becomes necessary to fix a small junction figure, composed of a single quadrilateral or of several quadrilaterals, at each of the intersections of the arcs. This will furnish the most probable length and azimuth of a line at the junction of two arcs which will be used in the adjustment of the adjacent sections. The ideal condition is found in those places where a Laplace azimuth and a measured base are included in this junction figure. (See fig. 2.) In places where this is not the case the length to be used as the controlling length of the junction figure is computed through the best distance angles from each of the nearest measured bases in the radiating sections. A mean of these various values weighted directly as the strength of the angles through which the various lengths are computed is then adopted as the length to be used in the junction figure.

In a few cases where the junction figure is somewhat complicated the lengths of two or more lines at the margins of the figure are derived from the contiguous bases. The several lines are then held fixed in length in the adjustment of the junction figure. The effect of making

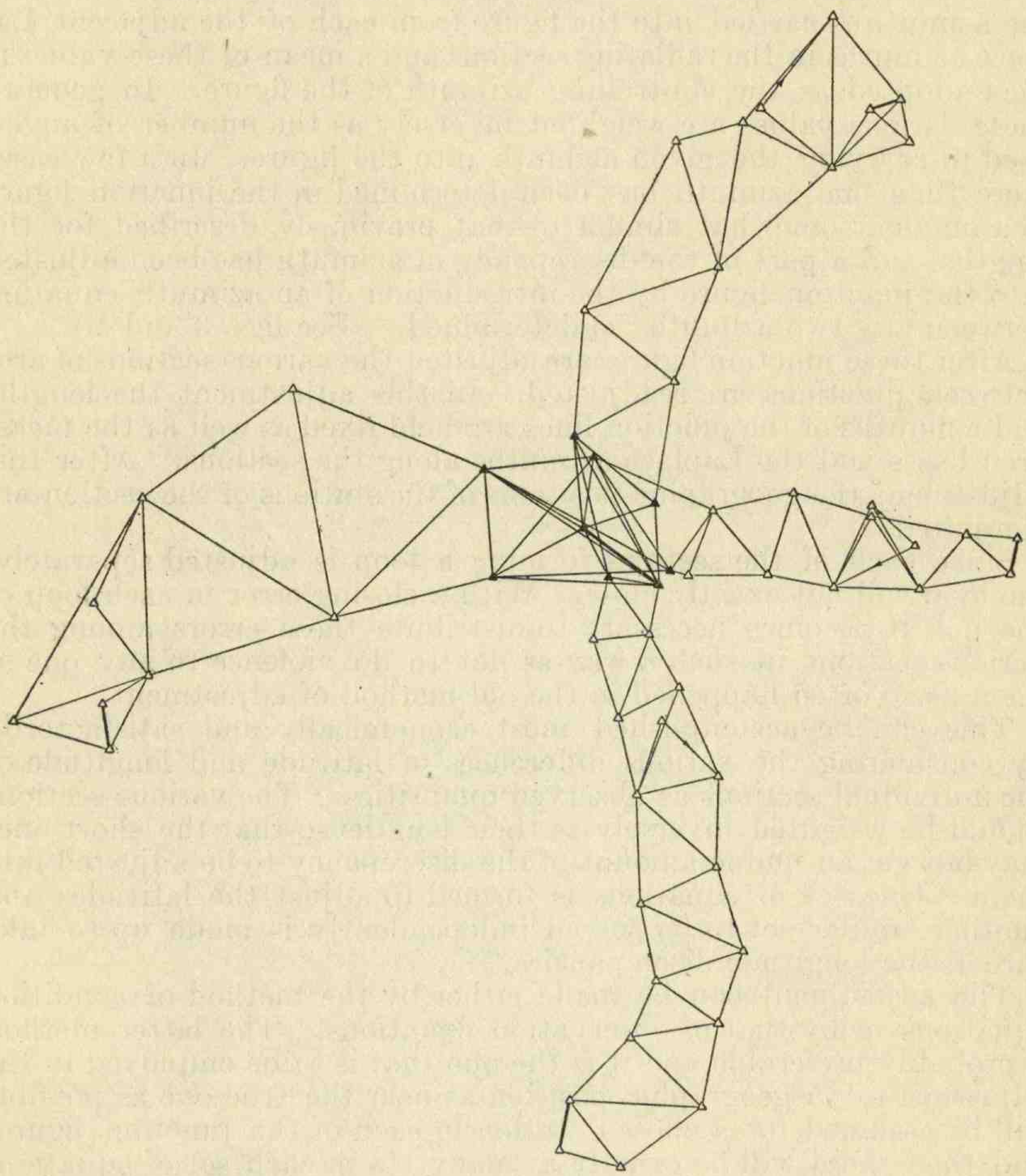

FIG. 4.—A junction figure without a base or a Laplace azimuth. The stations of the junction figure are shown by small black triangles. The length of the heavy line of the junction figure was determined by taking a weighted mean of the lengths computed from the four bases shown near the tips of the intersecting arcs. The azimuth of this line was computed in a similar manner from the nearest four Laplace azimuths. This junction is located at C, Figure 1

the adjustment in this way is to reduce the length discrepancies between the junction figure and the adjacent bases when the sections are adjusted.

A Laplace azimuth is an observed astronomic azimuth that has been corrected for the deflection of the vertical at the station. This correction can be computed if an astronomic longitude has been observed at the station as well as the astronomic azimuth. If we denote the astronomic by A and the geodetic by G, we have the relation,

$$(A-G) \text{ in azimuth} = -[(A-G) \text{ in longitude}] \sin \phi$$

in which ϕ is the latitude of the station.

If one of the Laplace stations is not found in the junction figure, the azimuth is carried into the figure from each of the adjacent Laplace azimuths in the radiating sections and a mean of these values is then adopted as the controlling azimuth of the figure. In general, these various values are weighted inversely as the number of angles used in carrying the given azimuth into the figure. In a few cases more than one azimuth has been determined in the junction figure in a manner somewhat similar to that previously described for the lengths, and a part of the discrepancy in azimuth has been adjusted into the junction figure by the introduction of an azimuth equation between any two azimuths so determined. (See figs. 3 and 4.)

After these junction figures are adjusted the various sections of arcs between junctions are readjusted. In this adjustment the lengths and azimuths of the junction lines are held fixed as well as the measured bases and the Laplace azimuths along the sections. After this adjustment the geographic positions of the stations of the section are computed.

Since each of the sections forming a loop is adjusted separately, the loop will not exactly close. With a closing error in each loop of the net, it becomes necessary to distribute these errors among the various sections in such a way as not to do violence to any one of them as so often happened in the old method of adjustment.

This can be accomplished most economically and satisfactorily by considering the various differences of latitude and longitude of the individual sections as observed quantities. The various sections should be weighted inversely as their lengths so that the short ones may not get an undue amount of the discrepancy to be adjusted into them. One set of equations is formed to adjust the latitudes and another similar set to be solved independently is made up to take care of the longitude discrepancies.

The adjustment can be made either by the method of condition equations or by that of observation equations. The latter method is probably preferable and it is the one that is to be employed in the adjustment. A geographic position as near the true one as possible will be assumed for a chosen station in each of the junction figures and then there will be exactly as many v's in each set of equations as there are sections in the net.

The normal equations for latitudes and longitudes will then be made up with the sections weighted inversely as their lengths. The solutions of the two sets of independent normal equations thus formed will give the most probable corrections to the assumed latitudes and longitudes in the junction figures. The final geographic positions of all of the stations in the junction figures can then be computed. These positions will be such that all of the separate sections can be adjusted into the general scheme without any undue strain or violence done to any part of the whole series of arcs. This object is the definite aim of the whole work of the readjustment. The matter of fitting in finally the various sections is nothing more than the usual routine adjustment of arcs in which a position closure is included.

In this readjustment and recomputation of the triangulation net, station Meades Ranch in central Kansas is taken as the starting point. At this station the latitude and longitude are adopted as follows:

$$\phi = 39^\circ\ 13'\ 26''.686$$
$$\lambda = 98^\circ\ 32'\ 30''.506$$

These values are identical with those that have been used for a number of years to define the United States standard datum and later the North Amercian datum. From these values the geodetic positions of the triangulation stations involved in the readjustment will be derived by continuous computation from this point.

The adjustment is well under way at the time of writing this report (May, 1926). The adjustments of the separate sections and of the junction figures are not held back by the lack of the two short arcs to be done this year, but necessarily the main adjustment can not be made until the new data are available. It is expected that the most probable values of the junction stations will be obtained by the end of this year.

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